

## حل مراجعة عامة وفق الهيكل الوزاري منهج ريفيل



### تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف التاسع المتقدم ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

تاريخ إضافة الملف على موقع المناهج: 22:16:07 2025-06-13

ملفات اكتب للمعلم اكتب للطالب ا اختبارات الكترونية ا اختبارات ا حلول ا عروض بوربوينت ا أوراق عمل  
منهج انجليزي ا ملخصات وتقارير ا مذكرات وبنوك ا الامتحان النهائي للمدرس

المزيد من مادة  
رياضيات:

### التواصل الاجتماعي بحسب الصف التاسع المتقدم



صفحة المناهج  
الإماراتية على  
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

المواد على تلغرام

### المزيد من الملفات بحسب الصف التاسع المتقدم والمادة رياضيات في الفصل الثالث

اختبار تجريبي الجزء الكتابي وفق الهيكل الوزاري

1

حل مراجعة امتحانية وفق الهيكل الوزاري منهج ريفيل

2

أسئلة اختبار تجريبي وفق الهيكل الوزاري منهج ريفيل بدون الحل

3

حل نموذج تدريبي للاختبار النهائي وفق الهيكل الوزاري

4

نموذج تدريبي للاختبار النهائي وفق الهيكل الوزاري

5



Math

# Grade 9 A EoT Part 1 - MCQ

School Name:



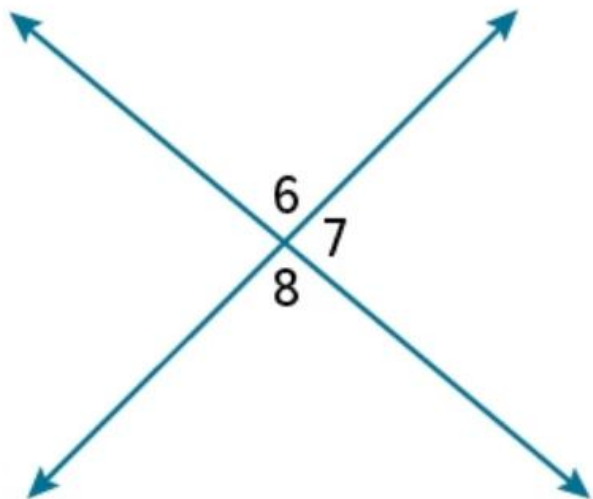
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# 1. Learning Outcome/lesson Name: Write proofs involving supplementary and complementary angles

Example/Exercise: 13-18

Find the measure of each numbered angle and name the theorem that you used to justify your work.

13.  $m\angle 6 = (2x - 21)^\circ$   $m\angle 7 = (3x - 34)^\circ$



*SOLUTION:*

$\angle 6$  and  $\angle 7$  form a linear pair.

$$m\angle 6 + m\angle 7 = 180^\circ$$

$$(2x - 21)^\circ + (3x - 34)^\circ = 180^\circ$$

$$(5x - 55)^\circ = 180^\circ$$

$$5x^\circ = 235^\circ$$

$$x = 47^\circ$$

Definition of linear pair

Supplement Theorem

Substitution Property of Equality

Substitution Property of Equality

Addition Property of Equality

Division Property of Equality

Therefore  $m\angle 6 = 2x - 21 = 2(47) - 21 = 73^\circ$  and  
 $m\angle 7 = 3x - 34 = 107^\circ$ .

$\angle 6$  and  $\angle 8$  are vertical angles.

Definition of vertical angles

$$\angle 6 \cong \angle 8$$

Vertical Angles Theorem

$$m\angle 6 = m\angle 8$$

Def. of congruent angles

$$73^\circ = m\angle 8$$

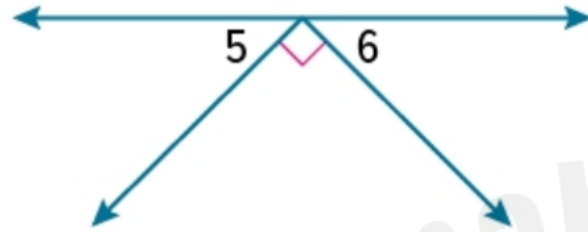
Substitution Property of Equality

*ANSWER:*

$$m\angle 6 = m\angle 8 = 73^\circ, m\angle 7 = 107^\circ$$

(Supp. Thm. and Vert.  $\angle$ s Thm.)

14.  $m\angle 5 = m\angle 6$



*SOLUTION:*

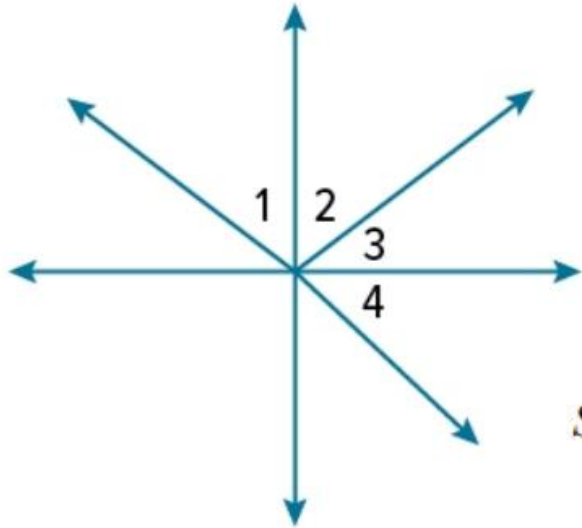
$\angle$ s 5 and 6 not only have the same measure, but when combined with a right angle, form a straight angle or supplementary angles.

$m\angle 5 = m\angle 6$	Given
$m\angle 5 + m\angle 6 + 90^\circ = 180^\circ$	Supplement Theorem
$m\angle 5 + m\angle 6 = 90^\circ$	Subtraction Property of Equality
$2m\angle 5 = 90^\circ$	Substitution Property of Equality
$m\angle 5 = 45^\circ$	Division Property of Equality
$m\angle 6 = 45^\circ$	Transitive Property of Congruence

*ANSWER:*

$m\angle 5 = m\angle 6 = 45^\circ$   
(Supp. Thm.)

15.  $\angle 2$  and  $\angle 3$  are complementary.  
 $\angle 1 \cong \angle 4$  and  $m\angle 2 = 28^\circ$ .



*SOLUTION:*

$$m\angle 2 + m\angle 3 = 90^\circ$$

$$m\angle 3 = 90^\circ - m\angle 2$$

$$m\angle 3 = 90^\circ - 28^\circ$$

$$m\angle 3 = 62^\circ$$

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180^\circ$$

$$m\angle 1 + 90^\circ + m\angle 4 = 180^\circ$$

$$m\angle 1 + m\angle 4 = 180^\circ$$

$$2m\angle 1 = 90^\circ$$

$$m\angle 1 = 45^\circ$$

$$m\angle 1 = m\angle 4 = 45^\circ$$

Definition of complementary

Subtraction Property of Equality

Substitution Property of Equality

Substitution Property of Equality

Supplement Theorem

Substitution Property of Equality

Subtraction Property of Equality

Substitution Property of Equality

Division Property of Equality

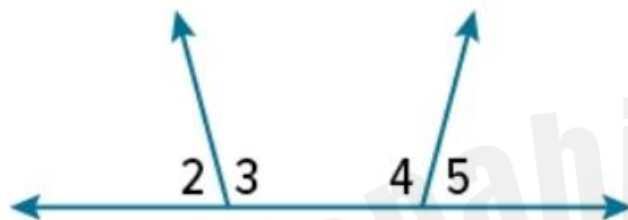
Transitive Property and definition of congruent angles

*ANSWER:*

$$m\angle 3 = 62^\circ, m\angle 1 = m\angle 4 = 45^\circ$$

(Supp. Thm.)

16.  $\angle 2$  and  $\angle 4$  and  $\angle 4$  and  $\angle 5$  are supplementary.  
 $m\angle 4 = 105^\circ$



*SOLUTION:*

Angles 2 and 5 are congruent because they both supplementary to angle 4, by the  $\cong$  Supp. Thm.

$$m\angle 4 + m\angle 5 = 180^\circ$$

Definition of supplementary

$$105^\circ + m\angle 5 = 180^\circ$$

Substitution Property of Equality

$$m\angle 5 = 75^\circ$$

Subtraction Property of Equality

$$m\angle 2 = m\angle 5 = 75^\circ$$

Transitive Property of Congruence

$$m\angle 2 + m\angle 3 = 180^\circ$$

Definition of a linear pair

$$75^\circ + m\angle 3 = 180^\circ$$

Substitution Property of Equality

$$m\angle 3 = 105^\circ$$

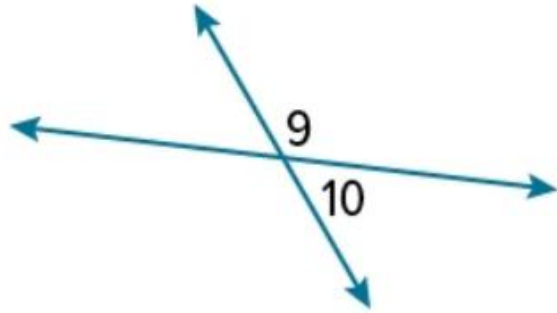
Subtraction Property of Equality

*ANSWER:*

$$m\angle 2 = 75^\circ, m\angle 3 = 105^\circ, m\angle 5 = 75^\circ$$

( $\cong$  Supp. Thm.)

17.  $m\angle 9 = (3x + 12)^\circ$   
 $m\angle 10 = (x - 24)^\circ$



*SOLUTION:*

$\angle 9$  and  $\angle 10$  form a linear pair.

$$m\angle 9 + m\angle 10 = 180^\circ$$

$$(3x + 12)^\circ + (x - 24)^\circ = 180^\circ$$

$$(4x - 12)^\circ = 180^\circ$$

$$4x^\circ = 192^\circ$$

$$x = 48^\circ$$

Definition of linear pair

Supplement Theorem

Substitution of Equality

Substitution of Equality

Addition Property of Equality

Division Property of Equality

$$m\angle 9 = 3x + 12 = 3(48) + 12 = 156^\circ$$

$$m\angle 10 = x - 24 = 48 - 24 = 24^\circ$$

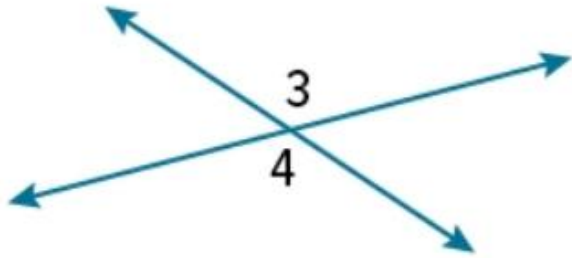
*ANSWER:*

$$m\angle 9 = 156^\circ, m\angle 10 = 24^\circ$$

(Supp. Thm.)



18.  $m\angle 3 = (2x + 23)^\circ$   
 $m\angle 4 = (5x - 112)^\circ$



*SOLUTION:*

$\angle 3$  and  $\angle 4$  are vertical angles.

$$\angle 3 \cong \angle 4$$

$$m\angle 3 = m\angle 4$$

$$(2x + 23)^\circ = 5x^\circ - 112^\circ$$

$$23^\circ = 3x^\circ - 112^\circ$$

$$135^\circ = 3x^\circ$$

$$45^\circ = x$$

Definition of vertical angles

Vertical Angles Theorem

Definition of congruent angles

Substitution of Equality

Subtraction Property of Equality

Addition Property of Equality

Division Property of Equality

$$m\angle 3 = 2x + 23 = 2(45) + 23 = 113^\circ$$

$$m\angle 3 = m\angle 4 = 113^\circ$$

*ANSWER:*

$$m\angle 3 = m\angle 4 = 113^\circ$$

(Vert.  $\angle$ s Thm.)



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## 2. Learning Outcome/lesson Name: Draw translations in the coordinate plane

Example/Exercise: 8-11

**Name the image of each point after the given translation vector.**

8.  $F(-3, 1); \langle 5, -1 \rangle$

*SOLUTION:*

The vector  $\langle 5, -1 \rangle$  means to map each point  $(x, y) \rightarrow (x + 5, y - 1)$ .

$(x, y) \rightarrow (x + 5, y - 1)$	Original mapping
$(-3, 1) \rightarrow (-3 + 5, 1 - 1)$	Substitution.
$F(-3, 1) \rightarrow F'(2, 0)$	Simplify.

*ANSWER:*

$F'(2, 0)$

9.  $Q(4, -2); \langle -2, -5 \rangle$

*SOLUTION:*

The vector  $\langle -2, -5 \rangle$  means to map each point  $(x, y) \rightarrow (x - 2, y - 5)$ .

$(x, y) \rightarrow (x - 2, y - 5)$	Original mapping
$(4, -2) \rightarrow (4 - 2, -2 - 5)$	Substitution.
$Q(4, -2) \rightarrow Q'(2, -7)$	Simplify.

*ANSWER:*

$Q'(2, -7)$

10.  $P(9, 1.5); \langle 3, -0.5 \rangle$

*SOLUTION:*

The vector  $\langle 3, -0.5 \rangle$  means to map each point  $(x, y) \rightarrow (x + 3, y - 0.5)$ .

$$(x, y) \rightarrow (x + 3, y - 0.5)$$

$$(9, 1.5) \rightarrow (9 + 3, 1.5 - 0.5)$$

$$P(9, 1.5) \rightarrow P'(12, 1)$$

Original mapping

Substitution.

Simplify.

*ANSWER:*

$$P'(12, 1)$$

11. The image of  $A(-3, -5)$  under a translation is  $A'(6, -1)$ . Find the image of  $B(3, -2)$  under the same translation.

*SOLUTION:*

Subtract the  $x$ - and  $y$ -values of  $A$  from  $A'$  to find the translation, then use this result to find the image of  $B$ .

$$x: 6 - (-3) = 9$$

$$y: -1 - (-5) = 4$$

So, the translation is  $(x, y) \rightarrow (x + 9, y + 4)$

Find  $B'$ .

$$(x, y) \rightarrow (x + 9, y + 4) \quad \text{Original mapping}$$

$$(3, -2) \rightarrow (3 + 9, -2 + 4) \quad \text{Substitution.}$$

$$B(3, -2) \rightarrow B'(12, 2) \quad \text{Simplify.}$$

*ANSWER:*

$$B'(12, 2)$$



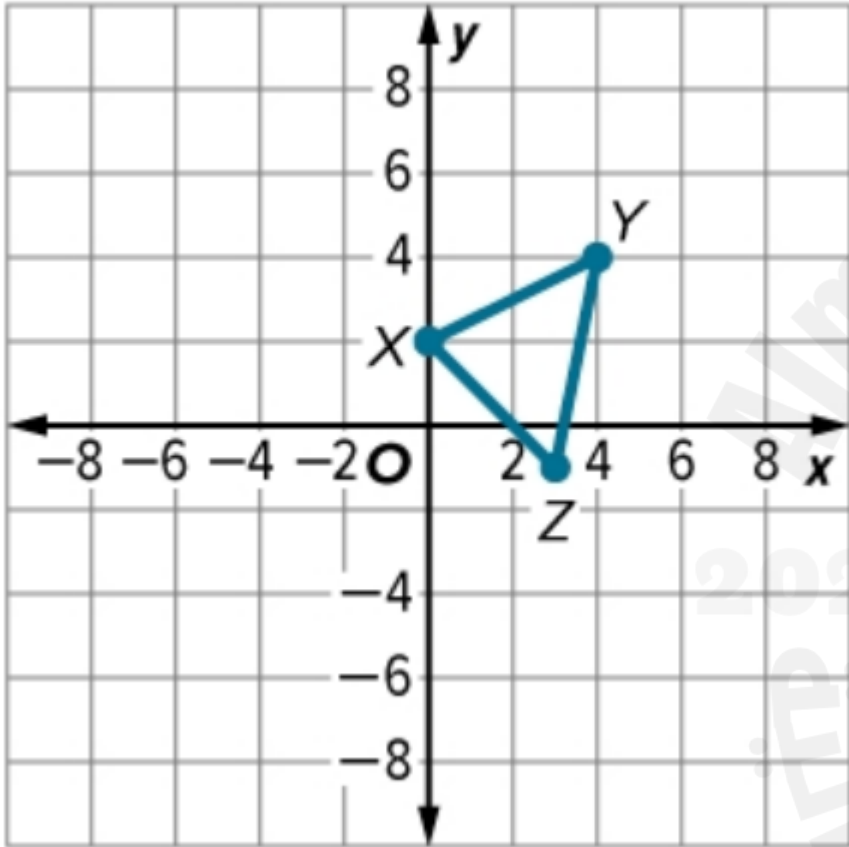
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### 3. Learning Outcome/lesson Name: Draw rotations in the coordinate plane

Example/Exercise: 1-7

1. Triangle XYZ has vertices  $X(0, 2)$ ,  $Y(4, 4)$ , and  $Z(3, -1)$ . Graph  $\triangle XYZ$  and its image after a rotation of  $180^\circ$  about  $(2, -3)$ .

Step 1: Graph  $\triangle XYZ$ .



**Step 2:** Map the center of rotation to the origin.

To map the center of rotation to the origin, translate the center of rotation along the vector  $\langle -2, 3 \rangle$ . Then translate the vertices of  $\triangle XYZ$  along the same vector.

$$(x, y) \rightarrow (x - 2, y + 3)$$

$$X(0, 2) \rightarrow (-2, 5)$$

$$Y(4, 4) \rightarrow (2, 7)$$

$$Z(3, -1) \rightarrow (1, 2)$$

**Step 3:** Rotate  $180^\circ$  about the origin.

$$(x, y) \rightarrow (-x, -y)$$

$$X(-2, 5) \rightarrow (2, -5)$$

$$Y(2, 7) \rightarrow (-2, -7)$$

$$Z(1, 2) \rightarrow (-1, -2)$$

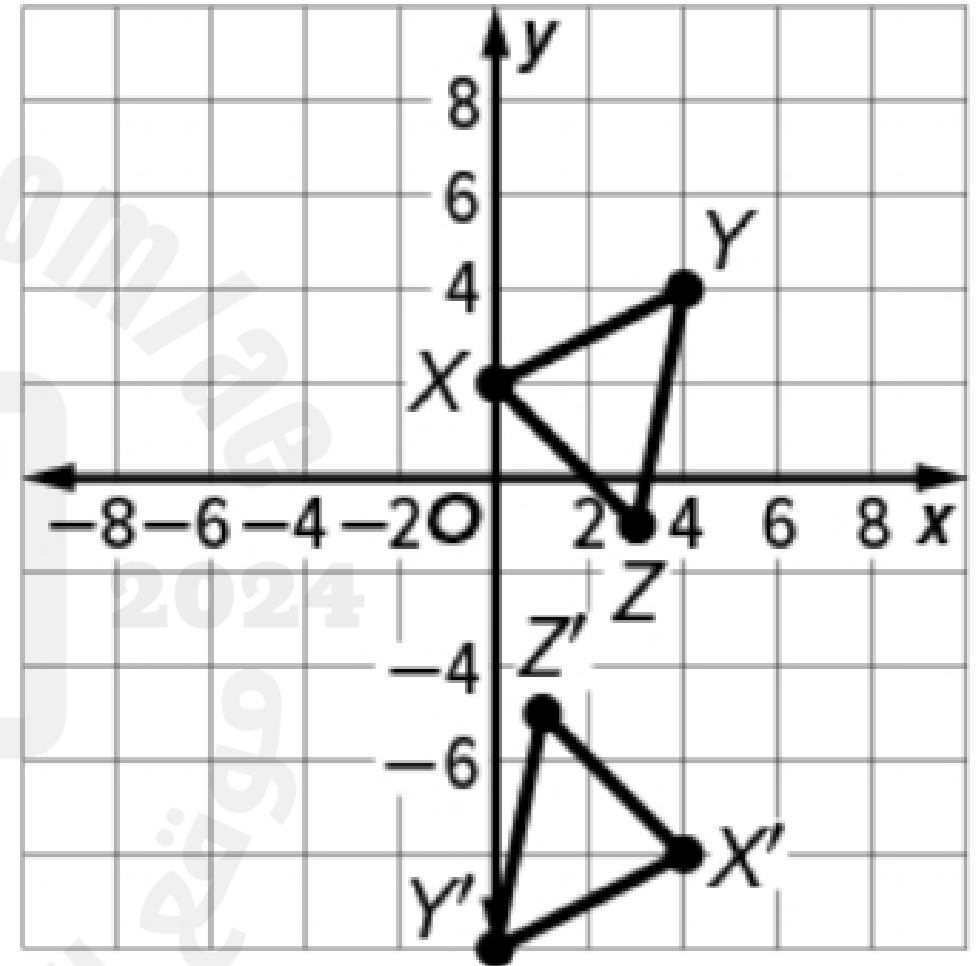
*ANSWER:*

- **Step 4:** Map the center of rotation to its original position.

To map the center of rotation to its original position, translate the center of rotation along the vector .

Then translate the vertices of the rotated triangle along the same vector.

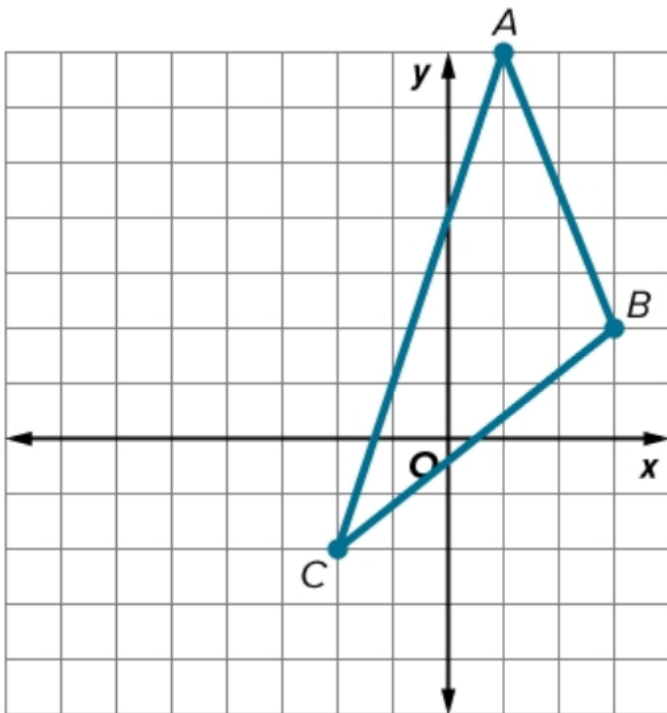
- $(x, y) \rightarrow (x + 2, y - 3)$
- $X(2, -5) \rightarrow X'(4, -8)$
- $Y(-2, -7) \rightarrow Y'(0, -10)$
- $Z(-1, -2) \rightarrow Z'(1, -5)$





2. Triangle ABC has vertices A(1, 7), B(3, 2), and C(-2, 2). Graph  $\triangle ABC$  and its image after a rotation of  $270^\circ$  counterclockwise about  $(-4, 2)$ .

Step 1: Graph  $\triangle ABC$



Step 2: Map the center of rotation to the origin.

To map the center of rotation to the origin, translate the center of rotation along the vector.

Then translate the vertices of  $\triangle ABC$  along the same vector.

$$(x, y) \rightarrow (x + 4, y - 2)$$

$$A(1, 7) \rightarrow (5, 5)$$

$$B(3, 2) \rightarrow (7, 0)$$

$$C(-2, -2) \rightarrow (2, -4)$$

Step 3: Rotate  $270^\circ$  counterclockwise about the origin.

$$(x, y) \rightarrow (y, -x)$$

$$A(5, 5) \rightarrow (5, -5)$$

$$B(7, 0) \rightarrow (0, -7)$$

$$C(2, -4) \rightarrow (-4, -2)$$

Step 4: Map the center of rotation to its original position.

To map the center of rotation to its original position, translate the center of rotation along the vector. Then translate the vertices of the rotated triangle along the same vector.

