

Math Coverage Module 12, 13 & 14



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف التاسع المتقدم ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

تاريخ إضافة الملف على موقع المناهج: 02:20:28 2026-06-21

ملفات اكتب للمعلم اكتب للطالب | اختبارات الكترونية | اختبارات | حلول | عروض بوربوينت | أوراق عمل
منهج انجليزي | ملخصات وتقارير | مذكرات وبنوك | الامتحان النهائي | للمدرس

المزيد من مادة
رياضيات:

إعداد: Saleh Abu Najla

التواصل الاجتماعي بحسب الصف التاسع المتقدم



صفحة المناهج
الإماراتية على
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

المواد على تلغرام

المزيد من الملفات بحسب الصف التاسع المتقدم والمادة رياضيات في الفصل الثالث

مراجعة محلولة للترم الثالث

1

مذكرة مراجعة للترم الثالث

2

الحقيبة التدريبية كراسة تدريبية

3

أسئلة متوقعة للامتحان 2026م بدون حل

4

Congruent Triangles - Module 14

5

GRADE 9 ADV
EOT 3

MATH
COVERAGE

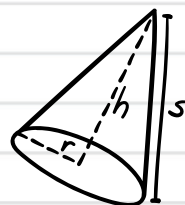
Module 12,13&14

Name:

Class:

Ms. Najla Abu Saleh

School Principal: Ms. Farah Al Gharbi



M12L1: Conjectures and Counterexamples



Counterexample in Math

A counter example in math is an example for which the conjecture is false.

Example 5 Find Counterexamples

Find a counterexample to show that each conjecture is false.

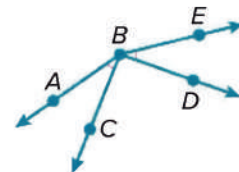
a. If n is a real number, then $-n$ is a negative number.

When n is -4 , $-n$ is $-(-4)$ or 4 , which is a positive number. Because $-n$ is not negative, this is a counterexample.

b. If $\angle ABC \cong \angle DBE$, then $\angle ABC$ and $\angle DBE$ are vertical angles.

When points A , B , and D are noncollinear and points E , B , and C are noncollinear, the conjecture is false.

In the figure, $\angle ABC \cong \angle DBE$, but $\angle ABC$ and $\angle DBE$ are not vertical angles.



1) Determine whether the conjecture is true or false. Find a counter example for any false conjecture.

“If n is a prime number, then $n + 1$ is not prime.”

a) True

b) False; a counterexample is $n = 2$

c) False; a counterexample is $n = 5$

d) Can't be determined

2) Determine whether the conjecture is true or false. Find a counter example for any false conjecture.

“If x is an integer, then $-x$ is positive.”

a) True

b) False; a counterexample is $x = 4$

c) False; a counterexample is $x = -3$

d) Can't be determined

3) Determine whether the conjecture is true or false. Find a counter example for any false conjecture.

“If $\angle 2$ and $\angle 3$ are supplementary angles, then $\angle 2$ and $\angle 3$ form a linear pair.”

a) True

b) False; supplementary angles must form a linear pair

c) False; two angles can be supplementary without being adjacent

d) Can't be determined

4) Determine whether the conjecture is true or false. Find a counter example for any false conjecture.

“If you have three points A , B , and C , then A , B , and C are noncollinear.”

a) True

b) False; a counterexample is three points on the same line

c) False; a counterexample is a triangle

d) Can't be determined

5) Determine whether the conjecture is true or false. Find a counter example for any false conjecture.

“If in $\triangle ABC$, $(AB)^2 + (BC)^2 = (AC)^2$, then $\triangle ABC$ is a right triangle. . “

a) True

b) False; a counterexample is an equilateral triangle

c) False; a counterexample is an isosceles triangle

d) Can't be determined

6) Determine whether the conjecture is true or false. Find a counter example for any false conjecture.

“If the area of a rectangle is 20 square meters, then the length is 10 meters and the width is 2 meters. “

a) True

b) False; a counterexample is a rectangle with length 5m and width 4m

c) False; a counterexample is a rectangle with length 7m and width 3m

d) Can't be determined

M12L2: Statements, Conditionals, and Biconditionals

CONDITIONAL STATEMENT

IMPORTANT

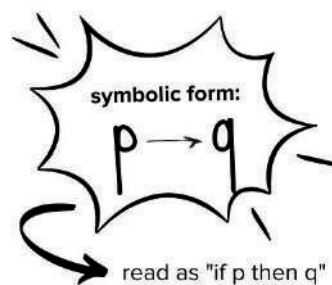
a statement that can be written in "if - then" form

in the sentence, after if comes the

hypothesis

and after then comes the

conclusion



Example 3 Identify the Hypothesis and Conclusion

Identify the hypothesis and conclusion of each conditional statement.

- a. If a polygon has six sides, then it is a hexagon.

Hypothesis: A polygon has six sides.

Conclusion: The polygon is a hexagon.

- b. Another performance will be scheduled if the first one is sold out.

Notice that the word *if* appears in the second portion of the sentence.

Hypothesis: The first performance is sold out.

Conclusion: Another performance will be scheduled.

123) Which option correctly identifies the hypothesis, conclusion, and if-then form?

Statement: Get a free water bottle with a one-year membership.

- a) **Hypothesis:** You get a free water bottle.

Conclusion: You sign up for a one-year membership.

If-then form: If you get a free water bottle, then you have a one-year membership.

- b) **Hypothesis:** You have a one-year membership.

Conclusion: You get a free water bottle.

If-then form: If you have a one-year membership, then you get a free water bottle.

- c) **Hypothesis:** A water bottle is free.

Conclusion: The membership lasts for one year.

If-then form: If a water bottle is free, then the membership is for one year.

124) Which option correctly identifies the hypothesis, conclusion, and if-then form?

Statement: Everybody at the party received a gift.

- a) **Hypothesis:** A person is at the party.

Conclusion: The person received a gift.

If-then form: If a person is at the party, then they received a gift.

- b) **Hypothesis:** A person received a gift.

Conclusion: The person is at the party.

If-then form: If a person received a gift, then they are at the party.

- c) **Hypothesis:** It is a party.

Conclusion: Everybody receives gifts.

If-then form: If it is a party, then everybody receives a gift.

125) Which option correctly identifies the hypothesis, conclusion, and if-then form?

Statement: The intersection of two planes is a line.

- a) **Hypothesis:** Two planes intersect.

Conclusion: Their intersection is a line.

If-then form: If two planes intersect, then their intersection is a line.

- b) **Hypothesis:** A geometric figure is a line.

Conclusion: It is formed by two intersecting planes.

If-then form: If a figure is a line, then it is the intersection of two planes.

- c) **Hypothesis:** Two lines intersect.

Conclusion: They form a plane.

If-then form: If two lines intersect, then they form a plane.

126) Which option correctly identifies the hypothesis, conclusion, and if-then form?

Statement: The area of a circle is $A = \pi r^2$.

- a) **Hypothesis:** The area is $A = \pi r^2$

Conclusion: The figure is a circle.

If-then form: If the area is $A = \pi r^2$ then the figure is a circle.

- b) **Hypothesis:** A figure has a radius r .

Conclusion: It is a circle.

If-then form: If a figure has a radius r , then it is a circle with area $A = \pi r^2$

- c) **Hypothesis:** A geometric figure is a circle with radius r .

Conclusion: Its area is $A = \pi r^2$

If-then form: If a geometric figure is a circle with radius r then its area is $A = \pi r^2$

126) Which option correctly identifies the hypothesis, conclusion, and if-then form?

Statement: The area of a circle is $A = \pi r^2$.

a) **Hypothesis:** The area is $A = \pi r^2$

Conclusion: The figure is a circle.

If-then form: If the area is $A = \pi r^2$ then the figure is a circle.

b) **Hypothesis:** A figure has a radius r .

Conclusion: It is a circle.

If-then form: If a figure has a radius r , then it is a circle with area $A = \pi r^2$

c) **Hypothesis:** A geometric figure is a circle with radius r .

Conclusion: Its area is $A = \pi r^2$

If-then form: If a geometric figure is a circle with radius r then its area is $A = \pi r^2$

128) Which option correctly identifies the hypothesis, conclusion, and if-then form?

Statement: A right angle measures 90 degrees.

a) **Hypothesis:** An angle is a right angle.

Conclusion: Its measure is 90 degrees.

If-then form: If an angle is a right angle, then its measure is 90 degrees.

b) **Hypothesis:** An angle measures 90 degrees.

Conclusion: It is a right angle.

If-then form: If an angle measures 90 degrees, then it is a right angle.

c) **Hypothesis:** The measure is 90 degrees.

Conclusion: The shape is an angle.

If-then form: If a measure is 90 degrees, then the shape is a right angle.

18

Write compound statements for conjunctions and disjunctions and determine truth values

Ex1(3,4,6)

711,717

Note: If a statement is represented by p , then **not p** is the **negation** of the statement. The symbol $\sim$ stands for **negation**.

The **negation** of a statement **has the opposite truth value**, of the original statement.

EXAMPLE

P : My name is Ms. Najla. (T)

$\sim p$: My name is **not** Ms. Najla. (F)

A compound statement using the word **and** is called a **conjunction**.

A conjunction **is true only**

when both statements that form it are true.

A conjunction is written as:

p **and** q , $p \wedge q$.

CONJUNCTION TRUTH TABLE		
P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

A compound statement using the word **or** is called a **disjunction**.

Adisjunction is **true if at least one of the statements is true**.

A disjunction is written as:

p **or** q , $p \vee q$.

DISJUNCTION TRUTH TABLE		
P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Example 1 Truth Values of Conjunctions

Use the statements to write a compound statement for each conjunction. Then find the truth values. Explain your reasoning.

p : The figure is a trapezoid.

q : The figure has four congruent sides.

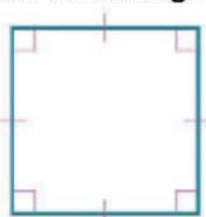
r : The figure has four right angles.

a. p and r

p and r : The figure is a trapezoid, and the figure has four right angles. Although r is true, p is false. So, p and r is false.

b. $\sim p \wedge q$

$\sim p \wedge q$: The figure is not a trapezoid, and the figure has four congruent sides. Both $\sim p$ and q are true, so $\sim p \wedge q$ is true.



82) Use the statements: p: $-3 - 2 = -5$ q: Vertical angles are congruent. r: $2 + 8 > 10$ Write a compound statement for : $q \vee \sim r$	
a) Vertical angles are congruent or $2 + 8 > 10$	b) Vertical angles are congruent or $2 + 8 \leq 10$
c) Vertical angles are congruent and $2 + 8 \leq 10$	d) Vertical angles are congruent and $2 + 8 > 10$

83) Use the statements: p: $-3 - 2 = -5$ q: Vertical angles are congruent. r: $2 + 8 > 10$ Find the truth value for: $q \vee \sim r$	
a) True	b) False
c) Neither true nor false	d) Can't be determined

84) Use the statements: p: $-3 - 2 = -5$ q: Vertical angles are congruent. r: $2 + 8 > 10$ Write a compound statement for : $r \vee q$	
a) $2 + 8 > 10$ and Vertical angles are congruent.	b) $2 + 8 > 10$ or Vertical angles are congruent.
c) $2 + 8 \leq 10$ or Vertical angles are congruent.	d) $2 + 8 \leq 10$ and Vertical angles are congruent.

85) Use the statements: p: $-3 - 2 = -5$ q: Vertical angles are congruent. r: $2 + 8 > 10$ Find the truth value for: $r \vee q$	
a) True	b) False
c) Neither true nor false	d) Can't be determined

86) Use the statements: p: $-3 - 2 = -5$ q: Vertical angles are congruent. r: $2 + 8 > 10$ Write a compound statement for : $\sim r \vee \sim p$	
a) $2 + 8 \leq 10$ or $-3 - 2 \neq -5$	b) $2 + 8 > 10$ or $-3 - 2 \neq -5$
c) $2 + 8 \leq 10$ and $-3 - 2 \neq -5$.	d) $2 + 8 > 10$ and $-3 - 2 = -5$

87) Use the statements: p: $-3 - 2 = -5$ q: Vertical angles are congruent. r: $2 + 8 > 10$ Find the truth value for: $\sim r \vee \sim p$	
a) True	b) False
c) Neither true nor false	d) Can't be determined



88) Use the statements:

$p: -3 - 2 = -5$

q : Vertical angles are congruent.

$r: 2 + 8 > 10$

Write a compound statement for : p and q

a) $-3 - 2 = -5$ or
Vertical angles are
congruent.

b) $-3 - 2 \neq -5$ and
Vertical angles are
congruent.

c) $-3 - 2 = -5$ and Vertical
angles are congruent.

d) $-3 - 2 \neq -5$ or Vertical
angles are congruent.

