

تم تحميل هذا الملف من موقع المناهج المصرية



" >

\* للحصول على أوراق عمل لجميع الصفوف وجميع المواد اضغط هنا

<https://almanahj.com/eg>

\* للحصول على أوراق عمل لجميع مواد الصف الثالث الإعدادي اضغط هنا

<https://almanahj.com/eg/9>

\* للحصول على جميع أوراق الصف الثالث الإعدادي في مادة رياضيات وجميع الفصول, اضغط هنا

<https://almanahj.com/eg/9>

\* للحصول على أوراق عمل لجميع مواد الصف الثالث الإعدادي في مادة رياضيات الخاصة بـ اضغط هنا

<https://almanahj.com/eg/9>

\* لتحميل كتب جميع المواد في جميع الفصول للـ الصف الثالث الإعدادي اضغط هنا

<https://almanahj.com/eg/grade9>



# Geometry

## نماذج امتحانات

الصف **3** الإعدادي

الفصل الدراسي الثاني ٢٠٢١

## Answer the following questions :

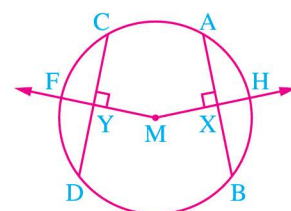
### 1 Choose the correct answer from those given :

- 1 The slope of the straight line  $3x + 2y = 1$  is .....
  - (a)  $\frac{2}{3}$
  - (b)  $-\frac{3}{2}$
  - (c)  $-\frac{2}{3}$
  - (d)  $\frac{3}{2}$
- 2 M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm. , then  $MN \in$  .....
  - (a)  $]8, \infty[$
  - (b)  $]3, 5[$
  - (c)  $]0, 2[$
  - (d)  $]2, 8[$
- 3 The measurement of any angle of the regular hexagon is .....
  - (a)  $90^\circ$
  - (b)  $108^\circ$
  - (c)  $120^\circ$
  - (d)  $135^\circ$
- 4 ABCD is a cyclic quadrilateral ,  $m(\angle A) = 70^\circ$  , then  $m(\angle C)$  equals .....
  - (a)  $25^\circ$
  - (b)  $20^\circ$
  - (c)  $110^\circ$
  - (d)  $100^\circ$
- 5 In  $\triangle ABC$  , if  $(AB)^2 = (AC)^2 + (BC)^2$  , then  $\angle B$  is .....
  - (a) acute.
  - (b) obtuse.
  - (c) right.
  - (d) reflex.
- 6 The measure of the inscribed angle drawn in a semicircle equals .....
  - (a)  $130^\circ$
  - (b)  $90^\circ$
  - (c)  $50^\circ$
  - (d)  $180^\circ$

### 2 [a] In the opposite figure :

$\overline{AB}$  and  $\overline{CD}$  are two chords equal in length in the circle M  
 $\overrightarrow{MX} \perp \overline{AB}$  ,  $\overrightarrow{MY} \perp \overline{CD}$

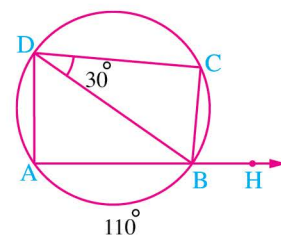
**Prove that :**  $HX = FY$



### [b] In the opposite figure :

$H \in \overrightarrow{AB}$  ,  $m(\widehat{AB}) = 110^\circ$   
 $m(\angle CDB) = 30^\circ$

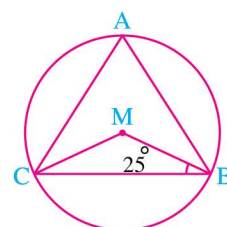
**Find :**  $m(\angle HBC)$



### 3 [a] In the opposite figure :

ABC is a triangle drawn in the circle M  
 $m(\angle MBC) = 25^\circ$

**Find :**  $m(\angle BAC)$

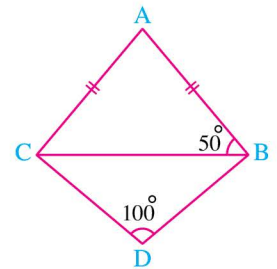


**[b] In the opposite figure :**

$$AB = AC, m(\angle D) = 100^\circ$$

$$, m(\angle ABC) = 50^\circ$$

**Prove that :** ABDC is a cyclic quadrilateral.



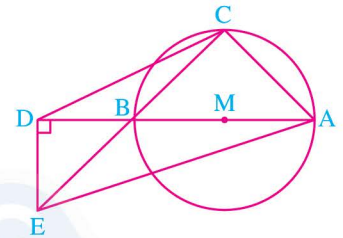
**4 [a] In the opposite figure :**

$\overline{AB}$  is a diameter in the circle M

,  $D \in \overline{AB}$ ,  $D \notin \overline{AB}$ ,  $\overline{DE} \perp \overline{AB}$