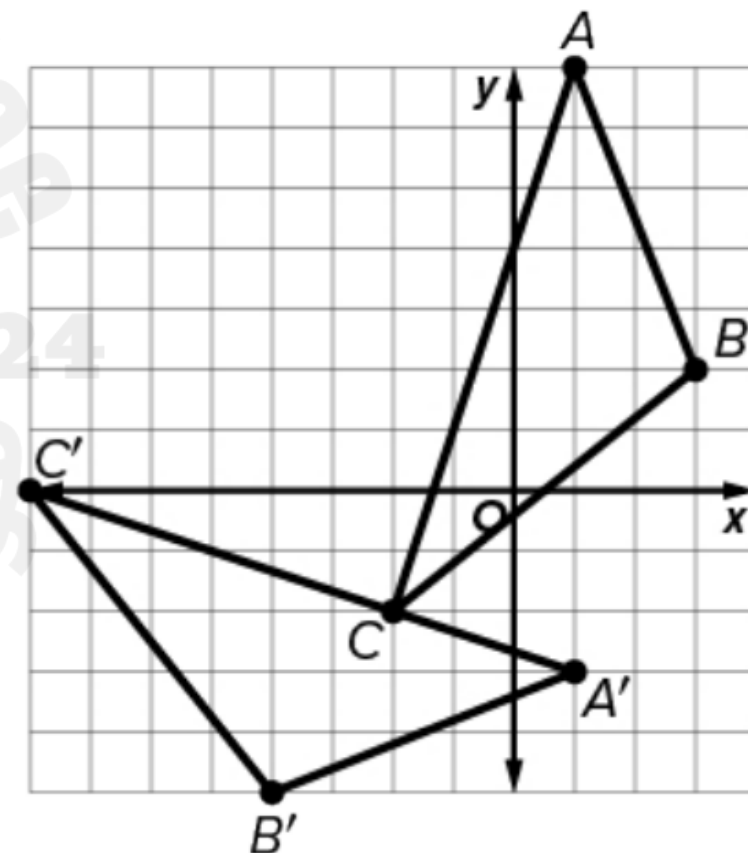
$$(x, y) \rightarrow (x - 4, y + 2)$$

$$A(5, -5) \rightarrow A'(1, -3)$$

$$B(0, -7) \rightarrow B'(-4, -5)$$

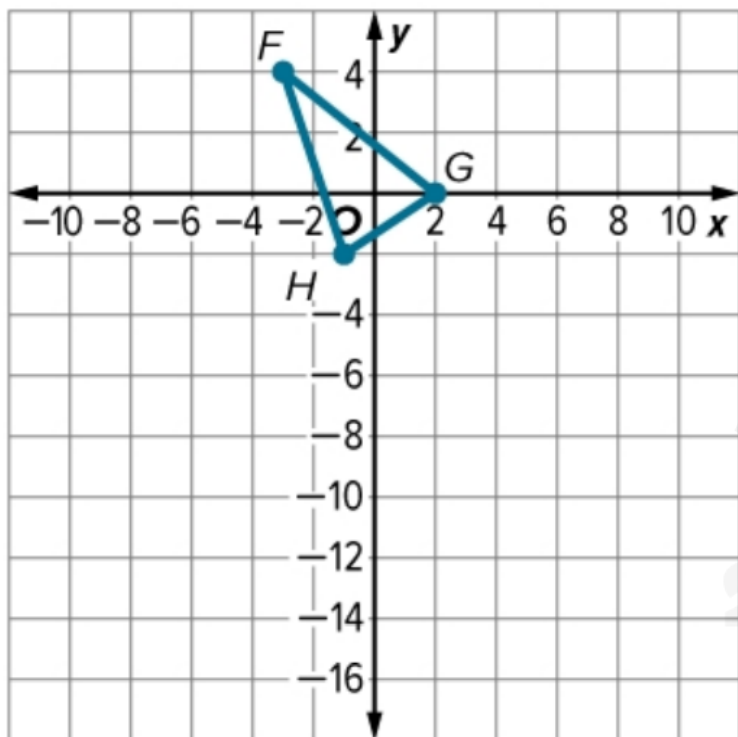
$$C(-4, -2) \rightarrow C'(-8, 0)$$

*ANSWER:*



3. Triangle FGH has vertices  $F(-3, 4)$ ,  $G(2, 0)$ , and  $H(-1, -2)$ . Graph  $\triangle FGH$  and its image after a rotation of  $180^\circ$  about  $(-3, -6)$ .

Step 1: Graph  $\triangle FGH$ .



Step 2: Map the center of rotation to the origin. To map the center of rotation to the origin, translate the center of rotation along the vector. Then translate the vertices of  $\triangle FGH$  along the same vector.

$$(x, y) \rightarrow (x + 3, y + 6)$$

$$F(-3, 4) \rightarrow (0, 10)$$

$$G(2, 0) \rightarrow (5, 6)$$

$$H(-1, -2) \rightarrow (2, 4)$$

Step 3: Rotate  $180^\circ$  about the origin.

$$(x, y) \rightarrow (-x, -y)$$

$$F(0, 10) \rightarrow (0, -10)$$

$$G(5, 6) \rightarrow (-5, -6)$$

$$H(2, 4) \rightarrow (-2, -4)$$

Step 4: Map the center of rotation to its original position.

To map the center of rotation to its original position, translate the center of rotation along the vector. Then translate the vertices of the rotated triangle along the same vector.

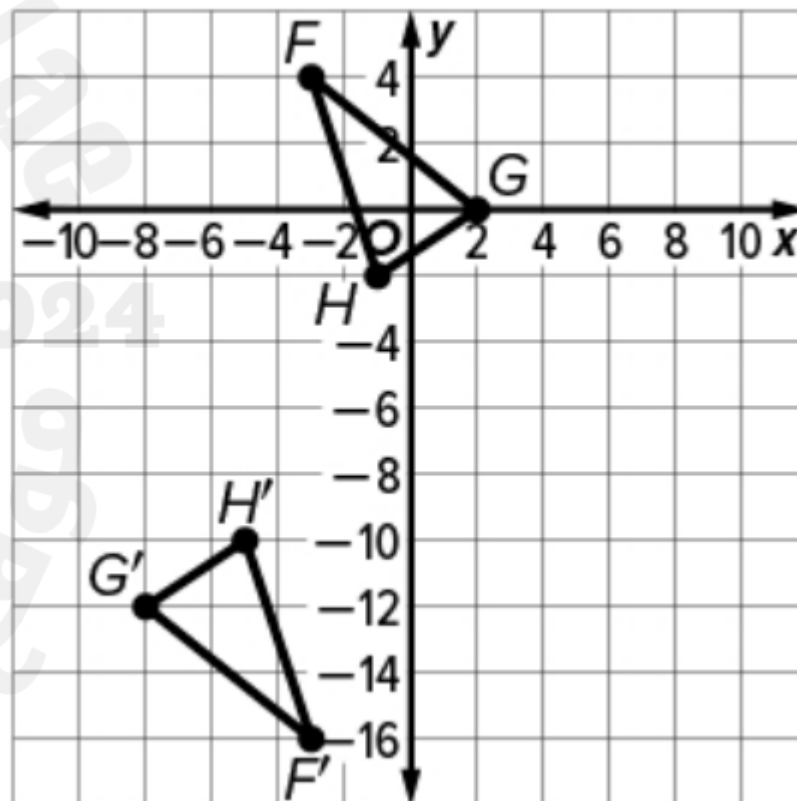
$$(x, y) \rightarrow (x - 3, y - 6)$$

$$F(0, -10) \rightarrow F'(-3, -16)$$

$$G(-5, -6) \rightarrow G'(-8, -12)$$

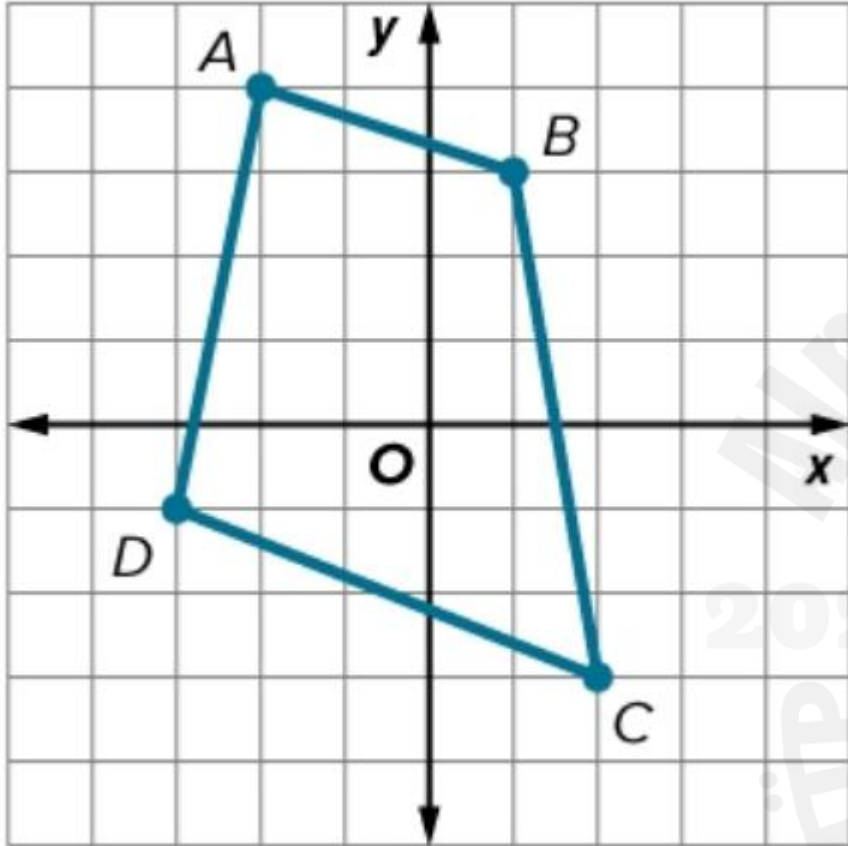
$$H(-2, -4) \rightarrow H'(-5, -10)$$

*ANSWER:*



4. Quadrilateral ABCD has vertices A(-2, 4), B(1, 3), C(2, -3), and D(-3, -1). Graph quadrilateral ABCD and its image after a rotation of  $90^\circ$  counterclockwise about (-1, 2).

Step 1: Graph quadrilateral ABCD.



Step 2: Map the center of rotation to the origin.  
To map the center of rotation to the origin, translate the center of rotation along the vector. Then translate the vertices of quadrilateral ABCD along the same vector.

$$(x, y) \rightarrow (x + 1, y - 2)$$

$$A(-2, 4) \rightarrow (-1, 2)$$

$$B(1, 3) \rightarrow (2, 1)$$

$$C(2, -3) \rightarrow (3, -5)$$

$$D(-3, -1) \rightarrow (-2, -3)$$

Step 3: Rotate  $90^\circ$  counterclockwise about the origin.

$$(x, y) \rightarrow (-y, x)$$

$$A(-1, 2) \rightarrow (-2, -1)$$

$$B(2, 1) \rightarrow (-1, 2) \quad C(3, -5) \rightarrow (5, 3)$$

$$D(-2, -3) \rightarrow (3, -2)$$

Step 4: Map the center of rotation to its original position.

To map the center of rotation to its original position, translate the center of rotation along the vector. Then translate the vertices of the rotated triangle along the same vector.

$$(x, y) \rightarrow (x - 1, y + 2)$$

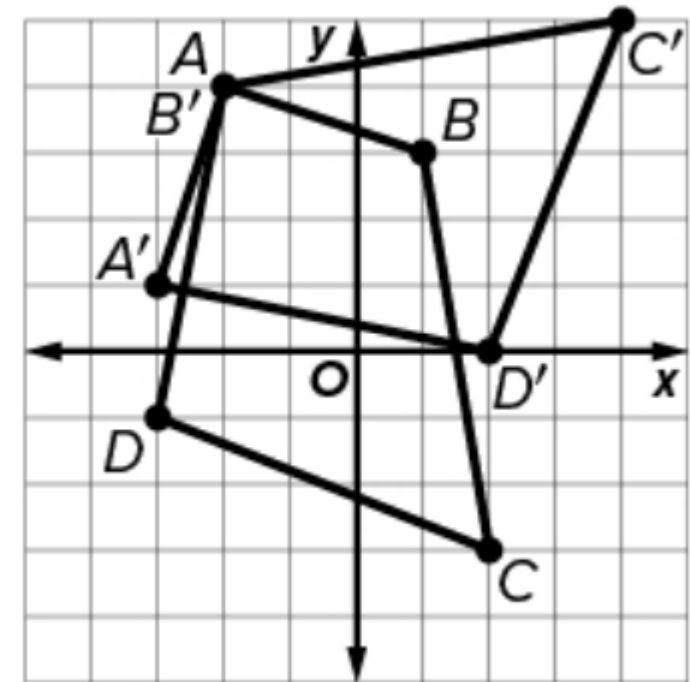
$$A(-2, -1) \rightarrow A'(-3, 1)$$

$$B(-1, 2) \rightarrow B'(-2, 4)$$

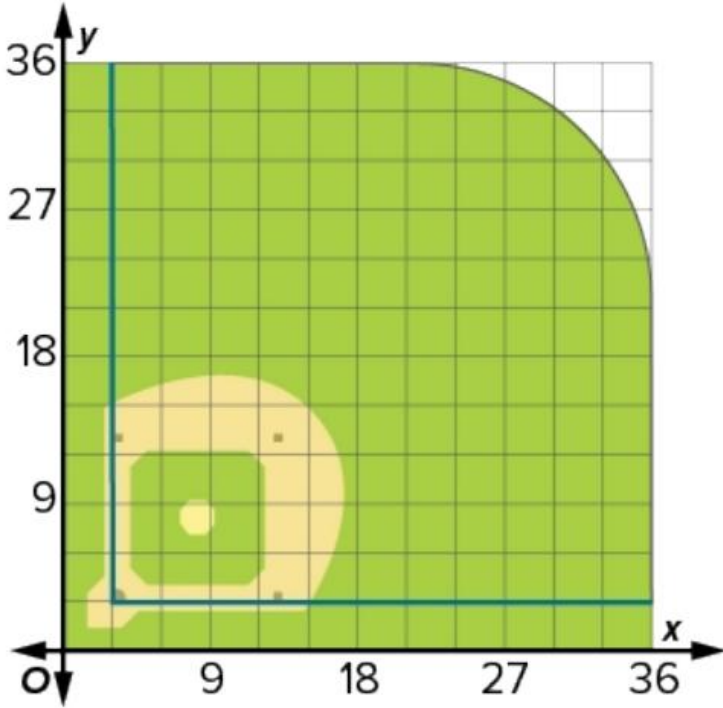
$$C(5, 3) \rightarrow C'(4, 5)$$

$$D(3, -2) \rightarrow D'(2, 0)$$

*ANSWER:*



5. BASEBALL A scale drawing of a baseball field is shown on the coordinate plane, where home plate is at (3, 3), first base is at (13, 3), second base is at (13, 13), and third base is at (3, 13). Suppose the baseball field is rotated  $270^\circ$  counterclockwise about second base, what are the coordinates of each base?



Map the center of rotation to the origin.  
To map the center of rotation to the origin, translate the center of rotation along the vector. Then translate the bases along the same vector.

$$(x, y) \rightarrow (x - 13, y - 13)$$

$$\text{home plate: } (3, 3) \rightarrow (-10, -10)$$

$$\text{first base: } (13, 3) \rightarrow (0, -10)$$

$$\text{second base: } (13, 13) \rightarrow (0, 0)$$

$$\text{third base: } (3, 13) \rightarrow (-10, 0)$$

Rotate  $270^\circ$  counterclockwise about the origin.

$$(x, y) \rightarrow (y, -x)$$

$$\text{home plate: } (-10, -10) \rightarrow (-10, 10)$$

$$\text{first base: } (0, -10) \rightarrow (-10, 0)$$

$$\text{second base: } (0, 0) \rightarrow (0, 0)$$

$$\text{third base: } (-10, 0) \rightarrow (0, 10)$$

Map the center of rotation to its original position.  
To map the center of rotation to its original position, translate the center of rotation along the vector. Then translate the vertices of the rotated triangle along the same vector.

$$(x, y) \rightarrow (x + 13, y + 13)$$

$$\text{home plate:}$$

$$(-10, 10) \rightarrow (3, 23)$$

$$\text{first base: } (-$$

$$10, 0) \rightarrow (3, 13)$$

$$\text{second base:}$$

$$(0, 0) \rightarrow (13, 13)$$

$$\text{third base:}$$

$$(0, 10) \rightarrow (13, 23)$$

ANSWER:

$$\text{home plate: } (3, 23),$$

$$\text{first base: } (3, 13),$$

$$\text{second base: } (13, 13),$$

$$\text{third base: } (13, 23)$$

6. Point Q with coordinates  $(4, -7)$  is rotated  $270^\circ$  clockwise about  $(5, 1)$ . What are the coordinates of its image?

Map the center of rotation to the origin.

To map the center of rotation to the origin, translate the center of rotation along the vector.

Then translate the point along the same vector.

$$(x, y) \rightarrow (x - 5, y - 1)$$

$$Q(4, -7) \rightarrow (-1, -8)$$

Rotate  $270^\circ$  clockwise about the origin which is the same as  $90^\circ$  counterclockwise.

$$(x, y) \rightarrow (-y, x)$$

$$Q(-1, -8) \rightarrow (8, -1)$$

Map the center of rotation to its original position.

To map the center of rotation to its original position, translate the center of rotation along the vector.

Then translate the point along the same vector.

$$(x, y) \rightarrow (x + 5, y + 1)$$

$$Q(8, -1) \rightarrow Q'(13, 0)$$

ANSWER:  $Q'(13, 0)$

7. Parallelogram JKLM has vertices J(2, 1), K(7, 1), L(6, -3), and M(1, -3). What are the coordinates of the image of K if the parallelogram is rotated  $270^\circ$  counterclockwise about  $(-2, -1)$ ?

Map the center of rotation to the origin.

To map the center of rotation to the origin, translate the center of rotation along the vector.

Then translate K along the same vector.

$$(x, y) \rightarrow (x + 2, y + 1)$$

$$K(7, 1) \rightarrow (9, 2)$$

Rotate  $270^\circ$  counterclockwise about the origin.

$$(x, y) \rightarrow (y, -x)$$

$$K(9, 2) \rightarrow (2, -9)$$

Map the center of rotation to its original position.

To map the center of rotation to its original position, translate the center of rotation along the vector .

Then translate K along the same vector.

$$(x, y) \rightarrow (x - 2, y - 1)$$

$$K(2, -9) \rightarrow (0, -10)$$

ANSWER:  $K'(0, -10)$



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## 4. Learning Outcome/lesson Name: Identify line and rotational symmetries in two- dimensional figures

Example/Exercise: 9-14



9. RECYCLING A waste management company offers recycling programs for its clients. Recycling is denoted by the symbol shown. Does the recycling symbol have rotational symmetry? Explain.



ANSWER: Yes; the symbol can map onto itself with a rotation that is less than  $360^\circ$ .

10. VACATION Annabel and her family went to a beach for vacation. While she was on the beach, Annabel collected seashells. Does the seashell shown have rotational symmetry? Explain.



ANSWER: No; no rotation less than  $360^\circ$  maps the seashell onto itself.

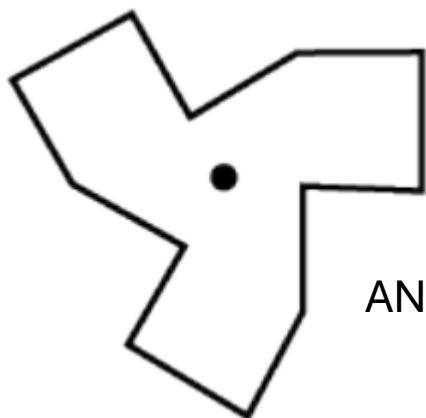


**Determine whether each figure has rotational symmetry. If so, locate the center of symmetry, and state the order and magnitude of symmetry**

11.



SOLUTION: Yes; the figure can be rotated onto itself 3 times from  $0^\circ$  to  $360^\circ$ , so the order is 3. The magnitude is  $360^\circ \div 3 = 120^\circ$ .  
The center of symmetry is drawn below.

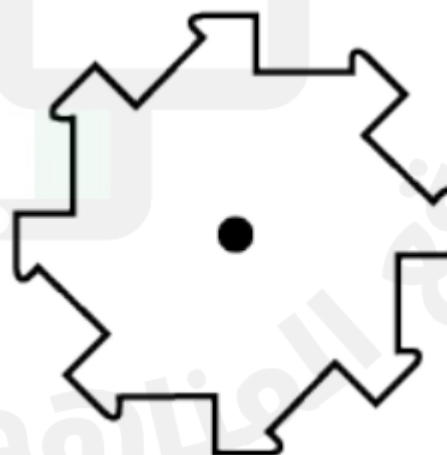


ANSWER: yes; 3;  $120^\circ$

12.



SOLUTION: Yes; the figure can be rotated onto itself 8 times from  $0^\circ$  to  $360^\circ$ , so the order is 8. The magnitude is  $360^\circ \div 8 = 45^\circ$ .  
The center of symmetry is drawn below.



ANSWER: yes; 8;  $45^\circ$

13.



SOLUTION: Yes; the figure can be rotated onto itself 2 times from  $0^\circ$  to  $360^\circ$ , so the order is 2. The magnitude is  $360^\circ \div 2 = 180^\circ$ .

The center of symmetry is drawn below.



ANSWER: yes; 2;  $180^\circ$

14.



SOLUTION: No; the figure cannot be rotated onto itself.

ANSWER: no



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## 5. Learning Outcome/lesson Name: Use properties of equilateral triangles

Example/Exercise: 6-15

6. Find SR, ST, RT,  $m\angle TRS$ , and  $m\angle RST$ . Round to the nearest tenth, if necessary.

SOLUTION:

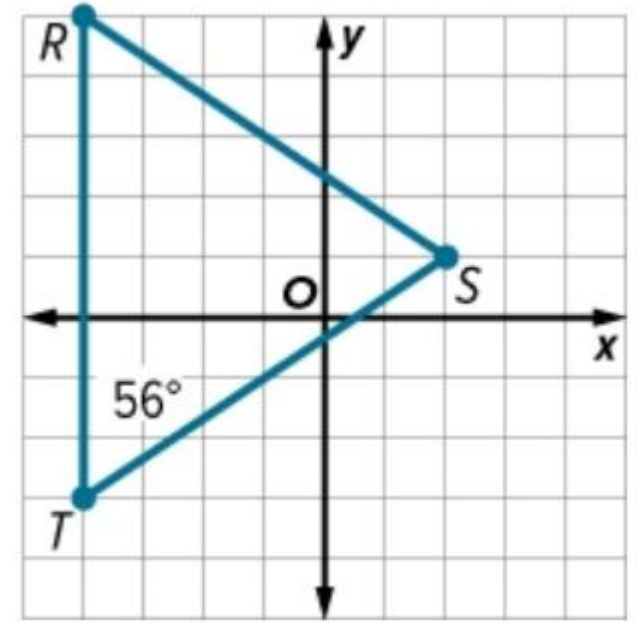
The coordinates of  $\triangle RST$  are  $R(-4, 5)$ ,  $S(2, 1)$ , and  $T(-4, -3)$ .

$$\begin{aligned} SR &= \sqrt{(-4-2)^2 + (5-1)^2} \\ &= \sqrt{36+16} \\ &= \sqrt{52} \text{ or about } 7.2 \text{ units} \end{aligned}$$

$$\begin{aligned} ST &= \sqrt{(-4-2)^2 + (-3-1)^2} \\ &= \sqrt{36+16} \\ &= \sqrt{52} \text{ or about } 7.2 \text{ units} \end{aligned}$$

$$\begin{aligned} RT &= \sqrt{[-4-(-4)]^2 + (-3-5)^2} \\ &= \sqrt{0+64} \\ &= \sqrt{64} \text{ or } 8 \text{ units} \end{aligned}$$

So,  $\triangle RST$  is an isosceles triangle with  $\overline{SR} \cong \overline{ST}$ . Because,  $\overline{SR} \cong \overline{ST}$ , we know that  $\angle R \cong \angle T$  by the Isosceles Triangle Theorem. We are given that  $m\angle T = 56^\circ$ , so by definition of congruent, we know  $m\angle R = 56^\circ$ .



$$m\angle R + m\angle S + m\angle T = 180^\circ$$

$$56 + m\angle S + 56 = 180^\circ$$

$$m\angle S + 112 = 180^\circ$$

$$m\angle S = 68^\circ$$

Triangle Angle – Sum Theorem

Substitution

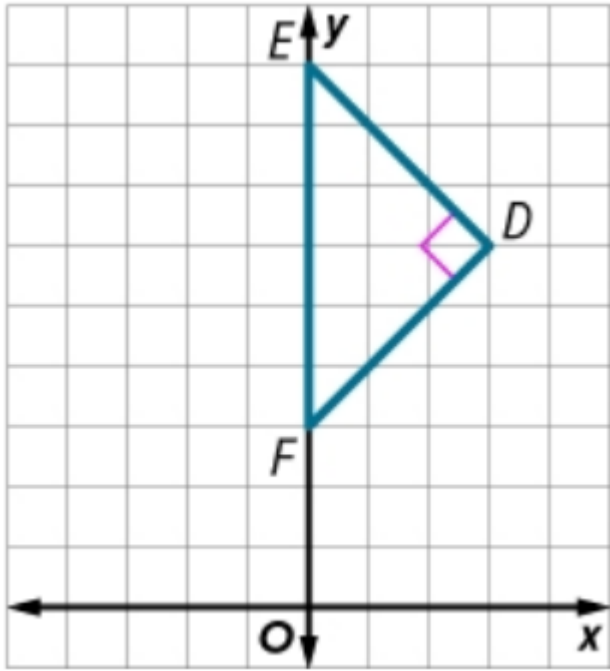
Simplify.

