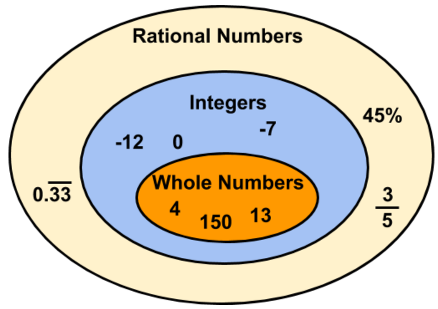
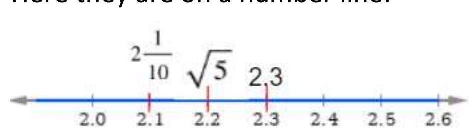
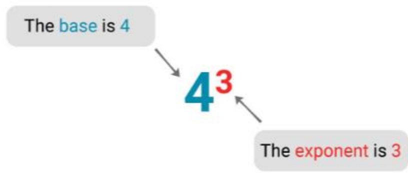
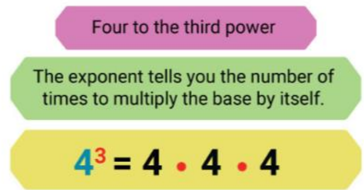
MATH

GRADE 8

Revision

Chapter 1: Real Numbers

Rational and Irrational Numbers • **Rational numbers** are numbers that can be expressed as a fraction where the denominator is not equal to zero. Examples: $2.1 = \frac{21}{10}$, $\quad -2 = \frac{-2}{1}$, $\quad 4\frac{3}{8} = \frac{35}{8}$, $45\% = \frac{45}{100} = \frac{9}{20}$	
• We use **bar notation** to write a repeating decimal	Examples: $0.41414141 \ldots = 0.\overline{41}$, $\quad 0.333333 \ldots = 0.\overline{3}$
• Write a fraction or mixed number as a decimal	Examples: $\frac{3}{8} = 3 \div 8 = 0.375$ $-2\frac{3}{10} = -\frac{23}{10} = -23 \div 10 = -2.3$
• Write a decimal as a fraction or mixed number	Example: *Write* $3.\overline{24}$ *as a mixed number*: $N = 3.242424 \ldots$ $100N = 324.2424 \ldots$ $100N - N = 324.2424 \ldots = 3.242424 \ldots$ $99N = 321$ $N = \frac{321}{99} = 3\frac{24}{99}$
• **Irrational numbers** are numbers that cannot be expressed as a ratio.	Examples: Decimals that do not terminate, but also do not have a repeating pattern, like 7.190233902…, π, $\sqrt{11}$
• The set of **real numbers** is the set of rational numbers and irrational numbers. • You can **compare** real numbers and show them on a number line.	Example: *Compare real numbers:* $\sqrt{5}$, $2\frac{1}{10}$, 2.3 Express all numbers as a **decimal**. $\sqrt{5} \approx 2.2$, $2\frac{1}{10} = 2.1$ $2.1 < 2.2 < 2.3$ $So,\ 2\frac{1}{10} < \sqrt{5} < 2.3$ Here they are on a number line: 

Powers and Exponents	
	
Write an **expression** using **exponents**	$(-4)\cdot(-4)\cdot 5\cdot 5\cdot 5=(-4)^2\cdot 5^3$ $m\cdot n\cdot m\cdot n\cdot m\cdot m\cdot n=m^4\cdot n^3$
Evaluate an expression that has an exponent	$(-2)^3=(-2)\cdot(-2)\cdot(-2)=-8$ $\left(\frac{1}{4}\right)^3=\left(\frac{1}{4}\right)\cdot\left(\frac{1}{4}\right)\cdot\left(\frac{1}{4}\right)=\frac{1}{64}$
Substitute values for the **variables**, then **evaluate**	$a^2-b^3\ \ if\ \ a=-0.5\ \ and\ b=\ 3$ $a^2-b^3=(-0.5)^2-(3)^3$ $=(-0.5)(-0.5)-(3\cdot 3\cdot 3)$ $=0.25-27=\ -26.75$

Monomials • A **monomial** is an algebraic expression with only one term.	$6a,\quad -m^2,$ $5,\quad 3x^2y$
• **Product of Powers** When **multiplying** monomials with the same base, keep the base and **add** the exponents. $a^m\cdot a^n=a^{m+n}$	$7^2\cdot 7^4=7^{4+2}=7^6$ $-4b^2(-2b^3)=(-4)(-2)b^{2+3}=8b^5$
• **Quotient of Powers** When **dividing** monomials with the same base, keep the base and **subtract** the exponents. $\frac{a^m}{a^n}=a^{m-n}$	$\frac{x^{10}}{x^3}=x^{10-3}=x^7$ $\frac{35a^6}{-5a}=-7a^{6-1}=-7a^5$
• **Power of a Power** To find the **power** of a **power**, **multiply** the exponents. $(a^m)^n=a^{m\cdot n}$	$(y^2)^5=y^{2\cdot 5}=y^{10}$ $(5^3)^2=5^{3\cdot 2}=5^6$ $[(4^2)^3]^5=[4^6]^5=4^{30}$
• **Power of a Product** To find the **power** of a **product**, find the product of each factor raised to the **power**. $(ab)^m=a^m\cdot b^m$	$(2x^2y^3)^3=2^3x^{2\cdot 3}y^{3\cdot 3}=8x^6y^9$ $(-4m^5)^3=(-4)^3m^{5\cdot 3}=-64m^{15}$

Scientific Notation • A number written as the product of a factor (greater than or equal to 1, and less than 10) and a power of 10. $a \times 10^n$ $1 \le a < 10$ n is an integer	$51{,}000 = 5.1 \times 10^4$ 5.1×10^4 Factor: • greater than or equal to 1 • less than 10 Power of 10: 10 raised to a integer power
• How to write numbers in **scientific notation**	$236{,}785 \rightarrow 2.36785 \times 10^5$ $0.00062 \rightarrow 6.2 \times 10^{-4}$
• How to write numbers in **standard** form	$3.2 \times 10^5 \rightarrow 320000$
• **Multiplying** and **dividing**: Use the commutative and associative properties.	$4 \times 10^3 \times 2 \times 10^6 = 8 \times 10^9$ $\dfrac{12 \times 10^{13}}{6 \times 10^5} = 2 \times 10^8$
• **Adding** and **subtracting**: Rewrite one number so that both numbers have the same exponent.	$5.1 \times 10^3 - 1.9 \times 10^2$ $= 5.1 \times 10^3 - 0.19 \times 10^3 = 4.91 \times 10^3$

Roots • **Finding square roots** Every positive number has a positive and negative square root. $If \;\; a^2 = b\,, \quad then \;\; a = \pm\sqrt{b}$	$3^2 = 9\,, \quad so \;\; \pm 3 = \pm\sqrt{9}$ $\pm\sqrt{144} = \pm 12$ $-\sqrt{\dfrac{9}{25}} = -\dfrac{3}{5}$
• **Finding cube roots** $If \;\; a^3 = b\,, \quad then \;\; a = \sqrt[3]{b}$	Solve this equation: $m^3 = 64$ $m = \sqrt[3]{64}$ $m = 4$
• **Estimating Roots** Use the perfect squares and perfect cubes you know to estimate the square root or cube root of a number that is not a perfect square or cube.	Estimate $\sqrt{70}$ You know $8^2 = 64$ and $9^2 = 81$. $64 < 70 < 81$ $8^2 < 70 < 9^2$ $\sqrt{8^2} < \sqrt{70} < \sqrt{9^2}$ $8 < \sqrt{70} < 9$ 70 is closer to 64 than 81, so the best integer estimate for $\sqrt{70}$ is 8.

