# نموذج أسئلة امتحانية وفق الهيكل

## Q1

**In exercises 3–6, find all critical numbers by hand. Use your knowledge of the type of graph (e.g., parabola or cubic) to determine whether the critical number represents a local maximum, local minimum or neither.**

3. (a) $f(x) = x^2 + 5x - 1$ (b) $f(x) = -x^2 + 4x + 2$

4. (a) $f(x) = x^3 - 3x + 1$ (b) $f(x) = -x^3 + 6x^2 + 2$

5. (a) $f(x) = x^3 - 3x^2 + 6x$ (b) $f(x) = -x^3 + 3x^2 - 3x$



6. (a) $f(x) = x^4 - 2x^2 + 1$ (b) $f(x) = x^4 - 3x^3 + 2$

## Q2+3



**In exercises 7–24, find all critical numbers by hand. If available, use graphing technology to determine whether the critical number represents a local maximum, local minimum or neither.**

7. $f(x) = x^4 - 3x^3 + 2$
8. $f(x) = x^4 + 6x^2 - 2$
9. $f(x) = x^{3/4} - 4x^{1/4}$
10. $f(x) = (x^{2/5} - 3x^{1/5})^2$
11. $f(x) = \sin x \cos x, [0, 2\pi]$
12. $f(x) = \sqrt{3} \sin x + \cos x$
13. $f(x) = \dfrac{x^2 - 2}{x + 2}$
14. $f(x) = \dfrac{x^2 - x + 4}{x - 1}$
15. $f(x) = \frac{1}{2}(e^x + e^{-x})$
16. $f(x) = xe^{-2x}$
17. $f(x) = x^{4/3} + 4x^{1/3} + 4x^{-2/3}$
18. $f(x) = x^{7/3} - 28x^{1/3}$
19. $f(x) = 2x\sqrt{x + 1}$
20. $f(x) = x/\sqrt{x^2 + 1}$
21. $f(x) = |x^2 - 1|$
22. $f(x) = \sqrt[3]{x^3 - 3x^2}$

**Q4**

**In exercises 9–14, find all critical numbers and use the Second Derivative Test to determine all local extrema.**

9. $f(x) = x^4 + 4x^3 - 1$
10. $f(x) = x^4 + 4x^2 + 1$
11. $f(x) = xe^{-x}$
12. $f(x) = e^{-x^2}$
13. $f(x) = \dfrac{x^2 - 5x + 4}{x}$
14. $f(x) = \dfrac{x^2 - 1}{x}$

**Q5**



**In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.**

11. $f(x) = x + \sin x$
12. $f(x) = \sin x - \cos x$
13. $f(x) = \quad x$
14. $f(x) = x1\ x^2$



15. $f(x) = \sqrt{x^2 + 1}$
16. $f(x) = \overline{x - 1}$
17. $f(x) = \overline{x^3 - 3x^2 + 2x}$
18. $f(x) = \overline{x^3 - 3x^2 + 2x}$
19. $f(x) = x^{5/3} - 5x^{2/3}$
20. $f(x) = x^3 - \dfrac{3}{400}x$
21. $f(x) = e^{-2/x}$
22. $f(x) = e^{1/x^2}$

**Q6**

**In exercises 1–22, graph the function and completely discuss the graph as in example 6.2.**

1. $f(x) = x^3 - 3x^2 + 3x$
2. $f(x) = x^4 - 3x^2 + 2$
3. $f(x) = x^5 - 2x^3 + 1$
4. $f(x) = x^4 + 4x^3 - 1$
5. $f(x) = x + \frac{4}{\;}$
6. $f(x) = \frac{x^2 \quad 1}{\;}$
7. $f(x) = \frac{x^2 \quad 4}{x^3}$
8. $f(x) = \frac{x \quad 4}{x^3}$
9. $f(x) = \frac{2x}{x^2 - 1}$
10. $f(x) = \frac{3x^2}{x^2 + 1}$