Subtraction Property of Equality

ANSWER:

SR = 7.2 units; ST = 7.2 units; RT = 8 units;  
 $m\angle TRS = 56^\circ$ ;  $m\angle RST = 68^\circ$

7. Find the measures of  $\angle DEF$  and  $\angle EFD$ . Round to the nearest tenth, if necessary.



SOLUTION:

The coordinates of  $\triangle DEF$  are  $D(3, 6)$ ,  $E(0, 9)$ , and  $F(0, 3)$ .

$$\begin{aligned} DE &= \sqrt{(9-6)^2 + (0-3)^2} \\ &= \sqrt{9+9} \\ &= \sqrt{18} \text{ or } 2\sqrt{2} \text{ units} \end{aligned}$$

$$\begin{aligned} DF &= \sqrt{(6-3)^2 + (3-0)^2} \\ &= \sqrt{9+9} \\ &= \sqrt{18} \text{ or } 2\sqrt{2} \text{ units} \end{aligned}$$

So,  $\triangle DEF$  is an isosceles right triangle with  $\overline{DE} \cong \overline{DF}$ .

Because,  $\overline{DE} \cong \overline{DF}$ , we know that

$\angle DEF \cong \angle EFD$  by the Isosceles Triangle Theorem.

Since  $m\angle EDF = 90^\circ$ , then  $m\angle DEF = 45^\circ$  and  $m\angle EFD = 45^\circ$ .

ANSWER:

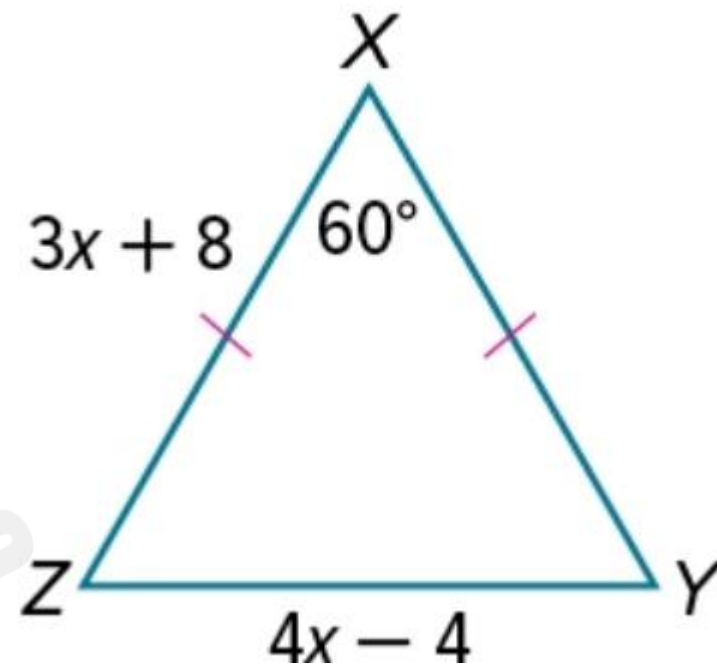
$m\angle DEF = 45^\circ$  and  $m\angle EFD = 45^\circ$

8. Find the value of  $x$ .

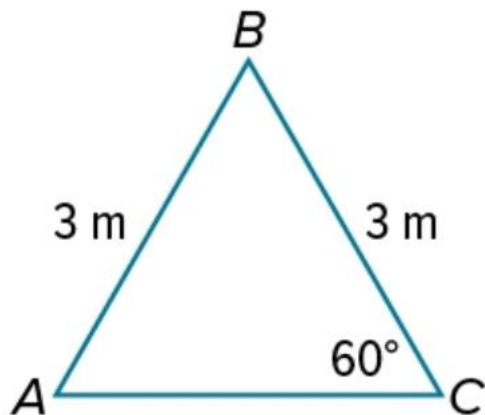
SOLUTION: Since the sides opposite angles  $Z$  and  $Y$  are congruent, then we know each angle of the triangle measures  $60^\circ$  by the Triangle Angle -Sum Theorem. Therefore, the triangle is an equilateral triangle by Corollary 5.3.

$$\begin{aligned} 3x + 8 &= 4x - 4 && \text{Corollary 5.3; definition of equilateral } \triangle \\ 8 &= x - 4 && \text{Subtract } 3x \text{ from each side.} \\ 12 &= x && \text{Add 4 to each side.} \end{aligned}$$

ANSWER: 12



9. Find  $m\angle B$  and AC.



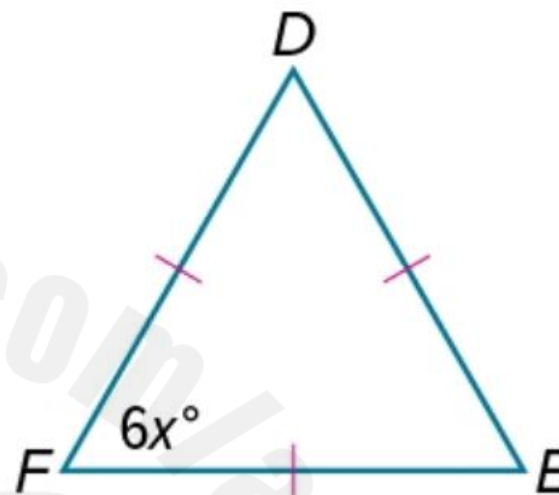
SOLUTION: Because  $AB = BC$ ,  $\overline{AB} \cong \overline{BC}$ .  
By the Isosceles Triangle Theorem, base angles A and C are congruent, so  $m\angle A = m\angle C$ .

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180^\circ && \text{Triangle Angle-Sum Theorem} \\ 60 + m\angle B + 60 &= 180^\circ && \text{Isosceles Triangle Theorem} \\ m\angle B &= 60^\circ && \text{Solve.} \end{aligned}$$

Because each angle of the triangle measures  $60^\circ$  by the Triangle Angle-Sum Theorem, the triangle is equilateral by Corollary 5.3.  
 $AB = BC = AC = 3 \text{ m}$ .

ANSWER:  $60^\circ$ ; 3 m

10. Find the value of x.

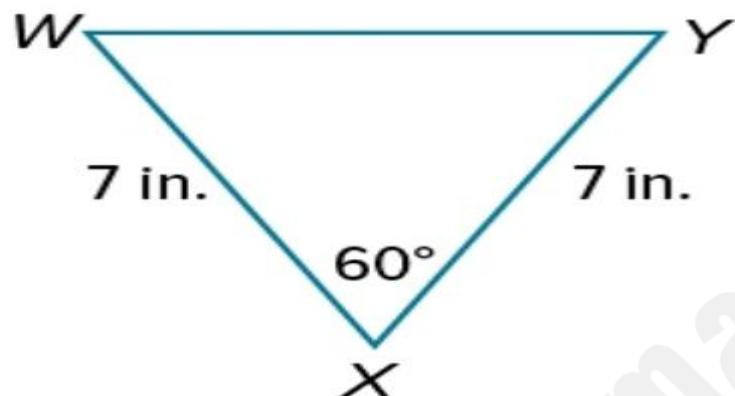


SOLUTION: Since all three sides are congruent,  $\triangle DEF$  is an equilateral triangle by definition. By Corollary 5.4, we know that each angle measures  $60^\circ$ .

$$\begin{aligned} 6x &= 60 && \text{Corollary 5.4; definition of equilateral } \triangle \\ x &= 10 && \text{Solve.} \end{aligned}$$

ANSWER: 10

11. Find  $m\angle Y$  and  $WY$ .



*SOLUTION:*

Because  $WX = XY$ ,  $\overline{WX} \cong \overline{XY}$ . By the Isosceles Triangle Theorem, base angles  $W$  and  $Y$  are congruent, so  $m\angle W = m\angle Y$ .

Use the Triangle Angle-Sum Theorem to write and solve an equation to find  $m\angle Y$ .

$$m\angle W + m\angle X + m\angle Y = 180^\circ \quad \text{Triangle Angle-Sum Theorem}$$

$$60 + 2m\angle Y = 180^\circ \quad \text{Substitute.}$$

$$2m\angle Y = 120^\circ \quad \text{Subtract 60 from each side.}$$

$$m\angle Y = 60^\circ \quad \text{Divide each side by 2.}$$

Because each angle of the triangle measures  $60^\circ$  by the Triangle Angle-Sum Theorem, the triangle is equilateral by Corollary 5.3.

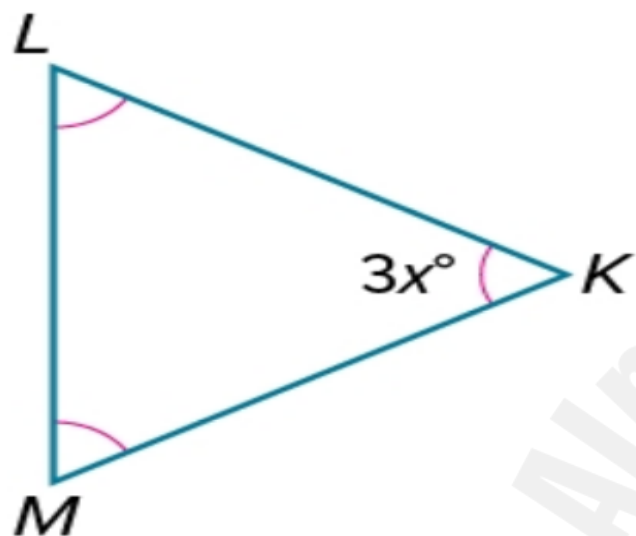
$$WX = XY = WY = 7 \text{ in.}$$

*ANSWER:*

$60^\circ$ ;  $7 \text{ in.}$



12. Find the value of  $x$ .



*SOLUTION:*

Since all three angles are congruent,  $\triangle MLK$  is an equiangular triangle by definition. By Corollary 5.3, we know that  $\triangle MLK$  is equilateral.

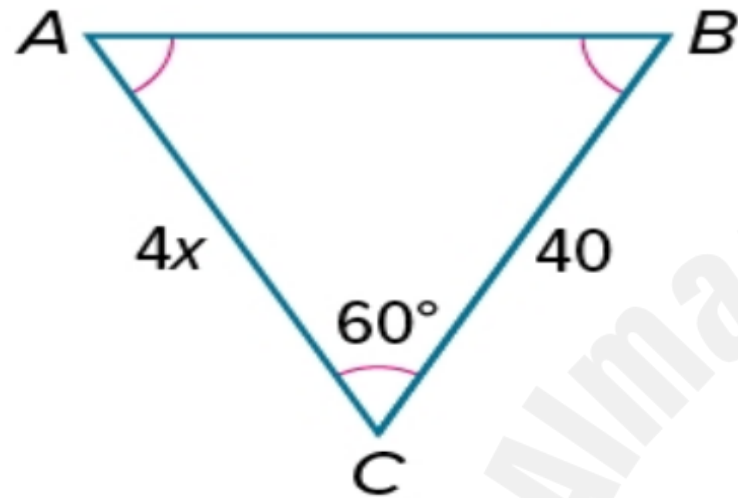
$$3x = 60 \quad \text{Corollary 5.4; definition of equilateral } \triangle$$

$$x = 20 \quad \text{Solve.}$$

*ANSWER:*

20

13. Find the value of  $x$ .



*SOLUTION:*

Since all three angles are congruent,  $\triangle ABC$  is an equiangular triangle by definition. By Corollary 5.3, we know that  $\triangle ABC$  is equilateral.

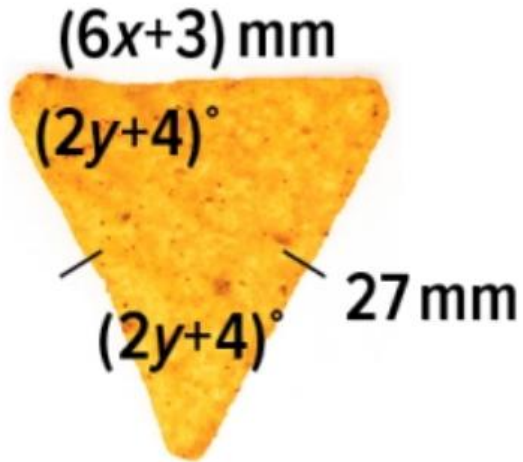
$$\begin{aligned} 4x &= 40 && \text{Corollary 5.4; definition of equilateral } \triangle \\ x &= 10 && \text{Solve.} \end{aligned}$$

*ANSWER:*

10

Find the value of each variable.

14. CHIPS Some tortilla chips can be modeled by a triangle.



- a. Solve for  $x$ .
- b. Solve for  $y$ .

**a.**

$$6x + 3 = 27$$

$$6x = 24$$

$$x = 4$$

Definition of equilateral  $\triangle$

Subtract 3 from each side.

Divide each side by 6.

**b.**

$$2y + 4 = 60$$

$$2y = 56$$

$$y = 28$$

Corollary 5.4

Subtract 4 from each side.

Divide each side by 2.

Since two sides of the chip are congruent, this chip is an isosceles triangle and by definition, its base angles are congruent.

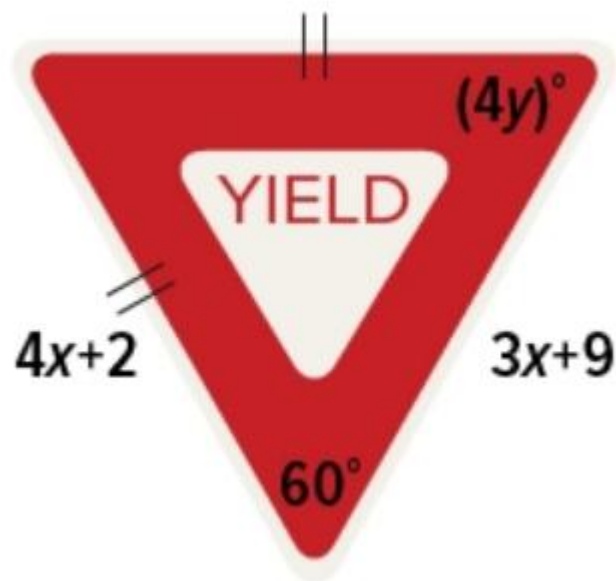
Since one of the base angles has a measure that is equal to the third angle of the triangle, then all three angles have the same measure. Since this chip is equiangular, then by Corollary 5.3, the chip is equilateral. We can use this information to find the values of  $x$  and  $y$ .

ANSWER:

a. 4

b. 28

15. SIGNS Yield signs notify drivers to slow down and allow oncoming vehicles to proceed first.



a. Solve for x.

b. Solve for y.

a.

$$4x + 2 = 3x + 9$$

$$x = 7$$

Definition of equilateral  $\triangle$

Solve.

b.

$$4y = 60$$

$$y = 15$$

Corollary 5.4

Divide each side by 4.

SOLUTION: Since two sides of the sign are congruent, this sign is an isosceles triangle and by definition, its base angles are congruent.

Since one of the base angles has a measure of  $60^\circ$ , then both base angles have a measure of  $60^\circ$ . By the Triangle Angle-Sum Theorem, we know the third angle also measures  $60^\circ$ .

By Corollary 5.3, the sign is an equilateral triangle.

We can use this information to find the values of x and y.

ANSWER:

a. 7

b. 15



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## 6. Learning Outcome/lesson Name: Draw reflections in the coordinate plane

Example/Exercise: 5-9

5. Determine the coordinates of  $S(-7, 1)$  after a reflection in the line  $y = 3$ .

SOLUTION: To reflect a point in the line  $y = 3$ , a horizontal line that is not the  $x$ -axis, the  $x$ -coordinates of the image remain the same as the preimage.

The distance from a point in the preimage to the line of reflection is the same as the distance from the corresponding point in the image to the line of reflection.

$$S(-7, 1) \rightarrow S'(-7, 5)$$

6. Determine the coordinates of  $Q(6, -4)$  after a reflection in the line  $x = 2$ .

SOLUTION: To reflect a point in the line  $x = 2$ , a vertical line that is not the  $y$ -axis, the  $y$ -coordinates of the image remain the same as the preimage. The distance from a point in the preimage to the line of reflection is the same as the distance from the corresponding point in the image to the line of reflection.

$$Q(6, -4) \rightarrow Q'(-2, -4)$$

7. BANNERS Fiona is making a banner in the shape of a triangle for a school project. She graphs the banner on a coordinate plane with vertices at  $P(0, 4)$ ,  $Q(2, 8)$ , and  $R(-3, 6)$ .

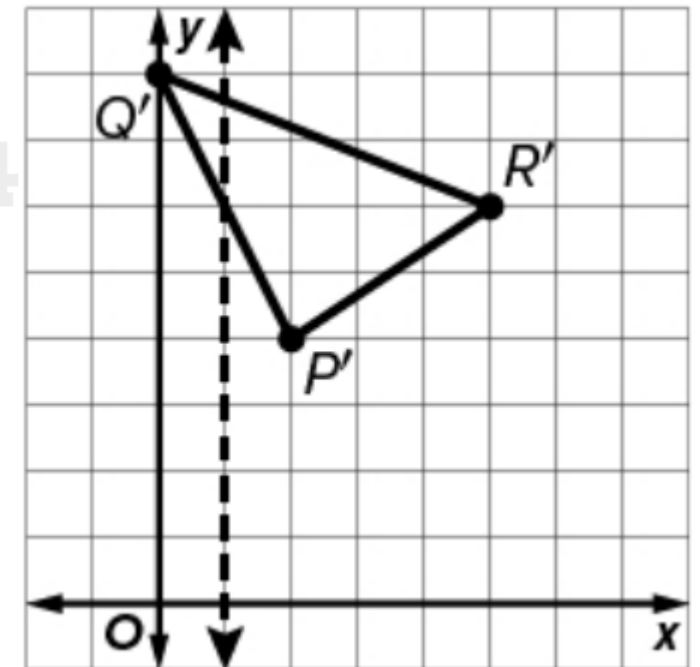
She wants to reflect the banner over the line  $x = 1$ .

Draw the image of the banner reflected in the line  $x = 1$ .

SOLUTION: To reflect a point in the line  $x = 1$ , a vertical line that is not the  $y$ -axis, the  $y$ -coordinates of the image remain the same as the preimage.

The distance from a point in the preimage to the line of reflection is the same as the distance from the corresponding point in the image to the line of reflection.

$$\begin{aligned} P(0, 4) &\rightarrow P'(2, 4) \\ Q(2, 8) &\rightarrow Q'(0, 8) \\ R(-3, 6) &\rightarrow R'(5, 6) \end{aligned}$$



8. SANDBOX Aliyah is drawing the top view of a square sandbox on a coordinate plane with vertices at  $D(1, 1)$ ,  $E(1, 6)$ ,  $F(6, 6)$ , and  $G(6, 1)$ .

She wants to change the location of the sandbox so that it is in the shade. She reflects the sandbox in the line  $x = 1$ .

Find the coordinates of the image of the sandbox.

SOLUTION:

To reflect a point in the line  $x = 1$ , a vertical line that is not the  $y$ -axis, the  $y$ -coordinates of the image remain the same as the preimage.

The distance from a point in the preimage to the line of reflection is the same as the distance from the corresponding point in the image to the line of reflection.

$$D(1, 1) \rightarrow D'(1, 1)$$

$$E(1, 6) \rightarrow E'(1, 6)$$

$$F(6, 6) \rightarrow F'(-4, 6)$$

$$G(6, 1) \rightarrow G'(-4, 1)$$

9. Determine the coordinates of  $W(-7, 4)$  after a reflection in the line  $y = 9$ .

SOLUTION:

To reflect a point in the line  $y = 9$ , a horizontal line that is not the  $x$ -axis, the  $x$ -coordinates of the image remain the same as the preimage.

The distance from a point in the preimage to the line of reflection is the same as the distance from the corresponding point in the image to the line of reflection.

$$W(-7, 4) \rightarrow W'(-7, 14)$$



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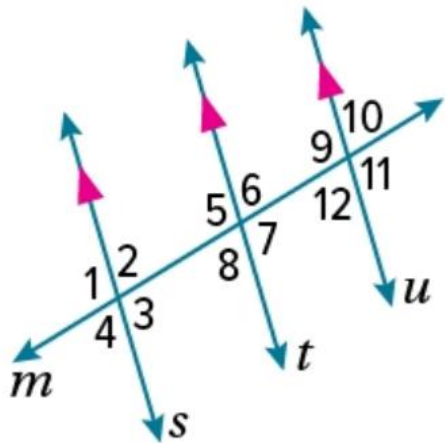
## 7. Learning Outcome/lesson

Name: Name angle pairs  
formed by two parallel lines  
cut by two transversals

Example/Exercise: 21-26



In the figure,  $m\angle 7 = 100^\circ$ .  
Find the measure of each angle.



21.  $\angle 9$

SOLUTION:

$$\begin{aligned}\angle 9 &\cong \angle 7 && \text{Alternate Interior Angles Theorem} \\ m\angle 9 &= m\angle 7 && \text{Definition of congruent angles} \\ m\angle 9 &= 100^\circ && \text{Substitution}\end{aligned}$$

The measure of  $\angle 9$  is  $100^\circ$ .

22.  $\angle 6$

SOLUTION:

$$\begin{aligned}m\angle 6 + m\angle 7 &= 180^\circ && \text{Consecutive Interior Angles Theorem} \\ m\angle 6 + 100^\circ &= 180^\circ && \text{Substitution} \\ m\angle 6 &= 80^\circ && \text{Subtract } 100^\circ \text{ from each side.}\end{aligned}$$

The measure of  $\angle 6$  is  $80^\circ$ .

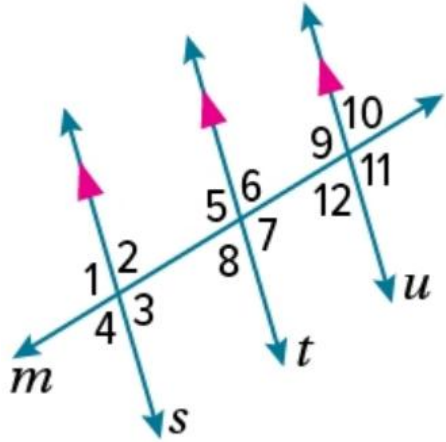
23.  $\angle 8$

SOLUTION:

$$\begin{aligned}m\angle 8 + m\angle 7 &= 180^\circ && \text{Consecutive Interior Angles Theorem} \\ m\angle 8 + 100^\circ &= 180^\circ && \text{Substitution} \\ m\angle 8 &= 80^\circ && \text{Subtract } 100^\circ \text{ from each side.}\end{aligned}$$

The measure of  $\angle 8$  is  $80^\circ$ .