Chapter 2: Expressions and Equations

Solving equations with **rational coefficients** (fractions and decimals). • For **fraction coefficients** use the **multiplicative inverse.** • The multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$. When you multiply them you get 1. • For **decimal coefficients**, divide both sides by the decimal. • **Check** the solution by substituting into the original equation.	Example: Solve $\frac{2}{3}x = 10$. Multiply both sides by $\frac{3}{2}$. $\left(\frac{3}{2}\right)\cdot\left(\frac{2}{3}\right)x = \left(\frac{3}{2}\right)\cdot\left(\frac{10}{1}\right)$ $\left(\frac{\cancel{3}^{1}}{\cancel{2}_{1}}\right)\cdot\left(\frac{\cancel{2}^{1}}{\cancel{3}_{1}}\right)x = \left(\frac{3}{2}\right)\cdot\left(\frac{10}{1}\right)$ $x = \frac{30}{2} = 15$ Check: $\frac{2}{3}(15) \stackrel{?}{=} 10$ $\frac{30}{3} = 10$ ✓ Solve $-2.5x = 50$. Divide both sides by -2.5 $\frac{-2.5x}{-2.5} = \frac{50}{-2.5}$ $x = -20$ Check: $-2.5(-20) \stackrel{?}{=} 50$ $50 = 50$ ✓
Two-step equations • Do two operations to isolate the variable.	Example: **Solve** $3x - 2 = 13$ The two operations are multiplication and subtraction. Do the **opposite** operations. First, **add** 2 to both sides: $3x - 2 + 2 = 13 + 2$ $3x = 15$ Next, **divide** both sides by 3. $\frac{3x}{3} = \frac{15}{3}$ You get: $x = 5$ Check: $3(5) - 2 \stackrel{?}{=} 13$ $15 - 2 = 13$ ✓
Write two-step equations • Read the description carefully. • Define the variable. • Translate the key math terms (quotient, less than, the same as, …) into symbols ($\div, -, =, \ldots$) in an equation.	Example: *One more than the quotient of a number and three is equal to five.* Let a be the number. (Choose any variable you like.) Quotient of a number and three: $\frac{a}{3}$ One more than that: $\frac{a}{3} + 1$ Is equal to five: $\frac{a}{3} + 1 = 5$ This equation, $\frac{a}{3} + 1 = 5$, can be solved in two steps.

Solve equations with variables on both sides • Use the properties of equality to bring all terms with variables to one side. • Then isolate that variable.	Example: Solve $\frac{5}{8}m + 1 = \frac{1}{4}m - 8$ Subtract 1 from both sides: $$\frac{5}{8}m + 1 - 1 = \frac{1}{4}m - 8 - 1$$ $$\frac{5}{8}m = \frac{1}{4}m - 9$$ Subtract $\frac{1}{4}m$ from both sides. $$\frac{5}{8}m - \frac{1}{4}m = \frac{1}{4}m - 9 - \frac{1}{4}m$$ Convert $\frac{1}{4}$ to $\frac{2}{8}$ so you can subtract. $$\frac{5}{8}m - \frac{2}{8}m = -9$$ $$\frac{3}{8}m = -9$$ Multiply both sides by $\frac{8}{3}$. $$\left(\frac{8}{3}\right)\frac{3}{8}m = \left(\frac{8}{3}\right)\left(\frac{-9}{1}\right)$$ $$m = -24$$ Check your answer by substituting -24 into the original equation.
Solve multi-step equations • When you see parentheses in an equation use the **distributive property**. $$a(b + c) = a \cdot b + a \cdot c$$	Example: Solve $10 - n = 2(3n - 16)$ First, use the distributive property on the right hand side. $$10 - n = 6n - 32$$ Bring the variable terms together on one side. $$10 - n + n = 6n + n - 32$$ $$10 = 7n - 32$$ $$10 + 32 = 7n - 32 + 32$$ $$42 = 7n$$ Divide both sides by 7 to isolate the variable, n. $$\frac{42}{7} = \frac{7n}{7}$$ $$6 = n$$ Remember to check the solution by substituting back into the original equation. Check: $10 - n = 2(3n - 16)$ $$10 - 6 \stackrel{?}{=} 2(3(6) - 16)$$ $$4 \stackrel{?}{=} 2(18 - 16)$$ $$4 \stackrel{?}{=} 2(2)$$ $$4 = 4 \checkmark$$

Chapter 3: Equations in Two Variables

Constant rate of change

A linear relationship follows this rule:

$$\boxed{\frac{change\ in\ y}{change\ in\ x} = \text{constant}}$$

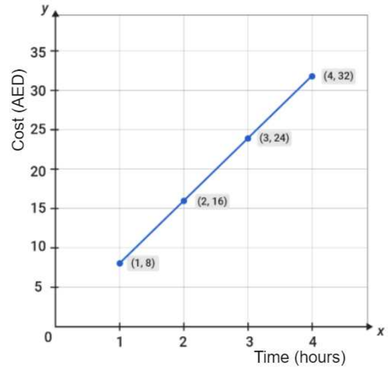
Linear relationships show a constant rate of change.
One quantity is changing constantly in relation to another quantity.

Liters of paint needed to make a mixture of pink paint	
Red (liters), *x*	White (liters), *y*
2	5
4	10
6	15
8	20

+2 +2 +2   +5 +5 +5

$\frac{change\ in\ y}{change\ in\ x} = \frac{5}{2}$ for each line. This table shows a linear relationship with a constant rate of change.

This graph also shows a constant rate of change



Choose two points, e.g. (4, 32) and (1, 8):
$\frac{change\ in\ y}{change\ in\ x} = \frac{32-8}{4-1} = \frac{24}{4} = 6$ for each pair of points. When the points are joined together they form a straight line.

If you extend this line it does not pass through the origin (0, 0) so the relationship is **not proportional**.

Slope

To find the slope divide the vertical change (the rise) by the horizontal change (the run).

$$\boxed{slope = \frac{rise}{run}}$$

or

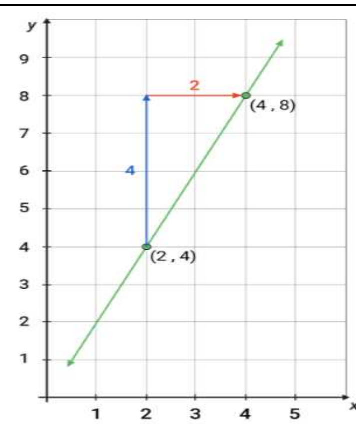
$$\boxed{slope = \frac{y_2 - y_1}{x_2 - x_1}}$$

You can find slope by using points from a table or from a graph.

In this graph, you have two points, (2,4) and (4,8).

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8-4}{4-2} = \frac{4}{2} = 2$$



Find the slope of the linear relationship in this table:

Number of Employees, *x*	0	3	6	9
Number of Hours, *y*	6	11	16	21

Use any two points, e.g. (6, 16) and (9, 21).

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{21-16}{9-6} = \frac{5}{3}$$

Equations in $y = mx$ form

- An equation written in the form, $y = mx$, represents a linear relationship that is a direct variation.
- The slope, m, is the constant of variation.
- $m = \frac{y}{x}$
- The graph of a direct variation always goes through the origin (0, 0).

Q. Can you write a direct variation equation if you have an x-value and y-value?

✓ Yes. For example:
A hot-air balloon rises 1,110 m in 5 minutes. Write and solve a direct variation equation to find how high it rises in 7 minutes.

$y = mx$, so $m = \frac{y}{x} = \frac{1{,}110}{5} = 222$
The direct variation equation is $y = 222x$.
How high will the balloon rise in 7 minutes?
Substitute:
$$y = 222x$$
$$y = 222(7)$$
$$y = 1{,}554$$

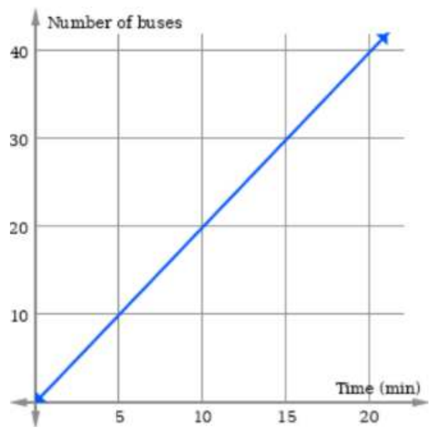
The hot-air balloon rises 1,554 m in 7 minutes.

