ملخص الدرس الخامس test derivative second the and Concavity من الوحدة الرابعة تطبيقات التفاضل منهج ريفيل

Math lessons

Simplification

اللهم إني أسألك فَهْمَ النَّبِيّين وَحِفْظَ الْمُرْسَلِينَ، وَالْمَلَائِكَةَ الْمُقَرَّبِينَ، اللهم اجعل لِسَانِي عَامِرًا بِذِكْرِكَ، وَقَلْبِي بِخَشْيَتِكَ، وَسِرِّي بِطَاعَتِكَ، إِنَّكَ عَلَىٰ كُلِّ شَيْءٍ قَدِيرٌ، وَحَسْبِيَ اللَّهُ وَنِعْمَ الْوَكِيلُ

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Some Rules from Term 1 You Should Know::

Chain Rule

$$\frac{d}{dx}f\big(g(x)\big) = f'\big(g(x)\big)\cdot g'(x)$$

Power Rule

$$\frac{d}{dx}x^n = nx^{n-1}$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Function	
Polynomials	Continuous Everywhere
$\sin x, \cos x, \tan^{-1}x$	Continuous Everywhere
$\sqrt[n]{x}$	Continuous Everywhere when n is <u>odd</u>
$\sqrt[n]{x}$	Continuous for $x > 0$ when n is <u>even</u>
$\ln x$	Continuous for $x > 0$
$\sin^{-1}x, \cos^{-1}x$	Continuous for $-1 < x < 1$

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Chpt 4: Applications of differentiation

Lesson 4.5: Concavity and the Second Derivative Test

DEFINITION 5.1

For a function f that is differentiable on an interval I, the graph of f is

(i) **concave up** on I if f' is increasing on I.
(ii) **concave down** on I if f' is decreasing on I.

THEOREM 5.1

Suppose that f'' exists on an interval I.

(i) If $f''(x) > 0$ on I, then the graph of f is concave up on I.
(ii) If $f''(x) < 0$ on I, then the graph of f is concave down on I.

** $f(x)=$..... original function

$f'(x)=$..... first derivative

$f''(x)=$..... second derivative

(i) **concave up** on I if f' is increasing on I.

(i) If $f''(x) > 0$ on I, then the graph of f is concave up on I.

In general, if the second derivative is positive, this means that the first derivative is increasing, and the original function is concave up.

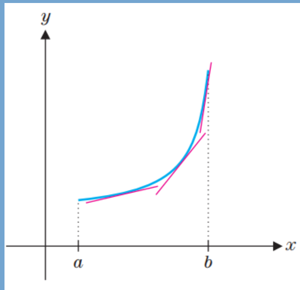

FIGURE 4.54a
Concave up, increasing

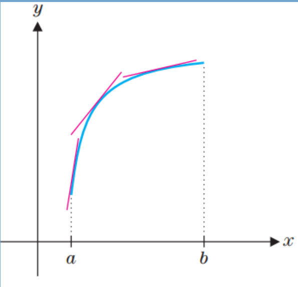

FIGURE 4.54b
Concave down, increasing

(ii) **concave down** on I if f' is decreasing on I.

(ii) If $f''(x) < 0$ on I, then the graph of f is concave down on I.

here if the second derivative is negative, this means that the first derivative is decreasing, and the original function is concave down.

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DEFINITION 5.2

Suppose that f is continuous on the interval (a, b) and that the graph changes concavity at a point $c \in (a, b)$ (i.e., the graph is concave down on one side of c and concave up on the other). Then, the point $(c, f(c))$ is called an **inflection point** of f.

**An inflection point is a point on a curve where the curve changes direction from being concave up to concave down, or vice versa. At an inflection point, the curve does not have a local maximum or minimum, but it changes its curvature.

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LET'S APPLY WHAT WE'VE LEARNED

DETERMINE THE INTERVALS WHERE THE GRAPH OF THE GIVEN FUNCTION IS CONCAVE UP AND CONCAVE DOWN, AND IDENTIFY INFLECTION POINTS. $f(x) = x^4 - 6x^2 + 2x + 3$

step 1: Determine the domain of the function.

since it's a polynomial function in this qn the domain will be $(-\infty,\infty)$

step 2: Find the first derivative of the original function.

$f'(x) = 4x^3 - 12x + 2$

step 3: Find the second derivative of the original function.

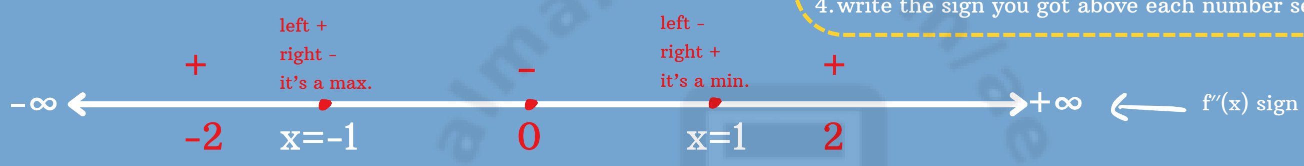
$f''(x) = 12x^3 - 12$

step 4: equate the second derivative function to 0 and find the value of x.

$f''(x) = 0 \;--->\; 12x^3 - 12 = 0$

$x=1$, $x=-1$

1. Draw a number line and locate both values of x you found.
2. take numbers close to both values
3. substitute the numbers you took in the f″(x) function.
4. write the sign you got above each number separately.



f(x) is concave up between the intervals $(-\infty,-1)\cup(2,\infty)$

f(x) is concave down between the interval $(-1,1)$

step 5: Identify inflection points

*substitute the value on the number line where the sign changes in the original function.