In the figure,  $m \angle 7 = 100^\circ$ .  
Find the measure of each angle.



25.  $\angle 5$

SOLUTION:

$$\begin{aligned}\angle 5 &\cong \angle 7 && \text{Vertical Angles Theorem} \\ m\angle 5 &= m\angle 7 && \text{Definition of congruent angles} \\ m\angle 5 &= 100^\circ && \text{Substitution}\end{aligned}$$

The measure of  $\angle 5$  is  $100^\circ$ .

24.  $\angle 2$

SOLUTION:

$$\begin{aligned}\angle 3 &\cong \angle 7 && \text{Corresponding Angles Theorem} \\ m\angle 3 &= m\angle 7 && \text{Definition of congruent angles} \\ m\angle 3 &= 100^\circ && \text{Substitution} \\ m\angle 2 + m\angle 3 &= 180^\circ && \text{Consecutive Interior Angles Theorem} \\ m\angle 2 + 100^\circ &= 180^\circ && \text{Substitution} \\ m\angle 2 &= 80^\circ && \text{Subtract } 100^\circ \text{ from each side.}\end{aligned}$$

The measure of  $\angle 2$  is  $80^\circ$ .

26.  $\angle 11$

SOLUTION:

$$\begin{aligned}\angle 11 &\cong \angle 7 && \text{Corresponding Angles Theorem} \\ m\angle 11 &= m\angle 7 && \text{Definition of congruent angles} \\ m\angle 11 &= 100^\circ && \text{Substitution}\end{aligned}$$

The measure of  $\angle 11$  is  $100^\circ$ .



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## 8. Learning Outcome/lesson Name: Use the slope to identify parallel and perpendicular lines

Example/Exercise: 22-25

## 12-8 Slope and Equations of Lines

22. passes through  $(-7, -4)$ ,

perpendicular to  $y = \frac{1}{2}x + 9$

SOLUTION: The slope of  $y = \frac{1}{2}x + 9$  is  $\frac{1}{2}$

so the slope of the line perpendicular to it is  $-2$ .

$$y = mx + b$$

Slope – intercept form

$$-4 = -2(-7) + b$$

$m = -2$  and  $(x, y) = (-7, -4)$

$$-4 = 14 + b$$

Simplify.

$$-18 = b$$

Subtract 14 from each side.

So, the equation is  $y = -2x - 18$ .

23. passes through  $(-1, -10)$ ,  
parallel to  $y = 7$

SOLUTION: The slope of  $y = 7$  is 0, so the slope of the line parallel to it is 0.

$$y = mx + b$$

Slope – intercept form

$$-10 = 0(-1) + b$$

$m = 0$  and  $(x, y) = (-1, -10)$

$$-10 = b$$

Simplify.

So, the equation is  $y = -10$ .

24. passes through (6, 2),

parallel to  $y = -\frac{2}{3}x + 1$

SOLUTION: The slope of  $y = -\frac{2}{3}x + 1$  is  $-\frac{2}{3}$ ,

so the slope of the line parallel to it is  $-\frac{2}{3}$ ,

$$y = mx + b$$

Slope – intercept form

$$2 = -\frac{2}{3}(6) + b \quad m = -\frac{2}{3} \text{ and } (x, y) = (6, 2)$$

$$2 = -4 + b$$

Simplify.

$$6 = b$$

Add 4 to each side.

So, the equation is  $y = -\frac{2}{3}x + 6$ .

25. passes through (-2, 2), perpendicular to  $y = -5x - 8$

SOLUTION: The slope of  $y = -5x - 8$  is  $-5$ , so the slope of

the line perpendicular to it is  $\frac{1}{5}$ .

$$y = mx + b$$

Slope – intercept form

$$2 = \frac{1}{5}(-2) + b \quad m = \frac{1}{5} \text{ and } (x, y) = (-2, 2)$$

$$2 = -\frac{2}{5} + b$$

Simplify.

$$\frac{12}{5} = b$$

Add  $\frac{2}{5}$  to each side.

So, the equation is  $y = \frac{1}{5}x + \frac{12}{5}$ .



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868,  
869

## 9. Learning Outcome/lesson Name: Use the AAS Postulate to test for triangle congruence

Example/Exercise: EX3, 10-15

### Example 3 Use AAS to Prove Triangles Congruent

Choose the correct statements and reasons to complete the flow proof.

Given:  $\overline{RQ} \cong \overline{ST}$  and  $\overline{RQ} \parallel \overline{ST}$

Prove:  $\triangle RUQ \cong \triangle TUS$



$\overline{RQ} \parallel \overline{ST}$

Given

$\angle RQS \cong \angle TSQ$

Alternate Interior Angles Theorem

$\overline{RQ} \cong \overline{ST}$

Given

$\triangle RUQ \cong \triangle TUS$

AAS

$\angle RUQ \cong \angle TUS$

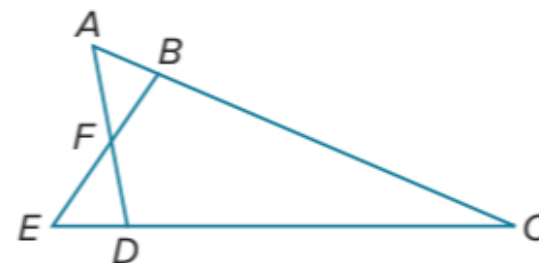
Vertical Angles Theorem

### Check

Choose the correct statements and reasons to complete the two-column proof.

Given:  $\angle DAC \cong \angle BEC$  and  $\overline{DC} \cong \overline{BC}$

Prove:  $\triangle ACD \cong \triangle ECB$



Statements

Reasons

1.  $\angle DAC \cong \angle BEC$

1. Given

2.  $\overline{DC} \cong \overline{BC}$

2.  $\overline{DC} \cong \overline{BC}$  Given

3.  $\angle C \cong \angle C$

3.  $\angle C \cong \angle C$  Reflexive property of congruence

4.  $\triangle ACD \cong \triangle ECB$

4.  $\triangle ACD \cong \triangle ECB$  AAS



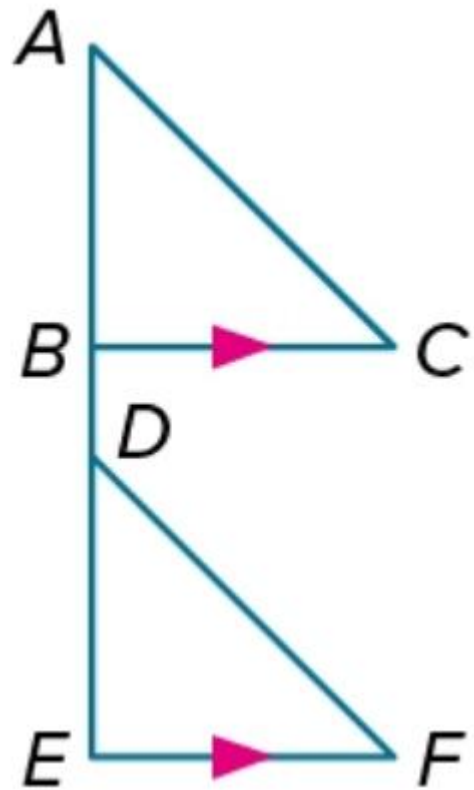
10. two-column proof

**Given:**  $\overline{BC} \parallel \overline{EF}$ ,  $\overline{AB} \cong \overline{DE}$ ,  $\angle C \cong \angle F$

**Prove:**  $\triangle ABC \cong \triangle DEF$

**Proof:**

Statements	Reasons
1. $\overline{BC} \parallel \overline{EF}$ , $\overline{AB} \cong \overline{DE}$ , $\angle C \cong \angle F$	1. Given
2. $\angle ABC \cong \angle DEF$	2. Corresponding Angles Theorem
3. $\triangle ABC \cong \triangle DEF$	3. AAS



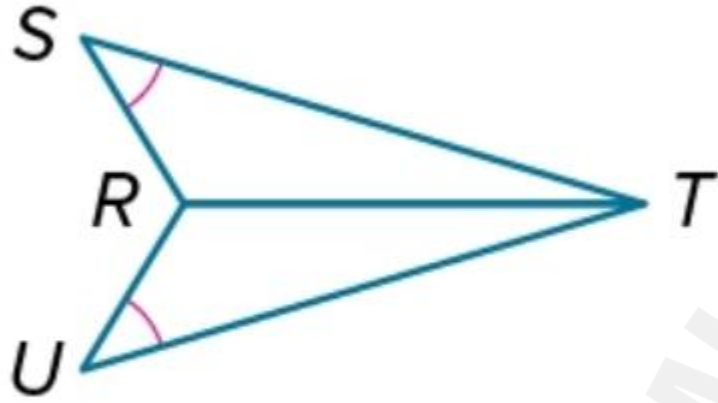
Since two angles and the nonincluded side of triangle ABC are congruent to the corresponding two angles and nonincluded side of triangle DEF, the triangles are congruent.



11. flow proof

**Given:**  $\angle S \cong \angle U$ , and  $\overline{TR}$  bisects  $\angle STU$ .

**Prove:**  $\triangle SRT \cong \triangle URT$



*SOLUTION:*

**Given:**  $\angle S \cong \angle U$ , and  $\overline{TR}$  bisects  $\angle STU$ .

**Prove:**  $\triangle SRT \cong \triangle URT$

**Proof:**

$\overline{TR}$  bisects  $\angle STU$ .

**Given**

$\angle STR \cong \angle UTR$

**Def. of  $\angle$  bisector**

$\angle S \cong \angle U$

**Given**

$\triangle SRT \cong \triangle URT$

**AAS**

$\overline{RT} \cong \overline{RT}$

**Reflexive Prop. of  $\cong$**

Since two angles and the nonincluded side of triangle SRT are congruent to the corresponding two angles and nonincluded side of triangle URT, the triangles are congruent.

12. flow proof

**Given:**  $\overline{JK} \cong \overline{MK}$ ,  $\angle N \cong \angle L$

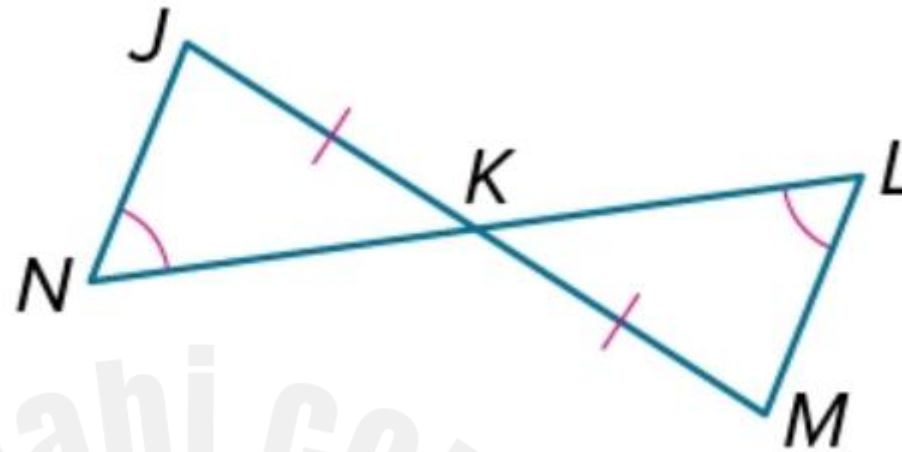
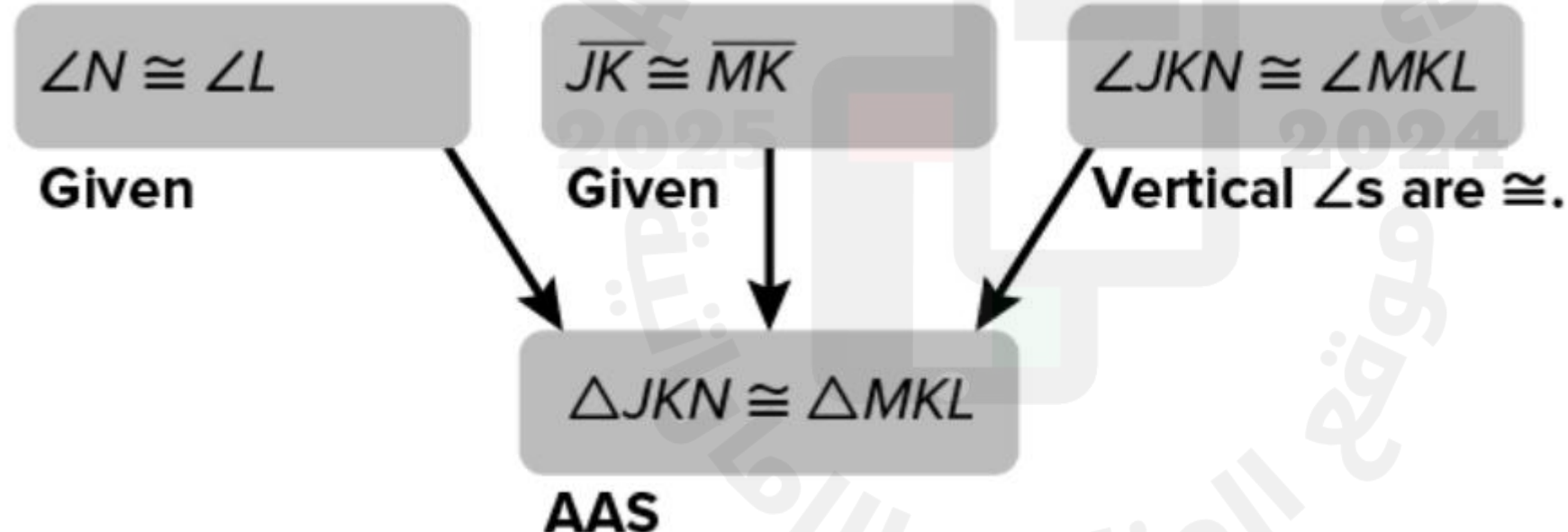
**Prove:**  $\triangle JKN \cong \triangle MKL$

*SOLUTION:*

**Given:**  $\overline{JK} \cong \overline{MK}$ ,  $\angle N \cong \angle L$

**Prove:**  $\triangle JKN \cong \triangle MKL$

**Proof:**

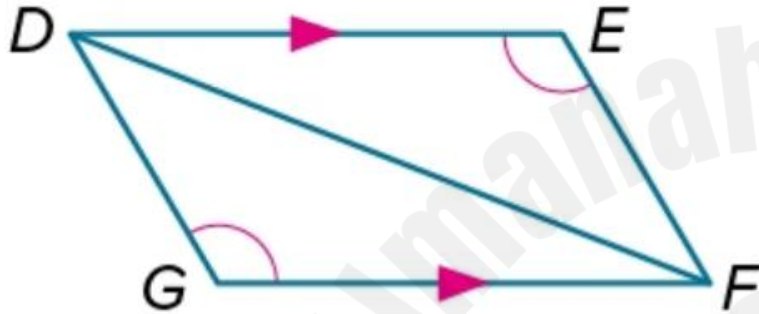


Since two angles and the nonincluded side of triangle JKN are congruent to the corresponding two angles and nonincluded side of triangle MKL, the triangles are congruent.

13. paragraph proof

**Given:**  $\overline{DE} \parallel \overline{FG}$ , and  $\angle E \cong \angle G$

**Prove:**  $\triangle DFG \cong \triangle FDE$



*SOLUTION:*

**Given:**  $\overline{DE} \parallel \overline{FG}$ , and  $\angle E \cong \angle G$

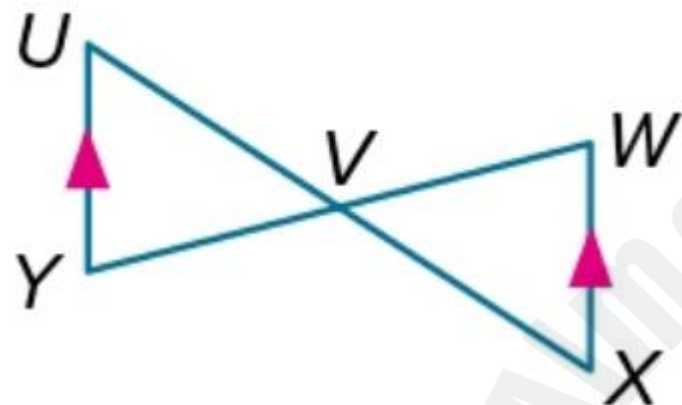
**Prove:**  $\triangle DFG \cong \triangle FDE$

**Proof:** It is given that  $\angle E \cong \angle G$  and  $\overline{DE} \parallel \overline{FG}$ . Considering  $\overline{DF}$  as the transversal that cuts through parallel segments  $\overline{DE}$  and  $\overline{FG}$ , by the Alternate Interior Angles Theorem,  $\angle DFG \cong \angle FDE$ .  $\overline{DF} \cong \overline{DF}$  by the Reflexive Property of Congruence. Therefore,  $\triangle DFG \cong \triangle FDE$  by AAS.

14. two-column proof

**Given:**  $V$  is the midpoint of  $\overline{YW}$ ;  $\overline{UY} \parallel \overline{XW}$ .

**Prove:**  $\triangle UVY \cong \triangle X VW$



**Given:**  $V$  is the midpoint of  $\overline{YW}$ ;  $\overline{UY} \parallel \overline{XW}$ .

**Prove:**  $\triangle UVY \cong \triangle X VW$

**Proof:**

Statements	Reasons
1. $V$ is the midpoint of $\overline{YW}$ ; $\overline{UY} \parallel \overline{XW}$	1. Given
2. $\overline{YV} \cong \overline{VW}$	2. Midpoint Theorem
3. $\angle VWX \cong \angle VYU$	3. Alternate Interior Angles Theorem
4. $\angle VUY \cong \angle VXW$	4. Alternate Interior Angles Theorem
5. $\triangle UVY \cong \triangle X VW$	5. AAS

15. two-column proof

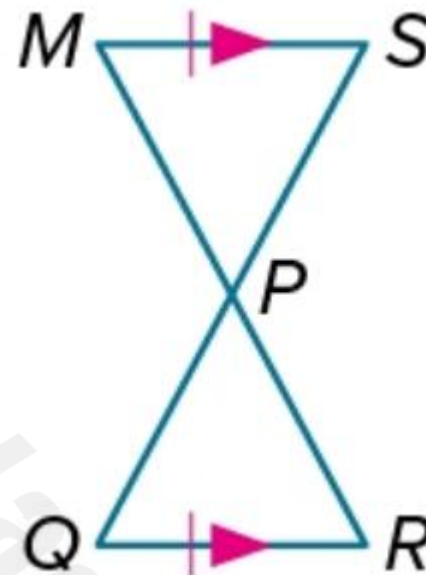
**Given:**  $\overline{MS} \cong \overline{RQ}$ ,  $\overline{MS} \parallel \overline{RQ}$

**Prove:**  $\triangle MSP \cong \triangle RQP$

*SOLUTION:*

**Given:**  $\overline{MS} \cong \overline{RQ}$ ,  $\overline{MS} \parallel \overline{RQ}$

**Prove:**  $\triangle MSP \cong \triangle RQP$



**Proof:**

Statements	Reasons
1. $\overline{MS} \cong \overline{RQ}$ , $\overline{MS} \parallel \overline{RQ}$	1. Given
2. $\angle SPM \cong \angle QPR$	2. Vertical Angles Theorem
3. $\angle SMP \cong \angle QRP$	3. Alternate Interior Angles Theorem
4. $\triangle MSP \cong \triangle RQP$	4. AAS



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# 10. Learning Outcome/lesson Name: Use the slope to identify parallel and perpendicular lines

Example/Exercise: EX3



### Example 3 Determine Line Relationships When Given Equations

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

a.  $y = 3x - 2$ ;  $y - 0 = -\frac{1}{3}(x - 2)$

slope-intercept form

point-slope form

$$y = 3x - 2$$

$$y - 0 = -\frac{1}{3}(x - 2)$$

↑ slope ↑

The two lines do not have the same slope, so the lines are not parallel. To determine whether the lines are perpendicular, find the product of the slopes.

$$3\left(-\frac{1}{3}\right) = -1$$

Product of slopes

Because the product of their slopes is  $-1$ , the two lines are perpendicular.

**b.**  $y = 3; x = 1$

$$y = 3$$

horizontal line

slope of 0

$$x = 1$$

vertical line

undefined slope

Vertical and horizontal lines are always perpendicular.

**c.**  $y - 5 = -\frac{3}{4}(x + 2); y = -\frac{3}{4}x + 2$

point-slope form

slope-intercept form

$$y - 5 = -\frac{3}{4}(x + 2)$$

$$y = -\frac{3}{4}x + 2$$




Because the slopes of both lines are  $-\frac{3}{4}$ , the lines are parallel.



d.  $y = 2x + 3$ ;  $y - 1 = \frac{1}{2}(x + 2)$

slope-intercept form

point-slope form

$$y = 2x + 3 \qquad y - 1 = \frac{1}{2}(x + 2)$$


The two lines do not have the same slope, so the lines are not parallel. To determine whether the lines are perpendicular, find the product of the slopes.

$$2\left(\frac{1}{2}\right) = 1$$

Product of slopes

Because the product of the slopes is not  $-1$ , the two lines are not perpendicular. So, the two lines are neither parallel nor perpendicular.

e.  $x = -2$ ;  $x = 4$

Both lines are vertical with undefined slope. Vertical lines are always parallel.

## Check

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

a.  $y = 3x - 9$ ;  $y = -\frac{1}{3}x + 2$  perpendicular,

b.  $y = \frac{9}{7}x - \frac{19}{7}$ ;  $y - 1 = \frac{9}{7}(x + 3)$  parallel,

c.  $x = -3$ ;  $x = 4$  parallel,



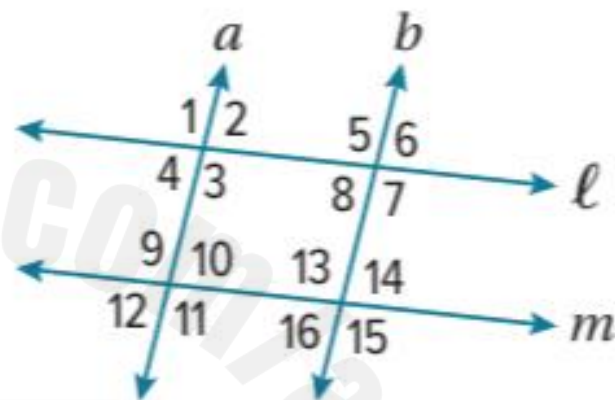
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# 11. Learning Outcome/lesson Name: Prove that two lines are parallel

Example/Exercise: EX1

### Example 1 Identify Parallel Lines

Use the given information to determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



a.  $\angle 2 \cong \angle 8$

$\angle 2$  and  $\angle 8$  are alternate interior angles of lines  $a$  and  $b$ .

Because  $\angle 2 \cong \angle 8$ ,  $a \parallel b$  by the Alternate Interior Angles Converse.

b.  $\angle 3 \cong \angle 11$

$\angle 3$  and  $\angle 11$  are corresponding angles of lines  $l$  and  $m$ . Because  $\angle 3 \cong \angle 11$ ,  $l \parallel m$  by the Converse of the Corresponding Angles Theorem.