Example 1: *Determine if the linear function represented in this table is a direct variation.*

Age, x	8	9	10	11
Grade, y	3	4	5	6

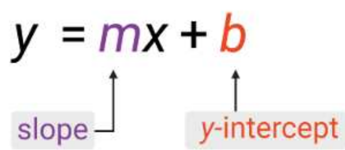
$m = \frac{y}{x} = \frac{3}{8}$ $\quad m = \frac{y}{x} = \frac{4}{9}$ $\quad m = \frac{y}{x} = \frac{5}{10} = \frac{1}{2}$
The value of m is different each time, so this is not a direct variation.

Example 2: *This graph below shows a direct variation. It passes through (0, 0). Find the constant of variation.*

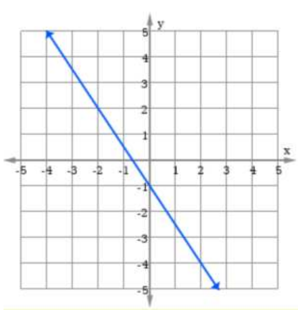


Use one point, for example (15, 30).
$$m = \frac{y}{x} = \frac{30}{15} = 2$$
This is the slope of the line.
There are two buses each minute.

Slope-intercept form



You can look at a graph and write the equation.

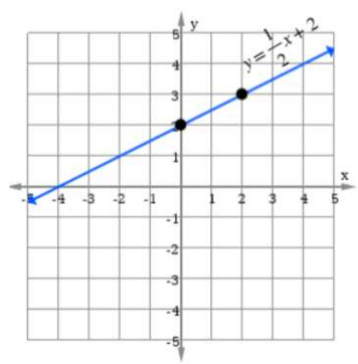


The y-intercept is -1.
So, $b = -1$
Find one more point on the line, for example (2, -2).
From $(0, -1)$ to $(2, -2)$ the rise is 3 and the run is -2.

The slope is $\frac{3}{-2} = -\frac{3}{2}$. So, $m = -\frac{3}{2}$.
The equation of the line is $y = -\frac{3}{2}x - 2$.

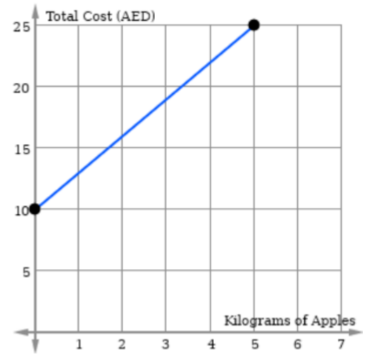
Example 1: *Graph a line with a slope of $\frac{1}{2}$ and a y-intercept of 2.*

First, plot the y-intercept at (0, 2).
$$slope = \frac{rise}{run} = \frac{1}{2}$$
From that point, rise 1 and run 2. Plot another point. Join them with an extended straight line.



- In a real-life problem, the y-intercept represents the initial value and the slope represents the rate of change.

Example 2:
Hamad buys apples for AED 3 per kilogram. The shop charges a delivery fee.
Find the slope and interpret the y-intercept.



The rate of change, AED 3 per kilogram, is the slope. So, $m = 3$. The y-intercept is the delivery charge of AED 10.
The equation of the line is $y = 3x + 10$

Graph a line using intercepts

- To find the ***x*-intercept** let $\boldsymbol{y = 0}$.
- To find the **y-intercept** let $\boldsymbol{x = 0}$.
- Use the two intercepts to graph the line.
- A **standard form** equation is written $\boldsymbol{Ax + By = C}$, where $A \geq 0$ and $A, B\ and\ C\ are\ integers$

You can interpret the x- and y-intercepts of a standard form equation:

You have AED 240 to buy gifts.
A box of chocolates, x, costs AED 40. A basket of fruit, y, costs AED 60.
This can be represented by the equation $40x + 60y = 240$.

You can find that the x-intercept is at $(6, 0)$, and the y-intercept is $(0, 4)$.

This means you can buy 6 boxes of chocolates and zero baskets of fruit for AED 240.

Or, you can buy zero boxes of chocolate and 4 baskets of fruit for AED 240.

Example 1: *Find the x- and y-intercepts of the equation. Use them to graph the equation.*

$y = \frac{2}{3}x - 4$

x-intercept: put $y = 0$

$0 = \frac{2}{3}x - 4$

$4 = \frac{2}{3}x$

$\left(\frac{3}{2}\right)\frac{4}{1} = \left(\frac{3}{2}\right)\frac{2}{3}x$

$6 = x$ The x-intercept is at $(6, 0)$.

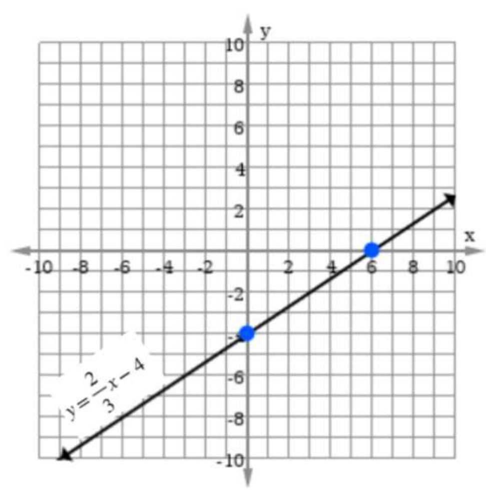
y-intercept: put $x = 0$

$y = \frac{2}{3}(0) - 4$

$y = 0 - 4$

$y = -4$ The y-intercept is at $(0, -4)$.

Plot the two points and join them with a straight line.



Write linear equations

- **Point-slope** form

$$y - y_1 = m(x - x_1)$$

m is the slope of the line, (x_1, y_1) is a point on the line.

- **Slope-intercept** form

$$y = mx + b$$

m is the slope and b is the y-intercept.

Q. How do I know which form to use?

✓ If you are given a point and the slope, use the point-slope form.

✓ If you are given the slope and the y-intercept, use the slope-intercept form.

✓ If you are given two points, use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slope. Then use the point-slope form.

Example 1:

Write an equation of a line that passes through $(8, -1)$ and has a slope of $\frac{1}{2}$.

$(x_1, y_1) = (8, -1)$ and $m = \frac{1}{2}$.

$y - y_1 = m(x - x_1)$

$y - -1 = \frac{1}{2}(x - 8)$

$\boxed{y + 1 = \frac{1}{2}(x - 8)}$ This is in point-slope form.

$y + 1 = \frac{1}{2}x - 4$

$\boxed{y = \frac{1}{2}x - 5}$ This is in slope-intercept form.

Example 2:

Write an equation of the line that passes through $(2, 3)$ and $(5, 5)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{5 - 2} = \frac{2}{3}$

$y - y_1 = m(x - x_1)$

$\boxed{y - 3 = \frac{2}{3}(x - 2)}$ Point-slope form

$y - 3 = \frac{2}{3}x - \frac{4}{3}$

$\boxed{y = \frac{2}{3}x + \frac{5}{3}}$ Slope-intercept form.

Solve systems of equations by graphing

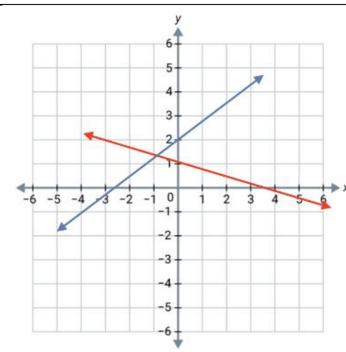
- A system of equations is two or more equations that have the same variables (usually x and y).
- When you are asked to solve a system of equations you are looking for a point of intersection of the two graphs.
- Draw graphs of both lines and look for a point of intersection.