,  $C \in \widehat{AB}$ ,  $\overline{CB} \cap \overline{DE} = \{E\}$

**Prove that :** ACDE is a cyclic quadrilateral



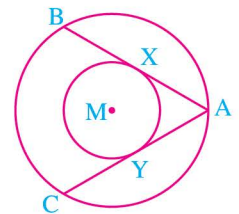
**[b] In the opposite figure :**

Two concentric circles of centre M

,  $\overline{AB}$  and  $\overline{AC}$  are two chords in the greater circle

and tangents to the smaller circle at X and Y respectively.

**Prove that :**  $AB = AC$



**5 [a] In the opposite figure :**

M and N are two intersecting circles at A and B

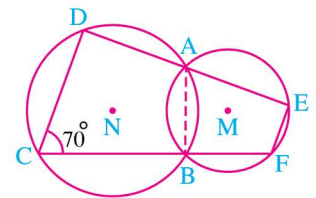
,  $\overline{AD}$  is drawn to intersect the circle M at E and

the circle N at D,  $\overline{AB}$  is drawn to intersect the circle M at

F and the circle N at C,  $m(\angle BCD) = 70^\circ$

**1 Find :**  $m(\angle EFB)$

**2 Prove that :**  $\overline{CD} \parallel \overline{EF}$



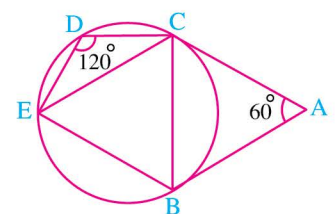
**[b] In the opposite figure :**

$\overline{AB}$  and  $\overline{AC}$  are tangent-segments to the circle at B and C

$$, m(\angle BAC) = 60^\circ, m(\angle CDE) = 120^\circ$$

**Prove that :** **1**  $\triangle BCE$  is an equilateral triangle.

**2**  $\overline{AC} \parallel \overline{BE}$



## Answer the following questions :

### 1 Choose the correct answer from those given :

- 1  $\angle A$  and  $\angle B$  are two complementary angles ,  $\angle B$  and  $\angle C$  are two supplementary angles ,  $m(\angle A) = 30^\circ$  , then  $m(\angle C) = \dots\dots\dots^\circ$

(a) 30 (b) 60 (c) 90 (d) 120

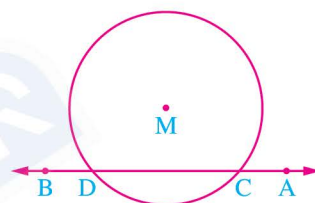
- 2 If the surface of the circle  $M \cap$  the surface of the circle  $N = \{A\}$  and the radius length of one of them equals 3 cm. and  $MN = 8$  cm. , then the radius length of the other circle equals  $\dots\dots\dots$  cm.

(a) 5 (b) 6 (c) 11 (d) 16

### 3 In the opposite figure :

$\overleftrightarrow{AB} \cap$  the surface of the circle  $M = \dots\dots\dots$

- (a)  $\{C, D\}$  (b)  $\overline{CD}$   
(c)  $\overleftrightarrow{CD}$  (d)  $\emptyset$



- 4 A circle can be drawn passing through the vertices of a  $\dots\dots\dots$

(a) rhombus. (b) parallelogram. (c) trapezium. (d) rectangle.

- 5 The rhombus whose two diagonal lengths are 12 cm. and 16 cm. , then its side length equals  $\dots\dots\dots$  cm.

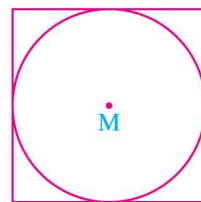
(a) 6 (b) 8 (c) 10 (d) 20

### 6 In the opposite figure :

If the side length of the square = 10 cm.