**Q7+8**



**In exercises 5–28, find the general antiderivative.**

5. $\int (3x^4 - 3x)\, dx$
6. $\int (x^3 - 2)\, dx$
7. $\int \left(3\sqrt{x} - \frac{1}{x^4}\right) dx$
8. $\int \left(2x^{-2} + \frac{1}{\sqrt{x}}\right) dx$
9. $\int \frac{x^{1/3} - 3}{x^{2/3}}\, dx$
10. $\int \frac{x + 2x^{3/4}}{x^{5/4}}\, dx$
11. $\int (2 \sin x + \cos x)\, dx$
12. $\int (3 \cos x - \sin x)\, dx$
13. $\int 2 \sec x \tan x\, dx$
14. $\int \frac{4}{\sqrt{1 - x^2}}\, dx$
15. $\int 5 \sec^2 x\, dx$
16. $\int 4 \frac{\cos x}{\sin^2 x}\, dx$
17. $\int (3e^x - 2)\, dx$
18. $\int (4x - 2e^x)\, dx$
19. $\int (3 \cos x - 1/x)\, dx$
20. $\int (2x^{-1} + \sin x)\, dx$

**Q9**

21. $\int \frac{4x}{x^2+4}\,dx$

22. $\int \frac{3}{4x^2+4}\,dx$

23. $\int \frac{\cos x}{\sin x}\,dx$

24. $\int \left(2\cos x - \sqrt{e^{2x}}\right)dx$

25. $\int \frac{e^x}{e^x+3}\,dx$

26. $\int \frac{e^x+3}{e^x}\,dx$

27. $\int x^{1/4}(x^{5/4}-4)\,dx$

28. $\int x^{2/3}(x^{-4/3}-3)\,dx$

**Q10**



**In exercises 9–14, evaluate the integral by computing the limit of Riemann sums.**

9. $\int_0^1 2x\,dx$

10. $\int_1^2 2x\,dx$

11. $\int_0^2 x^2\,dx$

12. $\int_0^3 (x^2+1)\,dx$

13. $\int_1^3 (x^2-3)\,dx$

14. $\int_{-2}^2 (x^2-1)\,dx$

**Q11**

**In exercises 3–6, find all critical numbers by hand. Use your knowledge of the type of graph (e.g., parabola or cubic) to determine whether the critical number represents a local maximum, local minimum or neither.**

**3.** (a) $f(x) = x^2 + 5x - 1$ (b) $f(x) = -x^2 + 4x + 2$

**4.** (a) $f(x) = x^3 - 3x + 1$ (b) $f(x) = -x^3 + 6x^2 + 2$

**5.** (a) $f(x) = x^3 - 3x^2 + 6x$ (b) $f(x) = -x^3 + 3x^2 - 3x$



**6.** (a) $f(x) = x^4 - 2x^2 + 1$ (b) $f(x) = x^4 - 3x^3 + 2$

**Q12**



**In exercises 7–24, find all critical numbers by hand. If available, use graphing technology to determine whether the critical number represents a local maximum, local minimum or neither.**

**7.** $f(x) = x^4 - 3x^3 + 2$

**8.** $f(x) = x^4 + 6x^2 - 2$

**9.** $f(x) = x^{3/4} - 4x^{1/4}$

**10.** $f(x) = (x^{2/5} - 3x^{1/5})^2$

**11.** $f(x) = \sin x \cos x, [0, 2\pi]$

**12.** $f(x) = \sqrt{3} \sin x + \cos x$

**Q13**

**In exercises 11–20, find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.**

11. $y = x^4 + 4x^3 - 2$
12. $y = x^5 - 5x^2 + 1$
13. $y = xe^{-2x}$
14. $y = x^2e^{-x}$
15. $y = \tan^{-1}(x^2)$
16. $y = \sin^{-1}\left(1 - \frac{1}{x^2}\right)$
17. $y = \dfrac{x}{1+x^3}$
18. $y = \dfrac{x}{1+x^4}$
19. $y = \sqrt{x^3 + 3x^2}$
20. $y = x^{4/3} + 4x^{1/3}$