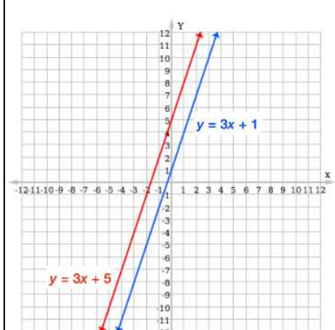
Q. How do I know how many solutions a system of equations has?

✓ If the lines have **different slopes**, they intersect at one point. There is one solution.

✓ If the lines have the **same slopes** and different y-intercepts, they are parallel and do not intersect. There is no solution.

Different slopes.
One solution
(the point of intersection).

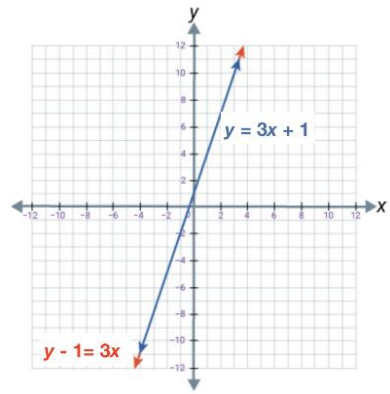




Same slopes, different y-intercepts.
No solution.

- ✓ If the lines are laying on top of each other, they are the **same line**. There are an infinite number of solutions.

Same line.
Infinite number of solutions.



Solve systems of equations algebraically

- Instead of graphing the two equations you can use **substitution** to find the point of intersection (if any).

Example 1:

Solve this system of equations:

$Equation\ 1\!:\ \ y = -2x - 6$

$Equation\ 1\!:\ \ y = 4x$

Substitute the *y*-value in equation 1 into equation 2, then solve for *x*.

$4x = -2x - 6$

$4x + 2x = -6$

$6x = -6$

$\boxed{x = -1}$ This is the *x*-value of the solution.

Substitute this x-value into equation 2 and solve for *y*.

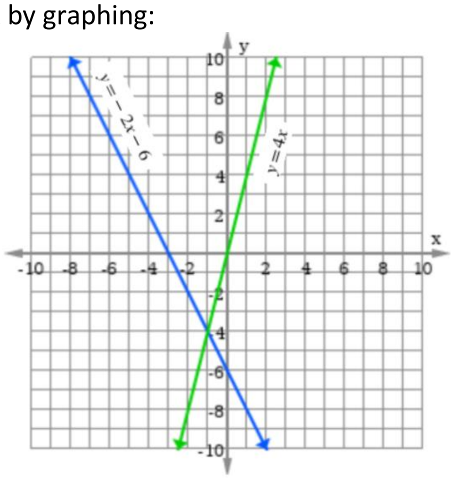
$y = 4x$

$y = 4(-1)$

$\boxed{y = -4}$ This is the *y*-value of the solution.

The solution is $(-1, -4)$.

Check by graphing:



You can see the point of intersection is $(-1, -4)$.

- You can solve real-world problems with this method.

Example 2:

Hessa made cheese rolls and zaatar rolls for her class. In total there were 28 rolls. She made three times as many cheese rolls as zaatar rolls.

Write and solve a systems of equations for this situation.

First, let *x* = number of cheese rolls, and *y* = number of zaatar rolls.

$Equation\ 1\!: x + y = 28$, because the total is 28.

$Equation\ 2\!:\ \ y = 3x$, because there were three times as many cheese rolls as zaatar rolls.

Substitute $y = 3x$ into equation 1.

$x + y = 28$

$x + 3x = 28$

$4x = 28$

$\boxed{x = 7}$

Substitute this x-value into equation 2 and solve for *y*.

$y = 3x$

$y = 3(7)$

$\boxed{y = 21}$

The solution is $(7,\ 28)$.

This means Hessa made 7 cheese rolls and 21 zaatar rolls.

You can check by graphing.

Chapter 4: Functions

Relations and representing relations

A linear equation can be represented in many ways, including in a **table**, in a set of **ordered pairs**, in **words** and in a **graph**.

- The **domain** of a relation is the set of *x*-coordinates.
- The **range** of a relation is the set of *y*-coordinates.

x	*y*
2	16
4	32
6	48
8	64

domain

range

In this table, the domain is {2, 4, 6, 8} and the range is {16, 32, 48, 64}.

- You can use relations to write equations.
- You can use equations to find unknown values.

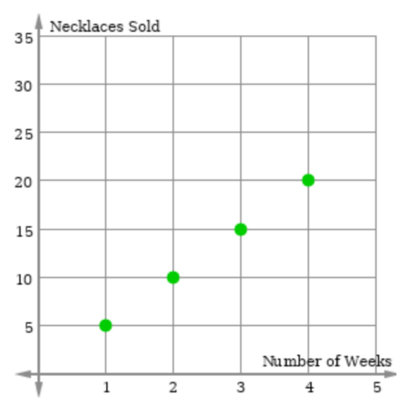
Example:

Sara sells 5 necklaces each week for 4 weeks. The set of ordered pairs is (1, 5), (2, 10), (3, 15), (4, 20).

*(a) Make a **table**. State the **domain** and the **range**. Then **graph** the ordered pairs.*

Number of Weeks, *x*	Necklaces Sold, *y*
1	5
2	10
3	15
4	20

domain = {1, 2, 3, 4}, range = {5, 10, 15, 20}



*(b) **Write an equation** to find the number of necklaces, y, sold in x weeks. Use the equation to find the number of necklaces sold in 9 weeks.*

Find the slope, m, using any two points. For example, (2, 10) and (4, 20).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 10}{4 - 2} = \frac{10}{2} = 5$$

Find the y-intercept using $y = mx + b$ (Use any point for example (3, 15) for x and y.)

$$y = mx + b$$
$$15 = 5(3) + b$$
$$15 = 15 + b$$
$$0 = b$$

So, the equation is $y = 5x + 0$, which is $\boxed{y = 5x}$.

In 9 weeks, she sold $y = 5(9) = 45$ necklaces.

Functions

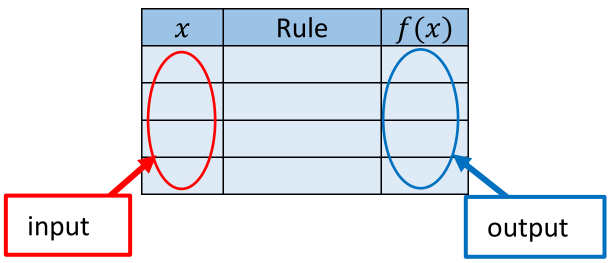
Here is an example of a function:

$$f(x) = 3x - 2$$

- The **input** is x.
- The **output** is $f(x)$.
- We **find $f(x)$** by substituting the value of x into the function.

Q. What is a **function table**?

✓ It is a table we use to find the outputs for a function. It looks like this:



The **rule** is this part of a function.

$f(x) = 3x - 2$

✓ The variable for the input, or **domain**, is the **independent variable**.

✓ The variable for the output, or **range**, is the **dependent variable**.

Example 1:

Find $f(3)$ if $f(x) = 5x + 1$

Substitute 3 for *x*.

$f(3) = 5(3) + 1$

$f(3) = 15 + 1$

$f(3) = 16$

Example 2:

Make a function table for $f(x) = 3x - 2$.

State the domain and range of the function.

Choose some values for the input. For example: $-2, 0, 2, 4$.

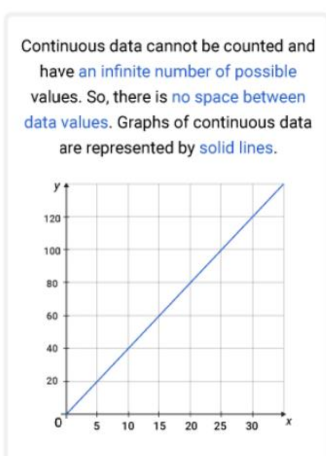
Substitute into the rule. Find the output, $f(x)$.

x	$3x - 2$	$f(x)$
-2	$3(-2) - 2$	-8
0	$3(0) - 2$	-2
2	$3(2) - 2$	4
4	$3(4) - 2$	10

The domain is $\{-2, 0, 2, 4\}$ and the range is $\{-8, -2, 4, 10\}$.

