

Congruent Triangles - Module 14



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف التاسع المتقدم ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

تاريخ إضافة الملف على موقع المناهج: 10:57:16 2026-06-10

ملفات اكتب للمعلم اكتب للطالب ا اختبارات الكترونية ا اختبارات ا حلول ا عروض بوربوينت ا أوراق عمل
منهج انجليزي ا ملخصات وتقارير ا مذكرات وبنوك ا الامتحان النهائي ا للمدرس

المزيد من مادة
رياضيات:

التواصل الاجتماعي بحسب الصف التاسع المتقدم



صفحة المناهج
الإماراتية على
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

المواد على تلغرام

المزيد من الملفات بحسب الصف التاسع المتقدم والمادة رياضيات في الفصل الثالث

9ADV MATH EOT TERM 3 2025-2026

1

الهيكل الوزاري لمادة الرياضيات

2

ملزمة وفق الهيكل

3

ملزمة هيكل واختياري وأسئلة

4

منهج ريفيل مراجعة هيكل تدريبات ومفاهيم متنوعة محلولة

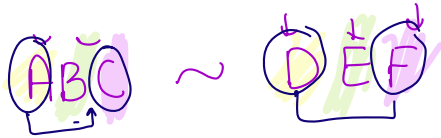
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Congruent Triangles

~ Congruent (Same)
Shape

~ Value

Order of letters Matter



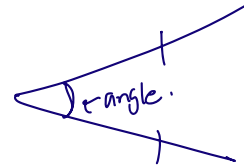
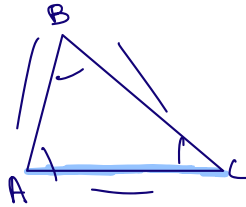
$$\angle A \cong \angle D$$

$$\angle B \cong \angle E$$

$$\angle C \cong \angle F$$

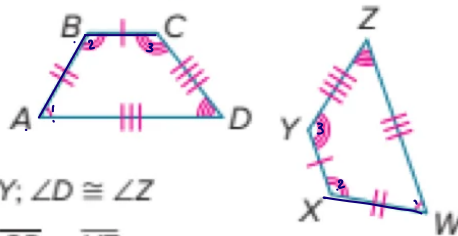
$$AC = DF$$

$$BC = EF$$



Example 1 Identify Corresponding Congruent Parts

Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.



Angles: $\angle A \cong \angle W$; $\angle B \cong \angle X$; $\angle C \cong \angle Y$; $\angle D \cong \angle Z$

Sides: $\overline{BC} \cong \overline{XY}$; $\overline{AB} \cong \overline{WX}$; $\overline{DA} \cong \overline{ZW}$; $\overline{CD} \cong \overline{YZ}$

Sides: $\overline{BC} \cong \overline{XY}$
 $\overline{AB} \cong \overline{WX}$
 $\overline{AD} \cong \overline{WZ}$
 $\overline{DC} \cong \overline{YZ}$

Angles: $\angle A \cong \angle W$
 $\angle B \cong \angle X$
 $\angle C \cong \angle Y$
 $\angle D \cong \angle Z$

$$ABCD \sim WXYZ$$

Study Tip

Congruence

Statements Valid
congruence statements for congruent polygons list corresponding vertices in the same order.

Study Tip

Use a Congruence Statement You can use a congruence statement to help you correctly identify corresponding sides.
 $\triangle RSV \cong \triangle TVS$
 $\overline{RS} \cong \overline{TV}$

Talk About It!

Suppose the congruence statement $\triangle RSV \cong \triangle TVS$ was not given. Would you be able to solve this problem? Explain.

Check

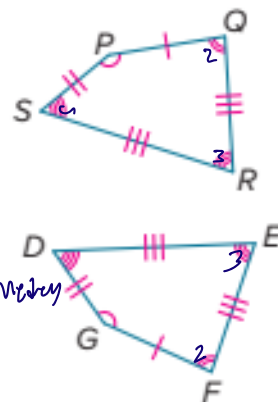
Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.

$$\angle P \cong \angle G; \angle Q \cong \angle F; \angle R \cong \angle E; \angle S \cong \angle D$$

$$\overline{PQ} \cong \overline{GF}; \overline{QR} \cong \overline{FE}; \overline{RS} \cong \overline{ED}; \overline{SP} \cong \overline{DG}$$