89) Use the statements:

$p: -3 - 2 = -5$

q : Vertical angles are congruent.

$r: 2 + 8 > 10$

Find the truth value for: p and q

a) True

b) False

c) Neither true nor false

d) Can't be determined

90) Use the statements:

$p: -3 - 2 = -5$

q : Vertical angles are congruent.

$r: 2 + 8 > 10$

Write a compound statement for : $p \wedge r$

a) $-3 - 2 = -5$ or
 $2 + 8 > 10$

b) $-3 - 2 \neq -5$ and
 $2 + 8 > 10$

c) $-3 - 2 = -5$ and
 $2 + 8 > 10$

d) $-3 - 2 \neq -5$ or
 $2 + 8 > 10$

91) Use the statements:

$p: -3 - 2 = -5$

q : Vertical angles are congruent.

$r: 2 + 8 > 10$

Find the truth value for: $p \wedge r$

a) True

b) False

c) Neither true nor false

d) Can't be determined

92) Use the statements:

$p: -3 - 2 = -5$

q : Vertical angles are congruent.

$r: 2 + 8 > 10$

Write a compound statement for : $\sim p \wedge \sim q$

a) $-3 - 2 \neq -5$ and
Vertical angles are **not**
congruent.

b) $-3 - 2 = -5$ and
Vertical angles are **not**
congruent.

c) $-3 - 2 \neq -5$ or Vertical
angles are **not**
congruent.

d) $-3 - 2 = -5$ or Vertical
angles are congruent.

93) Use the statements:

$p: -3 - 2 = -5$

q : Vertical angles are congruent.

$r: 2 + 8 > 10$

Find the truth value for: $\sim p \wedge \sim q$

a) True

b) False

c) Neither true nor false

d) Can't be determined

3	Prove theorems about angles by using the Angle Addition Postulate	(EX1,1-3)	747,753
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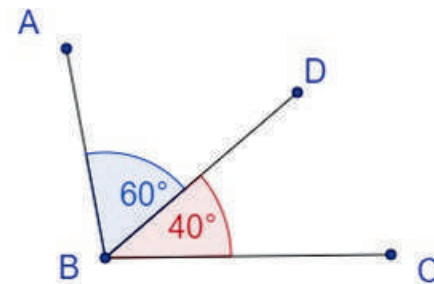
M12L6: Proving Angle Relationships

Angle Addition Postulate

The angle addition postulate states that if D is in the interior of ABC then

$$\angle ABD + \angle CBD = \angle ABC$$

$$60^\circ + 40^\circ = 100^\circ$$

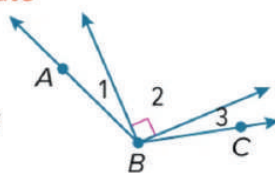


IMPORTANT

Example 1 Angle Addition Postulate

What is $m\angle 3$ if $m\angle 1 = 23^\circ$ and $m\angle ABC = 131^\circ$?

Choose from the reasons provided to justify each step.



$$m\angle 1 + m\angle 2 + m\angle 3 = m\angle ABC$$

$$23^\circ + 90^\circ + m\angle 3 = 131^\circ$$

$$113^\circ + m\angle 3 = 131^\circ$$

$$113^\circ + m\angle 3 - 113^\circ = 131^\circ - 113^\circ$$

$$m\angle 3 = 18^\circ$$

Angle Addition Postulate

Substitution Property

Substitution Property

Subtraction Property

Substitution Property

13) Calculate $m\angle ABC$. If $m\angle ABD = 70$ and $m\angle DBC = 43$

a) 27

c) 70

b) 113

d) 103

14) Find the $m\angle ABD$ if $m\angle ABC = 110^\circ$ and $m\angle 2 = 36^\circ$

(Note: From the diagram, $\angle 2$ represent $\angle DBC$)

a) 146°

c) 74°

b) 54°

d) 84°

15) $m\angle EBC = 55$ and $m\angle EBD = 20$, find $m\angle 2$

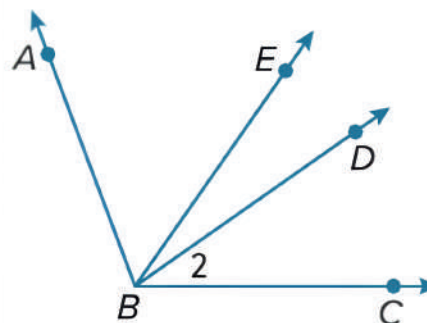
(Note: From the diagram, $\angle 2$ represent $\angle DBC$)

a) 35°

c) 75°

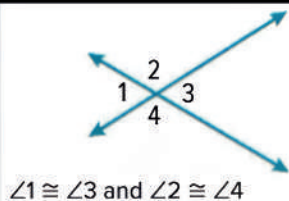
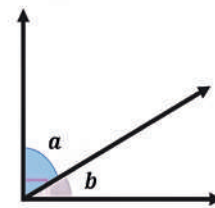
b) 20°

d) 45°



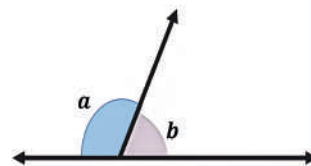
IMPORTANT**Angle Relationships****Theorem 12.8: Vertical Angles Theorem**

If two angles are vertical angles, then they are congruent.

**Complementary Angles**

$$m\angle a + m\angle b = 90^\circ$$

Angles that added up to 90°

Supplementary Angles

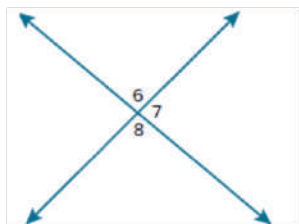
$$m\angle a + m\angle b = 180^\circ$$

Angles that added up to 180°

16) Find the measure of each numbered angle.

$$m\angle 6 = (2x - 21)^\circ$$

$$m\angle 7 = (3x - 34)^\circ$$



a) $\angle 6 = 73^\circ$, $\angle 7 = 107^\circ$,
 $\angle 8 = 73^\circ$

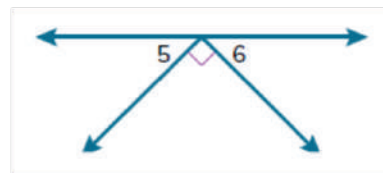
b) $\angle 6 = 107^\circ$, $\angle 7 = 73^\circ$,
 $\angle 8 = 107^\circ$

c) $\angle 6 = 90^\circ$, $\angle 7 = 90^\circ$,
 $\angle 8 = 90^\circ$

d) $\angle 6 = 117^\circ$, $\angle 7 = 63^\circ$,
 $\angle 8 = 117^\circ$

17) Find the measure of each numbered angle.

$$m\angle 5 = m\angle 6$$



a) $\angle 5 = 30^\circ$, $\angle 6 = 30^\circ$

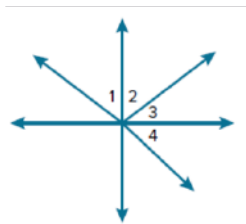
b) $\angle 5 = 90^\circ$, $\angle 6 = 90^\circ$

c) $\angle 5 = 60^\circ$, $\angle 6 = 60^\circ$

d) $\angle 5 = 45^\circ$, $\angle 6 = 45^\circ$

18) Find the measure of each numbered angle.

$\angle 2$ and $\angle 3$ are complementary.
 $\angle 1 \cong \angle 4$ and $m\angle 2 = 28^\circ$.



a) $\angle 1 = 62^\circ$, $\angle 3 = 62^\circ$,
 $\angle 4 = 62^\circ$

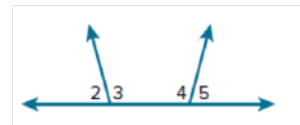
b) $\angle 1 = 28^\circ$, $\angle 3 = 62^\circ$,
 $\angle 4 = 28^\circ$

c) $\angle 1 = 34^\circ$, $\angle 3 = 62^\circ$,
 $\angle 4 = 34^\circ$

d) $\angle 1 = 45^\circ$, $\angle 3 = 62^\circ$,
 $\angle 4 = 45^\circ$

19) Find the measure of each numbered angle.

$\angle 2$ and $\angle 4$, and $\angle 4$ and $\angle 5$
are supplementary $m\angle 4 = 105^\circ$



a) $\angle 2 = 75^\circ$, $\angle 3 = 75^\circ$,
 $\angle 5 = 105^\circ$

b) $\angle 2 = 105^\circ$, $\angle 3 = 75^\circ$,
 $\angle 5 = 75^\circ$

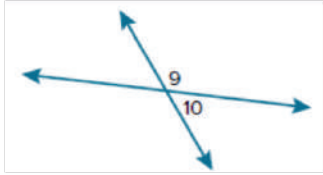
c) $\angle 2 = 75^\circ$, $\angle 3 = 105^\circ$,
 $\angle 5 = 75^\circ$

d) $\angle 2 = 105^\circ$, $\angle 3 = 105^\circ$,
 $\angle 5 = 75^\circ$

20) Find the measure of each numbered angle.

$$m\angle 9 = (3x + 12)^\circ$$

$$m\angle 10 = (x - 24)^\circ$$



a) $\angle 9 = 114^\circ$, $\angle 10 = 66^\circ$

b) $\angle 9 = 156^\circ$, $\angle 10 = 24^\circ$

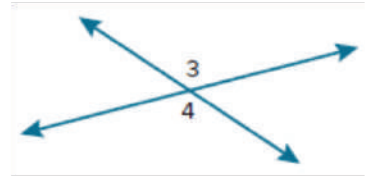
c) $\angle 9 = 90^\circ$, $\angle 10 = 90^\circ$

d) $\angle 9 = 120^\circ$, $\angle 10 = 60^\circ$

21) Find the measure of each numbered angle.

$$m\angle 3 = (2x + 23)^\circ$$

$$m\angle 4 = (5x - 112)^\circ$$



a) $\angle 3 = 113^\circ$, $\angle 4 = 113^\circ$

b) $\angle 3 = 113^\circ$, $\angle 4 = 67^\circ$

c) $\angle 3 = 90^\circ$, $\angle 4 = 90^\circ$

d) $\angle 3 = 75^\circ$, $\angle 4 = 105^\circ$

6	Classify lines as parallel, perpendicular, or neither by comparing the slopes of the lines	(1-6)	771
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M12L8: Slope and Equations of Lines

Remember this



Rate of change (Slope)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Parallel and Perpendicular Line

Parallel lines: Two lines are parallel if they have the **same slope**.

$$\text{slope 1} = \text{slope 2}$$

Perpendicular lines: Two lines are perpendicular if the **product of their slopes is -1**.

$$\text{Slope 1} \times \text{slope 2} = -1$$

If slope 1 is m , then slope 2 is $-\frac{1}{m}$

29) Determine whether $\overline{AB}$ and $\overline{CD}$ are *parallel*, *perpendicular*, or *neither*

$A(1, 5)$, $B(4, 4)$, $C(9, -10)$, $D(-6, -5)$

a) Parallel

b) Perpendicular

c) Neither

d) Can't be determined

30) Determine whether $\overline{AB}$ and $\overline{CD}$ are *parallel*, *perpendicular*, or *neither*

$A(-6, -9)$, $B(8, 19)$, $C(0, -4)$, $D(2, 0)$

a) Parallel

b) Perpendicular

c) Neither

d) Can't be determined

31) Determine whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel, perpendicular, or neither $A(4, 2), B(-3, 1), C(6, 0), D(-10, 8)$.	
a) Parallel	b) Perpendicular
c) Neither	d) Can't be determined

32) Determine whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel, perpendicular, or neither $A(8, -2), B(4, -1), C(3, 11), D(-2, -9)$	
a) Parallel	b) Perpendicular
c) Neither	d) Can't be determined

33) Determine whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel, perpendicular, or neither $A(8, 4), B(4, 3), C(4, -9), D(2, -1)$.	
a) Parallel	b) Perpendicular
c) Neither	d) Can't be determined

34) Determine whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel, perpendicular, or neither $A(4, -2), B(-2, -8), C(4, 6), D(8, 5)$	
a) Parallel	b) Perpendicular
c) Neither	d) Can't be determined

8	Classify lines as parallel, perpendicular, or neither by comparing the equations of the lines	(EX5,22-25)	770,772
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Example 5 Write Equations of Parallel and Perpendicular Lines

Write an equation in slope-intercept form for the line parallel to

$y = -\frac{3}{4}x + 3$ containing $(-3, 6)$.