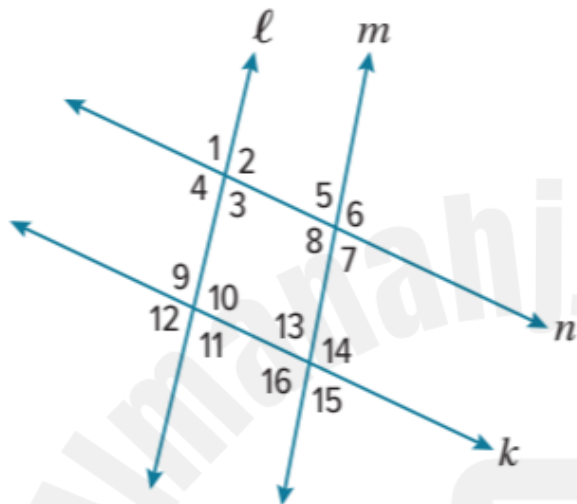
c.  $\angle 12 \cong \angle 14$

$\angle 12$  and  $\angle 14$  are alternate exterior angles of lines  $a$  and  $b$ .

Because  $\angle 12 \cong \angle 14$ ,  $a \parallel b$  by the Alternate Exterior Angles Converse.

## Check

Use the given information to determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



a.  $\angle 1 \cong \angle 15$

- A.  $\ell \parallel m$ ; Alternate Exterior Angles Converse
- B.  $n \parallel k$ ; Alternate Exterior Angles Converse
- C.  $\ell \parallel m$ ; Converse of Corresponding Angles Theorem
- D. It is not possible to determine whether the lines are parallel.

b.  $m\angle 3 + m\angle 10 = 180$

c.  $\angle 3 \cong \angle 5$

- A.  $\ell \parallel m$ ; Alternate Interior Angles Converse
- B.  $\ell \parallel m$ ; Consecutive Interior Angles Converse
- C.  $n \parallel k$ ; Alternate Interior Angles Converse
- D. It is not possible to determine whether the lines are parallel.

D. It is not possible to determine whether the lines are parallel.

$n \parallel k$ , Consecutive Interior Angle Converse

A.  $\ell \parallel m$ ; Alternate Interior Angles Converse



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## 12. Learning Outcome/lesson Name: Make conjectures based on inductive reasoning

Example/Exercise: 1-8

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next term in the sequence.

1. 4, 8, 12, 16, 20

SOLUTION:

$$4 + 4 = 8$$

$$8 + 4 = 12$$

$$12 + 4 = 16$$

$$16 + 4 = 20$$

Each term in the pattern is four more than the previous term.

The next number in the sequence is  $20 + 4 = 24$

2. 2, 22, 222, 2222

SOLUTION:

2 (1 twos digit)

22 (2 twos digits)

222 (3 twos digits)

2222 (4 twos digits)

Each term has an additional digit two as part of the number.

The next number in the sequence is 22222.

3.  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

SOLUTION:

$$1 \cdot \frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

Each term is one half the previous term.

The next number in the sequence is  $\frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$ .

4.  $6, \frac{11}{2}, 5, \frac{9}{2}, 4$

*SOLUTION:*

$$6 - \frac{1}{2} = \frac{11}{2}$$

$$\frac{11}{2} - \frac{1}{2} = 5$$

$$5 - \frac{1}{2} = \frac{9}{2}$$

$$\frac{9}{2} - \frac{1}{2} = 4$$

Each term is one half less than the previous term.

The next number in the sequence is  $4 - \frac{1}{2} = \frac{7}{2}$ .

5. Arrival times: 3:00 p.m., 12:30 p.m., 10:00 a.m., . . .

**SOLUTION:**

3:00 p.m. – 2 hours 30 minutes = 12:30 p.m.

12:30 p.m. – 2 hours 30 minutes = 10:00 a.m.

Each arrival time is 2 hours and 30 minutes prior to the previous arrival time. The next time in the sequence is

10:00 a.m. – 2 hours 30 minutes = 7:30 a.m.

6. Percent humidity: 100%, 93%, 86%, . . .

**SOLUTION:**

100% – 7% = 93%

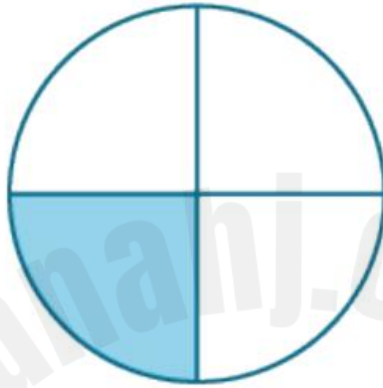
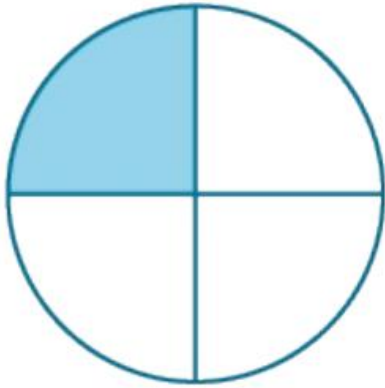
93% – 7% = 86%

Each percentage is 7% less than the previous percentage.

The next percentage in the sequence is 86% – 7% = 79%

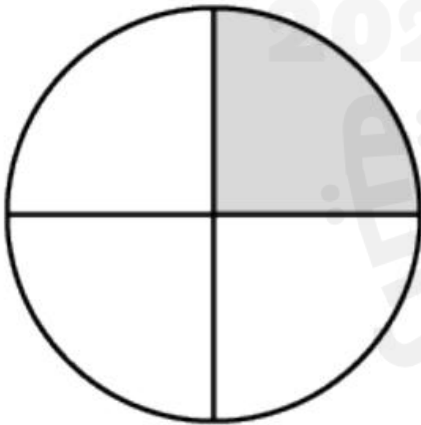


7.

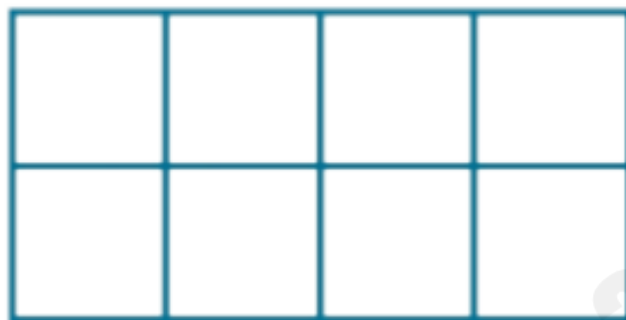


*SOLUTION:*

The shaded section in each circle has moved one section counterclockwise from its location in the previous circle.



8.



*SOLUTION:*

Each figure has one fewer column of squares than the previous figure.



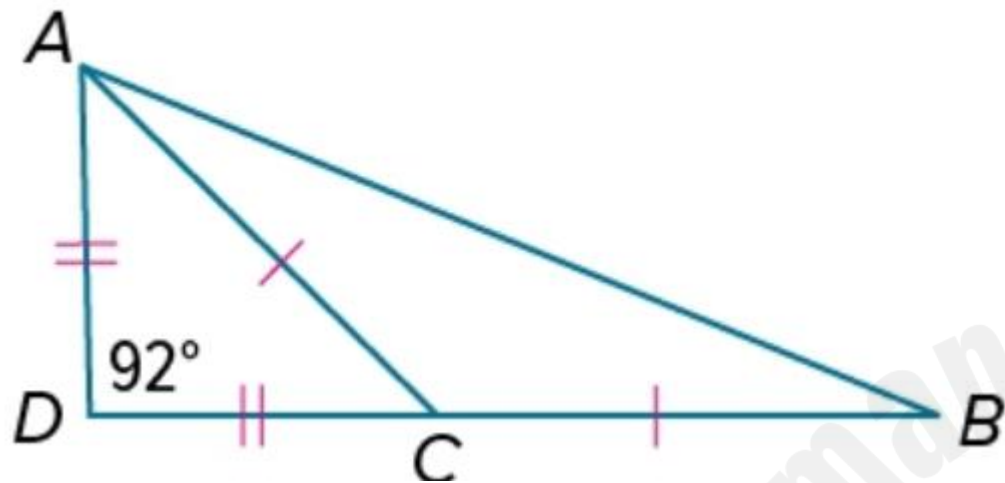


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# 13. Learning Outcome/lesson Name: Use properties of isosceles triangles

Example/Exercise: 20-24

**REGULARITY** Find each measure.



20.  $m\angle CAD$

**SOLUTION:**

$\triangle ACD$  is an isosceles triangle because it has two sides with the same measure. By definition of isosceles triangle, its base angles,  $\angle CAD$  and  $\angle ACD$  have the same measure.

By the Triangle Angle-Sum Theorem,  
 $m\angle CAD + m\angle ACD + 92 = 180^\circ$ .

By substitution,  $2m\angle CAD + 92 = 180^\circ$ .

By the Subtraction Property of Equality,  
 $2m\angle CAD = 88^\circ$ .

And by the Division Property of Equality,  
 $m\angle CAD = 44^\circ$ .

21.  $m\angle ACD$

**SOLUTION:**

$\triangle ACD$  is an isosceles triangle because it has two sides with the same measure. By definition of isosceles triangle, its base angles,  $\angle CAD$  and  $\angle ACD$  have the same measure. Since we found  $m\angle CAD = 44^\circ$ , then by the Transitive Property of Equality,  $m\angle ACD = 44^\circ$ .

22.  $m\angle ACB$

**SOLUTION:**  $\angle ACD$  and  $\angle ACB$  are supplementary angles.

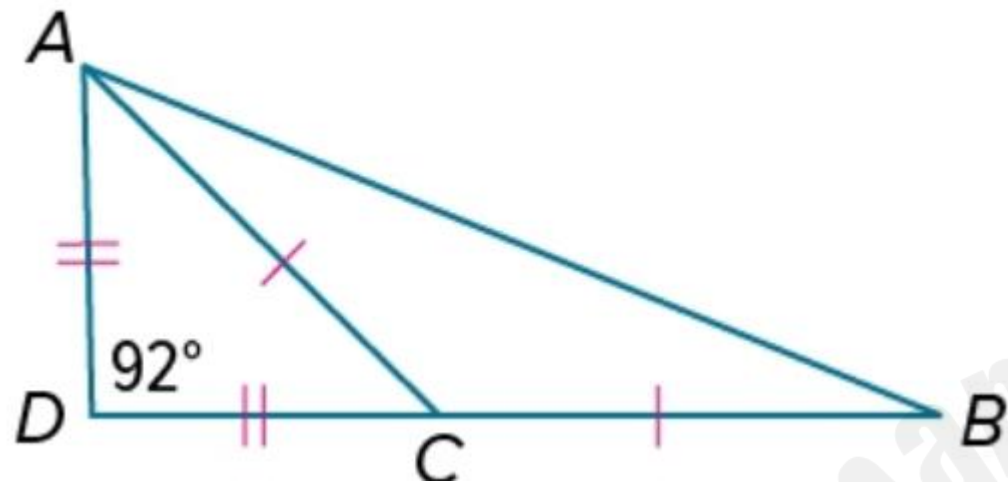
We already know  $m\angle ACD = 44^\circ$ .

Therefore,

$$m\angle ACB = 180^\circ - 44^\circ$$

$$m\angle ACB = 136^\circ.$$

**REGULARITY** Find each measure.



23.  $m\angle ABC$

SOLUTION:

$\triangle ABC$  is an isosceles triangle because it has two sides with the same measure. By definition of isosceles triangle, its base angles,  $\angle ABC$  and  $\angle BAC$  have the same measure.

By the Triangle Angle-Sum Theorem,

$$m\angle ABC + m\angle BAC + m\angle ACB = 180^\circ.$$

We already know  $m\angle ACB = 136^\circ$ .

By substitution,

$$2m\angle ABC + 136 = 180^\circ.$$

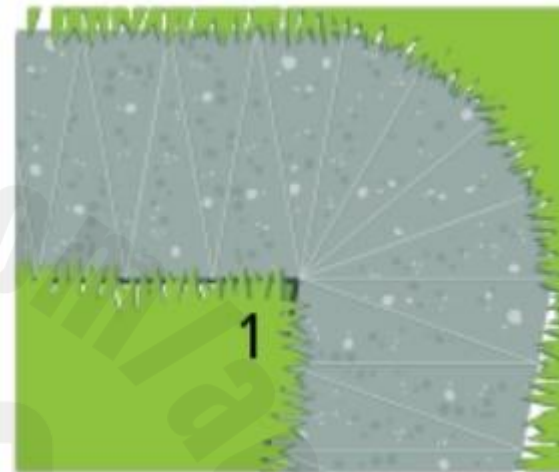
By the Subtraction Property of Equality,

$$2m\angle ABC = 44^\circ.$$

And by the Division Property of Equality,

$$m\angle ABC = 22^\circ.$$

24. PATHS A marble path, as shown, is constructed out of several congruent isosceles triangles. All the vertex angles measure  $20^\circ$ . What is the measure of angle 1 in the figure?



SOLUTION: The measure of angle 1, along with one base angle and one vertex angle of the isosceles triangles, form a straight angle.

If the vertex angle measures  $20^\circ$ , then the base angles measure  $80^\circ$ . Straight angles measure  $180^\circ$ . So,

$$m\angle 1 + 20 + 80 = 180^\circ.$$

$$m\angle 1 = 180^\circ - 20 - 80$$

$$m\angle 1 = 80^\circ$$



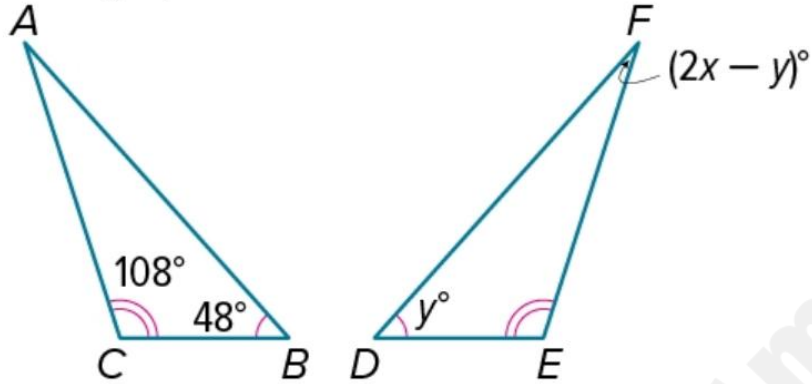
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# 14. Learning Outcome/lesson

Name: Name and use  
corresponding parts of  
congruent polygons

Example/Exercise: 6-11

In the diagram,  $\triangle ABC \cong \triangle FDE$ .



6. Find the value of  $x$ .

*SOLUTION:*

$$m\angle D + m\angle E + m\angle F = 180^\circ$$

$$\angle E \cong \angle C$$

$$m\angle E = m\angle C$$

$$m\angle E = 108^\circ$$

$$y^\circ + 108^\circ + (2x - y)^\circ = 180^\circ$$

$$(2x)^\circ + 108^\circ = 180^\circ$$

$$(2x)^\circ = 72^\circ$$

$$x^\circ = 36^\circ$$

The value of  $x$  is 36.

7. Find the value of  $y$ .

*SOLUTION:*

$$\angle B \cong \angle D \quad \text{CPCTC}$$

$$m\angle B = m\angle D \quad \text{Definition of congruence}$$

$$48^\circ = y^\circ \quad \text{Substitution}$$

The value of  $y$  is 48.

Triangle Angle – Sum Theorem

CPCTC

Definition of congruence

Transitive Property of Equality

Substitution

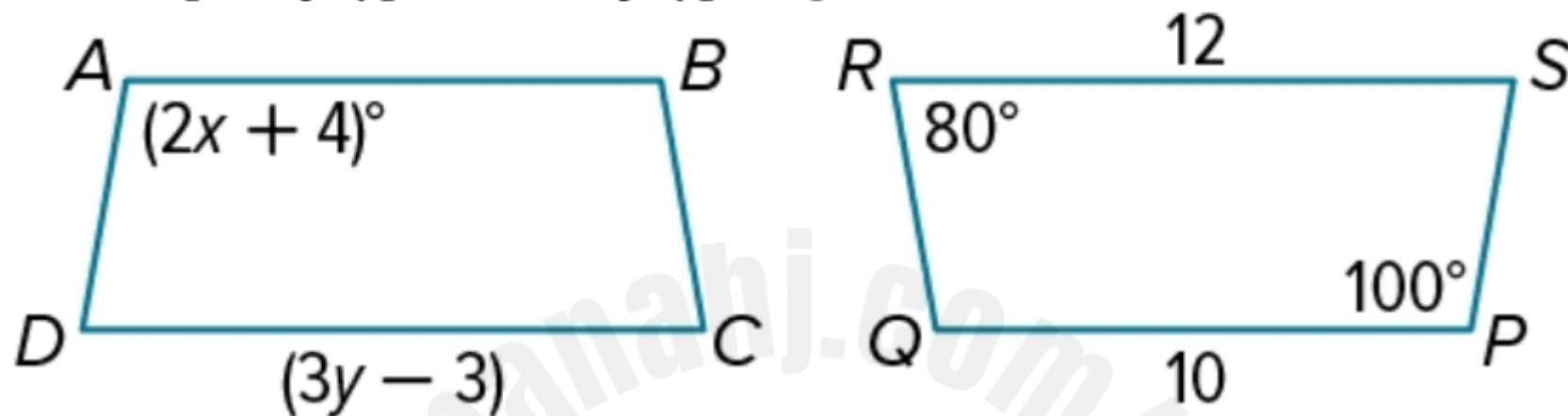
Simplify.

Subtract  $108^\circ$  from each side.

Divide each side by 2.



In the diagram, polygon  $ABCD \cong$  polygon  $PQRS$ .



8. Find the value of  $x$ .

*SOLUTION:*

$$\begin{aligned}\angle A &\cong \angle P && \text{CPCTC} \\ m\angle A &= m\angle P && \text{Definition of congruence} \\ (2x + 4)^\circ &= 100^\circ && \text{Substitution} \\ (2x)^\circ &= 96^\circ && \text{Subtract } 4^\circ \text{ from each side.} \\ x^\circ &= 48^\circ && \text{Divide each side by 2.}\end{aligned}$$

The value of  $x$  is 48.

9. Find the value of  $y$ .

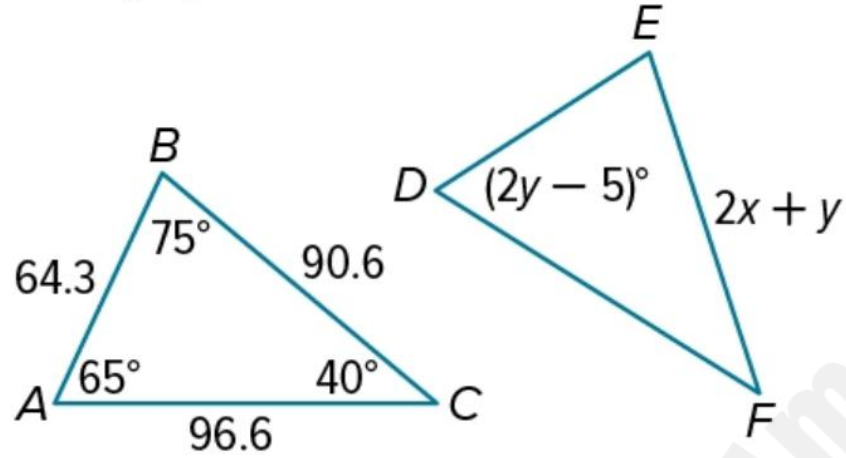
*SOLUTION:*

$$\begin{aligned}\overline{DC} &\cong \overline{SR} && \text{CPCTC} \\ DC &= SR && \text{Definition of congruence} \\ 3y - 3 &= 12 && \text{Substitution} \\ 3y &= 15 && \text{Add 3 to each side.} \\ y &= 5 && \text{Divide each side by 3.}\end{aligned}$$

The value of  $y$  is 5.



In the diagram,  $\triangle ABC \cong \triangle DEF$ .



11. Find the value of  $y$ .

*SOLUTION:*

$$\begin{aligned}\angle A &\cong \angle D && \text{CPCTC} \\ m\angle A &= m\angle D && \text{Definition of congruence} \\ 65^\circ &= (2y - 5)^\circ && \text{Substitution} \\ 70^\circ &= (2y)^\circ && \text{Add } 5^\circ \text{ to each side.} \\ 35^\circ &= y^\circ && \text{Divide each side by 2.}\end{aligned}$$

The value of  $y$  is 35.

10. Find the value of  $x$ .

*SOLUTION:*

$$\begin{aligned}\overline{EF} &\cong \overline{BC} && \text{CPCTC} \\ EF &= BC && \text{Definition of congruence} \\ 2x + y &= 90.6 && \text{Substitution} \\ 2x + 35 &= 90.6 && \text{Substitution} \\ 2x &= 55.6 && \text{Subtract 35 from each side.} \\ x &= 27.8 && \text{Divide each side by 2.}\end{aligned}$$

The value of  $x$  is 27.8.



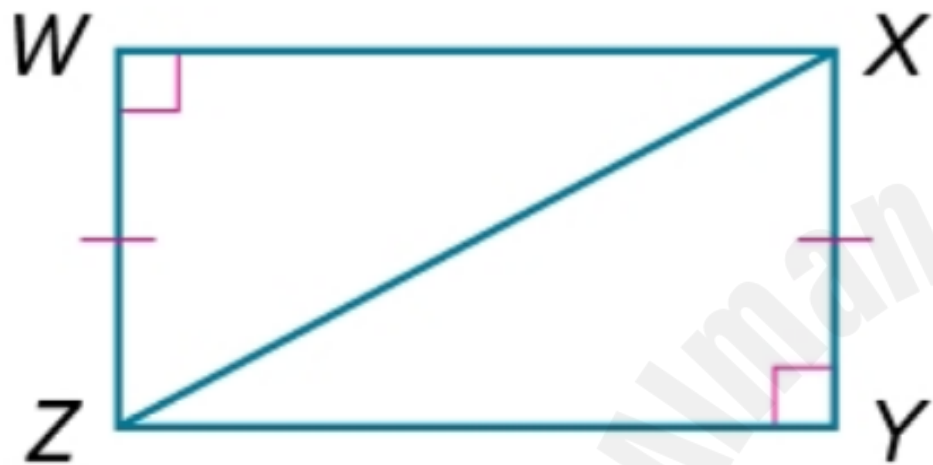
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# 15. Learning Outcome/lesson Name: Use the right triangle congruence theorems

Example/Exercise: 4-9

Determine whether each pair of triangles is congruent. If yes, include the theorem that applies.

4.



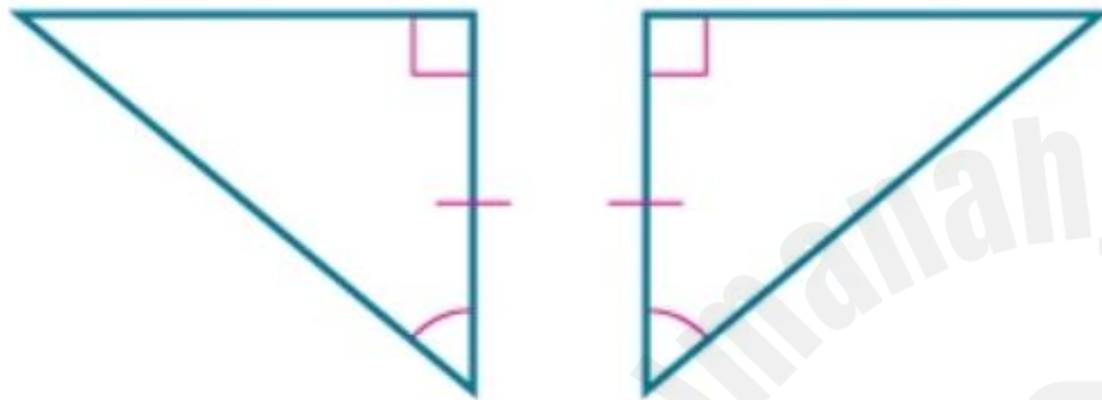
*SOLUTION:*

It is given that legs  $\overline{WZ} \cong \overline{YX}$ . The triangles share the same hypotenuse;  $\overline{XZ} \cong \overline{XZ}$  by the Reflexive Property of Congruence. Therefore, by the HL Theorem,  $\triangle WXZ \cong \triangle YZX$ .

*ANSWER:*

Yes; HL

5.