Linear Functions

- You can graph a linear function by plotting the ordered pairs on a coordinate plane.
- Data on a graph can be **continuous** or **discrete**.



Example:

A shop makes mobile phone covers with your name on them. They charge AED 15 for the cover and AED 2 for each letter of your name.

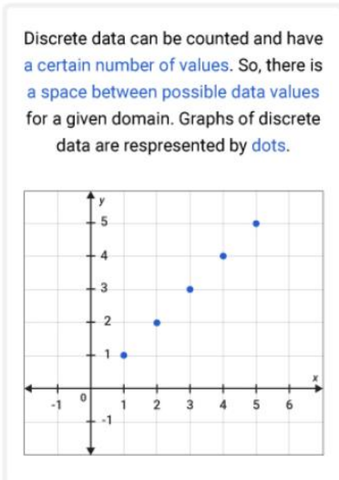
(a) Write a function to represent the total cost of any number of letters.

The **independent variable**, *x*, is the number of letters on the phone cover.

The **dependent variable**, *y*, is the total cost of the phone cover with letters.

The **initial value** (or *y*-intercept) is AED 15.

The function is $\boxed{y = 2x + 15}$



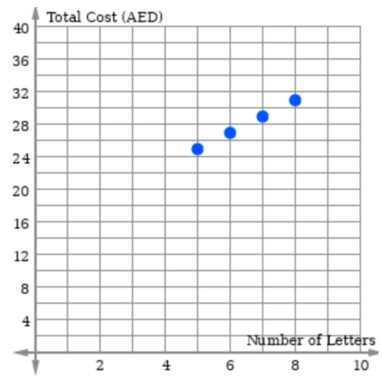
(b) Complete a function table to find the cost for 5, 6, 7, and 8 letters.

x	$2x + 15$	y
5	$2(5) + 15$	25
6	$2(6) + 15$	27
7	$2(7) + 15$	29
8	$2(8) + 15$	31

The ordered pairs are $(5, 25), (6, 27), (7, 29)$ *and* $(8, 31)$.

(c) Graph the function. Is the data continuous or discrete?

You cannot have half of a letter! So, the data is **discrete**.



Compare Properties of Functions and Construct Functions

- You know the slope-intercept form

$$y = mx + b$$

  m is the slope and b is the y-intercept.
- We also say m is the **rate of change** and b is **initial value**.
- You can **interpret** the rate of change and the initial value and use them to **compare** functions.

Q How do I interpret the rate of change?

In the example shown, if x is the number of kilograms of apples and y is the total cost, then the table represents a shop that has a greater unit rate. The apples cost more per kilogram in the shop represented by the table.

Q What does the initial value tell us?

It the example shown, the table has a y-intercept at $y = 4$. The graph has a y-intercept at $y = 15$. This could be the delivery charge, or the cost of packaging. The details are usually in the words of the question.

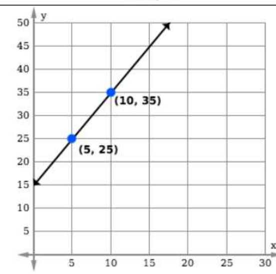
Example:

Compare the rate of change for each function.

Table

x	y
0	4
1	9
2	14
3	19

Graph



First, choose two points from the table to find the slope of the function represented by the table.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 4}{1 - 0} = \frac{5}{1} = 5$$

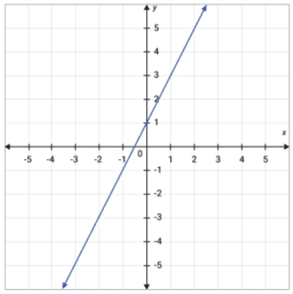
Then, use the two points in the graph to find the slope of that function.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{35 - 25}{10 - 5} = \frac{10}{5} = 2$$

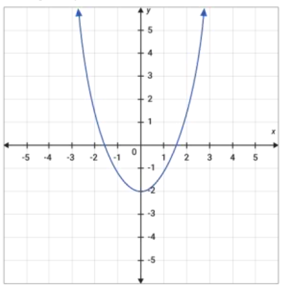
The function represented by the **table** has the **greatest** rate of change.

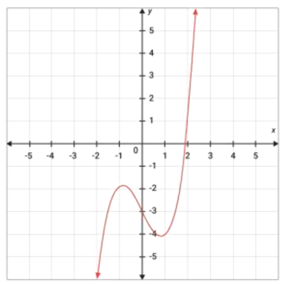
Linear and Nonlinear Functions

- You know a **linear** function is a function whose graph forms a **straight** line.



- A **nonlinear** function is a function whose graph does **not** form a **straight** line.





Example:

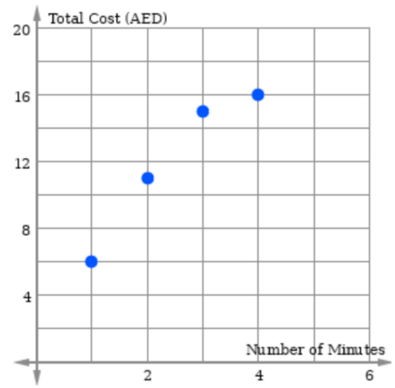
The charges for a massage chair in a mall are shown in the table.
Determine whether this represents a linear or a nonlinear function.

Number of Minutes, x	1	2	3	4
Total Cost (AED), y	6	11	15	16

First, check the rate of change. As x increases by 1, y increases by a smaller amount each time. The rate of change is **not constant**, so this function must be **nonlinear**.

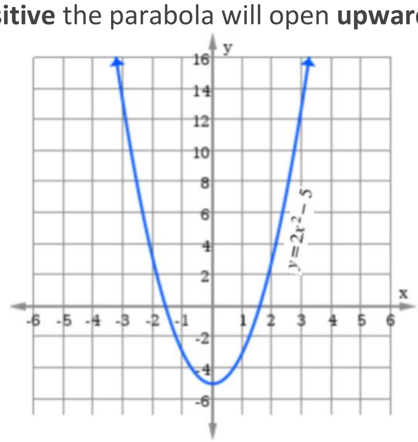
Check by graphing:

It is clearly not a linear function. It is a nonlinear function.



Quadratic Functions

- A **quadratic function** is a function that can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$.
- The graph of a quadratic function is called a **parabola**.
- If the coefficient of the squared term is **positive** the parabola will open **upward**.



Example:

The quadratic equation $p = 50 + 2r^2$ represents the profit, p, made by a factory that produces r ovens.

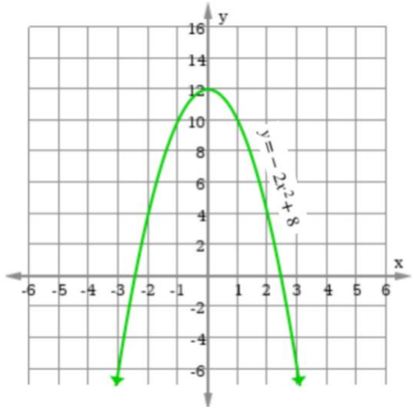
Graph this function. Then use your graph to estimate the profit for 5 ovens.

First, use a function table to find a set of ordered pairs.

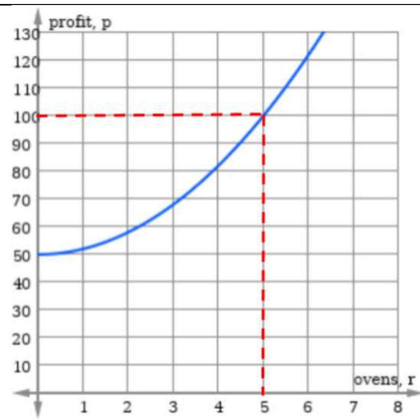
r	$50 + 2r^2$	p	(r, p)
0	50 + 2(0)	50	(0, 50)
2	50 + 2(4)	58	(2, 58)
4	50 + 2(16)	82	(4, 82)
6	50 + 2 (36)	122	(6, 122)

Then, plot the points and join them with a smooth curve to help find unknown values.