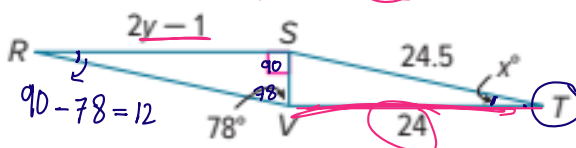
Complete the congruence statement.

$$\text{Polygon } PQRS \cong \text{Polygon } GFED$$



Example 2 Use Corresponding Parts of Congruent Triangles

In the diagram, $\triangle RSV \cong \triangle TVS$. Find the values of x and y .



$$\angle T = \angle R = 12$$

$$2y - 1 = 24$$

$$2y = 25$$

$$y = 12.5$$

Part A Find the value of x .

$$\angle T \cong \angle R$$

$$m\angle T = m\angle R$$

$$= 180^\circ - 90^\circ - 78^\circ$$

$$= 12^\circ$$

CPCTC

Definition of congruence

Triangle Angle-Sum Theorem

Solve.

The value of x is 12.

Part B Find the value of y .

$$\overline{RS} \cong \overline{TV}$$

$$RS = TV$$

$$2y - 1 = 24$$

$$y = 12.5$$

CPCTC

Definition of congruence

Substitution

Solve.

The value of y is 12.5.

Check

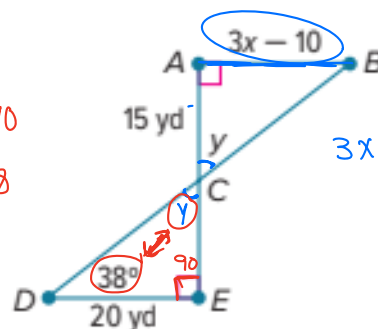
In the diagram, $\triangle ABC \cong \triangle EDC$. Find the values of x and y .

$$x = 10; y = 52$$

$$38 + 4 = 90$$

$$y = 90 - 38$$

$$y = 52$$



$$3x - 10 = 20$$

$$3x = 30$$

$$x = 10$$

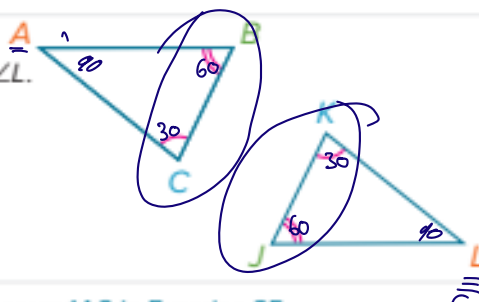
Go Online You can complete an Extra Example online.

Learn Third Angles Theorem and Triangle Congruence

Theorem 14.3: Third Angles Theorem

Words If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

Example If $\angle C \cong \angle K$ and $\angle B \cong \angle J$, then $\angle A \cong \angle L$.



You will prove Theorem 14.3 in Exercise 25.

Like congruence of segments and angles, congruence of triangles is reflexive, symmetric, and transitive.

Theorem 14.4: Properties of Triangle Congruence

Reflexive Property of Triangle Congruence

$$\triangle ABC \cong \triangle ABC$$

Symmetric Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$.

Transitive Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

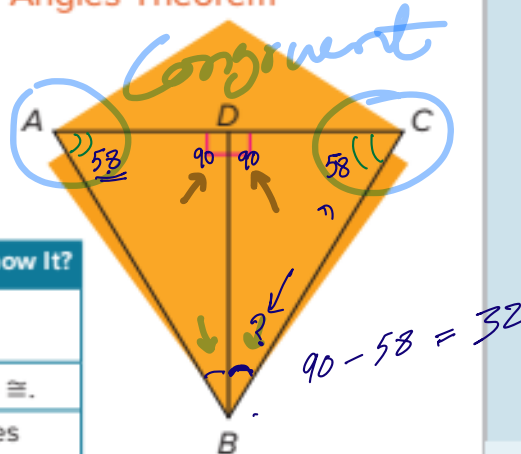
Go Online

Proofs of Theorem 14.4 are available.

Example 3 Use the Third Angles Theorem

ORIGAMI Aika is folding origami dragons for a party she is hosting.

If $\angle ABD \cong \angle CBD$ and $m\angle BAD = 58^\circ$, find $m\angle CBD$.