**Q14**

**In exercises 1–8, determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points.**

1. $f(x) = x^3 - 3x^2 + 4x - 1$
2. $f(x) = x^4 - 6x^2 + 2x + 3$
3. $f(x) = x + 1/x$
4. $f(x) = x + 3(1 - x)^{1/3}$
5. $f(x) = \sin x - \cos x$
6. $f(x) = \tan^{-1}(x^2)$
7. $f(x) = x^{4/3} + 4x^{1/3}$
8. $f(x) = xe^{-4x}$

**Q15**

**In exercises 22–19, compute sums of the form $\sum_{i=1}^{n} f(x_i)\Delta x$ for the given values of $x_i$.**

**19.** $f(x) = x^2 + 4x$; $x = 0.2, 0.4, 0.6, 0.8, 1.0$; $\Delta x = 0.2$; $n = 5$

**20.** $f(x) = 3x + 5$; $x = 0.4, 0.8, 1.2, 1.6, 2.0$; $\Delta x = 0.4$; $n = 5$

**21.** $f(x) = 4x^2 - 2$; $x = 2.1, 2.2, 2.3, 2.4, \ldots, 3.0$; $\Delta x = 0.1$; $n = 10$

**22.** $f(x) = x^3 + 4$; $x = 2.05, 2.15, 2.25, 2.35, \ldots, 2.95$; $\Delta x = 0.1$; $n = 10$



**Q16**

**In exercises 35–38, use the given function values to estimate the area under the curve using left-endpoint and right-endpoint evaluation.**

**35.**

| $x$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 2.0 | 2.4 | 2.6 | 2.7 | 2.6 | 2.4 | 2.0 | 1.4 | 0.6 |

**36.**

| $x$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 2.0 | 2.2 | 1.6 | 1.4 | 1.6 | 2.0 | 2.2 | 2.4 | 2.0 |

**37.**

| $x$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1.8 | 1.4 | 1.1 | 0.7 | 1.2 | 1.4 | 1.8 | 2.4 | 2.6 |

**38.**

| $x$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.0 | 0.4 | 0.6 | 0.8 | 1.2 | 1.4 | 1.2 | 1.4 | 1.0 |

**Q17**

**In exercises 15–20, write the given (total) area as an integral or sum of integrals.**

15. The area above the $x$-axis and below $y = 4 - x^2$

16. The area above the $x$-axis and below $y = 4x - x^2$

17. The area below the $x$-axis and above $y = x^2 - 4$

18. The area below the $x$-axis and above $y = x^2 - 4x$

19. The area between $y = \sin x$ and the $x$-axis for $0 \leq x \leq \pi$

20. The area between $y = \sin x$ and the $x$-axis for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{4}$.



**Q18**

**In exercises 29–32, use the Integral Mean Value Theorem to estimate the value of the integral.**

29. $\int_{\pi/3}^{\pi/2} 3\cos x^2 \, dx$

30. $\int_0^{1/2} e^{-x^2} \, dx$

31. $\int_0^2 \sqrt{2x^2+1} \, dx$

32. $\int_{-1}^1 \frac{3}{x^3+2} \, dx$

**In exercises 33 and 34, find a value of $c$ that satisfies the conclusion of the Integral Mean Value Theorem.**

33. $\int_0^2 3x^2 \, dx \, (= 8)$

34. $\int_{-1}^1 (x^2 - 2x) \, dx \, (= \frac{2}{3})$

**Q19**

In exercises 37 and 38, assume that $\int_1^3 f(x)\,dx = 3$ and $\int_1^3 g(x)\,dx = -2$ and find