- If the coefficient of the x^2 term is **negative** the parabola will open **downward**.



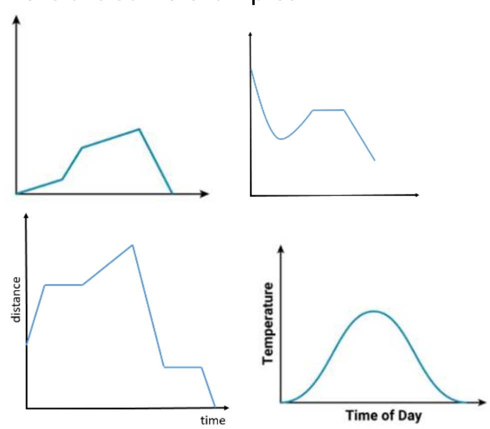
- You can use a parabola to find an unknown value.



To find the profit when they sell 5 ovens, draw a vertical line from $x = 5$ up to the graph, then draw a horizontal line to the profit axis.
The profit for 5 ovens will be AED 100.

Qualitative Graphs

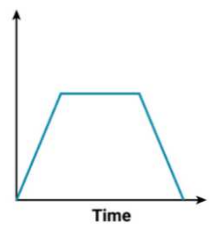
These are graphs that give you a **general idea** of the change in one variable in relation to another. They usually have **no numbers** on the axes.
Here are some examples:



- Use a **straight** line to show a constant rate of change. The steepness of a graph tells you how quick or slow the rate of change is.
- Use a **curved** line to show a rate of change that is not constant.
- Use a **horizontal** line to show a period of no change.
- You need to be able to **sketch** a qualitative graph.
- You need to be able to **analyze** a qualitative graph.

Example 1:

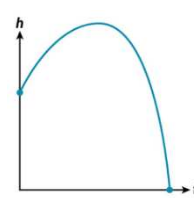
This graph shows how the noise level changed in a classroom. Describe the change in the noise level over time.



The **first part** of the graph shows a **constant increase** in the noise level (the line is increasing from left to fright).
Then, there was a period of **no change** in the noise level (the horizontal line tells us this).
The **last part** shows a **constant decrease** in the noise level (the line is decreasing from left to right).

Example 2:

This graph shows the height, h, of a ball.
Describe how the height changed over time.



The ball started from a height (the starting point is not zero). The height **increased** at a rate that was **not constant** (the upward curved line tells us that). Then the height **decreased** at a rate that was **not constant**, until the ball hit the ground (it ended at a height of zero).

Mock Exam-1

Part 1

Circle the letter corresponding to the correct answer.

1) Write 1.2 as a fraction or mixed number in simplest form.

a) $1\frac{2}{100}$

b) $1\frac{2}{5}$

c) $\frac{12}{100}$

d) $1\frac{1}{5}$

2) Simplify $(-2b)(3b^4)$ using the laws of exponents,

a) $-5b^4$

b) $-6b^5$

c) b^4

d) $6b^5$

3) Write 64,300 in scientific notation,

a) 64.3×10^3

b) 64.3×10^4

c) 6.43×10^4

d) 6.43×10^3

4) Find the cube root: $\sqrt[3]{-125}$

a) -5

b) 5

c) no real root

d) -25

5) Order the set of numbers $\{\sqrt{61},\ \frac{10}{3},\ \sqrt[3]{166},\ 7\frac{1}{2}\}$ from least to greatest.

a) $\{\frac{10}{3},\ \sqrt[3]{166},\ 7\frac{1}{2},\ \sqrt{61}\}$

b) $\{\sqrt{61},\ \frac{10}{3}, \sqrt[3]{166},\ 7\frac{1}{2}\}$

c) $\{\frac{10}{3}, \sqrt{61},\ \sqrt[3]{166},\ 7\frac{1}{2}\}$

d) $\{\sqrt{61},\ \frac{10}{3},\ 7\frac{1}{2}, \sqrt[3]{166}, \}$

6) Solve the equation, $1.4m = 3.5$

a) $m = 4.9$

b) $m = 2.5$

c) $m = 2.1$

d) $m = 2.3$

7) Identify the slope, m, in this equation: $y = -\frac{4}{5}x - 8$

a) $m = -8$

b) $m = \frac{4}{5}$

c) $m = -\frac{4}{5}$

d) $m = 8$

8) Translate the sentence into an equation:

The sum of three times a number and five is seven.

a) $3 + 5x = 7$

b) $3x + 5 = 7$

c) $3x - 5 = 7$

d) $5 - 3x = 7$

9) Write the equation for the direct variation represented in this table:

x	0	2	4	6
y	0	8	16	24

a) $y = 4x$

b) $y = \frac{1}{4}x$

c) $y = 4x + 2$

d) $y = 2x$

10) State the domain of the function represented in this table of values:

x	y
5	8
7	11
10	17
15	27

a) $\{5, 7, 8, 11\}$

b) $\{5, 7, 10, 15\}$

c) $\{8, 11, 17, 27\}$

d) $\{10, 15, 17, 27\}$

11) Find $f(2)$ if $f(x) = -3x - 8$

a) $f(2) = -14$

b) $f(2) = -2$

c) $f(2) = 18$

d) $f(2) = 0$

12) Which of these equations shows a nonlinear function?

a) $2x + 3y = 5$

b) $y = 2x^2 + 2$

c) $y = 2x + 2$

d) $y = \frac{x}{4}$

13) Write an equation in point-slope form of a line that passes through $(-2, 3)$ and has a slope of -2.

a) $y + 3 = -2(x + 2)$

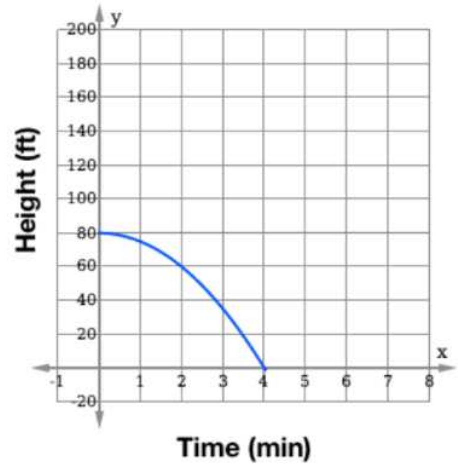
b) $y - 3 = -2(x - 2)$

c) $y - 3 = -2(x + 2)$

d) $y + 2 = -2(x - 3)$

14) A snowflake is falling from the top of a building. The change in height is represented in the graph.

Use the graph to estimate the height of the snowflake at 2 minutes.



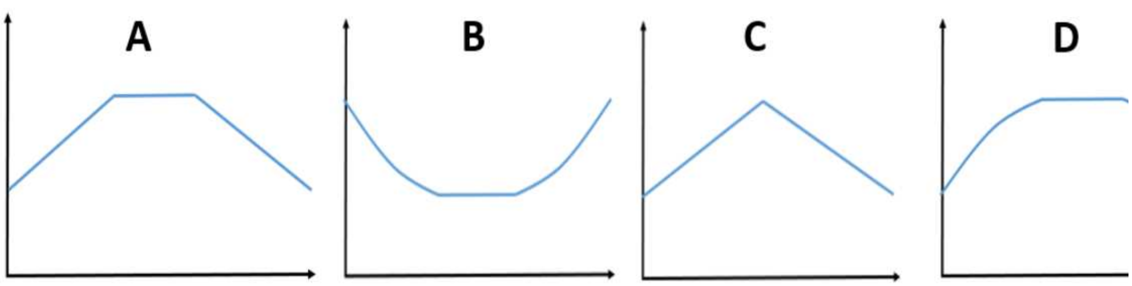
a) $75\ ft$

b) $59\ ft$

c) $40\ ft$

d) $47\ ft$

15) A car decreased its speed at a rate that was not constant, then it stayed at the same speed for a while and after that it increased its speed at a rate that was not constant. Which graph represent this situation?



a) Graph A

b) Graph B

c) Graph C

d) Graph D

Part 2

Show all your work when answering these questions.