What Do You Know?	How Do You Know It?
$\angle ABD \cong \angle CBD$, $m\angle BAD = 58^\circ$	Given
$\angle BDC \cong \angle BDA$	All rt. $\angle$ s are $\cong$.
$\angle BAD \cong \angle BCD$	Third Angles Theorem
$m\angle CBD + m\angle BCD = 90^\circ$	The acute $\angle$ s of a rt. $\triangle$ are compl.
$m\angle BCD = m\angle BAD$	Def. of congruence

$$m\angle BCD = 58^\circ.$$

Transitive Property

$$m\angle CBD + 58^\circ = 90^\circ$$

Substitute.

$$m\angle CBD = 32^\circ.$$

Solve.

The measure of $\angle CBD$ is 32° .

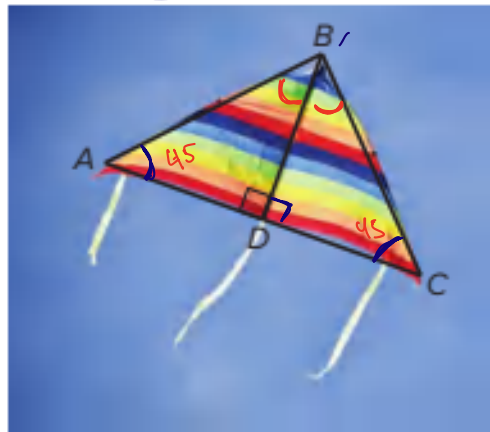
Go Online You can complete an Extra Example online.

Think About It!

How could you find the $m\angle CBD$ in a different way?

Check

KITES The kite shown is made of two congruent triangles. If $m\angle BAD = m\angle BCD = 45^\circ$, find $m\angle ABD$. $m\angle ABD = ?$



$$90 - 45 \\ = 45$$

Study Tip

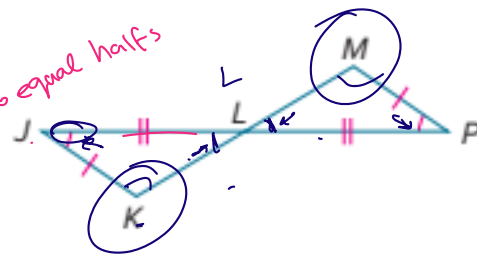
Symbols To indicate that two triangles are not congruent, write $\triangle ABC \not\cong \triangle EFG$. $\triangle ABC \not\cong \triangle EFG$ is read as triangle ABC is not congruent to triangle EFG.

Example 4 Prove that Two Triangles Are Congruent

Write a two-column proof.

Given: $\angle J \cong \angle P$, $\overline{JK} \cong \overline{PM}$,
 $\overline{JL} \cong \overline{PL}$, and
 L bisects $\overline{KM}$

Prove: $\triangle JLK \cong \triangle PLM$



Statements	Reasons
1. $\angle J \cong \angle P$, $\overline{JK} \cong \overline{PM}$, $\overline{JL} \cong \overline{PL}$ and L bisects $\overline{KM}$.	1. Given
2. $\angle JLK \cong \angle PLM$	2. Vertical angles are congruent.
3. $\overline{LK} \cong \overline{LM}$	3. Definition of segment bisector
4. $\angle K \cong \angle M$	4. Third Angles Theorem
5. $\triangle JLK \cong \triangle PLM$	5. Definition of congruent triangles

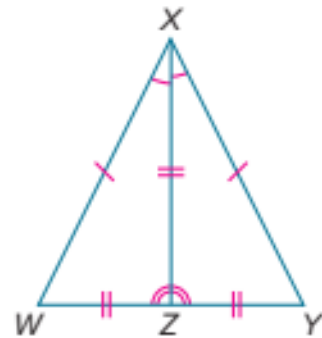
Check

Write a paragraph proof.

Given: $\angle WXZ \cong \angle YXZ$, $\angle XZW \cong \angle XZY$,
 $\overline{WX} \cong \overline{YX}$, $\overline{WZ} \cong \overline{YZ}$

Prove: $\triangle WXZ \cong \triangle YXZ$

It is given that $\overline{WX} \cong \overline{YX}$ and $\overline{WZ} \cong \overline{YZ}$. By the SSS Property, $\overline{XZ} \cong \overline{XZ}$. It is also given that $\angle WXZ \cong \angle YXZ$ and $\angle XZW \cong \angle XZY$. So, by ASA the Theorem, $\angle W \cong \angle Y$. By the definition of congruent triangles, $\triangle WXZ \cong \triangle YXZ$.



Go Online You can complete an Extra Example online.