Step 1: Find the slope (m)
 $m_1 = -\frac{3}{4}$
 $m_2 = -\frac{3}{4}$

Step 2: Find y-intercept (b)
 $y = mx + b$
 $6 = -\frac{3}{4}(-3) + b$
 $b = \frac{15}{4}$

Step 3: Write the Equation
 $y = mx + b$
 $y = -\frac{3}{4}x + \frac{15}{4}$

Write an Equation in Slope-intercept form

Step 1: Use the given equation and its relationship (parallel or perpendicular) to **determine the slope of your line.**

Step 2: Substitute the given point and the slope into the slope-intercept form equation ($y = mx + b$) to **find the y-intercept.**

Step 3: Write the final equation in slope-intercept form.



Perpendicular lines: Two lines are perpendicular if the **product of their slopes is -1**.

Slope 1 $\times$ slope 2 = -1

If slope 1 is m , then slope 2 is $-\frac{1}{m}$

Parallel lines: Two lines are parallel if they have the **same slope**.

slope 1 = slope 2

41) Find the equation of the line that passes through $(-7, -4)$ and is perpendicular to $y = \frac{1}{2}x + 9$.

a) $y = -2x - 18$

b) $y = 2x + 10$

c) $y = -21x - 215$

d) $y = -2x + 10$

42) Find the equation of the line that passes through $(-1, -10)$ and is parallel to $y = 7$.

a) $y = -1$

b) $y = -10$

c) $x = -1$

d) $y = 7x - 3$

43) Find the equation of the line that passes through $(6, 2)$ and is parallel to $y = -\frac{2}{3}x + 1$.

a) $y = -\frac{2}{3}x + 6$

b) $y = \frac{3}{2}x - 7$

c) $y = -\frac{2}{3}x - 2$

d) $y = -\frac{2}{3}x + 2$

44) Find the equation of the line that passes through $(-2, 2)$ and is perpendicular to $y = -5x - 8$.

a) $y = \frac{1}{5}x + \frac{12}{5}$

b) $y = -5x - 8$

c) $y = \frac{1}{5}x + 2$

d) $y = 5x + 12$

M12L9: Proving Lines Parallel

Alternate interior angles.

Nonadjacent interior angles that lie on opposite sides of transversal t .

Theorem 12.15: Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

$$\angle 4 \text{ and } \angle 6 \Rightarrow \angle 4 = \angle 6$$

$$\angle 3 \text{ and } \angle 5 \Rightarrow \angle 3 = \angle 5$$

Alternate exterior angles.

Nonadjacent exterior angles that lie on opposite sides of transversal t .

Theorem 12.17: Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

$$\angle 2 \text{ and } \angle 8 \Rightarrow \angle 2 = \angle 8$$

$$\angle 1 \text{ and } \angle 7 \Rightarrow \angle 1 = \angle 7$$

Corresponding angles.

Occur on the same side of transversal t and the same side of line q and r .

Theorem 12.14: Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

$$\angle 3 \text{ and } \angle 7 \Rightarrow \angle 3 = \angle 7$$

$$\angle 4 \text{ and } \angle 8 \Rightarrow \angle 4 = \angle 8$$

Consecutive interior angles.

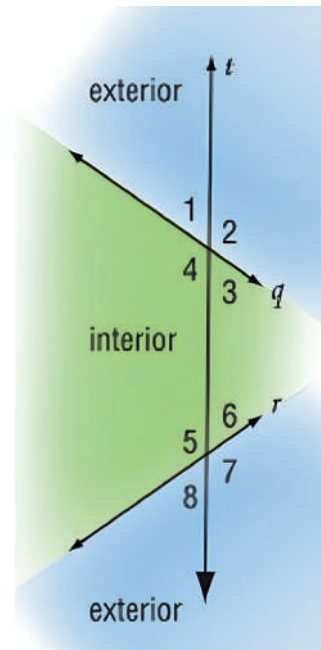
Interior angles that lie on the same side of transversal t .

Theorem 12.16: Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

$$\angle 4 \text{ and } \angle 5 \Rightarrow \angle 4 + \angle 5 = 180$$

$$\angle 3 \text{ and } \angle 6 \Rightarrow \angle 3 + \angle 6 = 180$$



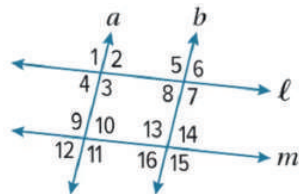
Example 1 Identify Parallel Lines

Use the given information to determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

a. $\angle 2 \cong \angle 8$

$\angle 2$ and $\angle 8$ are alternate interior angles of lines a and b .

Because $\angle 2 \cong \angle 8$, $a \parallel b$ by the Alternate Interior Angles Converse.



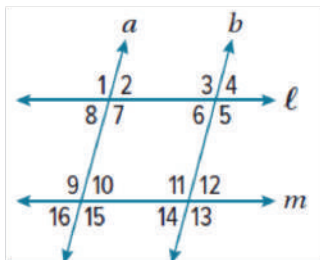
b. $\angle 3 \cong \angle 11$

$\angle 3$ and $\angle 11$ are corresponding angles of lines ℓ and m . Because $\angle 3 \cong \angle 11$, $\ell \parallel m$ by the Converse of the Corresponding Angles Theorem.

c. $\angle 12 \cong \angle 14$

$\angle 12$ and $\angle 14$ are alternate exterior angles of lines a and b . Because $\angle 12 \cong \angle 14$, $a \parallel b$ by the Alternate Exterior Angles Converse.

7) If $\angle 3 \cong \angle 7$, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



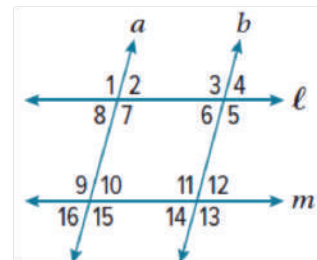
a) $a \parallel b$, Alternate Interior Angles Converse

b) $a \parallel b$, Corresponding Angles Converse

c) $a \parallel b$, Alternate Exterior Angles Converse

d) It is not possible to determine whether the lines are parallel.

8) If $\angle 9 \cong \angle 11$, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



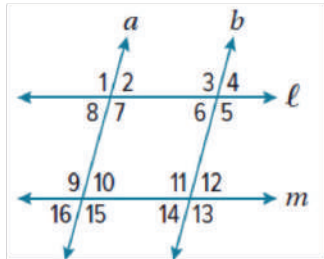
a) $a \parallel b$, Alternate Interior Angles Converse

b) $a \parallel b$, Corresponding Angles Converse

c) $\ell \parallel m$, Corresponding Angles Converse

d) It is not possible to determine whether the lines are parallel.

9) If $\angle 2 \cong \angle 16$, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



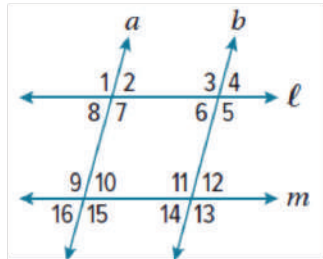
a) $a \parallel b$, Corresponding Angles Converse

b) $l \parallel m$, Alternate Exterior Angles Converse

c) $l \parallel m$, Alternate Interior Angles Converse

d) It is not possible to determine whether the lines are parallel.

10) If $m\angle 5 + m\angle 12 = 180^\circ$, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



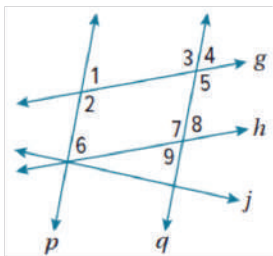
a) $a \parallel b$, Consecutive Interior Angles Converse

b) $l \parallel m$, Consecutive Interior Angles Converse

c) $l \parallel m$, Alternate Interior Angles Converse

d) It is not possible to determine whether the lines are parallel.

11) If $\angle 1 \cong \angle 6$, determine which lines, if any, are parallel. State the theorem that justifies your answer.



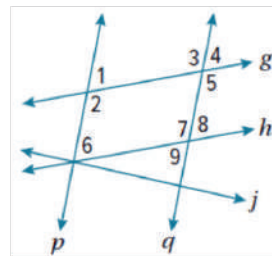
a) $g \parallel h$, Corresponding Angles Converse

b) $p \parallel q$, Alternate Interior Angles Converse

c) $g \parallel j$, Corresponding Angles Converse

d) It is not possible to determine whether the lines are parallel.

12) If $m\angle 7 + m\angle 6 = 180^\circ$, determine which lines, if any, are parallel. State the theorem that justifies your answer.



a) $g \parallel h$, Consecutive Interior Angles Converse

b) $p \parallel q$, Consecutive Interior Angles Converse

c) $g \parallel j$, Consecutive Interior Angles Converse

d) It is not possible to determine whether the lines are parallel.

22

Apply angle relationship theorems to identify parallel lines and find missing values

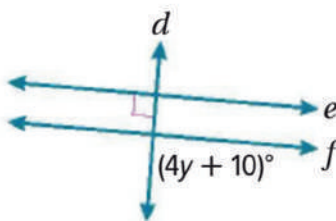
Ex2(7-12)

777,779

Example 2 Use Angle Relationships

Find the value of y so that $e \parallel f$.

From the figure, you know that line d is perpendicular to line e . For lines e and f to be parallel, line f must also be perpendicular to line d . If line f is perpendicular to line d , then $(4y + 10) = 90^\circ$. Solve for y .

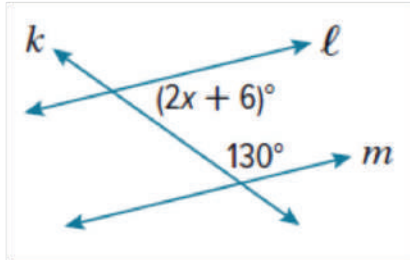


$$90 = 4y + 10 \quad \text{Definition of perpendicular}$$

$$80 = 4y \quad \text{Subtract 10 from each side.}$$

$$20 = y \quad \text{Divide each side by 4.}$$

107) Find the value of x so that $l \parallel m$.



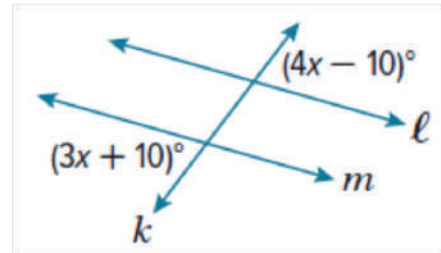
a) 52

b) 22

c) 56

d) 68

108) Find the value of x so that $l \parallel m$.



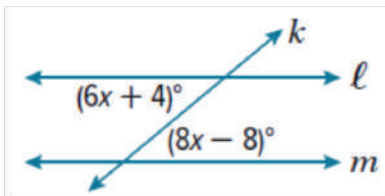
a) 10

b) 15

c) 20

d) 25

109) Find the value of x so that $l \parallel m$.



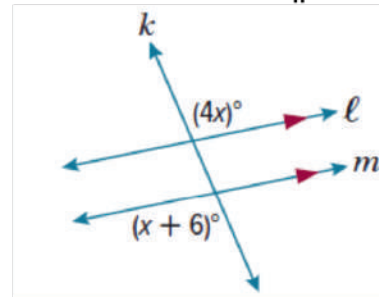
a) 4

b) 6

c) 8

d) 10

110) Find the value of x so that $l \parallel m$.



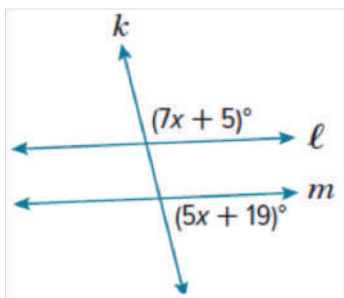
a) 2

b) 18

c) 24

d) 30

111) Find the value of x so that $l \parallel m$.