16) Evaluate the expression, $b^3 - (a+b)^2$, if $a = 3$ and $b = -2$.

    -9

17) A company has set aside AED 10^7 for employee bonuses for National Day.
    If the company has 10^4 employees and the money is divided equally among them, how much will each employee receive?

    10^3 AED

18) This table shows the population in some countries.
    How many more people live in Saudi Arabia than in the U.A.E?
    Give your answer in scientific notation.

    2.37×10^7

Country	Population
Australia	2.4×10^7
Egypt	9.7×10^7
Mongolia	3.1×10^6
Saudi Arabia	3.3×10^7
U.A.E	9.3×10^6

19) Khalifa wants to take some swimming lessons. It costs AED 150 to join the swimming club. Then it costs AED 45 for each lesson.
    (a) Write an equation to represent the total cost, *y*, for *x* lessons.

    $\boldsymbol{y = 150 + 45x}$

    (b) Khalifa has AED 420 to spend. Use the equation to find how many lessons he can attend.
    6 lessons

20) (a) Solve the equation:
    $$3(4x-1) + 13 = 5(2+2x) + 2x$$

    All real numbers are solutions

    (b) State if the equation has one solution, no solution or infinitely many solutions.

    infinitely many solutions

21) Determine whether the relationship between the two quantities described in the table is linear.
If so, find the constant rate of change.
If not, explain your reasoning.

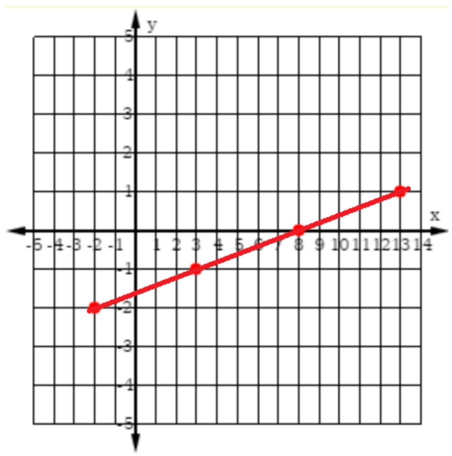
Linear, it has a constant rate of change

Time (min)	Temperature (°C)
9	60
10	64
11	68
12	72

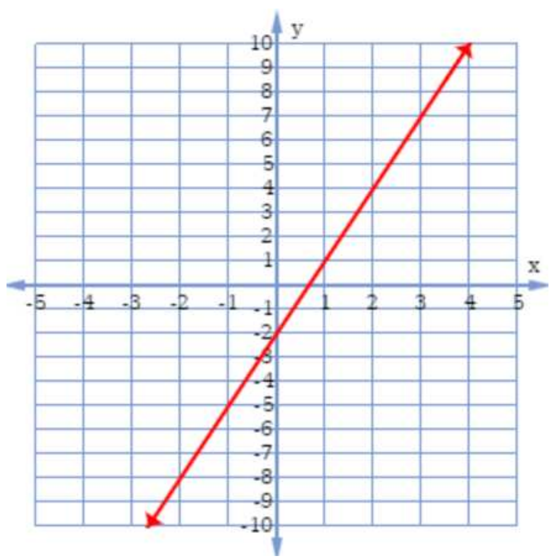
22) The points given in this table lie on a line. Find the slope of the line. Then graph the line.

x	–2	3	8	13
y	–2	–1	0	1

$m = \frac{1}{5}$



23) Graph a line with a slope of 3 and a y-intercept of -2.



24) Write an equation in slope-intercept form for the line that passes through $(5, -1)$ and $(-10, 8)$.

$\boldsymbol{y = -\frac{3}{5}x + 2}$

25) Noura has AED 48 to spend on pens and pencils.

A pen, x, costs AED 4.

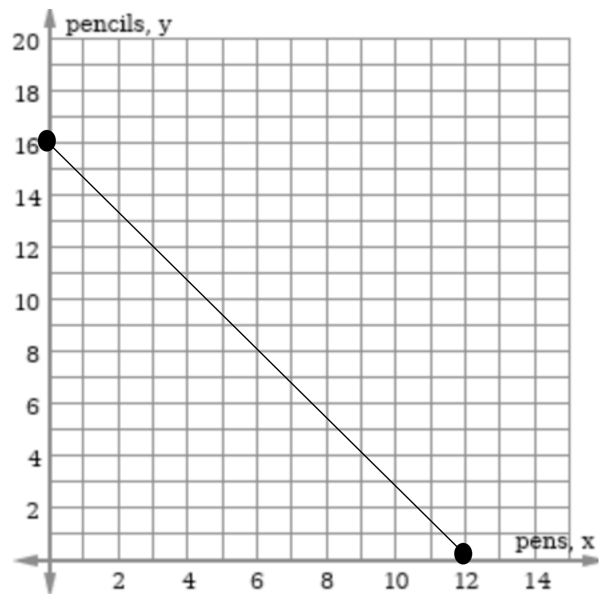
A pencil, y, costs AED 3.

The number of pens and pencils she can buy is represented by the equation $4x + 3y = 48$.

(a) Use the *x*- and *y*-intercepts to graph the equation.

$\boldsymbol{x\ intercept = 12}$

$\boldsymbol{y\ intercept = 16}$



(b) Interpret the *x*- and *y*-intercepts.

Number of pens she can buy = 12

Number of pencils she can buy = 16

End of Mock Test 1

Mock Exam-2

Part 1

Circle the letter corresponding to the correct answer.

26) Write 0.28 as a fraction in simplest form.

a) $2\frac{8}{10}$

b) $\frac{28}{10}$

c) $\frac{7}{25}$

d) $\frac{7}{50}$

27) Simplify $8m^5(2m^3)$ using the laws of exponents

a) $16m^{15}$

b) $10m^8$

c) $10m^{15}$

d) $16m^8$

28) Write 3.45×10^{-3} in standard form.

a) 0.0345

b) 0.00345

c) 0.000345

d) $3{,}450$

29) Find the cube root: $\sqrt[3]{-8}$

a) 2

b) -2

c) no real root

d) -4

30) Order the set of numbers $\{\sqrt{52},\ 4,\ \sqrt[3]{301},\ 8\frac{1}{10}\}$ from least to greatest.

a) $\{4,\ \sqrt[3]{301},\ \sqrt{52},\ 8\frac{1}{10}\}$

b) $\{4,\ \sqrt{52},\ \sqrt[3]{301},\ 8\frac{1}{10}\}$

c) $\{4,\ 8\frac{1}{10},\ \sqrt{52},\ \sqrt[3]{301}\}$

d) $\{4,\ 8\frac{1}{10},\ \sqrt[3]{301},\ \sqrt{52}\}$

31) Solve the equation, $\frac{2}{3}k = 1\frac{1}{3}$

a) $k = \frac{2}{3}$

b) $k = \frac{1}{3}$

c) $k = 2$

d) $k = 3$

32) Identify the slope, m, in this equation: $y = -\frac{2}{7}x - \frac{2}{3}$

a) $m = -\frac{2}{3}$

b) $m = \frac{2}{7}$

c) $m = -\frac{2}{7}$

d) $m = -\frac{7}{2}$

33) Translate the sentence into an equation:

The difference between 10 and $\frac{1}{4}$ of a number is 8.