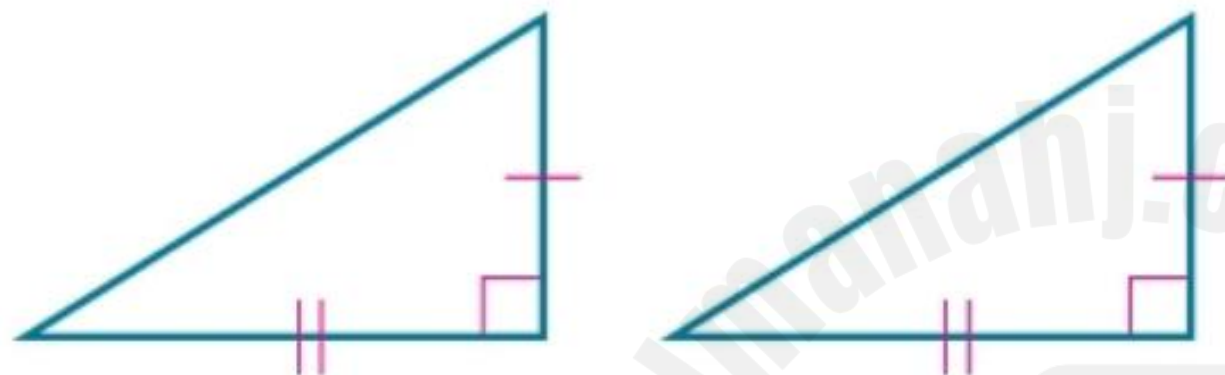
*SOLUTION:*

It is given that one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of the other right triangle, so they are congruent by the LA Theorem of Congruence.

*ANSWER:*

Yes; LA

6.



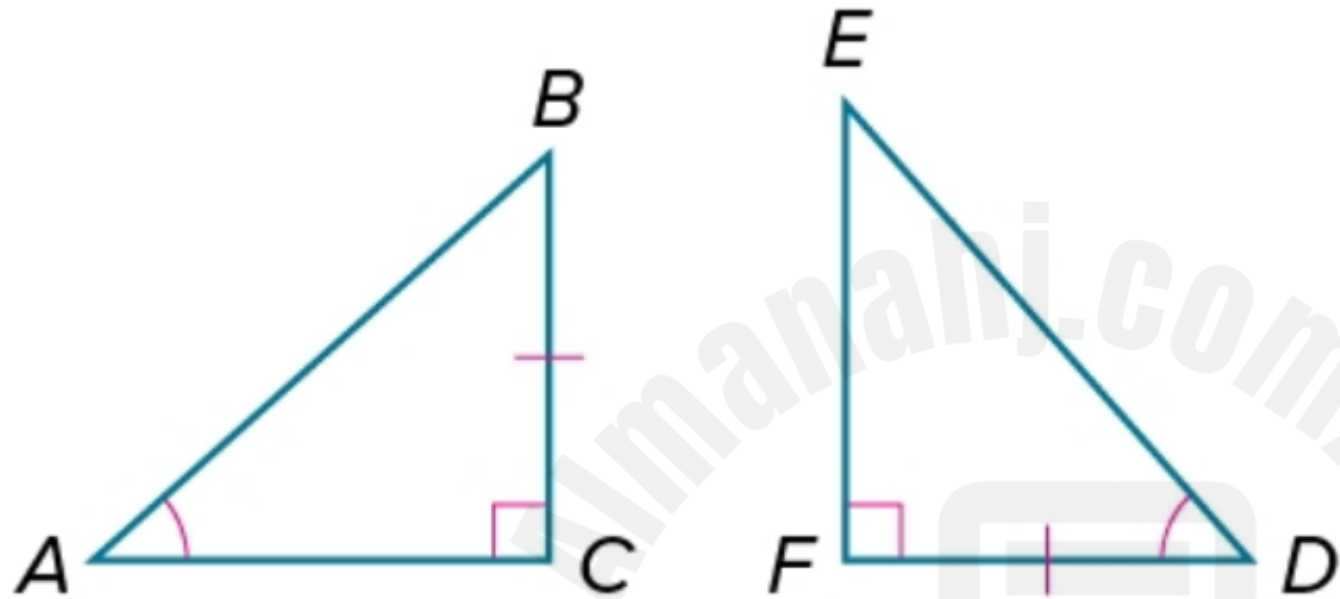
*SOLUTION:*

It is given that the legs of one right triangle are congruent to the corresponding legs of the other right triangle, so they are congruent by the LL Theorem of Congruence.

*ANSWER:*

Yes; LL

7.



*SOLUTION:*

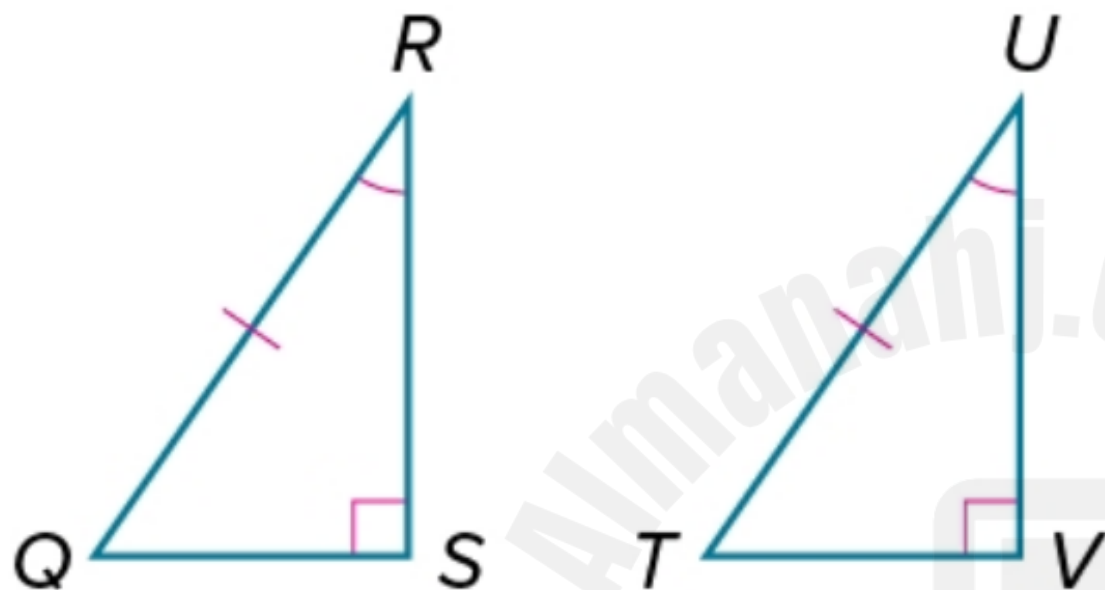
At first glance, you might be inclined to say that these triangles can be shown congruent by the LA Theorem of Congruence; however, the congruent legs and angles marked in the diagram are not corresponding. Therefore, there is not enough information provided to show that they are congruent.

*ANSWER:*

No; not enough information



8.



*SOLUTION:*

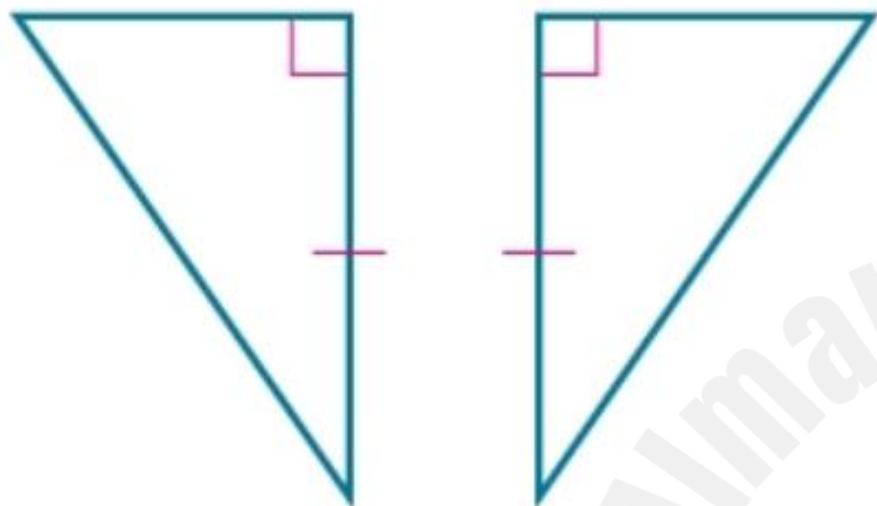
It is given that hypotenuses  $\overline{QR} \cong \overline{TU}$ . It is also given that corresponding acute angles  $R$  and  $U$  are congruent.

Therefore, by the HA Theorem,  $\triangle QRS \cong \triangle TUV$ .

*ANSWER:*

Yes; HA

9.



*SOLUTION:*

The only thing shown to be congruent with these two right triangles is one pair of corresponding legs. However, this is not enough information to apply any of the four theorems to prove them congruent.

*ANSWER:*

No; not enough information





Math

# Grade 9 A EoT Part 2 - FRQ

School Name:



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# 16. Learning Outcome/lesson Name: Draw reflections in the coordinate plane

Example/Exercise: 1-4

Graph the image of each figure under the given reflection.  
Determine the coordinates of the image. 1.  $\triangle ABC$  in the line  $y = x$

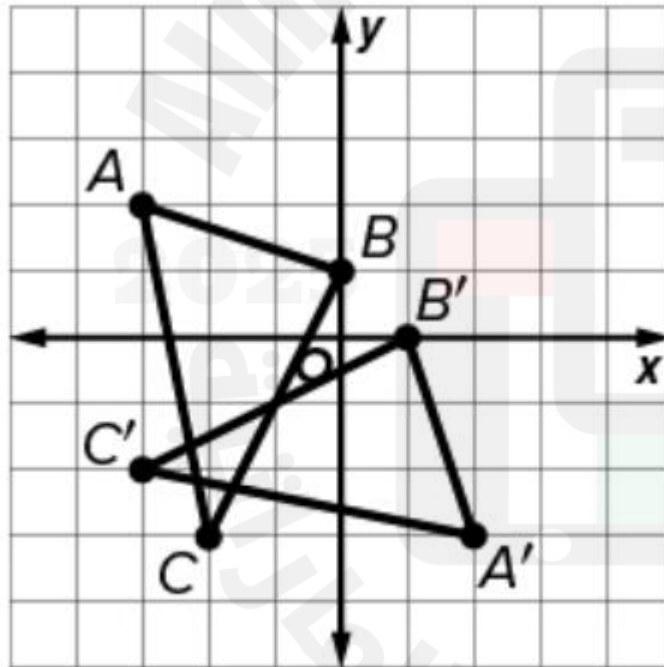
SOLUTION:

To reflect a point in the line  $y = x$  interchange the  $x$ - and  $y$ -coordinates;  
 $(x, y) \rightarrow (y, x)$ .

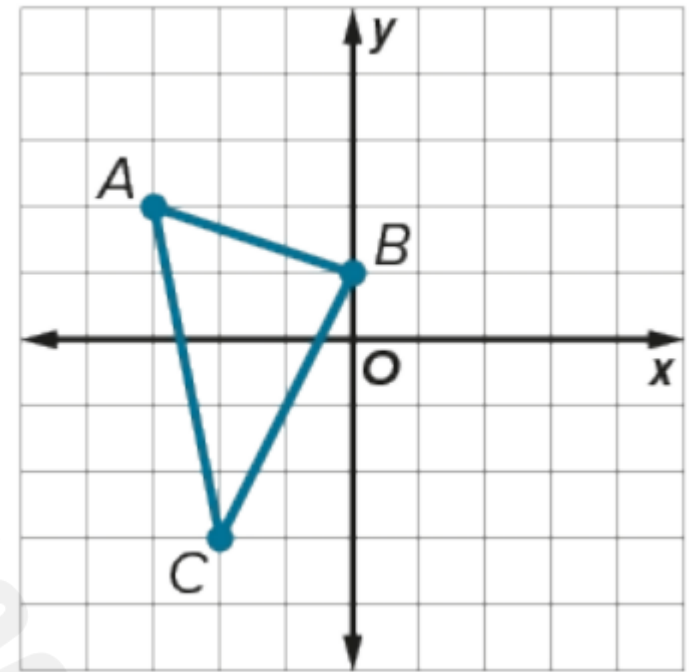
$$A(-3, 2) \rightarrow A'(2, -3)$$

$$B(0, 1) \rightarrow B'(1, 0)$$

$$C(-2, -3) \rightarrow C'(-3, -2)$$



$$A'(2, -3), B'(1, 0), C'(-3, -2)$$



### SOLUTION:

To reflect a point in the line  $x = -1$ , a vertical line that is not the  $y$ -axis, the  $y$ -coordinates of the image remain the same as the preimage. The distance from a point in the preimage to the line of reflection is the same as the distance from the corresponding point in the image to the line of reflection.

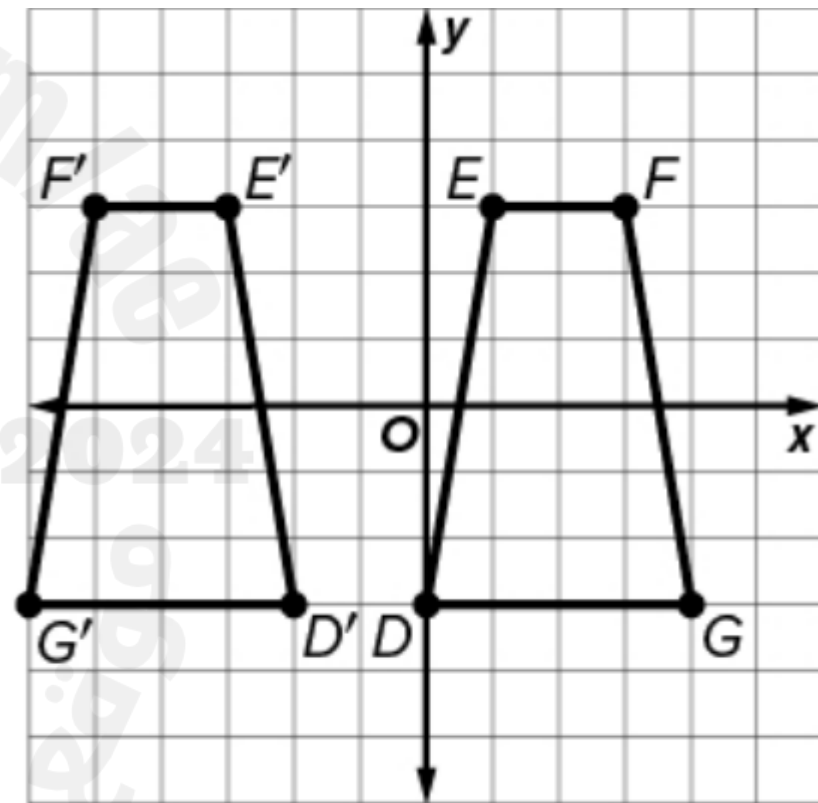
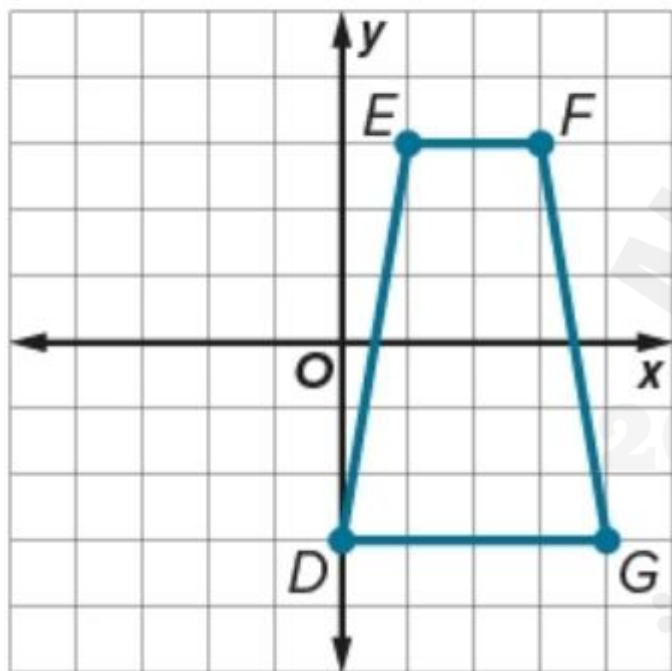
$$D(0, -3) \rightarrow D'(-2, -3)$$

$$E(1, 3) \rightarrow E'(-3, 3)$$

$$F(3, 3) \rightarrow F'(-5, 3)$$

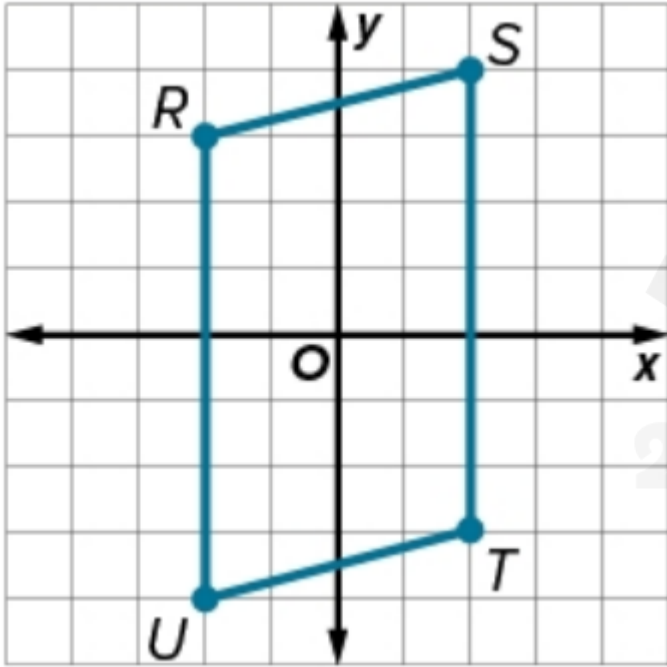
$$G(4, -3) \rightarrow G'(-6, -3)$$

2. trapezoid  $DEFG$  in the line  $x = -1$



$$D'(-2, -3), E'(-3, 3), F'(-5, 3), G'(-6, -3)$$

3. parallelogram RSTU in the line  $y = x$



SOLUTION:

To reflect a point in the line  $y = x$  interchange the x- and y-coordinates;

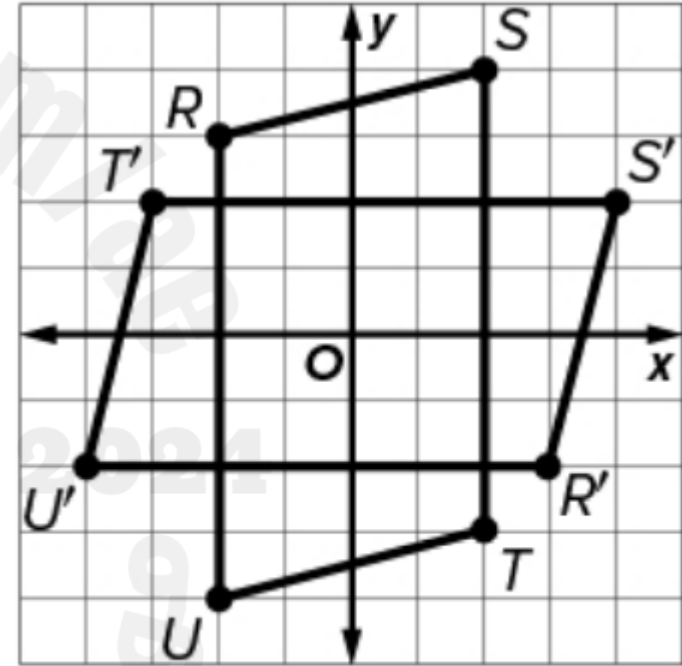
$(x, y) \rightarrow (y, x)$ .

$R(-2, 3) \rightarrow R'(3, -2)$

$S(2, 4) \rightarrow S'(4, 2)$

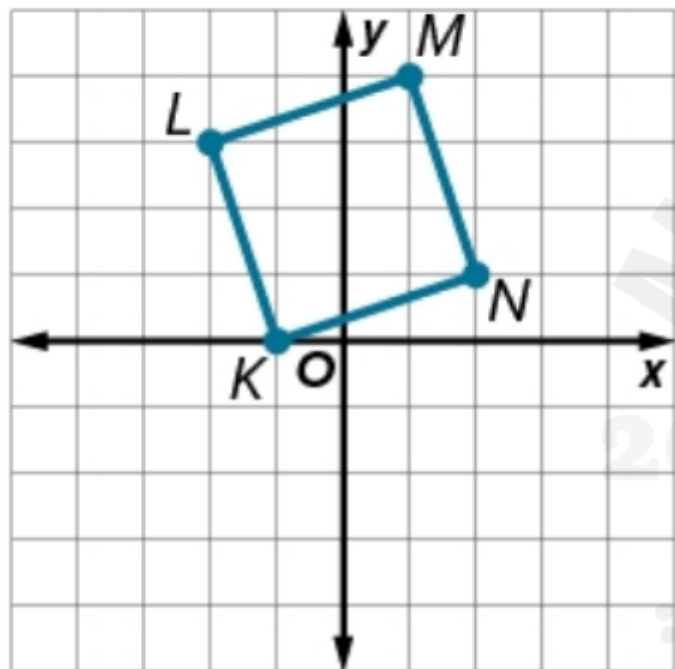
$T(2, -3) \rightarrow T'(-3, 2)$

$U(-2, -4) \rightarrow U'(-4, -2)$



$R'(3, -2), S'(4, 2), T'(-3, 2), U'(-4, -2)$

4. square  $KLMN$  in the line  $y = -2$



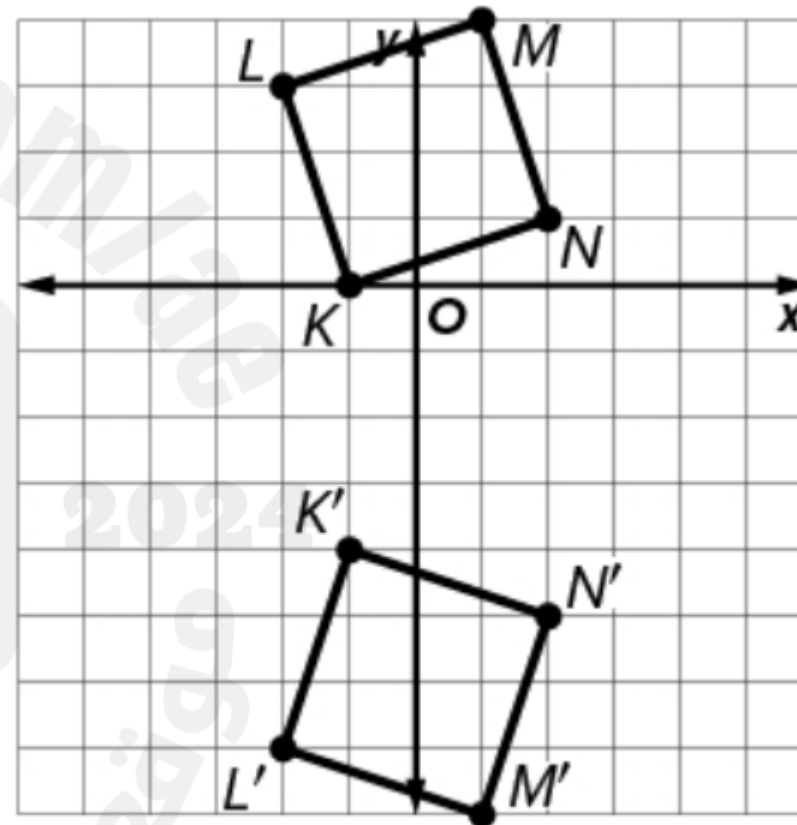
SOLUTION: To reflect a point in the line  $y = -2$ , a horizontal line that is not the x-axis, the x-coordinates of the image remain the same as the preimage. The distance from a point in the preimage to the line of reflection is the same as the distance from the corresponding point in the image to the line of reflection.

$$K(-1, 0) \rightarrow K'(-1, -4)$$

$$L(-2, 3) \rightarrow L'(-2, -7)$$

$$M(1, 4) \rightarrow M'(1, -8)$$

$$N(2, 1) \rightarrow N'(2, -5)$$



$$K'(-1, -4), L'(-2, -7), M'(1, -8), N'(2, -5)$$



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# 17. Learning Outcome/lesson Name: Determine truth values of negations, conjunctions, and disjunctions

Example/Exercise: 1-6

# Using Logic – Truth Value

p	q	Conjunction $p \wedge q$	Disjunction $p \vee q$	Negation $\sim p$	Negation $\sim q$
T	T	T	T	F	F
T	F	F	T	F	T
F	T	F	T	T	F
F	F	F	F	T	T

- A conjunction is written as **p and q** or  $p \wedge q$
- A disjunction is written as **p or q** or  $p \vee q$



Use the statements to write each conjunction or disjunction. Then find the truth values. Explain your reasoning.

p:  $-3 - 2 = -5$

q: Vertical angles are congruent.

r:  $2 + 8 > 10$

1. p and q

SOLUTION: The statement  $-3 - 2 = -5$  is true, so p is true. The statement vertical angles are congruent is true, so q is true. For p and q to be true, both p and q must be true. p is true and q is true, so p and q is true.