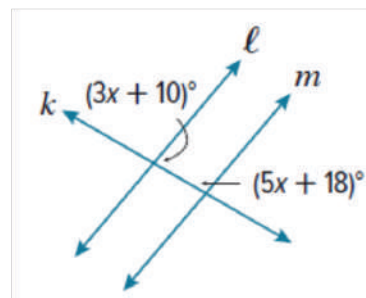
a) 3

b) 5

c) 7

D) 13

112) Find the value of x so that $l \parallel m$.



a) 4

b) 6

c) 19

d) 13

M12L10: Perpendiculars and Distance

IMPORTANT

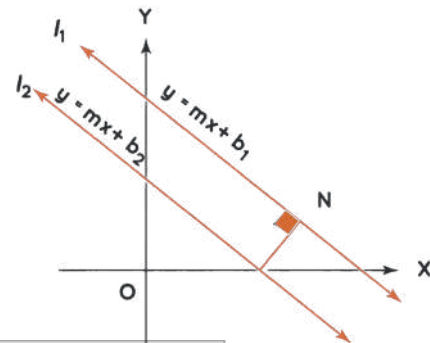
Distance Between Parallel Lines

Slope Intercept Form

$$y = mx + b$$

$$y = (\text{slope})x + y\text{-intercept}$$

$$d = \frac{|b_2 - b_1|}{\sqrt{1 + m^2}}$$



35) Find the distance between the parallel lines:

$$y = 7$$

$$y = -1$$

a) 6 units

b) 7 units

c) 8 units

d) 9 units

36) Find the distance between the parallel lines:

$$x = -6$$

$$x = 5$$

a) 1 units

b) 11 units

c) 13 units

d) 30 units

37) Find the distance between the parallel lines:

$$y = 3x$$

$$y = 3x + 10$$

a) 10 units

b) $3\sqrt{10}$ units

c) $\sqrt{10}$ units

d) $2\sqrt{10}$ units

38) Find the distance between the parallel lines:

$$y = -5x$$

$$y = -5x + 26$$

a) $\sqrt{26}$ units

b) 26 units

c) $\sqrt{13}$ units

d) $2\sqrt{26}$ units

39) Find the distance between the parallel lines:

$$y = x + 9$$

$$y = x + 3$$

a) 6 units

b) $3\sqrt{2}$ units

c) $6\sqrt{2}$ units

d) 3 units

40) Find the distance between the parallel lines:

$$y = -2x + 5$$

$$y = -2x - 5$$

a) $2\sqrt{5}$ units

b) 10 units

c) $\sqrt{5}$ units

d) $4\sqrt{5}$ units

M13L1: Reflections

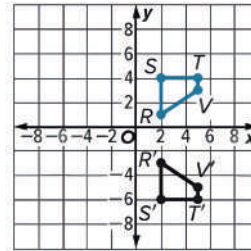
IMPORTANT

1) Reflection in a <u>Horizontal Line</u> $y = b$	$(x, y) \mapsto (x, 2b - y)$
2) Reflection in a <u>Vertical Line</u> $x = a$	$(x, y) \mapsto (2a - x, y)$
3) Reflection in a <u>Line</u> $y = x$	$(x, y) \mapsto (y, x)$

Example 1 Reflection in a Horizontal or Vertical Line

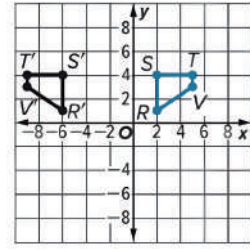
Consider quadrilateral $RSTV$ with vertices at $R(2, 1)$, $S(2, 4)$, $T(5, 4)$, and $V(5, 3)$. Graph the image of quadrilateral $RSTV$ under each reflection. Determine the coordinates of the image.

a. in the line $y = -1$



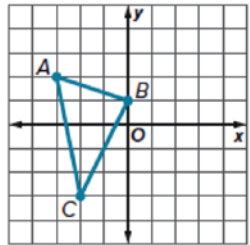
$R'(2, -3)$, $S'(2, -6)$,
 $T'(5, -6)$, $V'(5, -5)$

b. in the line $x = -2$



$R'(-6, 1)$, $S'(-6, 4)$,
 $T'(-9, 4)$, $V'(-9, 3)$

45) Triangle ABC has the vertices $A(-2, 2)$, $B(0, 1)$, and $C(-1, -3)$. Find the coordinates of the vertices after a reflection across the line $y = x$.



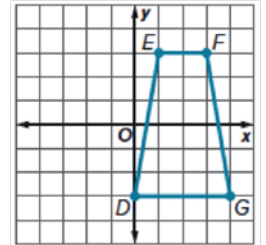
a) $A'(2, -3)$, $B'(1, 0)$,
 $C'(-3, -2)$

b) $A'(-2, -2)$, $B'(0, -1)$,
 $C'(-1, 3)$

c) $A'(2, -2)$, $B'(0, 1)$,
 $C'(1, -3)$

d) $A'(2, -2)$, $B'(1, 0)$,
 $C'(3, 1)$

46) Trapezoid $DEFG$ has the vertices $D(0, -3)$, $E(1, 3)$, $F(3, 3)$, and $G(4, -3)$. Find the coordinates of the vertices after a reflection in the line $x = -1$.



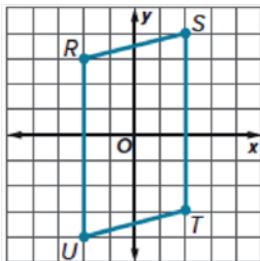
a) $D'(-2, -3)$, $E'(-3, 3)$,
 $F'(-5, 3)$, $G'(-6, -3)$

b) $D'(0, 1)$, $E'(1, -5)$,
 $F'(3, -5)$, $G'(4, 1)$

c) $D'(-1, -3)$, $E'(0, 3)$,
 $F'(-2, 3)$, $G'(-3, -3)$

d) $D'(2, -3)$, $E'(3, 3)$,
 $F'(5, 3)$, $G'(6, -3)$

47) $RSTU$ has the vertices $R(-2, 3)$, $S(2, 4)$, $T(2, -2)$, and $U(-2, -3)$. Find the coordinates of the vertices after a reflection in the line $y = x$.



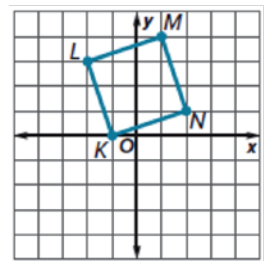
a) $R'(2, -3)$, $S'(-2, -4)$,
 $T'(-2, 2)$, $U'(2, 3)$

b) $R'(3, -2)$, $S'(4, 2)$,
 $T'(-3, 2)$, $U'(-4, -2)$

c) $R'(-2, -3)$, $S'(2, -4)$,
 $T'(2, 2)$, $U'(-2, 3)$

d) $R'(-3, 2)$, $S'(-4, -2)$,
 $T'(2, -2)$, $U'(3, 2)$

48) Square $KLMN$ has the vertices $K(-1, 0)$, $L(-2, 3)$, $M(1, 4)$, and $N(2, 1)$. Find the coordinates of the vertices after a reflection across the horizontal line $y = -2$.



a) $K'(-1, -4)$, $L'(-2, -7)$,
 $M'(1, -8)$, $N'(2, -5)$

b) $K'(-1, 4)$, $L'(-2, 7)$,
 $M'(1, 8)$, $N'(2, 5)$

c) $K'(-3, 0)$, $L'(-2, 3)$,
 $M'(-5, 4)$, $N'(-6, 1)$

d) $K'(-1, -2)$, $L'(-2, -5)$,
 $M'(1, -6)$, $N'(2, -3)$

49) Determine the coordinates of S(-7,1) after a reflection in the line $y = 3$

a) S'(-7,5)

b) S'(7,5)

c) S'(7,-5)

d) S'(-7,-5)

50) Determine the coordinates of Q(6,-4) after a reflection in the line $x = 2$

a) Q'(2,-4)

b) Q'(-2,-4)

c) Q'(-6,-4)

d) Q'(6,4)

10

Determine the translation vector

(EX1,1-2)

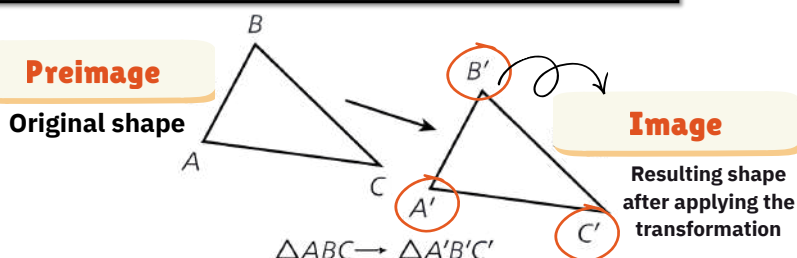
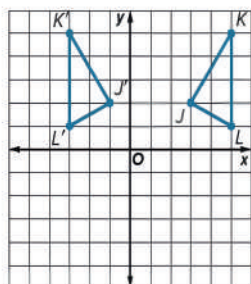
803,805

M13L2: Translations

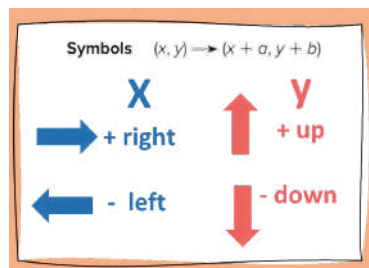
Example 1 Determine a Translation Vector

Determine whether a translation maps $\triangle JKL$ onto $\triangle J'K'L'$. If so, find the translation vector. If not, explain why.

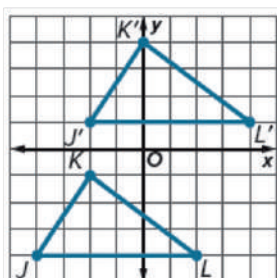
$\triangle J'K'L'$ is not a translation of $\triangle JKL$.
The distances between corresponding vertices are not equal.



Keep in mind that the **vector** (a, b) is equivalent to the mapping $(x, y) \rightarrow (x + a, y + b)$.



51) Determine whether a translation maps $\triangle JKL$ onto $\triangle J'K'L'$. If so, find the translation vector.



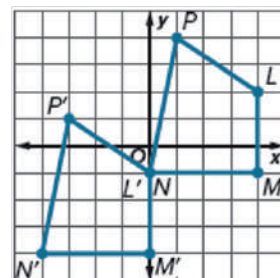
a) Yes; $\langle -2, 5 \rangle$

b) Yes; $\langle 2, 5 \rangle$

c) No; the orientation of the triangle changed, so it is a reflection or rotation instead of a translation.

d) No; the triangle changed size, meaning it is a dilation.

52) Determine whether a translation maps quadrilateral $LMNP$ onto quadrilateral $L'M'N'P'$. If so, find the translation vector.



a) Yes; $\langle -4, -4 \rangle$

b) No; the figure was rotated, not translated

c) No; the figure was reflected across the y-axis.

d) Yes; $\langle -4, -3 \rangle$

Module 13: Transformations and Symmetry

11

Use rigid motions to rotate figures about points that are not the origin and describe the effects of the rotations

(EX1,1-4)

807-809

M13L3: Rotations

IMPORTANT

ROTATION ABOUT THE ORIGIN

Rule

90 degrees clockwise /

270 degrees counterclockwise

$$(x, y) \rightarrow (y, -x)$$

180 degrees

$$(x, y) \rightarrow (-x, -y)$$

270 degrees clockwise /

90 degrees counterclockwise

$$(x, y) \rightarrow (-y, x)$$

ROTATING AROUND A POINT OTHER THAN THE ORIGIN (H, K)

Points	Translate $(x - h, y - k)$	Rotation Rule	Translate back $(x' + h, y' + k)$
(x, y)	1) Subtract the coordinates of the center of rotation from the point.	2) Apply the rotation rule	3) Add the center of rotation back to the rotated point

Example 1 Rotation About a Point That Is Not the Origin

Triangle ABC has vertices A(-8, 5), B(-6, 9), and C(-3, 6). Graph $\triangle ABC$ and its image after a rotation of 180° about (-5, 3).

Step 1 Graph $\triangle ABC$.

Step 2 Map the center of rotation to the origin.

To map the center of rotation to the origin, translate the center of rotation along the vector $\langle 5, -3 \rangle$. Then translate the vertices of $\triangle ABC$ along the same vector.

$$(x, y) \rightarrow (x + 5, y - 3)$$

$$A(-8, 5) \rightarrow (-3, 2) \quad B(-6, 9) \rightarrow (-1, 6) \quad C(-3, 6) \rightarrow (2, 3)$$

Step 3 Rotate 180° about the origin.

$$(x, y) \rightarrow (-x, -y)$$

$$A(-3, 2) \rightarrow (3, -2)$$

$$B(-1, 6) \rightarrow (1, -6)$$

$$C(2, 3) \rightarrow (-2, -3)$$

Step 4 Map the center of rotation to its original position.

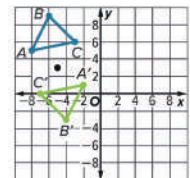
To map the center of rotation to its original position, translate the center of rotation along the vector $\langle -5, 3 \rangle$. Then translate the vertices of the rotated triangle along the same vector.

$$(x, y) \rightarrow (x - 5, y + 3)$$

$$A(3, -2) \rightarrow A'(-2, 1)$$

$$B(1, -6) \rightarrow B'(-4, -3)$$

$$C(-2, -3) \rightarrow C'(-7, 0)$$



53) Triangle XYZ has vertices X(0, 2), Y(4, 4), and Z(3, -1). Graph $\triangle XYZ$ and its image after a rotation of 180° about (2, -3).

a) X'(4, -8), Y'(0, -10), and Z'(1, -5).

b) X'(2, -5), Y'(-2, -7), and Z'(-1, -2).

c) X'(0, -2), Y'(-4, -4), and Z'(-3, 1).

d) X'(4, -1), Y'(0, -2), and Z'(1, 2).