, then the surface area of the circle =  $\dots\dots\dots \text{cm}^2$

- (a)  $100\pi$  (b)  $25\pi$   
(c)  $50\pi$  (d)  $40\pi$

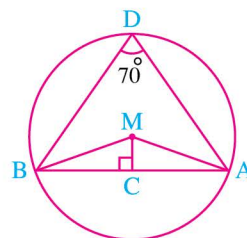


### 2 [a] In the opposite figure :

$\overline{AB}$  is a chord in the circle  $M$

,  $\overline{MC} \perp \overline{AB}$  ,  $m(\angle ADB) = 70^\circ$

Find :  $m(\angle AMC)$

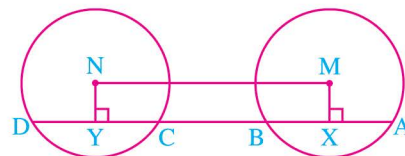


### [b] In the opposite figure :

$M$  and  $N$  are two congruent circles

,  $AB = CD$  ,  $\overline{MX} \perp \overline{AB}$  and  $\overline{NY} \perp \overline{CD}$

Prove that : The figure  $MXYN$  is a rectangle.



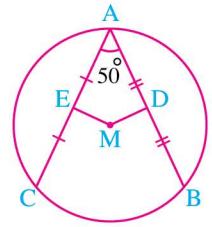
**3 [a] In the opposite figure :**

$\overline{AB}$  and  $\overline{AC}$  are two chords

in the circle M , D is the midpoint of  $\overline{AB}$

, E is the midpoint of  $\overline{AC}$  and  $m(\angle BAC) = 50^\circ$

**Find :**  $m(\angle DME)$



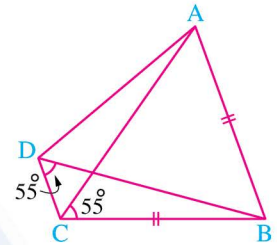
**[b] In the opposite figure :**

$AB = BC$

,  $m(\angle ACB) = 55^\circ$

and  $m(\angle BDC) = 55^\circ$

**Prove that :** The figure ABCD is a cyclic quadrilateral.



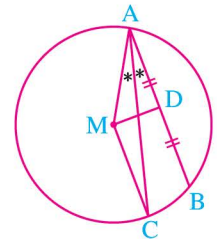
**4 [a] In the opposite figure :**

$\overline{AB}$  is a chord in the circle M

,  $\overline{AC}$  bisects  $\angle BAM$  and intersects the circle M at C

If D is the midpoint of  $\overline{AB}$

, **prove that :**  $\overline{DM} \perp \overline{CM}$



**[b]**  $\overline{AB}$  is a diameter in the circle M ,  $\overline{AC}$  and  $\overline{BD}$  are two tangents to the circle M ,  $\overline{CM}$  intersects the circle M at X and Y respectively and intersects  $\overline{BD}$  at E **Prove that :**  $CX = YE$

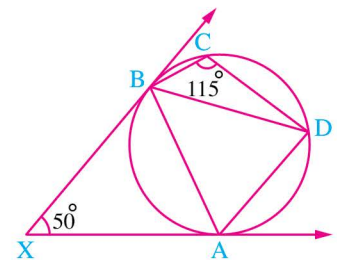
**5 [a] In the opposite figure :**

$\overline{XA}$  and  $\overline{XB}$  are two tangents to the circle at A and B

,  $m(\angle AXB) = 50^\circ$  ,  $m(\angle DCB) = 115^\circ$

**Prove that :** 1  $\overline{AB}$  bisects  $\angle DAX$

2  $BD = BA$

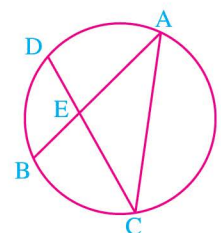


**[b] In the opposite figure :**

$\overline{AB}$  and  $\overline{CD}$  are two equal chords in length in the circle