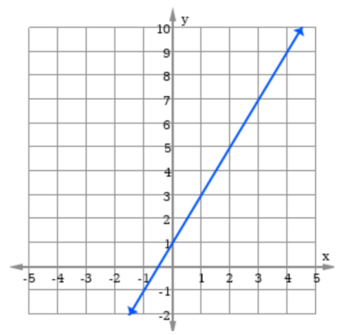
a) $10 - 8x = \frac{1}{4}$

b) $\frac{1}{4}x - 8 = 10$

c) $\frac{1}{4}x - 10 = 8$

d) $10 - \frac{1}{4}x = 8$

34) Write the equation for the function represented in this graph:



a) $y = 3x - 1$

b) $y = \frac{1}{2}x + 1$

c) $y = 2x + 1$

d) $y = 2x$

35) State the range of the function represented in this table of values:

x	y
5	8
7	11
10	17
15	27

a) $\{5, 7, 8, 11\}$

b) $\{5, 7, 10, 15\}$

c) $\{8, 11, 17, 27\}$

d) $\{10, 15, 17, 27\}$

36) Find $f(-3)$ if $f(x) = 10 - 7x$

a) $f(-3) = -11$

b) $f(-3) = -9$

c) $f(-3) = 21$

d) $f(-3) = 31$

37) Which of these equations shows a linear function?

a) $x = 4$

b) $y = 2x^2 + 3x - 2$

c) $y = 2x + 2$

d) $y = \frac{2}{3}x^2$

38) Write an equation in point-slope form of a line that passes through (1, 5) and has a slope of -3.

a) $y + 5 = 3(x + 1)$

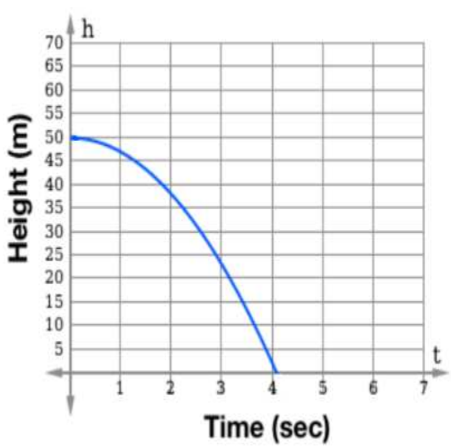
b) $y + 5 = -3(x + 1)$

c) $y - 1 = -3(x - 5)$

d) $y - 5 = -3(x - 1)$

39) A ball is dropped from a height.
The change in height is shown in the graph.

Use the graph to estimate the time when the ball was at 35 meters.



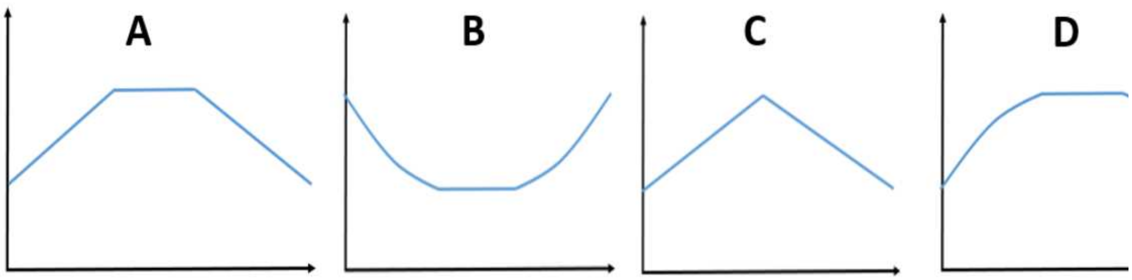
a) $0.2\ seconds$

b) $1.2\ seconds$

c) $2.2\ seconds$

d) $3.2\ seconds$

40) A car increased its speed at a constant rate, then decrease its speed at a constant rate.
Which graph represent this situation?



a) Graph A

b) Graph B

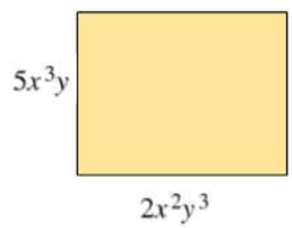
c) Graph C

d) Graph D

Part 2

<u>Show all your work when answering these questions.</u>

41) The length of this rectangle is $2x^2y^3$.
The width of this rectangle is $5x^3y$.
Write the area of the rectangle as a monomial.



$\boldsymbol{A = (2x^2y^3) \times (5x^3y)}$

$\boldsymbol{A = 10x^5y^4}$

42) This table shows the population in some countries.
How many times bigger is the population of the U.A.E. than the population of Mongolia?
$\boldsymbol{6.2 \times 10^6}$

Country	Population
Australia	2.4×10^7
Egypt	9.7×10^7
Mongolia	3.1×10^6
Saudi Arabia	3.3×10^7
U.A.E	9.3×10^6

43) Khalifa paid AED 250 to join a Falconry Club. He is learning how to handle a falcon. Each lesson costs AED 75.

(a) Write an equation to represent the total cost, y, for x lessons.

$y = 250 + 75x$

(b) Use the equation to find the total amount Khalifa pays if he attends 8 lessons.

$y = 850$ **AED**

44) (a) Solve the equation:

$$7 + 2(m - 1) = 3(2 + m) - m$$

There are no solutions

(b) State if the equation has one solution, no solution or infinitely many solutions.

No solution

45) Determine whether the relationship between the two quantities described in the table is linear.
If so, find the constant rate of change.
If not, explain your reasoning.

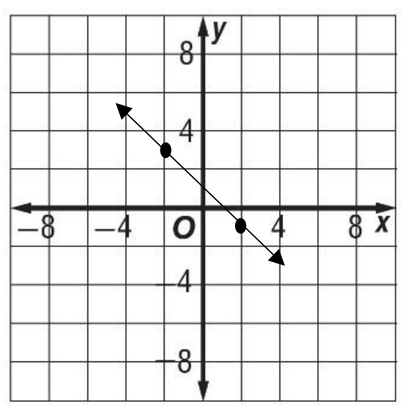
Linear, rate of change = 20

Number of Trees	Number of Apples
5	100
10	200
15	300
20	400

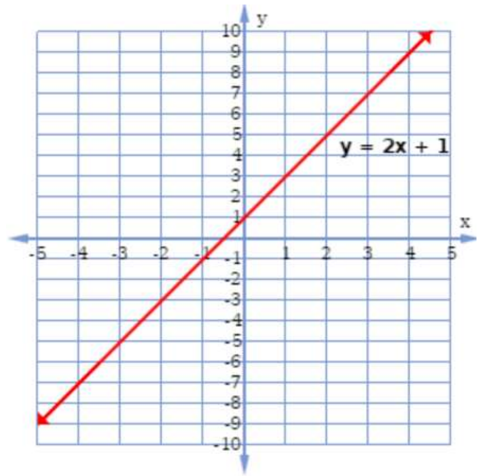
46) The points given in this table lie on a line. Find the slope of the line. Then graph the line.

x	–1	2	5	8
y	3	–1	–5	–9

$m = -\frac{4}{3}$



47) (a) Graph a line with a slope of 2 and a y-intercept of 1.



(b) Label the line with the equation of the line written in slope-intercept form.

$\boldsymbol{y = 2x + 1}$

48) Write an equation in point-slope form for the line that passes through $(-1, 5)$ and $(2, 7)$.

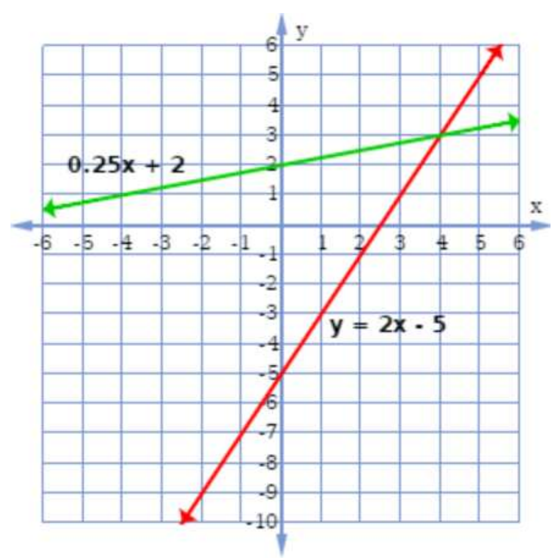
$\boldsymbol{y - 7 = \frac{2}{3}\,(x - 5)}$

49) Solve this system of equations by graphing:

$y = 2x - 5$

$y = \frac{1}{4}x + 2$

Solution: (4,3)



End of Mock Test 2