54) Triangle ABC has vertices A(1, 7), B(3, 2), and C(-2, -2). Graph $\triangle ABC$ and its image after a rotation of 270° counterclockwise about (-4, 2).

a) A'(1, -3), B'(-4, -5), and C'(-8, 0).

b) A'(-9, -3), B'(-4, -5), and C'(-8, 0).

c) A'(1, 7), B'(4, 2), and C'(-2, -2).

d) A'(1, -3), B'(-4, -5), and C'(-2, 4).

Step 1

Step 2

Step 3

Points	Translate $(x - h, y - k)$	Rotation Rule	Translate back $(x' + h, y' + k)$

Step 1

Step 2

Step 3

Points	Translate $(x - h, y - k)$	Rotation Rule	Translate back $(x' + h, y' + k)$



55) Triangle FGH has vertices F(-3, 4), G(2, 0), and H(-1, -2). Graph $\triangle FGH$ and its image after a rotation of 180° about (-3, -6).

a) F'(-3, -16), G'(-8, -12), and H'(-5, -10).

b) F'(3, -4), G'(-2, 0), and H'(1, 2).

c) F'(-3, -16), G'(8, -12), and H'(-5, -6).

d) F'(0, -10), G'(-5, -6), and H'(-2, -4).

56) Quadrilateral ABCD has vertices A(-2, 4), B(1, 3), C(2, -3), and D(-3, -1). Graph quadrilateral ABCD and its image after a rotation of 90° counterclockwise about (-1, 2).

a) A'(-3, 1), B'(-2, 4), and C'(4, 5), D'(2, 0).

b) A'(-3, 1), B'(-2, 4), and C'(4, 5), D'(2, -1)

c) A'(-3, 1), B'(-2, 4), and C'(4, 5), D'(-2, 0).

d) A'(-4, 2), B'(-1, 3), and C'(2, -3), D'(-3, -1).

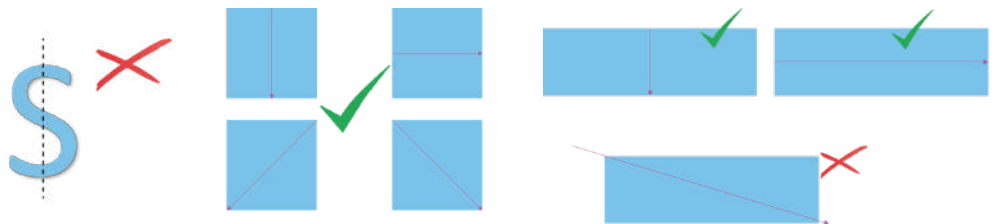
Points	Translate ($x - h, y - k$)	Rotation Rule	Translate back ($x' + h, y' + k$)

Points	Translate ($x - h, y - k$)	Rotation Rule	Translate back ($x' + h, y' + k$)

20	Use rotational symmetry to describe the rotations that carry a figure onto itself	(1-6)	829
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M13L6: Symmetry

To find the number of lines of symmetry of a shape:
"Count the number of lines that divide the shape into matching halves."



94) Determine whether the figure has a line of symmetry. If so, state how many lines of symmetry it has.



- a) 2 b) 3
c) 4 d) 1

95) Determine whether the figure has a line of symmetry. If so, state how many lines of symmetry it has.



- a) 2 b) 3
c) 4 d) 1

96) Determine whether the figure has a line of symmetry. If so, state how many lines of symmetry it has.



- a) 2 b) 3
c) 4 d) 0

97) Determine whether the figure has a line of symmetry. If so, state how many lines of symmetry it has.



- a) 2 b) 3
c) 4 d) 0

98) Determine whether the figure has a line of symmetry. If so, state how many lines of symmetry it has.



- a) 4 b) 5
c) 6 d) 7

For a **regular polygon** (all sides and angles equal):
Number of lines of symmetry = Number of sides

99) Determine whether the figure has a line of symmetry. If so, state how many lines of symmetry it has.

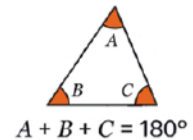


- a) 5 b) 6
c) 7 d) 8

M14L1: Angles of Triangles

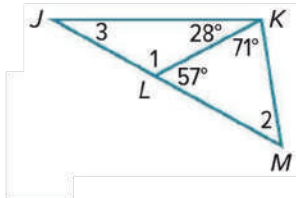
TRIANGLE ANGLE-SUM THEOREM

The **sum** of the measures of the interior angles of a triangle is **180°** .



Apply Example 1 Use the Triangle Angle-Sum Theorem.

118) Find the measure of each numbered angle.



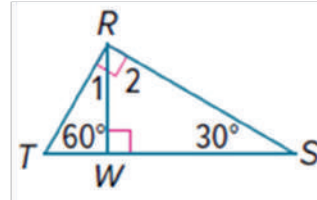
a) $\angle 1 = 95^\circ$, $\angle 2 = 81^\circ$, $\angle 3 = 57^\circ$

b) $\angle 1 = 57^\circ$, $\angle 2 = 81^\circ$, $\angle 3 = 95^\circ$

c) $\angle 1 = 123^\circ$, $\angle 2 = 52^\circ$, $\angle 3 = 29^\circ$

d) $\angle 1 = 85^\circ$, $\angle 2 = 81^\circ$, $\angle 3 = 67^\circ$

119) Find the measure of each numbered angle.



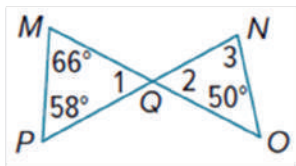
a) $\angle 1 = 30^\circ$, $\angle 2 = 60^\circ$

b) $\angle 1 = 60^\circ$, $\angle 2 = 30^\circ$

c) $\angle 1 = 90^\circ$, $\angle 2 = 30^\circ$

d) $\angle 1 = 30^\circ$, $\angle 2 = 90^\circ$

120) Find the measure of each numbered angle.



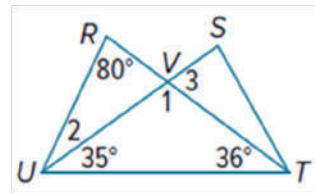
a) $\angle 1 = 56^\circ$, $\angle 2 = 50^\circ$, $\angle 3 = 80^\circ$

b) $\angle 1 = 56^\circ$, $\angle 2 = 56^\circ$, $\angle 3 = 74^\circ$

c) $\angle 1 = 66^\circ$, $\angle 2 = 50^\circ$, $\angle 3 = 64^\circ$

d) $\angle 1 = 56^\circ$, $\angle 2 = 80^\circ$, $\angle 3 = 50^\circ$

121) Find the measure of each numbered angle.



a) $\angle 1 = 99^\circ$, $\angle 2 = 64^\circ$, $\angle 3 = 81^\circ$

b) $\angle 1 = 80^\circ$, $\angle 2 = 64^\circ$, $\angle 3 = 99^\circ$

c) $\angle 1 = 109^\circ$, $\angle 2 = 29^\circ$, $\angle 3 = 71^\circ$

d) $\angle 1 = 81^\circ$, $\angle 2 = 64^\circ$, $\angle 3 = 99^\circ$

122) Find the measure of each numbered angle.



a) $\angle 1 = 17^\circ$, $\angle 2 = 17^\circ$

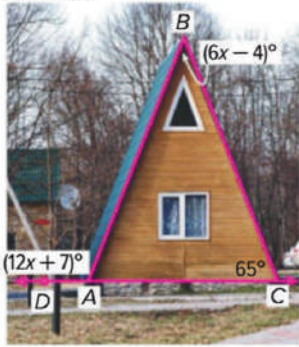
b) $\angle 1 = 34^\circ$, $\angle 2 = 34^\circ$

c) $\angle 1 = 17^\circ$, $\angle 2 = 34^\circ$

d) $\angle 1 = 34^\circ$, $\angle 2 = 17^\circ$

Example 2 Use the Exterior Angle Theorem

ARCHITECTURE Find the measure of $\angle DAB$ in the front face of the building.



$$m\angle DAB = m\angle ABC + m\angle BCA$$

$$12x + 7 = 6x - 4 + 65$$

$$x = 9$$

$$m\angle DAB = 12(9) + 7 \text{ or } 115^\circ$$

Exterior Angle Theorem

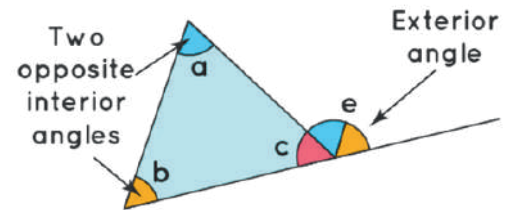
Substitution

Solve.

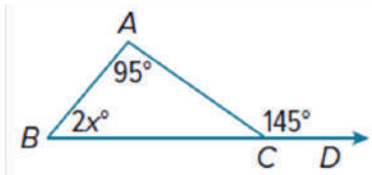
EXTERIOR ANGLE THEOREM

The measure of an **exterior angle** of a **triangle** is equal to the **sum** of the measures of the **two remote (opposite) interior angles**.

$$\angle e = \angle a + \angle b$$



57) Find $m\angle ABC$.



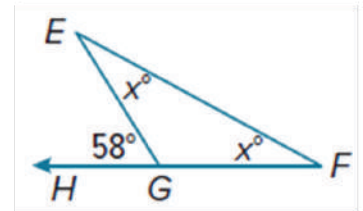
a) 50°

b) 95°

c) 25°

d) 145°

58) Find $m\angle F$.



a) 61°

b) 58°

c) 29°

d) 90°

59) A lookout tower sits on a network of struts and posts. Leslie measured two angles on the tower. If $m\angle 1 = (7x - 7)^\circ$, $m\angle 2 = (4x + 2)^\circ$, and $m\angle 3 = (2x + 6)^\circ$, what is $m\angle 1$?



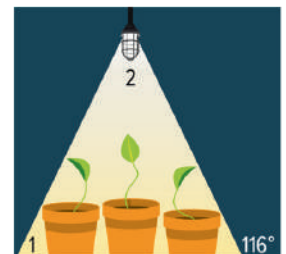
a) 89°

b) 15°

c) 98°

d) 36°

60) A gardener uses a grow light to grow vegetables indoors. If $m\angle 1 = (8x)^\circ$ and $m\angle 2 = (7x - 4)^\circ$, what is $m\angle 1$?



a) 52°

b) 116°

c) 64°

d) 36°

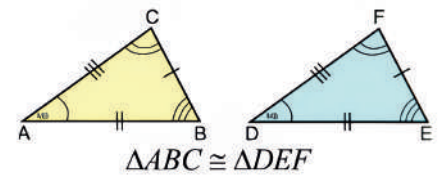
M14L2: Statements, Conditionals, and Biconditionals

Order is Important for
your Congruence Statement



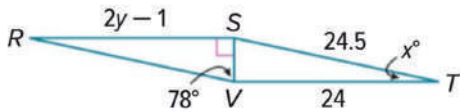
CONGRUENT TRIANGLES

Two triangles are congruent if and only if their corresponding parts are congruent. These corresponding parts include corresponding **angles** and corresponding **sides**.



Example 2 Use Corresponding Parts of Congruent Triangles

In the diagram, $\triangle RSV \cong \triangle TVS$. Find the values of x and y .



Part A Find the value of x .

$$\begin{aligned}\angle T &\cong \angle R \\ m\angle T &= m\angle R \\ &= 180^\circ - 90^\circ - 78^\circ \\ &= 12^\circ\end{aligned}$$

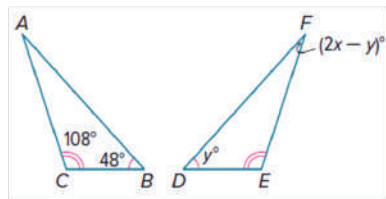
The value of x is 12.

Part B Find the value of y .

$$\begin{aligned}\overline{RS} &\cong \overline{TV} \\ RS &= TV \\ 2y - 1 &= 24 \\ y &= 12.5\end{aligned}$$

The value of y is 12.5.

61i) In the diagram, $\triangle ABC \cong \triangle FDE$. Find the value for x and y .



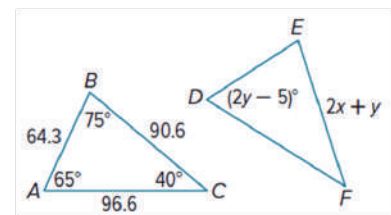
a) $x=36, y=48$

b) $x=48, y=36$

c) $x=24, y=48$

d) $x=36, y=24$

61ii) In the diagram, $\triangle ABC \cong \triangle DEF$. Find the value for x and y .