,  $\overline{AB} \cap \overline{CD} = \{E\}$

**Prove that :** The triangle ACE is an isosceles triangle.





## Answer the following questions :

### 1 Choose the correct answer from those given :

- 1 The measure of the inscribed angle is ..... the measure of the central angle subtended by the same arc.  
(a) half                      (b) twice                      (c) quarter                      (d) third
- 2 The length of the side opposite to the angle of measure  $30^\circ$  in the right-angled triangle equals ..... the length of the hypotenuse.  
(a)  $\frac{1}{2}$                       (b)  $\frac{\sqrt{3}}{2}$                       (c)  $\sqrt{2}$                       (d) 2
- 3 Two distant circles M and N with radii lengths 6 cm. and 8 cm. respectively, then MN ..... 14 cm.  
(a) <                      (b) >                      (c) =                      (d)  $\leq$
- 4 The angle of measure  $40^\circ$  is the complemented angle of the angle of measure .....  
(a) 320                      (b) 140                      (c) 60                      (d) 50
- 5 The area of the rhombus with diagonal lengths 6 cm. , 8 cm. is .....  $\text{cm}^2$   
(a) 2                      (b) 14                      (c) 24                      (d) 48
- 6 In the cyclic quadrilateral ABCD , if  $m(\angle A) = \frac{1}{2} m(\angle C)$  , then  $m(\angle A) = \dots\dots\dots^\circ$   
(a) 20                      (b) 30                      (c) 60                      (d) 120

### 2 [a] In the opposite figure :

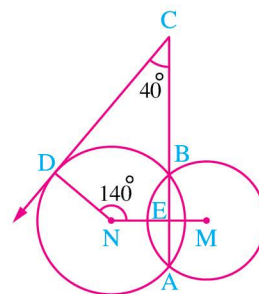
M and N are two intersecting circles at A and B

,  $C \in \overrightarrow{AB}$  ,  $\overrightarrow{AC} \cap \overrightarrow{MN} = \{E\}$

,  $D \in \text{the circle N}$  ,  $m(\angle DNM) = 140^\circ$

and  $m(\angle C) = 40^\circ$

**Prove that :**  $\overrightarrow{CD}$  is a tangent to the circle N at D



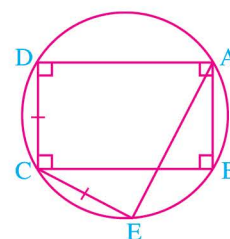
### [b] In the opposite figure :

ABCD is a rectangle inscribed in a circle

, the chord  $\overline{CE}$  is drawn

where  $CE = CD$

**Prove that :**  $AE = BC$



**3 [a]** State two cases of the cyclic quadrilateral.

**[b] In the opposite figure :**

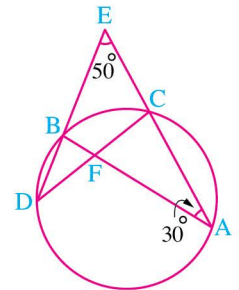
$$\overline{AB} \cap \overline{CD} = \{F\}, \overline{AC} \cap \overline{DB} = \{E\}$$

$$, m(\angle A) = 30^\circ$$

$$, m(\angle E) = 50^\circ$$

**Find :** 1  $m(\widehat{AD})$

2  $m(\angle AFD)$

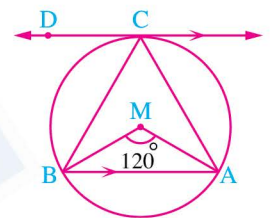


**4 [a] In the opposite figure :**

$\overrightarrow{CD}$  is a tangent to the circle at C

$$, \overline{CD} \parallel \overline{AB}, m(\angle AMB) = 120^\circ$$

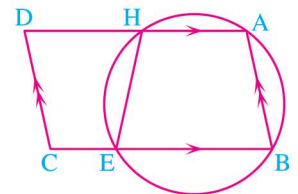
**Prove that :** The triangle CAB is an equilateral triangle.



**[b] In the opposite figure :**

ABCD is a parallelogram.

**Prove that :** HDCE is a cyclic quadrilateral.

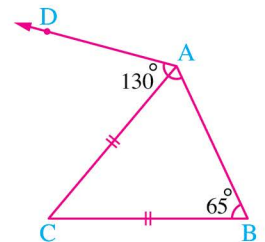


**5 [a] In the opposite figure :**

$$AC = BC, m(\angle ABC) = 65^\circ$$

$$, m(\angle DAB) = 130^\circ$$

**Prove that :**  $\overrightarrow{AD}$  is a tangent to the circle passing through the vertices of the triangle ABC



**[b] In the opposite figure :**

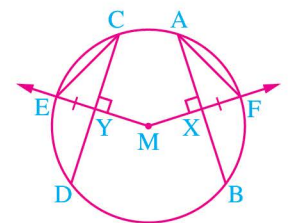
$\overline{AB}$  and  $\overline{CD}$  are two chords in the circle M

$\overrightarrow{MX} \perp \overline{AB}$  and intersects the circle at F

$\overrightarrow{MY} \perp \overline{CD}$  and intersects the circle at E,  $FX = EY$