37. (a) $\int_1^3 [f(x) + g(x)]\,dx$ (b) $\int_1^3 [2f(x) - g(x)]\,dx$

**Q20**



In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

5. $\int_1^4 \left(x\sqrt{x} + \frac{3}{x}\right) dx$

6. $\int_1^2 \left(4x - \frac{2}{x^2}\right) dx$

7. $\int_0^1 (6e^{-3x} + 4)\,dx$

8. $\int_0^2 \left(\frac{e^{2x} - 2e^{3x}}{e^{3x}}\right) dx$

9. $\int_{\pi/2}^{\pi} (2\sin x - \cos x)\,dx$

10. $\int_{\pi/4}^{\pi/2} 3\csc x \cot x\,dx$

11. $\int_0^{\pi/4} \sec t \tan t\,dt$

12. $\int_0^{\pi/4} \sec^2 t\,dt$

13. $\int_0^{1/2} \frac{3}{\sqrt{1 - x^2}}\,dx$

14. $\int_{-1}^{1} \frac{4}{1 + x^2}\,dx$

15. $\int_1^4 \frac{t - 3}{t}\,dt$

16. $\int_0^4 t(t - 2)\,dt$

17. $\int_0^t (e^{x/2})^2\,dx$

18. $\int_0^t (\sin^2 x + \cos^2 x)\,dx$

## Q21

**In exercises 5–30, evaluate the indicated integral.**

5. $\int x^3\sqrt{x^4+3}\,dx$

6. $\int \sqrt{1+10x}\,dx$

7. $\int \frac{\sin x}{\sqrt{\cos x}}\,dx$

8. $\int \sin^3 x\cos x\,dx$

9. $\int t^2\cos t^3\,dt$

10. $\int \sin t(\cos t+3)^{3/4}\,dt$

11. $\int xe^{x^2+1}\,dx$

12. $\int e^x\sqrt{e^x+4}\,dx$

13. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

14. $\int \frac{\cos(1/x)}{x^2}\,dx$



## Q22

15. $\int \frac{\sqrt{\ln x}}{x}\,dx$

16. $\int \sec^2 x\sqrt{\tan x}\,dx$

17. $\int \frac{1}{\sqrt{u}\,(\sqrt{u}+1)}\,du$

18. $\int \frac{v}{v^2+4}\,dv$

19. $\int \frac{4}{x(\ln x+1)^2}\,dx$

20. $\int \tan 2x\,dx$

21. $\int \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}\,dx$

22. $\int x^2\sec^2 x^3\,dx$

23. (a) $\int \frac{x}{\sqrt{1-x^4}}\,dx$ (b) $\int \frac{x^3}{\sqrt{1-x^4}}\,dx$

24. (a) $\int \frac{x^2}{1+x^6}\,dx$ (b) $\int \frac{x^5}{1+x^6}\,dx$

25. (a) $\int \frac{1+x}{1+x^2}\,dx$ (b) $\int \frac{1+x}{1-x^2}\,dx$

26. (a) $\int \frac{3\sqrt{x}}{1+x^3}\,dx$ (b) $\int \frac{x\sqrt{x}}{1+x^5}\,dx$

**Q23**

1. A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. The enclosed area is to equal 1800 $ft^2$. Find the minimum perimeter and the dimensions of the corresponding enclosure.

2. A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. There is 96 feet of fencing available. Find the maximum enclosed area and the dimensions of the corresponding enclosure.

3. A two-pen corral is to be built. The outline of the corral forms two identical adjoining rectangles. If there is 120 ft of fencing available, what dimensions of the corral will maximize the enclosed area?

4. A showroom for a department store is to be rectangular with walls on three sides, 6-ft door openings on the two facing sides and a 10-ft door opening on the remaining wall. The showroom is to have 800 $ft^2$ of floor space. What dimensions will minimize the length of wall used?

5. Show that the rectangle of maximum area for a given perimeter $P$ is always a square.

6. Show that the rectangle of minimum perimeter for a given area $A$ is always a square.

7. A box with no top is to be built by taking a 6 in-by-10 in sheet of cardboard, cutting $x$-in squares out of each corner and folding up the sides. Find the value of $x$ that maximizes the volume of the box.

**Q24**

45. Determine the position function if the velocity function is $v(t) = 3 - 12t$ and the initial position is $s(0) = 3$.

46. Determine the position function if the velocity function is $v(t) = 3e^{-t} - 2$ and the initial position is $s(0) = 0$.