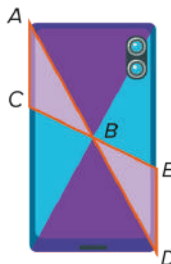
a) $x=27.8, y=30$

b) $x=27.8, y=35$

c) $x=10, y=35$

d) $x=10, y=30$

62) Camila is designing a new image for her cell phone case. If $m\angle ABC = 35^\circ$, $m\angle BAC = 29^\circ$, and $\angle ACB \cong \angle DEB$, what is $m\angle DEB$?



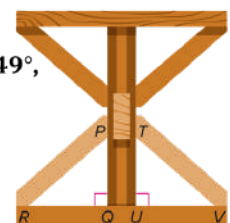
a) 29°

b) 30°

c) 35°

d) 116°

63) Mr. Lewis is building a rustic dining table. Instead of having four legs, the table has a set of supports at each end. If $\angle PRQ \cong \angle TVU$ and $m\angle RPQ = 49^\circ$, what is $m\angle TVU$?



a) 49°

b) 90°

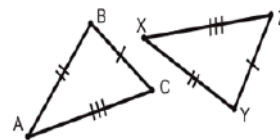
c) 41°

d) 30°

M14L3: Proving Triangles Congruent – SSS, SAS

SIDE-SIDE-SIDE (SSS) CONGRUENCE

If **three sides** of one triangle are **congruent to three sides** of a **second triangle**, then the triangles are congruent.



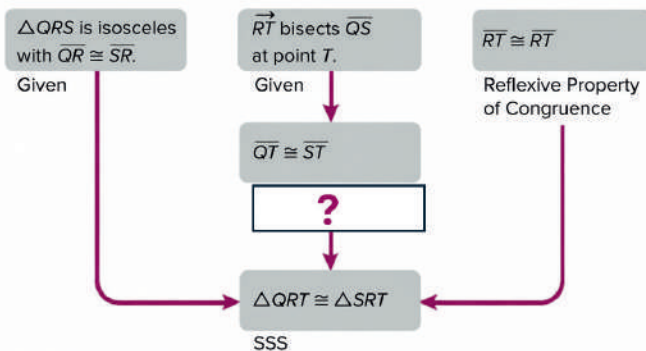
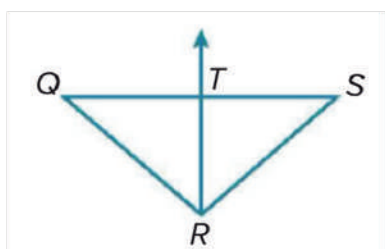
Example 1 Use SSS to Prove Triangles Congruent

113) Complete the flow proof.

Given: $\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$. $\overline{RT}$ bisects $\overline{QS}$ at point T .

Prove: $\triangle QRT \cong \triangle SRT$

Which reason is justified by statement “?”



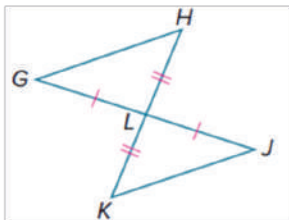
a) All Right Angles are Congruent

b) Vertical Angles Theorem

c) Definition of segment Bisector

d) Given

114) Determine which congruence theorem can be used to prove the triangles congruent.



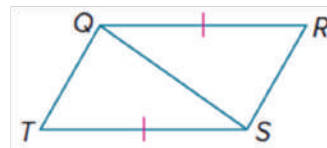
a) SSS

b) SAS

c) ASA

d) AAS

115) Determine which congruence theorem can be used to prove the triangles congruent.



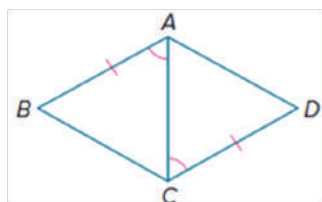
a) SSS

b) SAS

c) ASA

d) Can't be determined

116) Determine which congruence theorem can be used to prove the triangles congruent.



a) SSS

b) SAS

c) ASA

d) Can't be determined

117) Determine which congruence theorem can be used to prove the triangles congruent.



a) SSS

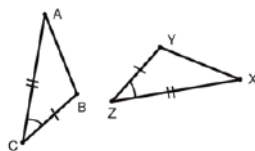
b) SAS

c) ASA

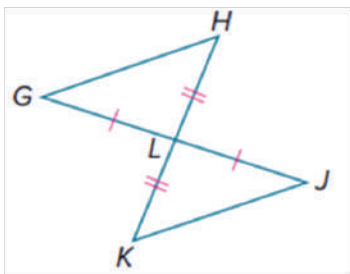
d) Can't be determined

SIDE-ANGLE-SIDE (SAS) CONGRUENCE

If **two sides and the included angle** of one triangle are congruent to **two sides and the included angle** of a second triangle, then the triangles are congruent.



70) Determine which congruence theorem can be used to prove the triangles congruent .



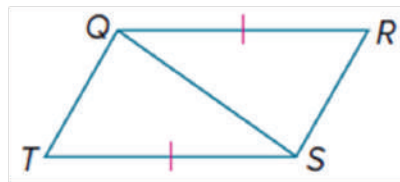
a) SSS

b) SAS

c) ASA

d) AAS

71) Determine which congruence theorem can be used to prove the triangles congruent .



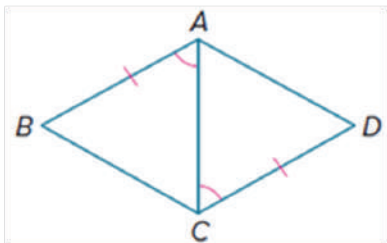
a) SSS

b) SAS

c) ASA

d) Cannot be determined

72) Determine which congruence theorem can be used to prove the triangles congruent .



a) SSS

b) SAS

c) ASA

d) Cannot be determined

73) Determine which congruence theorem can be used to prove the triangles congruent .



a) SSS

b) SAS

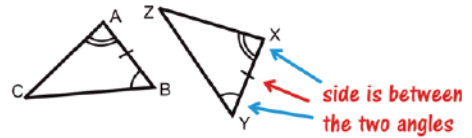
c) ASA

d) Cannot be determined

M14L4: Proving Triangles Congruent – ASA, AAS

ANGLE-SIDE-ANGLE (ASA) CONGRUENCE

If **two angles and the included side** of one triangle are congruent **to the two angles and the included side** of a second triangle, then the triangles are congruent.



REASONS FOR PROVING SIDES CONGRUENT IN TRIANGLES

1) **Given:** The side is stated directly in the question.

Example: Given: $\overline{RQ} \cong \overline{ST}$

2) **Reflexive Property of Congruence:** A side is congruent to itself when two triangles share the same side.

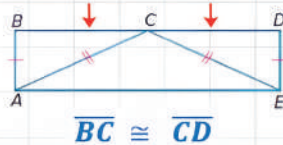
Example:

$$\overline{XZ} \cong \overline{ZX}$$

3) **Definition of Segment Bisector :**

A segment bisector divides a segment into two congruent parts.

Example: Given: $\overline{AC}$ bisects $\overline{BD}$

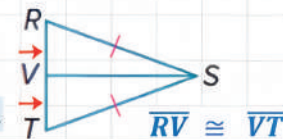


$$\overline{BC} \cong \overline{CD}$$

4) **Definition of Midpoint:**

A midpoint divides a segment into two congruent segments.

Example: Given: V is the midpoint of $\overline{RT}$.



$$\overline{RV} \cong \overline{VT}$$



Remember: Use the correct reason to justify each step and build a strong proof!



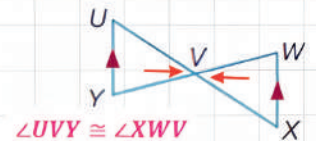
REASONS FOR PROVING ANGLES CONGRUENT IN TRIANGLES

1) **Given:** The angle is stated directly in the question.

Example: Given: $\angle N \cong \angle L$

2) **Vertical Angles Theorem:** Vertical angles are congruent.

Example:

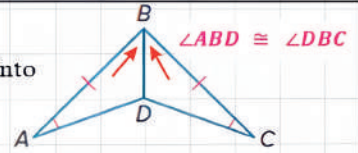


$$\angle U \cong \angle X$$

3) **Definition of Angle Bisector :**

An angle bisector divides an angle into two congruent angles.

Example: Given: $\overline{DB}$ bisects $\angle ABC$

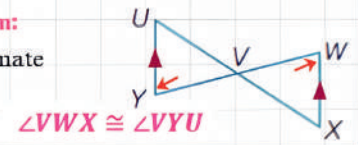


$$\angle ABD \cong \angle DBC$$

4) **Alternate Interior Angles Theorem:**

Parallel lines create congruent alternate interior angles.

Example: Given: $\overline{UY} \parallel \overline{XW}$

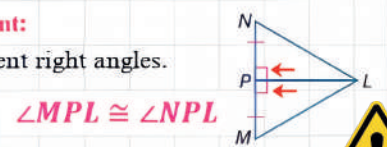


$$\angle V \cong \angle W$$

5) **All Right Angles Are Congruent:**

Perpendicular lines form congruent right angles.

Example: Given: $\overline{NP} \perp \overline{PL}$



$$\angle MPL \cong \angle NPL$$



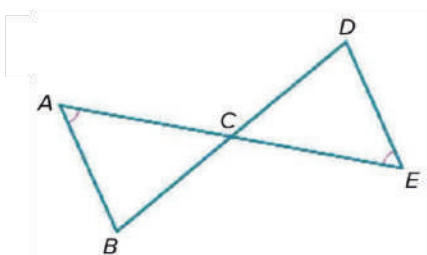
Example 1 Use ASA to Prove Triangles Congruent

100) Complete the two-column proof.

Given: $\angle BAC \cong \angle DEC$; $\overline{BD}$ bisects $\overline{AE}$.

Prove: $\triangle ACB \cong \triangle ECD$

Which reason is justified by statement “?”



Statements	Reasons
1. $\angle BAC \cong \angle DEC$	1. Given
2. $\overline{BD}$ bisects $\overline{AE}$	2. Given
3. $\overline{AC} \cong \overline{EC}$	3. Definition of segment bisector
4. $\angle ACB \cong \angle ECD$	4. ?
5. $\triangle ACB \cong \triangle ECD$	5. ASA

a) Reflexive Property

c) Definition of Angle Bisector

b) Vertical Angles Theorem

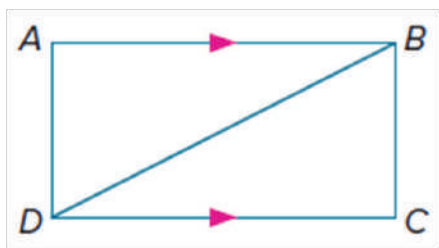
d) Alternate interior angle

101) Complete the two-column proof.

Given: $\overline{AB} \parallel \overline{CD}$, $\angle CBD \cong \angle ADB$.

Prove: $\triangle ABD \cong \triangle CDB$.

Which reason is justified by statement “?”



Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	1. Given
2. $\angle CBD \cong \angle ADB$	2. Given
3. $\angle ABD \cong \angle CDB$	3. ?
4. $\overline{BD} \cong \overline{BD}$	4. Reflexive Property of Congruence
5. $\triangle ABD \cong \triangle CDB$	5. ASA

a) Reflexive Property

b) Vertical Angles Theorem

c) Definition of Angle Bisector

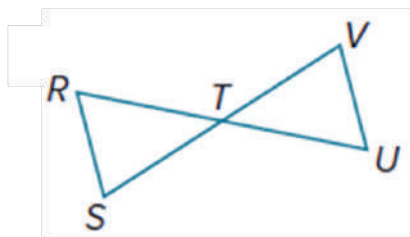
d) Alternate interior angle

102) Complete the two-column proof.

Given: $\angle S \cong \angle V$, T is the midpoint of $\overline{SV}$.

Prove: $\triangle RTS \cong \triangle UTV$

Which reason is justified by statement “?”