**Prove that :** 1  $AB = CD$

2  $AF = CE$





# إجابات نماذج امتحانات

الصف 3 الإعدادي

الفصل الدراسي الثاني ٢٠٢١

### Answers of model 1

1

- 1 b                      2 d                      3 c  
4 c                      5 a                      6 b

2

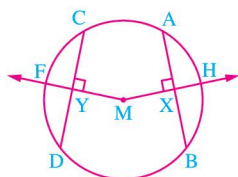
[a]  $\because AB = CD, \overline{MX} \perp \overline{AB}$   
 $\overline{MY} \perp \overline{CD}$

$$\therefore MX = MY$$

$$\therefore MH = MF = r$$

$$\therefore HX = FY$$

(Q.E.D.)

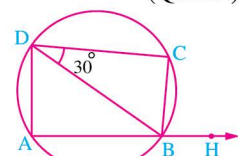


[b]  $\because m(\angle ADB) = \frac{1}{2} m(\widehat{AB})$   
 $= \frac{1}{2} \times 110^\circ$   
 $= 55^\circ$

$\therefore ABCD$  is a cyclic quadrilateral.  $110^\circ$

$$\therefore m(\angle HBC) = m(\angle CDB) + m(\angle ADB)$$

$$= 30^\circ + 55^\circ = 85^\circ \quad (\text{The req.})$$



3

[a] In  $\triangle BMC$ :

$$\therefore MB = MC = r$$

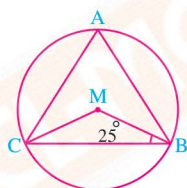
$$\therefore m(\angle MCB)$$

$$= m(\angle MBC) = 25^\circ$$

$$\therefore m(\angle BMC) = 180^\circ - (25^\circ + 25^\circ) = 130^\circ$$

$\therefore m(\angle BAC) = \frac{1}{2} m(\angle BMC)$   
(inscribed and central angles subtended by  $\widehat{BC}$ )

$$\therefore m(\angle BAC) = \frac{1}{2} \times 130^\circ = 65^\circ \quad (\text{The req.})$$



[b] In  $\triangle ABC$ :

$$\therefore AB = AC$$

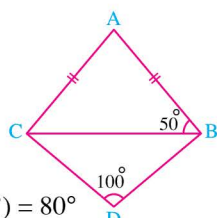
$$\therefore m(\angle ACB) = m(\angle ABC)$$

$$= 50^\circ$$

$$\therefore m(\angle A) = 180^\circ - (50^\circ + 50^\circ) = 80^\circ$$

$$\therefore m(\angle A) + m(\angle D) = 80^\circ + 100^\circ = 180^\circ$$

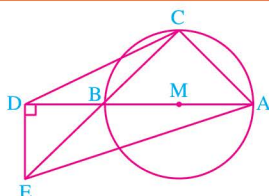
$\therefore ABDC$  is a cyclic quadrilateral. (Q.E.D.)



4

[a]  $\because \overline{AB}$  is a diameter of the circle.

$$\therefore m(\angle ACB) = 90^\circ$$



$$\therefore m(\angle ACE) = m(\angle ADE)$$

and they are drawn on  $\overline{AE}$  and on one side of it

$\therefore ACDE$  is a cyclic quadrilateral. (Q.E.D.)

[b] Construction:

Draw  $\overline{MX}, \overline{MY}$

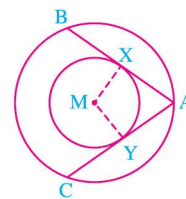
Proof:

$\because \overline{AB}, \overline{AC}$  are two tangents to the smaller circle at  $X, Y$  respectively

$$\therefore \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{AC}$$

$$\therefore MX = MY = r \quad (\text{radius length of the smaller circle})$$

$$\therefore AB = AC \quad (\text{Q.E.D.})$$



5

[a]  $\because ABCD$  is a cyclic quadrilateral

$$\therefore m(\angle BAD)$$

$$= 180^\circ - 70^\circ = 110^\circ$$

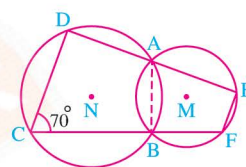
$\therefore ABFE$  is a cyclic quadrilateral and  $\angle BAD$  is exterior of it.