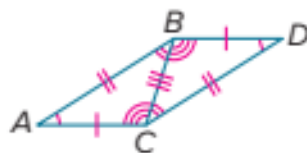
Practice

Go Online You can complete your homework online.

Example 1

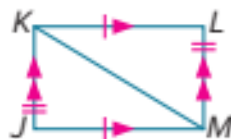
Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.

1.



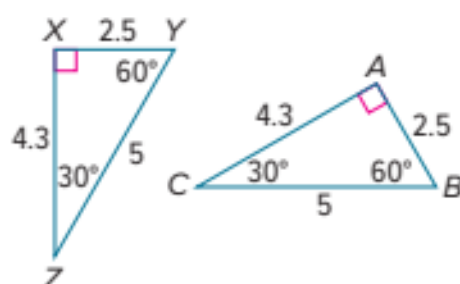
$\angle A \cong \angle D$; $\angle ABC \cong \angle DCB$;
 $\angle ACB \cong \angle DBC$; $\overline{AC} \cong \overline{DB}$;
 $\overline{AB} \cong \overline{DC}$; $\triangle ABC \cong \triangle DCB$

2.



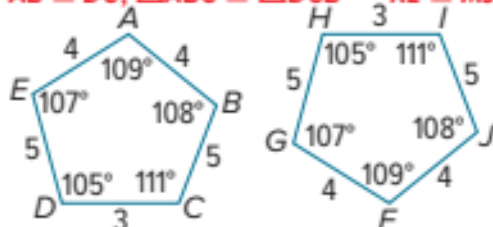
$\angle J \cong \angle L$; $\angle JKM \cong \angle LMK$;
 $\angle KMJ \cong \angle MLK$; $\overline{KJ} \cong \overline{ML}$;
 $\overline{KL} \cong \overline{MJ}$; $\triangle JKM \cong \triangle LMK$

3.



$\angle X \cong \angle A$; $\angle Y \cong \angle B$; $\angle Z \cong \angle C$;
 $\overline{XY} \cong \overline{AB}$; $\overline{XZ} \cong \overline{AC}$; $\overline{YZ} \cong \overline{BC}$;
 $\triangle XYZ \cong \triangle ABC$

4.



$\angle A \cong \angle F$; $\angle B \cong \angle J$; $\angle C \cong \angle I$; $\angle D \cong \angle H$;
 $\angle E \cong \angle G$; $\overline{AB} \cong \overline{FJ}$; $\overline{BC} \cong \overline{JI}$; $\overline{CD} \cong \overline{IH}$;
 $\overline{DE} \cong \overline{HG}$; $\overline{AE} \cong \overline{FG}$;
 polygon $ABCDE \cong$ polygon $FJIHG$

5.



$\angle R \cong \angle J$; $\angle T \cong \angle K$; $\angle S \cong \angle L$;
 $\overline{RT} \cong \overline{JK}$; $\overline{TS} \cong \overline{KL}$; $\overline{RS} \cong \overline{JL}$; $\triangle RTS \cong \triangle JKL$

Example 2

In the diagram, $\triangle ABC \cong \triangle FDE$.

6. Find the value of x . **36**

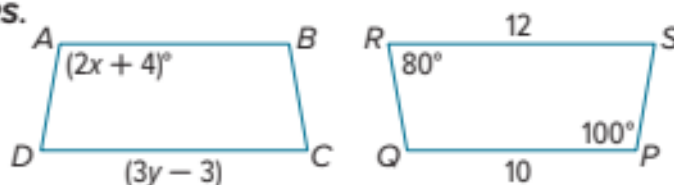
7. Find the value of y . **48**



In the diagram, polygon $ABCD \cong$ polygon $PQRS$.

8. Find the value of x . **48**

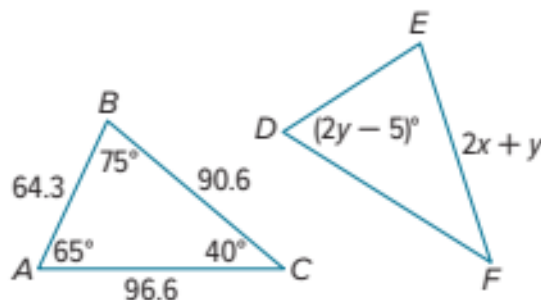
9. Find the value of y . **5**



In the diagram, $\triangle ABC \cong \triangle DEF$.