ANSWER:

$-3 - 2 = -5$ , and vertical angles are congruent; p is true, and q is true, so p and q is true.

2.  $p \wedge r$

SOLUTION:

The statement  $-3 - 2 = -5$  is true, so p is true. The statement  $2 + 8 > 10$  is not true, so r is false. For  $p \wedge r$  to be true, both p and r must be true. p is true and r is false, so  $p \wedge r$  is false.

ANSWER:

$-3 - 2 = -5$ , and  $2 + 8 > 10$ ; p is true, and r is false, so  $p \wedge r$  is false.

3.  $q \vee \sim r$

SOLUTION:

The statement vertical angles are congruent is true, so p is true. The negation of  $2 + 8 > 10$  is  $2 + 8 \leq 10$ . The statement  $2 + 8 \leq 10$  is true. so  $\sim r$  is true. For  $q \vee \sim r$  to be true, either q or  $\sim r$  must be true. q is true and  $\sim r$  is true, so  $q \vee \sim r$  is true.

ANSWER: Vertical angles are congruent, or  $2 + 8 \leq 10$ ; q is true, and  $\sim r$  is true, so  $q \vee \sim r$  is true.

Use the statements to write each conjunction or disjunction. Then find the truth values. Explain your reasoning.

$p: -3 - 2 = -5$

$q$ : Vertical angles are congruent.

$r: 2 + 8 > 10$

4.  $r \vee q$

SOLUTION:

The statement  $2 + 8 > 10$  is false, so  $r$  is false.

The statement vertical angles are congruent is true, so  $q$  is true.

For  $r \vee q$  to be true, either  $r$  or  $q$  must be true.

$r$  is false and  $q$  is true, so  $r \vee q$  is true.

ANSWER:

$2 + 8 > 10$ , or vertical angles are congruent;  $r$  is false, and  $q$  is true so  $r \vee q$  is true.

5.  $\sim p \wedge \sim q$

SOLUTION:

The statement  $-3 - 2 \neq -5$  is the negation of statement  $p$ .

The statement  $-3 - 2 \neq -5$  is true, so  $\sim p$  is false.

The statement not all vertical angles are congruent is the negation of statement  $q$ .

The statement not all vertical angles are congruent is false, so  $\sim q$  is false.

For  $\sim p \wedge \sim q$  to be true, both  $\sim p$  and  $\sim q$  must be true.

$\sim p$  is false, and  $\sim q$  is false, so  $\sim p \wedge \sim q$  is false.

ANSWER:

$-3 - 2 \neq -5$ , and not all vertical angles are congruent;  $\sim p$  is false, and  $\sim q$  is false, so  $\sim p \wedge \sim q$  is false.

6.  $\sim r \vee \sim p$

SOLUTION:

The statement  $2 + 8 \leq 10$  is the negation of statement  $r$ .

The statement  $2 + 8 \leq 10$  is true, so  $\sim r$  is true.

The statement  $-3 - 2 \neq -5$  is the negation of statement  $p$ .

The statement  $-3 - 2 \neq -5$  is false, so  $\sim p$  is false.

For  $\sim r \vee \sim p$  to be true, either  $\sim r$  or  $\sim p$  must be true.

$\sim r$  is true and  $\sim p$  is false, so  $\sim r \vee \sim p$  is true.

ANSWER:

$2 + 8 \leq 10$  or  $-3 - 2 \neq -5$ ;  $\sim r$  is true, and  $\sim p$  is false, so  $\sim r \vee \sim p$  is true.



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# 18. Learning Outcome/lesson Name: Write proofs involving supplementary and complementary angles

Example/Exercise: 9-12

PROOF Write a two-column proof.

9. Given:  $m\angle ABC = m\angle DEF$

$\angle ABC$  and  $\angle DEF$  are supplementary.

*SOLUTION:*

**Given:**  $m\angle ABC = m\angle DEF$

$\angle ABC$  and  $\angle DEF$  are supplementary.

**Prove:**  $\angle ABC$  and  $\angle DEF$  are right angles.

**Proof:**

**Prove:**  $\angle ABC$  and  $\angle DEF$  are right angles.



Statements	Reasons
1. $m\angle ABC = m\angle DEF$	1. Given
2. $\angle ABC \cong \angle DEF$	2. Definition of $\cong$ angles
3. $\angle ABC$ and $\angle DEF$ are supplementary.	3. Given
4. $\angle ABC$ and $\angle DEF$ are right angles.	4. If two $\angle$ s are $\cong$ and supplementary, then each $\angle$ is a right $\angle$ .

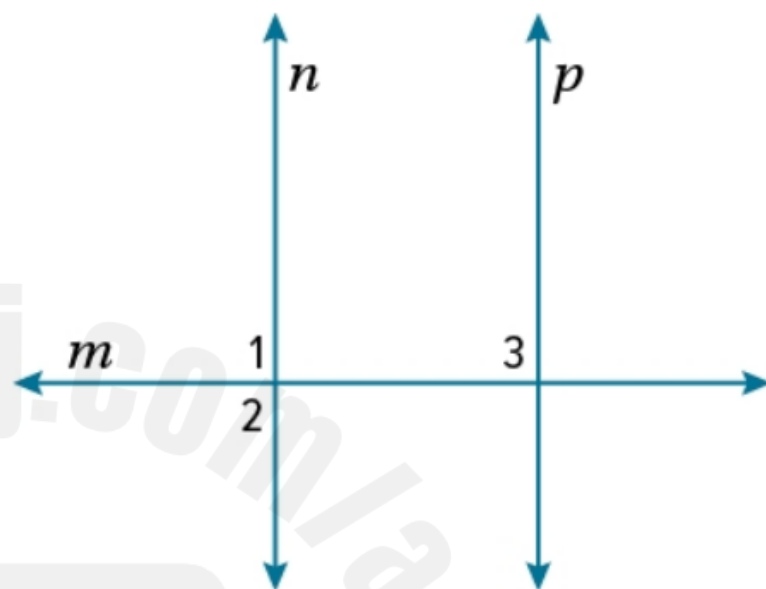
10. **Given:**  $\angle 1 \cong \angle 2$ ;  $m \perp p$

**Prove:**  $\angle 2 \cong \angle 3$

*SOLUTION:*

**Given:**  $\angle 1 \cong \angle 2$ ;  $m \perp p$

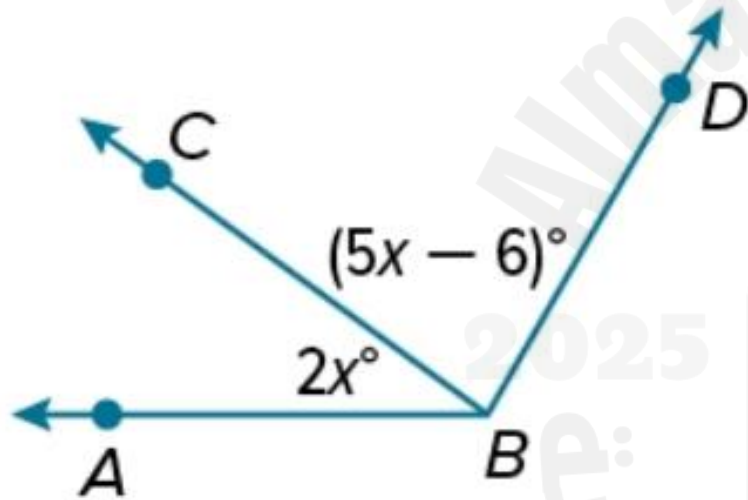
**Prove:**  $\angle 2 \cong \angle 3$



**Proof:**

Statements	Reasons
1. $\angle 1 \cong \angle 2$ ; $m \perp p$	1. Given
2. $\angle 1$ and $\angle 2$ form a linear pair	2. Definition of linear pair
3. $\angle 1$ and $\angle 2$ are right angles.	3. If 2 $\cong$ angles form a linear pair, they are right $\angle$ s.
4. $\angle 3$ is a right angle.	4. $\perp$ lines form 4 rt. angles.
5. $\angle 2 \cong \angle 3$	5. All rt. $\angle$ s are congruent.

11. Find  $m \angle ABC$  and  $m \angle CBD$  if  $m \angle ABD = 120^\circ$ .



*SOLUTION:*

$$m \angle ABC + m \angle CBD = m \angle ABD$$

$$2x^\circ + (5x - 6)^\circ = 120^\circ$$

$$(7x - 6)^\circ = 120^\circ$$

$$7x^\circ = 126^\circ$$

$$x = 18$$

$$m \angle ABC = 2x^\circ$$

$$m \angle ABC = 2(18)^\circ$$

$$m \angle ABC = 36^\circ$$

$$m \angle CBD = (5x - 6)^\circ$$

$$m \angle CBD = (5(18) - 6)^\circ$$

$$m \angle CBD = 84^\circ$$

Angle Addition Postulate

Substitution Property of Equality

Substitution Property of Equality

Addition Property of Equality

Division Property of Equality

Given

Substitution Property of Equality

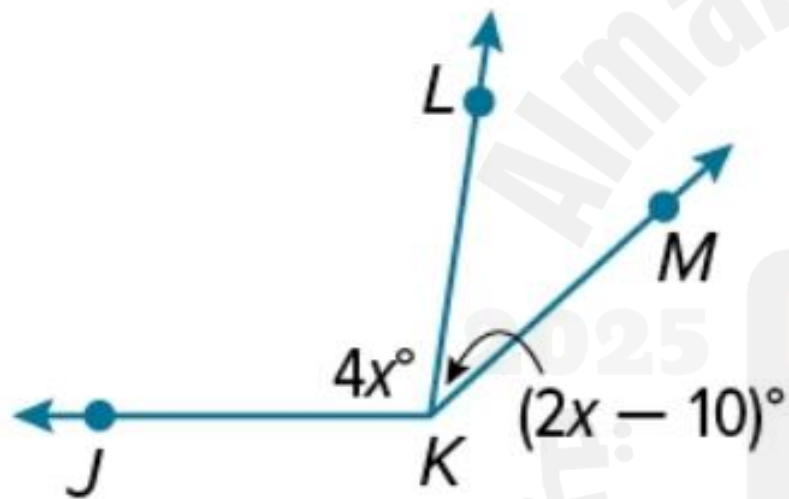
Substitution Property of Equality

Given

Substitution Property of Equality

Substitution Property of Equality

12. Find  $m\angle JKL$  and  $m\angle LKM$  if  $m\angle JKM = 140^\circ$ .



*SOLUTION:*

$$m\angle JKL + m\angle LKM = m\angle JKM$$

$$4x^\circ + (2x - 10)^\circ = 140^\circ$$

$$(6x - 10)^\circ = 140^\circ$$

$$6x^\circ = 150^\circ$$

$$x = 25^\circ$$

$$m\angle JKL = 4x^\circ$$

$$m\angle JKL = 4(25)^\circ$$

$$m\angle JKL = 100^\circ$$

$$m\angle LKM = (2x - 10)^\circ$$

$$m\angle LKM = (2(25) - 10)^\circ$$

$$m\angle LKM = 40^\circ$$

Angle Addition Postulate

Substitution Property of Equality

Substitution Property of Equality

Addition Property of Equality

Division Property of Equality

Given

Substitution Property of Equality

Substitution Property of Equality

Given

Substitution Property of Equality

Substitution Property of Equality





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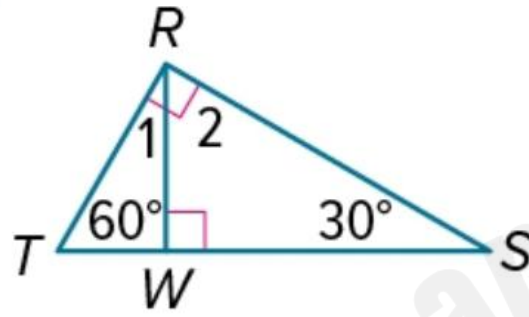
# 19. Learning Outcome/lesson Name: Apply the Exterior Angle Theorem

Example/Exercise: 1-8 & 32-25



Find the measure of each numbered angle.

1.



*SOLUTION:*

Because  $\angle TWR$  and  $\angle RWS$  form a linear pair (definition of linear pair), and  $m\angle RWS = 90^\circ$  (definition of right angle), then  $m\angle TWR = 90^\circ$  by the Supplement Theorem.

$$m\angle 1 + 60^\circ + 90^\circ = 180^\circ \quad \text{Triangle Angle - Sum Theorem}$$

$$m\angle 1 + 150^\circ = 180^\circ \quad \text{Simplify.}$$

$$m\angle 1 = 30^\circ \quad \text{Subtract } 150^\circ \text{ from each side.}$$

$$m\angle 1 + m\angle 2 = 90^\circ \quad \text{Definition of right angle}$$

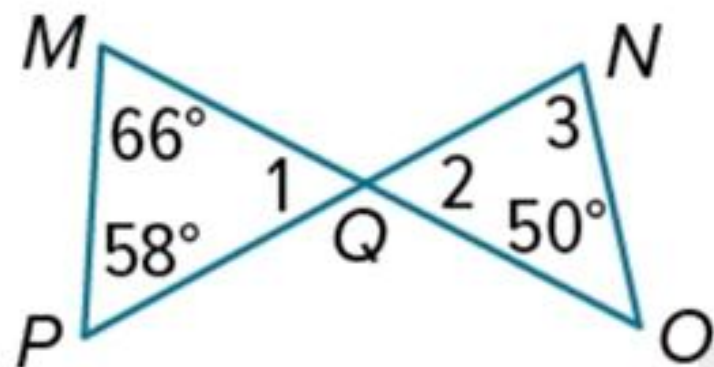
$$30^\circ + m\angle 2 = 90^\circ \quad \text{Simplify.}$$

$$m\angle 2 = 60^\circ \quad \text{Subtract } 30^\circ \text{ from each side.}$$

*ANSWER:*

$$m\angle 1 = 30^\circ; m\angle 2 = 60^\circ$$

2.



*SOLUTION:*

$$m\angle 1 + 66^\circ + 58^\circ = 180^\circ$$

Triangle Angle – Sum Theorem

$$m\angle 1 + 124^\circ = 180^\circ$$

Simplify.

$$m\angle 1 = 56^\circ$$

Subtract  $124^\circ$  from each side.

$$\angle 1 \cong \angle 2$$

Vertical Angles Theorem

$$m\angle 1 = m\angle 2$$

Definition of congruent

$$56^\circ = m\angle 2$$

Substitution

*ANSWER:*

$$m\angle 1 = 56^\circ; m\angle 2 = 56^\circ, m\angle 3 = 74^\circ$$

$$m\angle 2 + m\angle 3 + 50^\circ = 180^\circ$$

Triangle Angle – Sum Theorem

$$56^\circ + m\angle 3 + 50^\circ = 180^\circ$$

Substitution

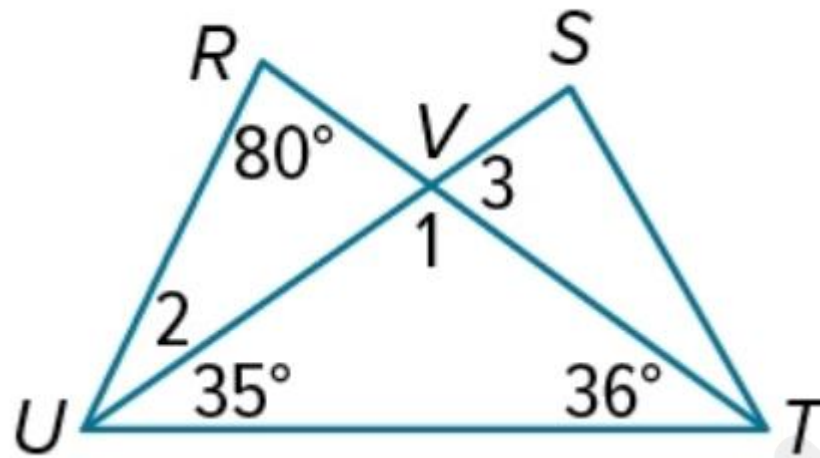
$$m\angle 3 + 106^\circ = 180^\circ$$

Simplify.

$$m\angle 3 = 74^\circ$$

Subtract  $106^\circ$  from each side.

3.



*SOLUTION:*

$$m\angle 1 + 35^\circ + 36^\circ = 180^\circ \quad \text{Triangle Angle – Sum Theorem}$$

$$m\angle 1 + 71^\circ = 180^\circ \quad \text{Simplify.}$$

$$m\angle 1 = 109^\circ \quad \text{Subtract } 71^\circ \text{ from each side.}$$

$$m\angle 1 + m\angle 3 = 180^\circ \quad \text{Defn linear pair and Supplement Theorem}$$

$$109^\circ + m\angle 3 = 180^\circ \quad \text{Substitution}$$

$$m\angle 3 = 71^\circ \quad \text{Subtract } 109^\circ \text{ from each side.}$$

$$\angle 3 \cong \angle RVU \quad \text{Vertical Angles Theorem}$$

$$m\angle 3 = m\angle RVU \quad \text{Definition of congruent}$$

$$71^\circ = m\angle RVU \quad \text{Substitution}$$

$$m\angle 2 + m\angle RVU + 80^\circ = 180^\circ \quad \text{Triangle Angle – Sum Theorem}$$

$$m\angle 2 + 71^\circ + 80^\circ = 180^\circ \quad \text{Substitution}$$

$$m\angle 2 + 151^\circ = 180^\circ \quad \text{Simplify.}$$

$$m\angle 2 = 29^\circ \quad \text{Subtract } 151^\circ \text{ from each side.}$$

*ANSWER:*

$$m\angle 1 = 109^\circ; m\angle 2 = 29^\circ, m\angle 3 = 71^\circ$$

4.



*SOLUTION:*

$$\angle 1 \cong \angle 2$$

Given

$$m\angle 1 = m\angle 2$$

Definition congruent angles

$$m\angle 1 + m\angle 2 + 146^\circ = 180^\circ$$

Triangle Angle – Sum Theorem

$$2m\angle 1 + 146^\circ = 180^\circ$$

Simplify.

$$2m\angle 1 = 34^\circ$$

Subtract  $146^\circ$  from each side.

$$m\angle 1 = 17^\circ$$

Divide each side by 2.

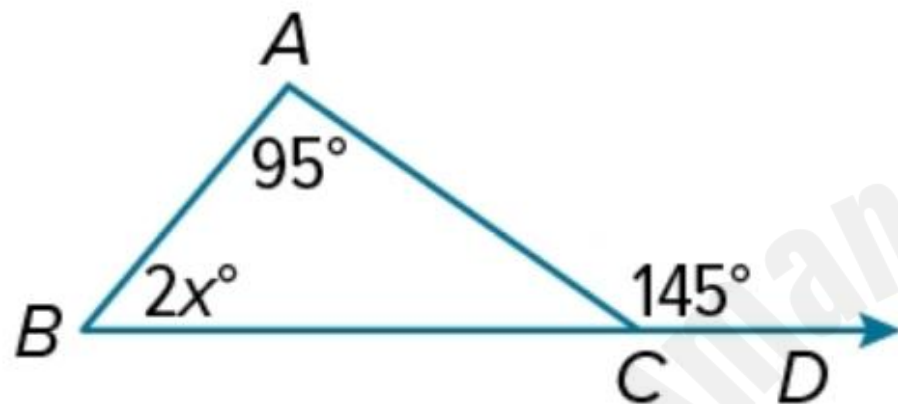
$$m\angle 2 = 17^\circ$$

Substitution

*ANSWER:*

$$m\angle 1 = m\angle 2 = 17^\circ$$

5.  $m\angle ABC$



*SOLUTION:*

$$2x^\circ + 95^\circ = 145^\circ$$

Exterior Angle Theorem

$$2x^\circ = 50^\circ$$

Subtract  $95^\circ$  from each side.

$$m\angle ABC = 2x^\circ$$

Given

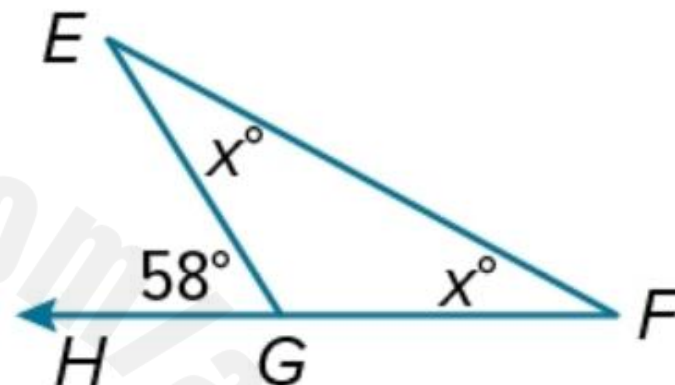
$$m\angle ABC = 50^\circ$$

Transitive Property of Equality

*ANSWER:*

$$50^\circ$$

6.  $m\angle F$



*SOLUTION:*

$$x^\circ + x^\circ = 58^\circ$$

Exterior Angle Theorem

$$2x^\circ = 58^\circ$$

Simplify.

$$x^\circ = 29^\circ$$

Divide each side by 2.

$$m\angle F = x^\circ$$

Given

$$m\angle F = 29^\circ$$

Transitive Property of Equality

*ANSWER:*

$$29^\circ$$





- 7. TOWERS A lookout tower sits on a network of struts and posts. Leslie measured three angles on the tower. If  $m\angle 1 = (7x - 7)^\circ$ ,  $m\angle 2 = (4x + 2)^\circ$ , and  $m\angle 3 = (2x + 6)^\circ$ , what is  $m\angle 1$ ?

*SOLUTION:*

$$m\angle 2 + m\angle 3 = m\angle 1$$

$$(4x + 2)^\circ + (2x + 6)^\circ = (7x - 7)^\circ$$

$$(6x + 8)^\circ = (7x - 7)^\circ$$

$$8^\circ = (x - 7)^\circ$$

$$15^\circ = x^\circ$$

$$m\angle 1 = (7x - 7)^\circ$$

$$m\angle 1 = (105 - 7)^\circ$$

$$m\angle 1 = 98^\circ$$

Exterior Angle Theorem

Substitution

Simplify.

Subtract  $6x^\circ$  from each side.

Add  $7^\circ$  to each side.

Given

Substitution

Simplify.

*ANSWER:*

$98^\circ$

8. GARDENING A gardener uses a grow light to grow vegetables indoors. If  $m\angle 1 = 8x^\circ$  and  $m\angle 2 = (7x - 4)^\circ$ , what is  $m\angle 1$ ?



*SOLUTION:*

$$m\angle 1 + m\angle 2 = 116^\circ$$

$$(8x)^\circ + (7x - 4)^\circ = 116^\circ$$

$$(15x - 4)^\circ = 116^\circ$$

$$15x^\circ = 120^\circ$$

$$8^\circ = x^\circ$$

$$m\angle 1 = (8x)^\circ$$

$$m\angle 1 = (8 \cdot 8)^\circ$$

$$m\angle 1 = 64^\circ$$

Exterior Angle Theorem

Substitution

Simplify.