$$\therefore m(\angle EFB) = m(\angle BAD) = 110^\circ \quad (\text{First req.})$$

$$\therefore m(\angle EFB) + m(\angle BCD) = 110^\circ + 70^\circ = 180^\circ$$

and they are interior angles in the same side of  $\overleftrightarrow{FC}$

$$\therefore \overline{CD} \parallel \overline{EF} \quad (\text{Second req.})$$



[b]  $\because \overline{AB}, \overline{AC}$

are tangent-segments to the circle

$$\therefore AB = AC$$

$$\therefore m(\angle ACB) = \frac{180^\circ - 60^\circ}{2} = 60^\circ \quad (1)$$

$$\therefore m(\angle BEC) \text{ (inscribed)}$$

$$= m(\angle ACB) \text{ (tangency)} = 60^\circ \quad (2)$$

$\therefore EBCD$  is a cyclic quadrilateral

$$\therefore m(\angle EBC) = 180^\circ - 120^\circ = 60^\circ \quad (3)$$

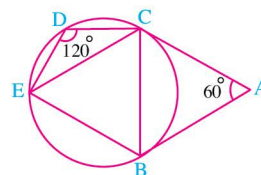
$\therefore$  From (2), (3) in  $\triangle EBC$ :

$$\therefore m(\angle BCE) = 60^\circ$$

$\therefore \triangle BCE$  is equilateral. (Q.E.D. 1)

From (1), (3):  $\therefore m(\angle ACB) = m(\angle EBC)$  and they are alternate angles

$$\therefore \overline{AC} \parallel \overline{BE} \quad (\text{Q.E.D. 2})$$



### Answers of model 2

1

- 1 d                      2 a                      3 b  
4 d                      5 c                      6 b

2

[a]  $\therefore m(\angle AMB) = 2m(\angle ADB)$   
 $= 2 \times 70^\circ = 140^\circ$

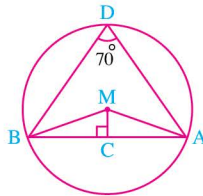
(central and inscribed angles subtended by  $\widehat{AB}$ )

In  $\triangle ABM$  :  $\therefore \overline{MC} \perp \overline{AB}$

$\therefore MA = MB = r$

$\therefore \overline{MC}$  bisects  $\angle AMB$

$\therefore m(\angle AMC) = \frac{1}{2}m(\angle AMB) = \frac{1}{2} \times 140^\circ = 70^\circ$   
(The req.)



[b]  $\therefore M, N$  are two congruent circles

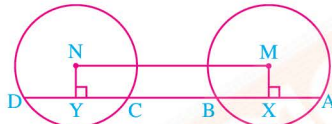
$\therefore AB = CD$

$\therefore \overline{MX} \perp \overline{AB}$

$\therefore \overline{NY} \perp \overline{CD}$

$\therefore MX = NY$  ,  $\overline{MX} \parallel \overline{NY}$

$\therefore$  MXYN is a rectangle. (Q.E.D.)



3

[a]  $\therefore D$  is the midpoint of  $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

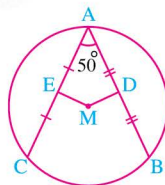
$\therefore m(\angle ADM) = 90^\circ$

$\therefore E$  is the midpoint of  $\overline{AC}$

$\therefore \overline{ME} \perp \overline{AC}$                        $\therefore m(\angle AEM) = 90^\circ$

From the quadrilateral ADME :

$\therefore m(\angle DME) = 360^\circ - (90^\circ + 90^\circ + 50^\circ) = 130^\circ$   
(The req.)



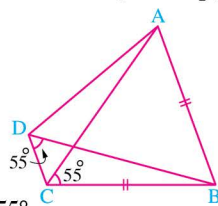
[b] In  $\triangle ABC$  :

$\therefore AB = BC$

$\therefore m(\angle BAC)$   
 $= m(\angle ACB) = 55^\circ$

$\therefore m(\angle BDC) = m(\angle BAC) = 55^\circ$   
and they are drawn on  $\overline{BC}$  and on one side of it

$\therefore$  ABCD is a cyclic quadrilateral. (Q.E.D.)



4

[a] In  $\triangle AMC$  :

$\therefore AM = MC = r$

$\therefore m(\angle MAC) = m(\angle ACM)$

$\therefore m(\angle BAC) = m(\angle MAC)$

$\therefore m(\angle BAC) = m(\angle ACM)$   
and they are alternate angles.

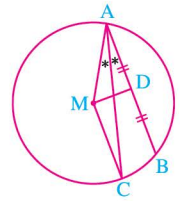
$\therefore \overline{AB} \parallel \overline{CM}$

$\therefore D$  is the midpoint of  $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

$\therefore \overline{DM} \perp \overline{CM}$

$\therefore \overline{AB} \parallel \overline{CM}$   
(Q.E.D.)