Statements	Reasons
1. $\angle S \cong \angle V$	1. Given
2. T is the midpoint of $\overline{SV}$.	2. Given
3. $\overline{ST} \cong \overline{TV}$	3. ?
4. $\angle RTS \cong \angle VTU$	4. Vertical Angles Theorem
5. $\triangle RTS \cong \triangle UTV$	5. ASA

a) Given

b) Definition of Midpoint

c) Reflexive Property

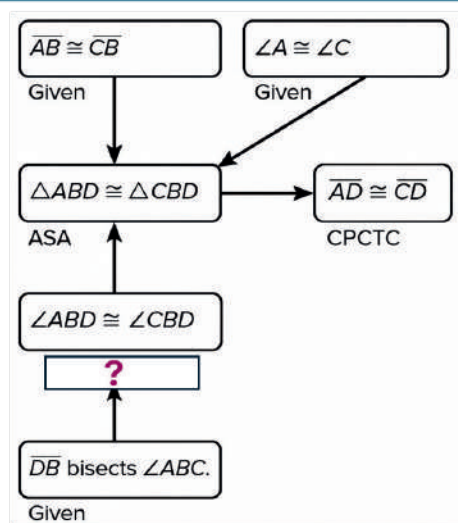
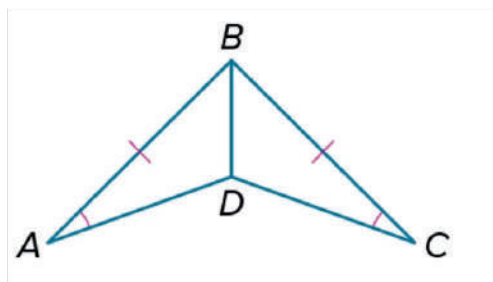
d) Alternate interior angle

103) Complete the flow proof.

Given: $\overline{AB} \cong \overline{CB}$, $\angle A \cong \angle C$, and $\overline{DB}$ bisects $\angle ABC$.

Prove: $\overline{AD} \cong \overline{CD}$

Which reason is justified by statement “?”



a) Reflexive Property

b) Vertical Angles Theorem

c) Definition of Angle Bisector

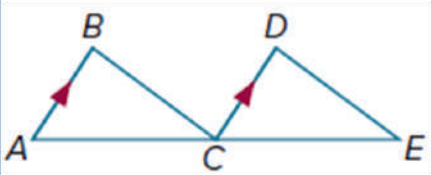
d) Alternate interior angle

104) Complete the paragraph proof.

Given: $\overline{CD}$ bisects $\overline{AE}$, $\overline{AB} \parallel \overline{CD}$, and $\angle E \cong \angle BCA$

Prove: $\triangle ABC \cong \triangle CDE$

Which reason is justified by statement “?”



Proof: It is given that $\angle E \cong \angle BCA$ and $\overline{CD}$ bisects $\overline{AE}$. Since $\overline{CD}$ bisects $\overline{AE}$, by the definition of bisector, $\overline{AC} \cong \overline{CE}$. We are also given that $\overline{AB} \parallel \overline{CD}$. By considering segment $\overline{AE}$ as the transversal, we can use this to determine that $\angle A$ is congruent to $\angle DCE$ by the ?. From this we know that $\triangle ABC \cong \triangle CDE$ by the ASA Congruence Postulate.

a) Corresponding angles

b) Vertical Angles Theorem

c) Definition of Angle Bisector

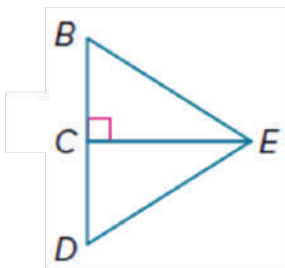
d) Alternate interior angle

105) Complete the paragraph proof.

Given: $\overline{CE}$ bisects $\angle BED$; $\angle BCE$ and $\angle ECD$ are right angles.

Prove: $\triangle ECB \cong \triangle ECD$

Which reason is justified by statement “?”



Proof: We are given that $\overline{CE}$ bisects $\angle BED$ and that $\angle BCE$ and $\angle ECD$ are right angles. Because all right angles are congruent, $\angle BCE \cong \angle ECD$. By the definition of angle bisector, $\angle BEC \cong \angle DEC$. The ? tells us that $\overline{EC} \cong \overline{EC}$. By Angle-Side-Angle Congruence Postulate, $\triangle ECB \cong \triangle ECD$.

a) Reflexive Property

b) Vertical Angles Theorem

c) Definition of segment Bisector

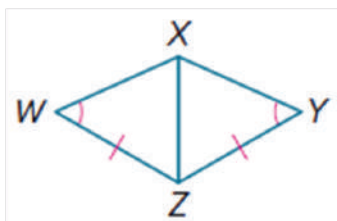
d) Definition of Midpoint

106) Complete the paragraph proof.

Given: $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, and $\overline{XZ}$ bisects $\angle WZY$

Prove: $\triangle XWZ \cong \triangle XYZ$

Which reason is justified by statement “?”



Proof: It is given that $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, and $\overline{XZ}$ bisects $\angle WZY$. Therefore, $\angle WZX \cong \angle YZX$ by the ?. We can now state that $\triangle XWZ \cong \triangle XYZ$ by applying the Angle-Side-Angle Congruence Postulate.

a) Reflexive Property

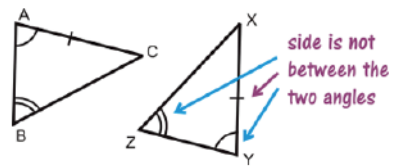
b) Vertical Angles Theorem

c) Definition of angle Bisector

d) Alternate interior angle

ANGLE-ANGLE-SIDE (AAS) CONGRUENCE

If **two angles and the non-included side** of one triangle are congruent to **the corresponding two angles and non-included side** of a second triangle, then the two triangles are congruent



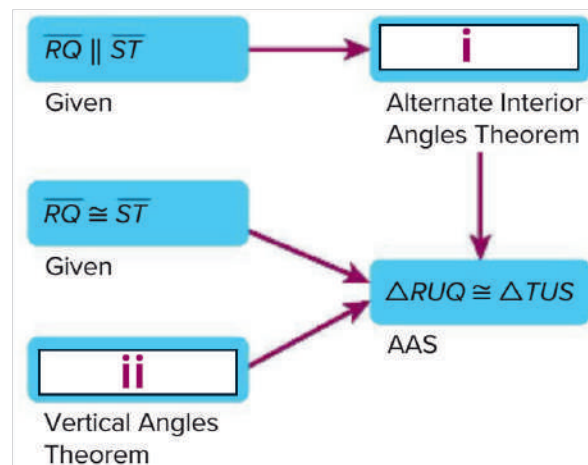
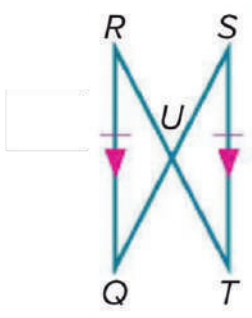
Example 3 Use AAS to Prove Triangles Congruent

74) Complete the flow proof.

Given: $RQ \cong ST$, $RQ \parallel ST$

Prove: $\triangle RUQ \cong \triangle TUS$

Which statement is justified by reason | and ||?



- a) I: $\angle RQS \cong \angle TSQ$
II: $\angle RUQ \cong \angle TUS$
- c) I: $\angle RUQ \cong \angle TUS$
II: $\angle RQS \cong \angle TSQ$

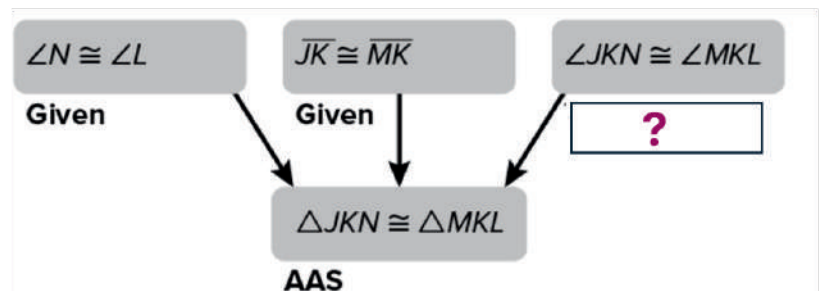
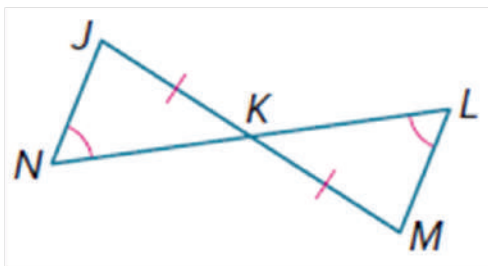
- b) I: $\angle RQS \cong \angle RTS$
II: $\angle TSQ \cong \angle QRT$
- d) I: $\angle TSQ \cong \angle QRT$
II: $\angle RQS \cong \angle RTS$

75) Complete the flow proof.

Given: $\overline{JK} \cong \overline{MK}$, $\angle N \cong \angle L$

Prove: $\triangle JKN \cong \triangle MKL$

Which reason is justified by statement “?”



- a) Reflexive Property

- b) Vertical Angles Theorem

- c) Definition of Angle Bisector

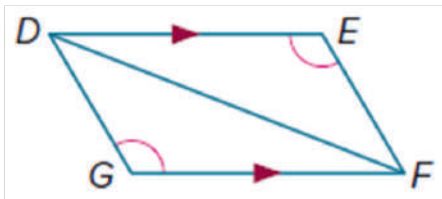
- d) Alternate interior angle

76) Complete the paragraph proof.

Given: $\overline{DE} \parallel \overline{FG}$, $\angle E \cong \angle G$

Prove: $\triangle DFG \cong \triangle FDE$

Which reason is justified by statement “?”



Proof: It is given that $\angle E \cong \angle G$ and $\overline{DE} \parallel \overline{FG}$. Considering $\overline{DF}$ as the transversal that cuts through parallel segments $\overline{DE}$ and $\overline{FG}$, by the Alternate Interior Angles Theorem, $\angle DFG \cong \angle FDE$. $\overline{DF} \cong \overline{DF}$ by the ?. Therefore, $\triangle DFG \cong \triangle FDE$ by AAS.

a) Reflexive Property

b) Vertical Angles Theorem

c) Definition of segment Bisector

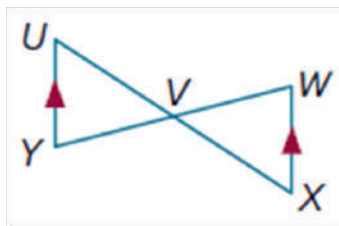
d) Definition of Midpoint

77) Complete the two-column proof

Given: V is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$.

Prove: $\triangle UYV \cong \triangle XVW$

Which reason is justified by statement “?”



Statements	Reasons
1. V is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$	1. Given
2. $\overline{YV} \cong \overline{VW}$	2. Midpoint Theorem
3. $\angle VWX \cong \angle VYU$	3. ?
4. $\angle VUY \cong \angle VXW$	4. ?
5. $\triangle UYV \cong \triangle XVW$	5. AAS

a) Reflexive Property

b) Vertical Angles Theorem

c) Definition of Angle Bisector

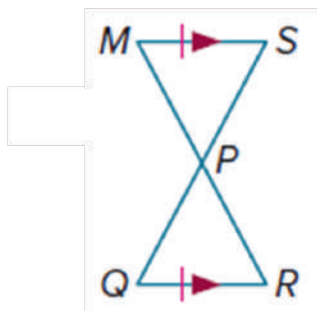
d) Alternate interior angle

78) Complete the two-column proof

Given: $\overline{MS} \cong \overline{RQ}$, $\overline{MS} \parallel \overline{RQ}$

Prove: $\triangle MSP \cong \triangle RQP$

Which reason is justified by statement “?”



Statements	Reasons
1. $\overline{MS} \cong \overline{RQ}$, $\overline{MS} \parallel \overline{RQ}$	1. Given
2. $\angle SPM \cong \angle QPR$	2. ?
3. $\angle SMP \cong \angle QRP$	3. Alternate Interior Angles Theorem
4. $\triangle MSP \cong \triangle RQP$	4. AAS

a) Reflexive Property

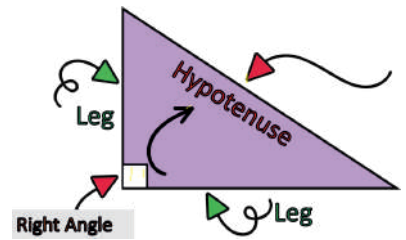
b) Vertical Angles Theorem

c) Definition of Angle Bisector

d) Alternate interior angle

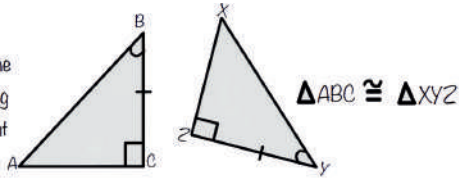
M14L5: Proving Right Triangles Congruent

RIGHT TRIANGLES CONGRUENCE THEOREMS



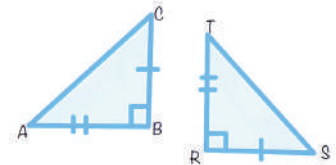
Leg-Angle (LA)

If the leg and an acute angle of one right triangle are congruent to a leg and an acute angle of another right triangle, then the triangles are congruent.