10. Find the value of x . **27.8**

11. Find the value of y . **35**



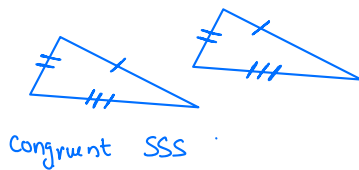
proving triangles Congruence :

S → Side

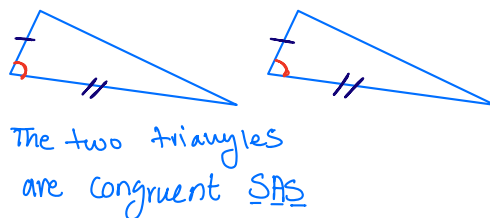
A → Angle

14.3

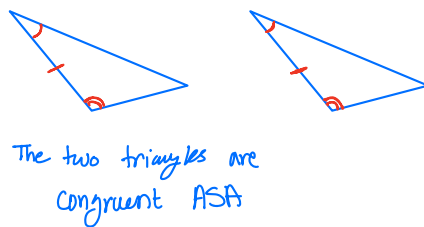
SSS : Three Congruent Sides
Side Side Side



SAS : Two sides with Angle between the side
Side Angle Side

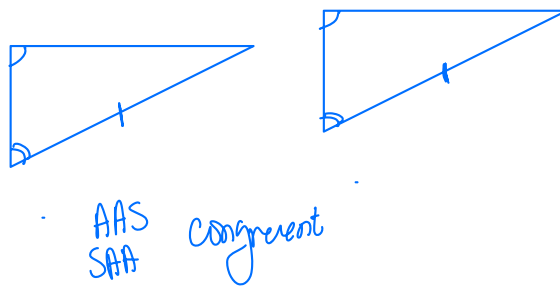


ASA : Two angles with the Side between them
Angle Side Angle



14.4

AAS : Two following angles and a side after.
Angle Angle Side
SAA



Proving Triangles Congruent: SSS, SAS

Explore Conditions That Prove Triangles Congruent

Online Activity Use dynamic geometry software to complete the Explore.

INQUIRY What conditions can be used to identify whether two triangles are congruent?

Learn Proving Triangles Congruent: SSS

You can prove two triangles congruent by showing that all six pairs of corresponding parts are congruent. However, it is possible to prove two triangles congruent using fewer pairs of corresponding parts.

If two triangles have the same three side lengths, then there is a series of rigid motions that will show the two triangles congruent. This leads to the postulate below.

Postulate 14.1: Side-Side-Side (SSS) Congruence

If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

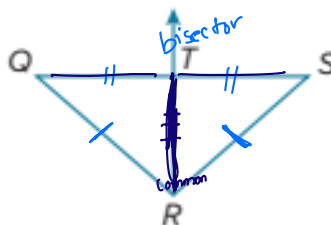
Example 1 Use SSS to Prove Triangles Congruent

Write a flow proof to show that $\triangle QRT \cong \triangle SRT$.

Given: $\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$, $\overline{RT}$ bisects $\overline{QS}$ at point T .

Prove: $\triangle QRT \cong \triangle SRT$

Proof:



$\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$.

Given

Given

$\overline{RT}$ bisects $\overline{QS}$ at point T .

Given

$\overline{QT} \cong \overline{ST}$

Definition of segment bisector

$\overline{RT} \cong \overline{RT}$

Reflexive Property of Congruence

$\triangle QRT \cong \triangle SRT$

SSS

Go Online You can complete an Extra Example online.

Today's Goals

- Use the SSS Congruence criterion for triangles to solve problems and prove relationships in geometric figures.
- Use the SAS Congruence criterion for triangles to solve problems and prove relationships in geometric figures.

Today's Vocabulary

included angle

Talk About It!

Will two equilateral triangles always be congruent by SSS? Justify your argument.

Go Online

An alternate method is available for this example.

Statement

$\overline{QR} \cong \overline{SR}$

$\overline{RT}$ bisects $\overline{QS}$

$\overline{QT} \cong \overline{ST}$

$\overline{TR} \cong \overline{TR}$

$\triangle QRT \cong \triangle SRT$

Reason

Given

Given

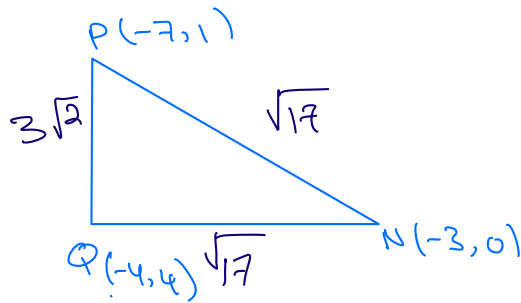
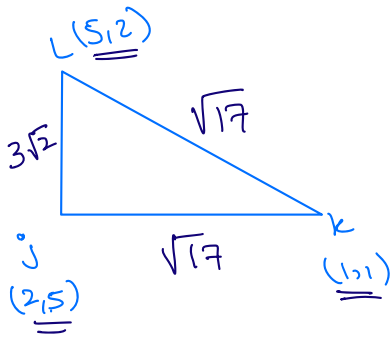
Definition of a bisector