[b]  $\therefore \overline{AC}$  is a tangent to the circle M at A

$\therefore \overline{MA} \perp \overline{AC}$

$\therefore m(\angle CAM) = 90^\circ$

$\therefore \overline{BD}$  is a tangent to the circle M at B

$\therefore \overline{MB} \perp \overline{BD}$

$\therefore m(\angle EBM) = 90^\circ$

$\therefore$  In  $\triangle CAM, EBM$  :

$$\begin{cases} m(\angle CAM) = m(\angle EBM) = 90^\circ \\ m(\angle AMC) = m(\angle BME) \text{ (V.O.A.)} \\ MA = MB \text{ (lengths of two radii)} \end{cases}$$

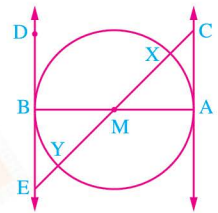
$\therefore$  The two triangles are congruent and we deduce that  $CM = EM$

$\therefore XM = YM$  (lengths of two radii)

$\therefore$  by subtracting

$\therefore CX = YE$

(Q.E.D.)



5

[a]  $\therefore \overline{XA}, \overline{XB}$

are two tangents to the circle

$\therefore XA = XB$

$\therefore$  In  $\triangle ABX$

$m(\angle XAB) = m(\angle XBA) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$

$\therefore$  ABCD is a cyclic quadrilateral

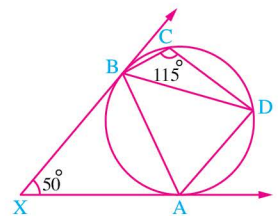
$\therefore m(\angle BAD) + m(\angle DCB) = 180^\circ$

$\therefore m(\angle BAD) = 180^\circ - 115^\circ = 65^\circ$

$\therefore m(\angle XAB) = m(\angle BAD)$

$\therefore \overline{AB}$  bisects  $\angle DAX$

(Q.E.D.1)



$$\begin{aligned} \therefore \angle ADB & \text{ (inscribed)} \\ & = \angle XAB \text{ (tangency)} = 65^\circ \end{aligned}$$

$$\therefore \angle BAD = \angle ADB$$

$$\therefore \text{In } \triangle ABD : BD = BA \quad (\text{Q.E.D.2})$$

$$[b] \therefore AB = CD$$

$$\therefore m(\widehat{AB}) = m(\widehat{CD})$$

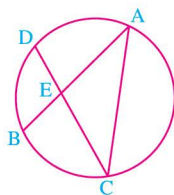
Subtracting  $m(\widehat{BD})$  from both sides

$$\therefore m(\widehat{AD}) = m(\widehat{BC})$$

$$\therefore \angle ACD = \angle BAC$$

$$\therefore \text{In } \triangle ACE : AE = CE$$

$$\therefore \triangle ACE \text{ is an isosceles triangle.} \quad (\text{Q.E.D.})$$



### Answers of model 3

1

$$[1] a$$

$$[2] a$$

$$[3] b$$

$$[4] d$$

$$[5] c$$

$$[6] c$$

2

$$[a] \therefore \overline{MN} \text{ is the line of centres}$$

$$\therefore \overline{AB} \text{ is the common chord.}$$

$$\therefore \overline{AB} \perp \overline{MN}$$

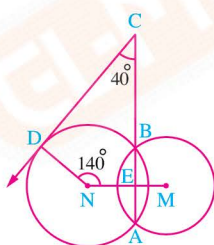
$$\therefore \angle BEN = 90^\circ$$

In the quadrilateral CDNE :

$$\therefore \angle CDN = 360^\circ - (140^\circ + 40^\circ + 90^\circ) = 90^\circ$$

$$\therefore \overline{ND} \perp \overline{CD}$$

$$\therefore \overline{CD} \text{ is a tangent to the circle N at D} \quad (\text{Q.E.D.})$$



$$[b] \therefore AB = CD$$

(properties of the rectangle)