Add  $4^\circ$  to each side.

Divide each side by 15.

Given

Substitution

Simplify.

*ANSWER:*

$64^\circ$

32. ANALYZE In  $\triangle ABC$ , if an exterior angle adjacent to  $\angle A$  is acute, is the triangle acute, right, or obtuse, or can its classification not be determined? Explain your reasoning.

ANSWER:

Obtuse; because the exterior angle is acute, the sum of the remote interior angles must be acute, which means that the third angle must be obtuse. Therefore, the triangle must be obtuse.

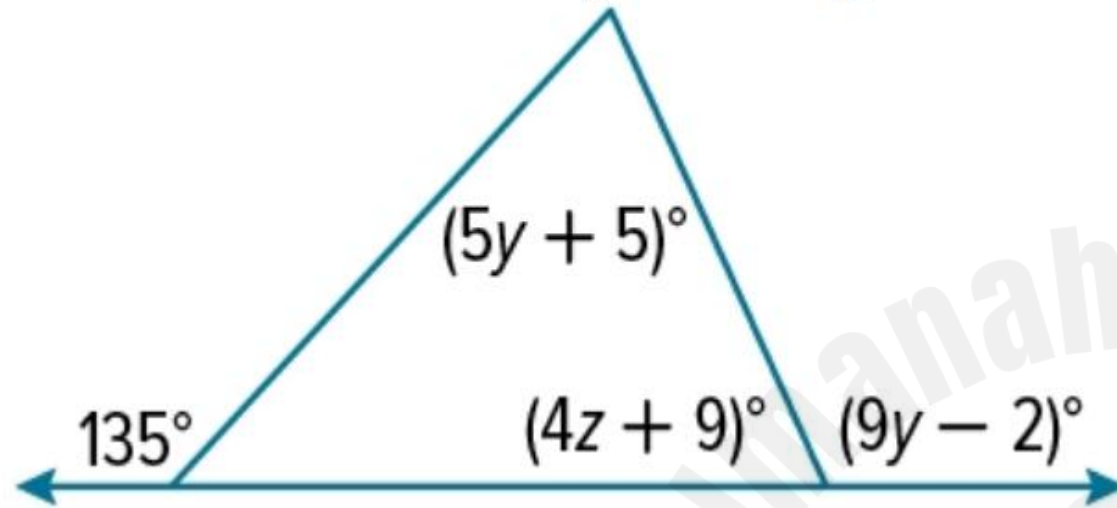
33. WRITE Explain why a triangle cannot have an obtuse, acute, and a right exterior angle.

ANSWER:

Sample answer: Because if an exterior angle is acute, the adjacent angle must be obtuse. Because if another exterior angle is right, the adjacent angle must be right. A triangle cannot contain a right angle and an obtuse angle because the sum would be greater than  $180^\circ$ . Therefore, a triangle cannot have an obtuse, an acute, and a right exterior angle.



34. **PERSEVERE** Find the values of  $y$  and  $z$  in the figure.



**SOLUTION:**

$$\begin{aligned}(5y + 5)^\circ + (4z + 9)^\circ &= 135^\circ && \text{Exterior Angle Theorem} \\(5y + 4z + 14)^\circ &= 135^\circ && \text{Simplify.} \\(5y + 4z)^\circ &= 121^\circ && \text{Subtract } 14^\circ \text{ from each side.}\end{aligned}$$

$$\begin{aligned}(9y - 2)^\circ + (4z + 9)^\circ &= 180^\circ && \text{Defn linear pair and Supplement Theorem} \\(9y + 4z + 7)^\circ &= 180^\circ && \text{Simplify.} \\(9y + 4z)^\circ &= 173^\circ && \text{Subtract } 7^\circ \text{ from each side.}\end{aligned}$$

Now you have two equations in two variables that can be solved by elimination using subtraction.

$$\begin{aligned}9y + 4z &= 173 \\(-) 5y + 4z &= 121 \\ \hline 4y &= 52 \\ y &= 13\end{aligned}$$

Subtract to eliminate  $4z$ .

Simplify.

Divide each side by 4 to solve for  $y$ .

$$\begin{aligned}9y + 4z &= 173 \\9(13) + 4z &= 173 \\117 + 4z &= 173 \\4z &= 56 \\z &= 14\end{aligned}$$

Original equation.

Substitute 13 for  $y$  original equation.

Simplify.

Subtract 117 from each side.

Divide each side by 4.

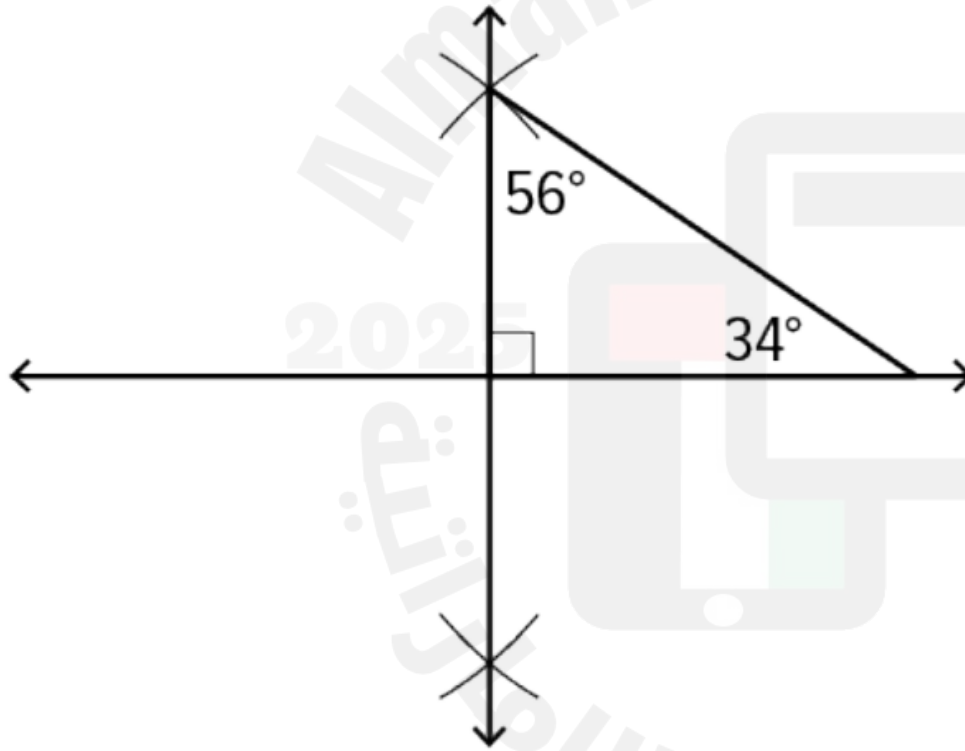
**ANSWER:**

$$y = 13, z = 14$$

35. **CREATE** Construct a right triangle and measure one of the acute angles. Calculate the measure of the second acute angle and explain your method. Confirm your result using a protractor.

*SOLUTION:*

You can find the measure of the second angle by subtracting the first angle from  $90^\circ$  because the acute angles of a right triangle are complementary.





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## 20. Learning Outcome/lesson Name: Use the SSS Postulate to test for triangle congruence

Example/Exercise: 1-6

**PROOF** Write the specified type of proof.

1. two-column proof

**Given:**  $\overline{AB} \cong \overline{XY}$ ,  $\overline{AC} \cong \overline{XZ}$ ,  $\overline{BC} \cong \overline{YZ}$

**Prove:**  $\triangle ABC \cong \triangle XYZ$

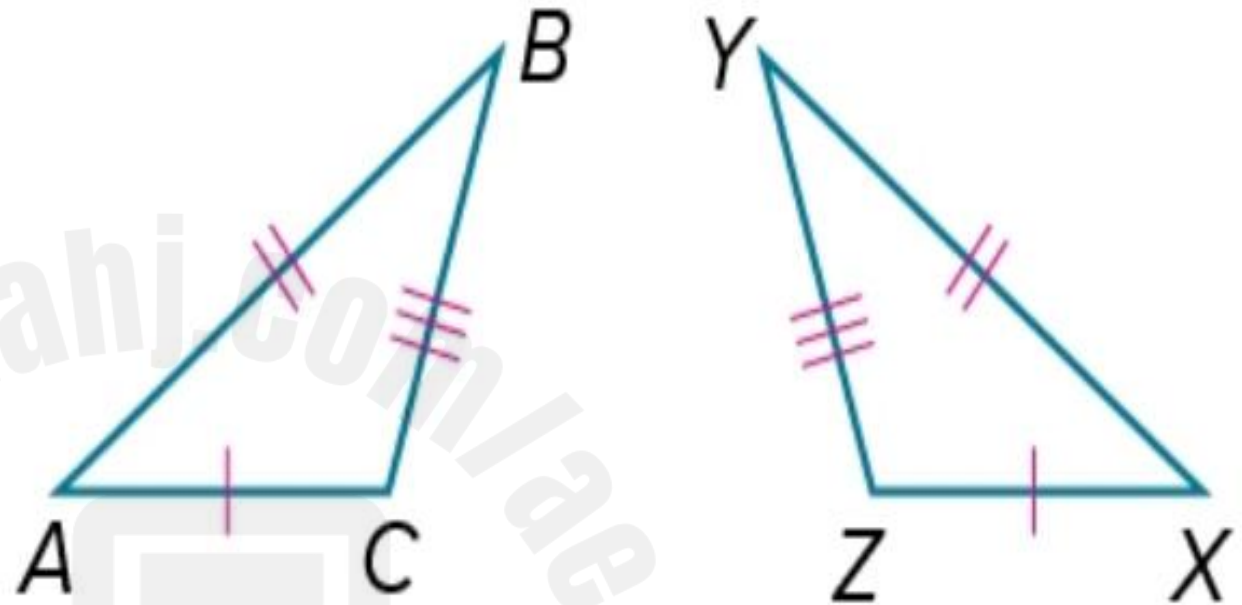
*SOLUTION:*

**Given:**  $\overline{AB} \cong \overline{XY}$ ,  $\overline{AC} \cong \overline{XZ}$ ,  $\overline{BC} \cong \overline{YZ}$

**Prove:**  $\triangle ABC \cong \triangle XYZ$

**Proof:**

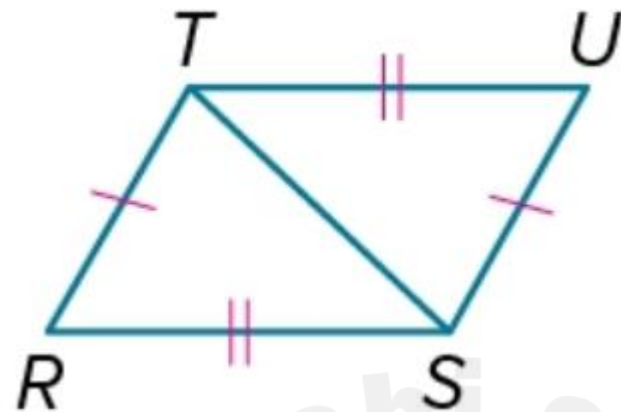
Statements	Reasons
1. $\overline{AB} \cong \overline{XY}$ , $\overline{AC} \cong \overline{XZ}$ , $\overline{BC} \cong \overline{YZ}$	1. Given
2. $\triangle ABC \cong \triangle XYZ$	2. SSS Postulate



2. flow proof

**Given:**  $\overline{RS} \cong \overline{UT}$ ,  $\overline{RT} \cong \overline{US}$

**Prove:**  $\triangle RST \cong \triangle UTS$



*SOLUTION:*

**Given:**  $\overline{RS} \cong \overline{UT}$ ,  $\overline{RT} \cong \overline{US}$

**Prove:**  $\triangle RST \cong \triangle UTS$

**Proof:**

$$\overline{RS} \cong \overline{UT}$$

Given

$$\overline{RT} \cong \overline{US}$$

Given

$$\overline{ST} \cong \overline{TS}$$

Reflexive Prop.

$$\triangle RST \cong \triangle UTS$$

SSS

3. two-column proof

**Given:**  $\overline{AB} \cong \overline{CD}$ ,  $D$  is the midpoint of  $\overline{AC}$ .

**Prove:**  $\triangle ABD \cong \triangle CBD$

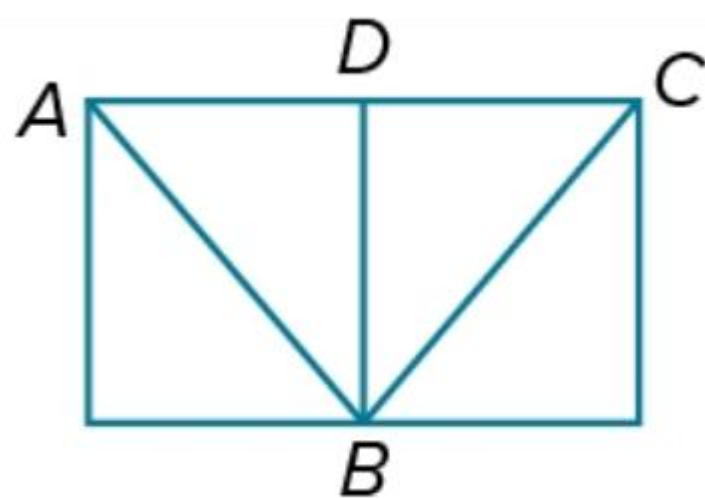
*SOLUTION:*

**Given:**  $\overline{AB} \cong \overline{CD}$ ,  $D$  is the midpoint of  $\overline{AC}$ .

**Prove:**  $\triangle ABD \cong \triangle CBD$

**Proof:**

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$ , $D$ is the midpoint of $\overline{AC}$ .	1. Given
2. $\overline{AD} \cong \overline{DC}$	2. Definition of midpoint
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive Property of Congruence
4. $\triangle ABD \cong \triangle CBD$	4. SSS Postulate

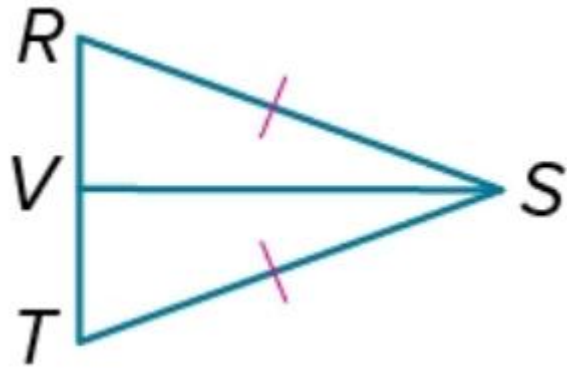


**Proof:**

4. flow proof

**Given:**  $\overline{RS} \cong \overline{TS}$ ,  $V$  is the midpoint of  $\overline{RT}$

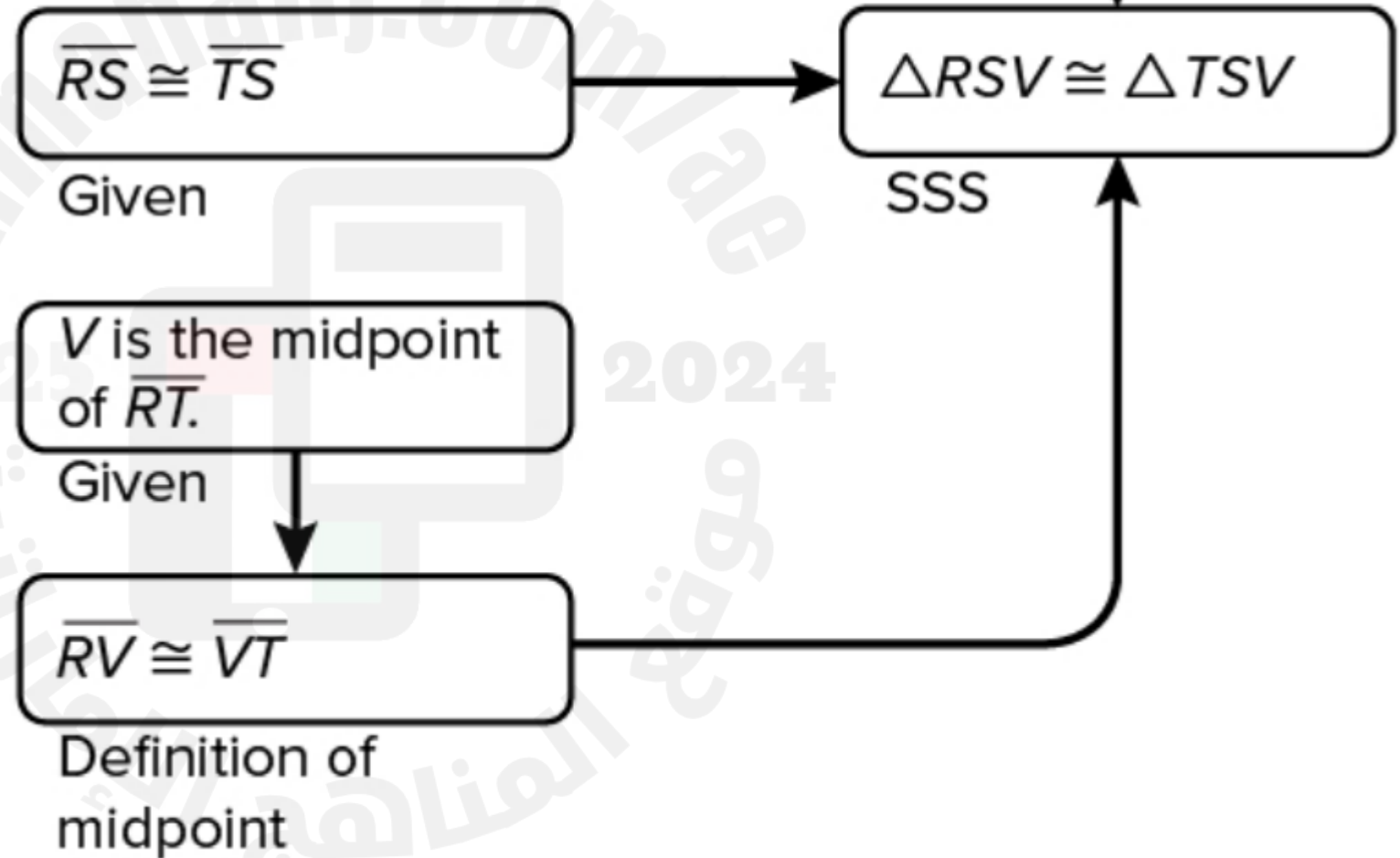
**Prove:**  $\triangle RSV \cong \triangle TSV$



**SOLUTION:**

**Given:**  $\overline{RS} \cong \overline{TS}$ ,  $V$  is the midpoint of  $\overline{RT}$ .

**Prove:**  $\triangle RSV \cong \triangle TSV$





5. paragraph proof

**Given:**  $\overline{QR} \cong \overline{SR}$ ,  $\overline{ST} \cong \overline{QT}$

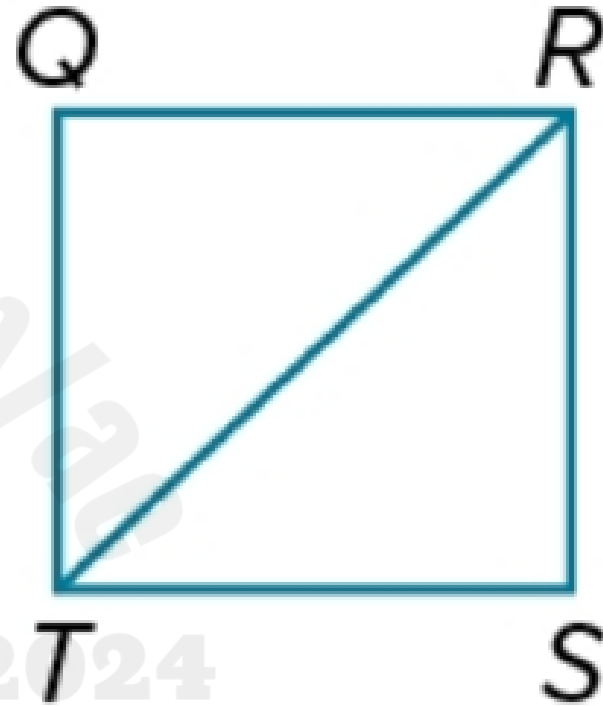
**Prove:**  $\triangle QRT \cong \triangle SRT$

*SOLUTION:*

paragraph proof

**Given:**  $\overline{QR} \cong \overline{SR}$ ,  $\overline{ST} \cong \overline{QT}$

**Prove:**  $\triangle QRT \cong \triangle SRT$



**Proof:** We are given that  $\overline{QR} \cong \overline{SR}$  and  $\overline{ST} \cong \overline{QT}$ . We can also state  $\overline{RT} \cong \overline{RT}$  by the Reflexive Property. Because  $\overline{QR} \cong \overline{SR}$ ,  $\overline{ST} \cong \overline{QT}$ , and  $\overline{RT} \cong \overline{RT}$ ,  $\triangle QRT \cong \triangle SRT$  by SSS.



6. two-column proof

**Given:**  $\overline{AB} \cong \overline{ED}$ ,  $\overline{CA} \cong \overline{CE}$ ,  $\overline{AC}$  bisects  $\overline{BD}$ .

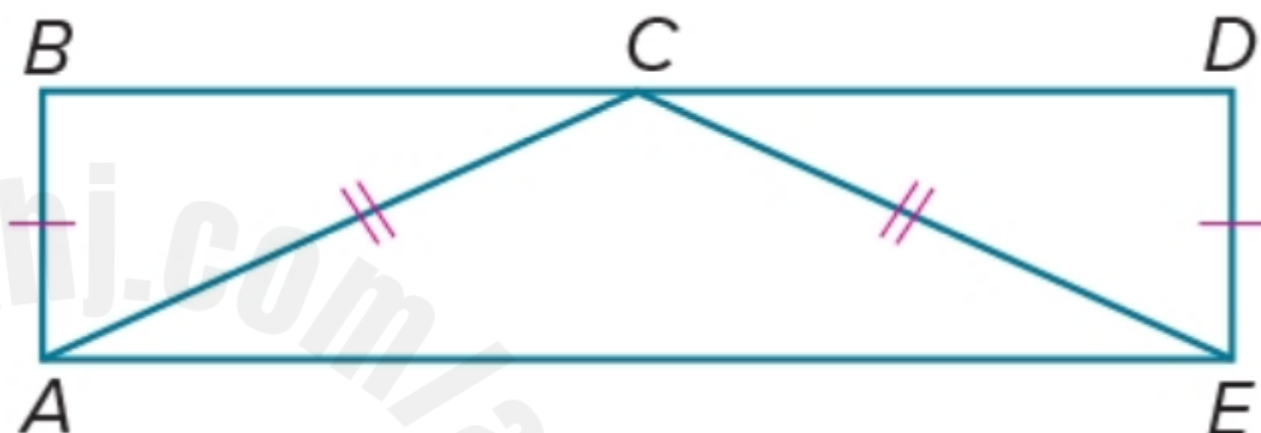
**Prove:**  $\triangle ABC \cong \triangle EDC$

*SOLUTION:*

**Given:**  $\overline{AB} \cong \overline{ED}$ ,  $\overline{CA} \cong \overline{CE}$ ,  $\overline{AC}$  bisects  $\overline{BD}$ .

**Prove:**  $\triangle ABC \cong \triangle EDC$

**Proof:**



Statements	Reasons
1. $\overline{AB} \cong \overline{ED}$ , $\overline{CA} \cong \overline{CE}$ , $\overline{AC}$ bisects $\overline{BD}$ .	1. Given
2. $C$ is the midpoint of $\overline{BD}$ .	2. Definition of segment bisector
3. $\overline{BC} \cong \overline{CD}$	3. Midpoint Theorem
4. $\triangle ABC \cong \triangle EDC$	4. SSS Postulate