Reflexive property

SSS

Example 2 Use SSS on the Coordinate Plane

Triangle JKL has vertices J(2, 5), K(1, 1), and L(5, 2). Triangle QNP has vertices Q(-4, 4), N(-3, 0), and P(-7, 1). Is $\triangle JKL \cong \triangle QNP$?



$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$\begin{array}{l} \overline{LJ} = \sqrt{(5-2)^2 + (2-5)^2} = 3\sqrt{2} \\ \overline{JK} = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{17} \\ \overline{LK} = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{17} \end{array} \left\{ \begin{array}{l} \overline{PQ} = \sqrt{(-7-(-4))^2 + (1-4)^2} = 3\sqrt{2} \\ \overline{QN} = \sqrt{(-4-(-3))^2 + (4-0)^2} = \sqrt{17} \\ \overline{PN} = \sqrt{(-7-(-3))^2 + (1-0)^2} = \sqrt{17} \end{array} \right.$$

$\triangle JKL \cong \triangle QNP$ by SSS

Think About It!

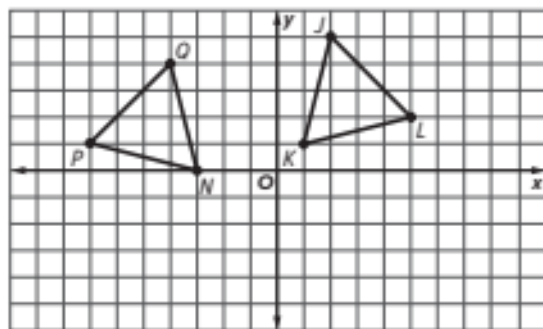
Is the following statement *true* or *false*? Justify your argument.

If the congruent sides in one isosceles triangle have the same measure as the congruent sides in another isosceles triangle, then the triangles are congruent.

Example 2 Use SSS on the Coordinate Plane

Triangle JKL has vertices $J(2, 5)$, $K(1, 1)$, and $L(5, 2)$. Triangle QNP has vertices $Q(-4, 4)$, $N(-3, 0)$, and $P(-7, 1)$. Is $\triangle JKL \cong \triangle QNP$?

Part A Graph the triangles.



Part B Make a conjecture.

Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.

From the graph, it appears that the triangles have the same shape and size, so we can conjecture that they are congruent.

Part C Support your conjecture.

Use the Distance Formula to show that all corresponding sides have the same measure.

$$\begin{aligned} JL &= \sqrt{(5 - 2)^2 + (2 - 5)^2} \\ &= \sqrt{9 + 9} \text{ or } 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} QP &= \sqrt{[-7 - (-4)]^2 + (1 - 4)^2} \\ &= \sqrt{9 + 9} \text{ or } 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} LK &= \sqrt{(1 - 5)^2 + (1 - 2)^2} \\ &= \sqrt{16 + 1} \text{ or } \sqrt{17} \end{aligned}$$

$$\begin{aligned} PN &= \sqrt{[-3 - (-7)]^2 + (0 - 1)^2} \\ &= \sqrt{16 + 1} \text{ or } \sqrt{17} \end{aligned}$$

$$\begin{aligned} KJ &= \sqrt{(2 - 1)^2 + (5 - 1)^2} \\ &= \sqrt{1 + 16} \text{ or } \sqrt{17} \end{aligned}$$

$$\begin{aligned} NQ &= \sqrt{[-4 - (-3)]^2 + (4 - 0)^2} \\ &= \sqrt{1 + 16} \text{ or } \sqrt{17} \end{aligned}$$

$JL = QP$, $LK = PN$, and $KJ = NQ$. By the definition of congruent segments, all corresponding segments are congruent. Therefore, $\triangle JKL \cong \triangle QNP$ by SSS.

 **Go Online** You can complete an Extra Example online.