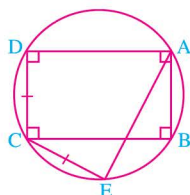
$$\therefore CE = CD$$

$$\therefore AB = CE$$

$$\therefore m(\widehat{AB}) = m(\widehat{CE}) \text{ and adding } m(\widehat{BE}) \text{ to both sides.}$$

$$\therefore m(\widehat{AE}) = m(\widehat{BC})$$

$$\therefore AE = BC \quad (\text{Q.E.D.})$$



3

$$[a] \text{ State by yourself.}$$

$$\begin{aligned} [b] \therefore m(\widehat{BC}) & = 2m(\angle A) \\ & = 2 \times 30^\circ = 60^\circ \end{aligned}$$

$$\begin{aligned} \therefore m(\angle E) & = \frac{1}{2} [m(\widehat{AD}) - m(\widehat{BC})] \\ \therefore 50^\circ & = \frac{1}{2} [m(\widehat{AD}) - 60^\circ] \end{aligned}$$

$$\therefore 100^\circ = m(\widehat{AD}) - 60^\circ$$

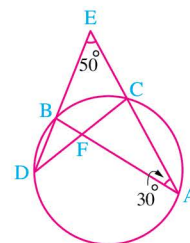
$$\therefore m(\widehat{AD}) = 160^\circ$$

$$\therefore m(\angle AFD) = \frac{1}{2} [m(\widehat{AD}) + m(\widehat{BC})]$$

$$\therefore m(\angle AFD) = \frac{1}{2} [160^\circ + 60^\circ] = 110^\circ$$

(First req.)

(Second req.)



4

$$\begin{aligned} [a] \therefore m(\angle ACB) & = \frac{1}{2} m(\angle AMB) = 60^\circ \end{aligned}$$

(inscribed and central angles subtended the same arc  $\widehat{AB}$ ) (1)

$$\therefore \overline{CD} \parallel \overline{AB}$$

$$\therefore m(\widehat{AC}) = m(\widehat{BC})$$

$$\therefore AC = BC$$

(2)

From (1) and (2) :

$$\therefore \triangle CAB \text{ is equilateral.} \quad (\text{Q.E.D.})$$

$$[b] \therefore \overline{AB} \parallel \overline{DC}, \overline{AD}$$

is a transversal to them.

$$\begin{aligned} \therefore m(\angle A) + m(\angle D) & = 180^\circ \end{aligned} \quad (1)$$

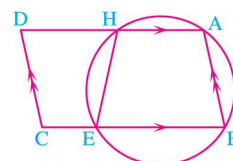
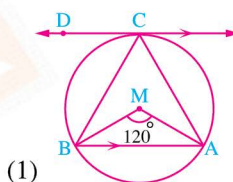
but  $\angle CEH$  is an exterior angle of the cyclic quadrilateral ABEH

$$\therefore m(\angle CEH) = m(\angle A) \quad (2)$$

From (1) and (2) :

$$\therefore m(\angle CEH) + m(\angle D) = 180^\circ$$

$$\therefore HDCE \text{ is a cyclic quadrilateral.} \quad (\text{Q.E.D.})$$



5

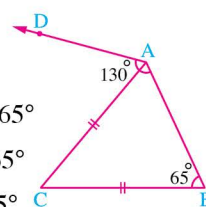
$$[a] \text{ In } \triangle ABC :$$

$$\therefore AC = BC$$

$$\therefore m(\angle BAC) = m(\angle ABC) = 65^\circ$$

$$\therefore m(\angle CAD) = 130^\circ - 65^\circ = 65^\circ$$

$$\therefore m(\angle B) = m(\angle CAD) = 65^\circ$$



$\therefore \overrightarrow{AD}$  is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b]  $\therefore MF = ME$   
(lengths of two radii)

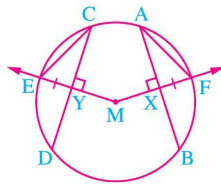
$\therefore XF = YE$

$\therefore MX = MY$

$\therefore \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{CD}$

$\therefore AB = CD$

(Q.E.D.1)



$\therefore \overline{MX} \perp \overline{AB}$

$\therefore X$  is the midpoint of  $\overline{AB}$

$\therefore AX = \frac{1}{2} AB$  ,  $\therefore \overline{MY} \perp \overline{CD}$

$\therefore Y$  is the midpoint of  $\overline{CD}$

$\therefore CY = \frac{1}{2} CD$  ,  $\therefore AB = CD$

$\therefore AX = CY$

$\therefore$  In  $\Delta \Delta AXF, CYE$

$$\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90^\circ \end{cases}$$

$\therefore \Delta AXF \cong \Delta CYE$   $\therefore AF = CE$  (Q.E.D.2)