$f(-1) = x^4 - 6x^2 + 2x + 3 = -4$ (-1,-4)

$f(1) = x^4 - 6x^2 + 2x + 3 = 0$ (1,0)

Summary of the steps::

- step 1: Determine the domain of the function.
- step 2: Find the first derivative of the original function.
- step 3: Find the second derivative.
- step 4: equate the resulting function to 0 and find the value of x.
- step 5: Find the intervals where the function is increasing and decreasing by using a number line.
- step 6: Identify inflection points by substituting the values on the number line where the sign changes in the original function.

READ THE STEPS CAREFULLY AND ATTENTIVELY. ONCE YOU ARE SURE YOU UNDERSTAND, GO TO THE NEXT SLIDE TO SOLVE ANOTHER QUESTION ON YOUR OWN.

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CHECK YOUR UNDERSTANDING

DETERMINE THE INTERVALS WHERE THE GRAPH OF THE GIVEN FUNCTION IS CONCAVE UP AND CONCAVE DOWN, AND IDENTIFY INFLECTION POINTS. $f(x) = x^4$

step 1: Determine the domain of the function.

since it's a polynomial function in this qn the domain will be $(-\infty,\infty)$

step 2: Find the first derivative of the original function.

$f'(x)= 4x^3$

step 3: Find the second derivative of the original function.

$f''(x)= 12x^2$

step 4: equate the second derivative function to 0 and find the value of x.

$f''(x)=0$ ---> $12x^2 = 0$

$x=0$

1. Draw a number line and locate both values of x you found.
2. take numbers close to 0
3. substitute the numbers you took in the $f''(x)$ function.
4. write the sign you got above each number separately.

f(x) is concave up between the intervals $(-\infty,\infty)$

~~$(-\infty,0)\cup(0,\infty)$~~
this is wrong because f(x) is defined at x=0

step 5: Identify inflection points

*substitute the value on the number line where the sign changes in the original function.

No inflection points.. Because there is no sign change..

READ THE STEPS CAREFULLY AND ATTENTIVELY. ONCE YOU ARE SURE YOU UNDERSTAND, GO TO THE NEXT SLIDE TO SOLVE ANOTHER QUESTION ON YOUR OWN.

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OTHER IDEA

DETERMINE THE INTERVALS WHERE THE GRAPH OF THE GIVEN FUNCTION IS CONCAVE UP AND CONCAVE DOWN, AND IDENTIFY INFLECTION POINTS. $f(x) = x + \frac{25}{x}$, $x \neq 0$

step 1: Determine the domain of the function.

since $x \neq 0$, the domain will be $(-\infty, 0) \cup (0, \infty)$

step 2: Find the first derivative of the original function.

$$f'(x) = 1 - \frac{25}{x^2}$$

step 3: Find the second derivative of the original function.

$$f''(x) = \frac{50}{x^3}$$

step 4: equate the second derivative function to 0 and find the value of x.

$$f''(x) = \frac{50}{x^3}$$, $f''(x) \neq 0$

$f''(x)$ is undefined at x=0

1. Draw a number line and locate both values of x you found.
2. take numbers close to 0
3. substitute the numbers you took in the $f''(x)$ function.
4. write the sign you got above each number separately.

f(x) is concave up between the intervals $(0, \infty)$

f(x) is concave down between the interval $(-\infty, 0)$

step 5: Identify inflection points

*substitute the value on the number line where the sign changes in the original function.

There is a sign change on the number line at 0, BUT WE CAN'T SAY THAT THERE IS AN INFLECTION POINT AT 0 because the function is undefined at x=0..

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THEOREM 5.2 (Second Derivative Test)

Suppose that f'' is continuous on the interval (a, b) and $f'(c) = 0$, for some number $c \in (a, b)$.

(i) If $f''(c) < 0$, then $f(c)$ is a local maximum.
(ii) If $f''(c) > 0$, then $f(c)$ is a local minimum.

LAST QN

Use the Second Derivative Test to find the local extrema of $f(x) = x^4 - 8x^2 + 10$.

step 1: Determine the domain of the function.

since it's a polynomial function in this qn the domain will be (-∞,∞)

step 2: Find the first derivative of the original function.

$f'(x)= 4x^3 - 16x$

step 3: Find the second derivative of the original function.

$f''(x)= 12x^2 - 16$

step 4: equate the first derivative function to 0 and find the value of x.

$f''(x)=0$ ---> $4x^2 - 16x = 0$

x=0, x=2, x=-2

step 5: Find both local maximum and local minimum.

*substitute the critical numbers you found in the SECOND DERIVATIVE FUNCTION

$f''(0)= 12x^2 - 16 = -16$ ---> local maximum

$f''(2)= 12x^2 - 16 = 32$ ---> local minimum

$f''(-2)= 12x^2 - 16 = 32$ ---> local minimum

$f(0) = x^4 - 8x^2 + 10 = 10$ (0,10)

$f(2) = x^4 - 8x^2 + 10 = -6$ (2,-6)

$f(-2) = x^4 - 8x^2 + 10 = -6$ (-2,-6)

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Done with lesson 4.5..
Hope you understand!

اللهم إني أستودعُكَ ما قرأتُ وما حفظتُ وما تعلمتُ، فردّه إليَّ عند حاجتي إليه، إنك على كل شيءٍ قدير.

اللهم ذكرني منه ما نسيتُ، وعلمّني منه ما جهلتُ

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