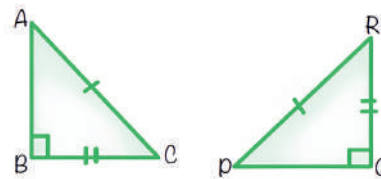
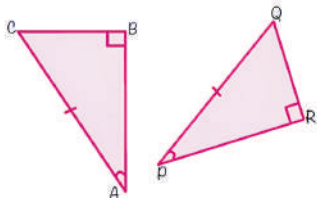
Leg-Leg (LL)

If the two legs of one right triangle are congruent to the two legs of another right triangle, then the triangles are congruent.



Hypotenuse-Angle (HA)

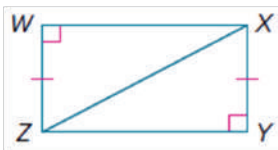
If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.



Hypotenuse-Leg (HL)

If the hypotenuse and leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.

64) Determine whether each pair of triangles is congruent. If yes, include the theorem that applies.



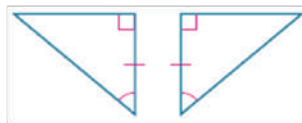
a) LL

b) HA

c) LA

d) HL

65) Determine whether each pair of triangles is congruent. If yes, include the theorem that applies.



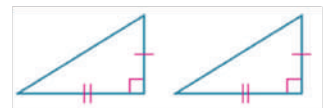
a) LL

b) HA

c) LA

d) HL

66) Determine whether each pair of triangles is congruent. If yes, include the theorem that applies.



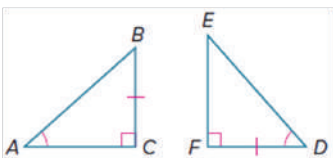
a) LL

b) HA

c) LA

d) HL

67) Determine whether each pair of triangles is congruent. If yes, include the theorem that applies.



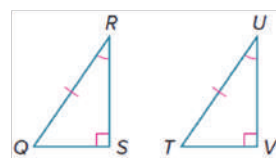
a) LL

b) HA

c) LA

d) Not congruent

68) Determine whether each pair of triangles is congruent. If yes, include the theorem that applies.



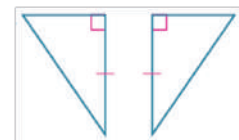
a) LL

b) HA

c) LA

d) HL

69) Determine whether each pair of triangles is congruent. If yes, include the theorem that applies.



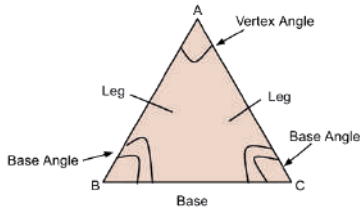
a) LL

b) HA

c) LA

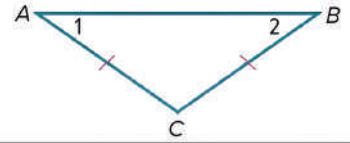
d) Not congruent

M14L6: Isosceles and Equilateral Triangles



ISOSCELES TRIANGLE THEOREM

Words	If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
Example	If $\overline{AC} \cong \overline{BC}$, then $\angle 2 \cong \angle 1$.



Example 2 Find Missing Measures in Isosceles Triangles

Find $m\angle B$ and $m\angle C$.

Part A Determine side relationships.

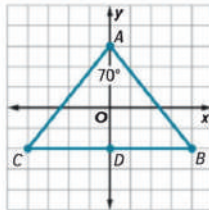
Use the Distance Formula to determine the measures of the sides of $\triangle ABC$. The coordinates of $\triangle ABC$ are $A(0, 3)$, $B(4, -2)$, and $C(-4, -2)$.

$$AB = \sqrt{(0 - 4)^2 + [3 - (-2)]^2} \text{ or } \sqrt{41} \text{ units}$$

$$AC = \sqrt{[0 - (-4)]^2 + [3 - (-2)]^2} \text{ or } \sqrt{41} \text{ units}$$

$$BC = \sqrt{[4 - (-4)]^2 + [-2 - (-2)]^2} \text{ or } 8 \text{ units}$$

So, $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{AC}$.



REMEMBER

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Part B Determine the angle measures.

Because $\overline{AB} \cong \overline{AC}$, we know that $\angle C \cong \angle B$ by the Isosceles Triangle Theorem.

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$m\angle A + 2m\angle B = 180^\circ$$

$$70^\circ + 2m\angle B = 180^\circ$$

$$m\angle B = m\angle C = 55^\circ$$

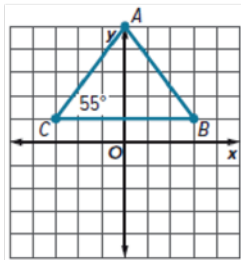
Triangle Angle-Sum Theorem

Definition of congruent

Substitute.

Solve.

79) Refer to the figure. Find $m\angle A$.



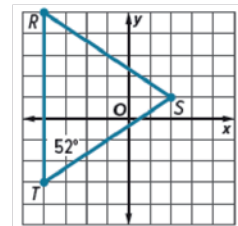
a) 55°

b) 70°

c) 110°

d) 125°

80) Find the measures of $m\angle TRS$, and $m\angle RST$. Round to the nearest tenth, if necessary.



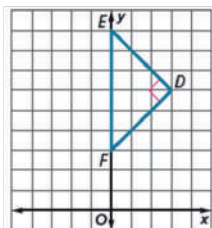
a) $m\angle TRS = 38^\circ$,
 $m\angle RST = 90^\circ$

b) $m\angle TRS = 52^\circ$,
 $m\angle RST = 38^\circ$

c) $m\angle TRS = 38^\circ$,
 $m\angle RST = 52^\circ$

d) $m\angle TRS = 52^\circ$,
 $m\angle RST = 76^\circ$

81) Find the measures of $\angle DEF$ and $\angle EFD$. Round to the nearest tenth, if necessary.



a) $\angle DEF = 45^\circ$, $\angle EFD = 45^\circ$

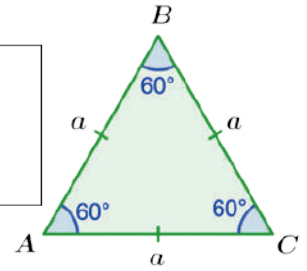
b) $\angle DEF = 67.5^\circ$,
 $\angle EFD = 67.5^\circ$

c) $\angle DEF = 22.5^\circ$,
 $\angle EFD = 22.5^\circ$

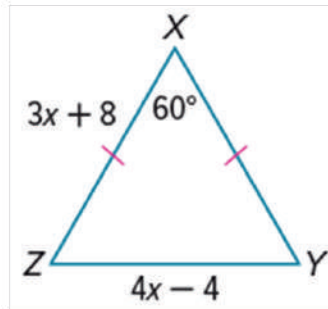
d) $\angle DEF = 90^\circ$,
 $\angle EFD = 45^\circ$

EQUILATERAL TRIANGLE

An **equilateral triangle** is a triangle in which **all three sides have the same length**, and **all three angles are equal**, each interior angle must measure 60° .



22) Find the value of x .



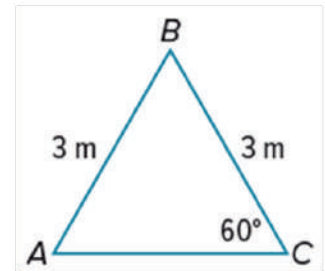
a) $x=8$

b) $x=10$

c) $x=12$

d) $x=15$

23) Find $m\angle B$ and AC .



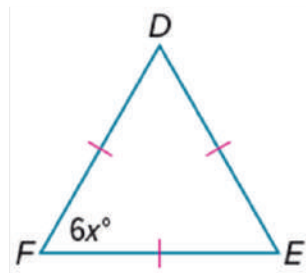
a) $m\angle B=60^\circ$, $AC=3$ m

b) $m\angle B=120^\circ$, $AC=3$ m

c) $m\angle B=60^\circ$, $AC=6$ m

d) $m\angle B=120^\circ$, $AC=6$ m

24) Find the value of x .



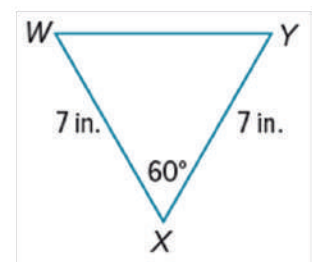
a) $x=5$

b) $x=10$

c) $x=15$

d) $x=20$

25) Find $m\angle Y$ and WY .



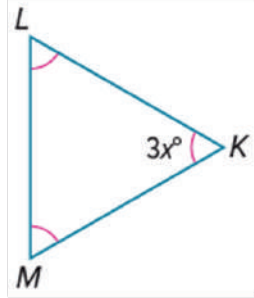
a) $m\angle Y=60^\circ$, $WY=7$ in.

b) $m\angle Y=60^\circ$, $WY=14$ in.

c) $m\angle Y=30^\circ$, $WY=7$ in.

d) $m\angle Y=30^\circ$, $WY=14$ in.

26) Find the value of x .



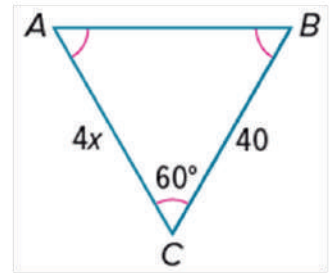
a) $x=15$

b) $x=20$

c) $x=30$

d) $x=45$

27) Find the value of x .



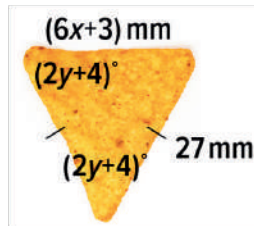
a) $x=5$

b) $\angle x=8$

c) $x=10$

d) $x=12$

28i) Some tortilla chips can be modeled by a triangle. Find the value for x and y .



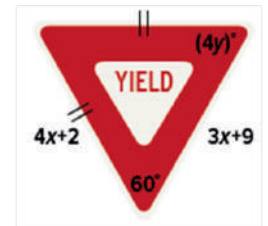
a) $x=3, y=18$

b) $x=4, y=28$

c) $x=5, y=18$

d) $x=3, y=20$

28ii) Yield signs notify drivers to slow down and allow oncoming vehicles to proceed first. Find the value for x and y .



a) $x = 7, y = 240$

b) $x = 15, y = 7$

c) $x = 7, y = 15$

d) $x = 11, y = 15$

مع ختام العام الدراسي، أتمنى أن تكون هذه الملزمة عوناً لكم في مراجعة ما تعلمتموه، وأن تحققوا النجاح والتفوق الذي تستحقونه. أثق بقدراتكم وبما بذلتموه من جهد، وأسأل الله أن يوفقكم في امتحاناتكم ويكمل مسيرتكم بالتميز.



معلمتكم
نجلاء أبو صالح



Answer key

L.O	Question	Answer
1	1	B
	2	B
	3	C
	4	B
	5	A
	6	B
2	7	A
	8	B
	9	B
	10	B
	11	A
	12	B

L.O	Question	Answer
3	13	B
	14	C
	15	A

L.O	Question	Answer
4	16	A
	17	D
	18	D
	19	C
	20	B
	21	A
5	22	C
	23	A
	24	B
	25	A
	26	B
	27	C
	28i	B
	28ii	C

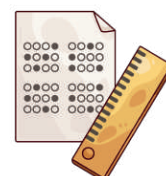
L.O	Question	Answer
6	29	A
	30	A
	31	C
	32	B
	33	B
	34	C
7	35	C
	36	B
	37	C
	38	A
	39	B
	40	A

L.O	Question	Answer
8	41	A
	42	B
	43	A
	44	A
9	45	A
	46	A
	47	B
	48	A
	49	A
	50	B

L.O	Question	Answer
10	51	B
	52	D

L.O	Question	Answer
11	53	A
	54	A
	55	A
	56	A
12	57	A
	58	C
	59	C
	60	C
13	61i	A
	61ii	B
	62	D
	63	C

L.O	Question	Answer
14	64	D
	65	C
	66	A
	67	C
	68	B
	69	D
15	70	B
	71	D
	72	B
	73	D





Answer key

L.O	Question	Answer
16	74	A
	75	B
	76	A
	77	D
	78	B
17	79	B
	80	D
	81	A
18	82	B
	83	A
	84	B
	85	A
	86	A
	87	A
19	88	C
	89	A
	90	C
	91	B
	92	A
	93	B

L.O	Question	Answer
20	94	B
	95	A
	96	D
	97	D
	98	B
	99	D

L.O	Question	Answer
21	100	B
	101	D
	102	B
	103	C
	104	A
	105	A
	106	C
22	107	B
	108	C
	109	B

L.O	Question	Answer
22	110	A
	111	D
	112	C
23	113	C
	114	B
	115	D
	116	B
	117	D
24	118	C
	119	A
25	123	B
	124	A
	125	A
	126	C
	127	B
	128	A