47. Determine the position function if the acceleration function is $a(t) = 3 \sin t + 1$, the initial velocity is $v(0) = 0$ and the initial position is $s(0) = 4$.

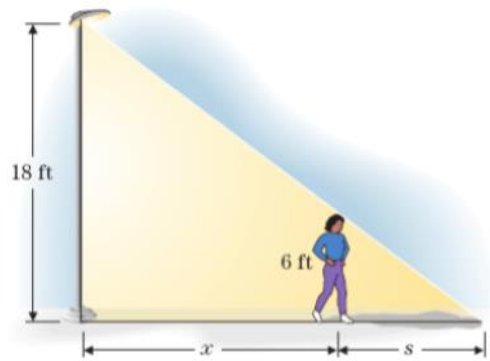
48. Determine the position function if the acceleration function is $a(t) = t^2 + 1$, the initial velocity is $v(0) = 4$ and the initial position is $s(0) = 0$.



49. Sketch the graph of two functions $f(x)$ corresponding to the given graph of $y = f'(x)$.

## Q25

19. Suppose a 6 ft-tall person is 12 ft away from an -18 ft-tall lamppost (see the figure). (a) If the person is moving away from he lamppost at a rate of 2 ft/s $^2$ at what rate is the length of the shadow changing? (Hint: Show that $\frac{x+s}{18} = \frac{s}{6}$.) (b) Repeat with the person 6 ft away from the lamppost and walking toward the lamppost at a rate of 3 ft/s.



Exercise 19

20. Boyle's law for a gas at constant temperature is $PV = c$, where $P$ is pressure, $V$ is volume and $c$ is a constant. Assume that both $P$ and $V$ are functions of time. (a) Show that $P'(t)/V'(t) = -c/V^2$. (b) Solve for $P$ as a function of $V$. Treating $V$ as an independent variable, compute $P'(V)$. Compare $P'(V)$ and $P'(t)/V'(t)$ from parts (a) and (b).

21. A dock is 6 ft above water. Suppose you stand on the edge of the dock and pull a rope attached to a boat at the constant rate of 2 ft/s. Assume that the boat remains at water level. At what speed is the boat approaching the dock when it is 20 feet from the dock? 10 feet from the dock? Isn't it surprising that the boat's speed is not constant?

22. Sand is poured into a conical pile with the height of the pile equalling the diameter of the pile. If the sand is poured at a constant rate of 5 $m^3/s$, at what rate is the height of the pile increasing when the height is 2 meters?



23. The frequency at which a guitar string vibrates (which determines the pitch of the note we hear) is related to the tension $T$ to which the string is tightened, the density $\rho$ of the string and the effective length $L$ of the string by the equation $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$. By running his finger along a string, a guitarist can change $L$ by changing the distance between the bridge and his finger. Suppose that $L = \frac{1}{2}$ ft and $\sqrt{\frac{T}{\rho}} = 220$ ft/s so that the units of $f$ are Hertz (cycles per second). If the guitarist's hand slides so that $L'(t) = -4$, find $f'(t)$. At this rate, how long will it take to raise the pitch one octave (that is, double $f$)?

24. Suppose that you are blowing up a balloon by adding air at the rate of 1 $ft^3/s$. If the balloon maintains a spherical shape, the volume and radius are related by $V = \frac{4}{3}\pi r^3$. Compare the rate at which the radius is changing when $r = 0.01$ ft versus when $r = 0.1$ ft. Discuss how this matches the experience of a person blowing up a balloon.

25. Water is being pumped into a spherical tank of radius 60 feet at the constant rate of 10 $ft^3/s$. (a) Find the rate at which the radius of the top level of water in the tank changes when the tank is half full. (b) Find the height at which the height of the water in the tank changes at the same rate as the radius.

26. Sand is dumped such that the shape of the sandpile remains a cone with height equal to twice the radius. (a) If the sand is dumped at the constant rate of 20 $ft^3/s$, find the rate at which the radius is increasing when the height reaches 6 feet.