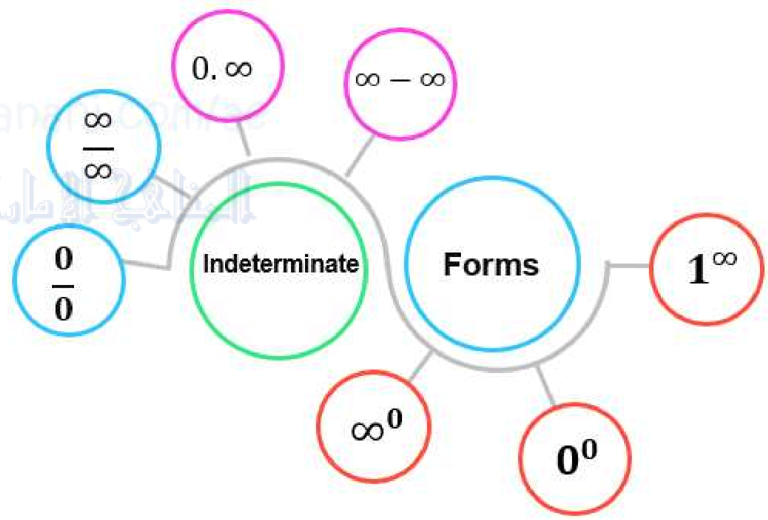
الملف أوراق عمل الدرس الثاني من الوحدة الرابعة Differentiation of Application

Lesson 2: Indeterminate form and L'Hôpital's Rule

Indeterminate Forms



Theorem: L'Hopital's Rule

Suppose that f and g are differentiable on the interval (a, b), except possibly at the point $c \in (a, b)$ and that $g'(x) \neq 0$ on (a, b), except possibly at c.

Suppose further that $\lim_{x\to c}\frac{f(x)}{g(x)}$ has the indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ and that $\lim_{x\to c}\frac{f'(x)}{g'(x)} = L(\text{or} \pm \infty)$. Then,

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

So, L'Hopital's Rule tells us that if we have an indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ all we need to do is differentiate the top and differentiate the bottom and then take the limit.

Check → Differentiate → Calculate

Remarks:

1) L'Hopital's Rule also holds if $\lim\limits_{x\to c}\frac{f(x)}{g(x)}$ is replaced with any of the limits

$$\lim_{x\to c^+}\frac{f(x)}{g(x)},\ \lim_{x\to c^-}\frac{f(x)}{g(x)},\ \lim_{x\to \infty}\frac{f(x)}{g(x)} \text{ or } \lim_{x\to -\infty}\frac{f(x)}{g(x)}.$$

2) You must check that the limit of $\frac{f(x)}{g(x)}$ is an indeterminate form $\frac{0}{0}, or\ \frac{\infty}{\infty}$,then apply L'Hopital's Rule.

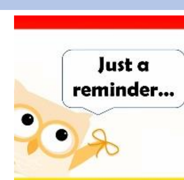
3) If you get $(0.\infty)$, $(\infty - \infty)$, ∞^{0}, 0^{0}, $or\ 1^{\infty}$ **change the function to get** $\frac{0}{0}, or\ \frac{\infty}{\infty}$ then you can apply L'Hopital's Rule

4) You can apply L'Hopital's Rule many times.

Remember:

1) $\frac{c}{\pm\infty} = 0 \quad , c \neq 0$
2) $\frac{\pm c}{0} = \pm\infty \quad , c \neq 0$

- $\lim\limits_{x\to 0^+} \ln x = -\infty$
- $\lim\limits_{x\to \infty} \ln x = \infty$



Determinate Forms

$\frac{0}{\pm\infty} = 0$

$\infty \cdot \infty = \infty$

$-\infty - \infty = -\infty$

$\infty + \infty = \infty$

$0^{\infty} = 0$

$0^{-\infty} = \infty$

$\boldsymbol{\lim\limits_{x\to \infty} e^{x} = \infty}$

$\boldsymbol{\lim\limits_{x\to -\infty} e^{x} = 0}$

- $\lim\limits_{x\to \infty} \tan^{-1} x = \frac{\pi}{2}$
- $\lim\limits_{x\to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

Exercises page 248: name the method by determining whether L'Hopital's Rule should be used or not.

Q45) $\lim_{x \to 0^+} \frac{\csc x}{\sqrt{x}}$

Q46) $\lim_{x \to 0^+} \frac{x^{-3/2}}{\ln x}$

Q47) $\lim_{x \to \infty} \frac{x^2 - 3x + 1}{\tan^{-1} x}$

Q48) $\lim_{x \to \infty} \frac{\ln(x^2)}{e^{x/3}}$

$\lim_{x \to \infty} (1 + \frac{1}{x})^x$

$\lim_{x \to \infty} (\ln x)^x$

Exercises page 247: find the indicated limits.

Q1) $\lim_{x \to -2} \frac{x + 2}{x^2 - 4}$

Q4) $\lim_{x \to \infty} \frac{x+1}{x^2+4x+3}$

Q8) $\lim_{t \to 0} \frac{\sin t}{\sin^{-1} t}$

Q9) $\lim_{x \to \pi} \frac{\sin 2x}{\sin x}$

Q10) $\lim_{x \to -1} \frac{\cos^{-1} x}{x^2 - 1}$

Q13) $\lim_{t \to 1} \frac{\sqrt{t} - 1}{t - 1}$

Q29) $\lim_{x \to 0^+} \frac{\ln x}{\cot x}$

Q22) $\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$

Q36) $\lim_{x \to 1} \frac{\sqrt{5 - x} - 2}{\sqrt{10 - x} - 3}$

Q30) $\lim_{x \to 0^+} \frac{\sqrt{x}}{\ln x}$

We can apply L'Hopital's Rule many times.

Q2) $$\lim_{x \to \infty} \frac{x^2 - 4}{x^2 - 3x + 2}$$

Q3) $$\lim_{x \to \infty} \frac{3x^2 + 2}{x^2 - 4}$$

Q11) $\lim_{x\to 0}\frac{\sin x - x}{x^3}$

Q15) $\lim_{x\to \infty}\frac{x^3}{e^x}$

Exercises page 248: find all error(s).

Q41) $$\lim_{x\to 0}\frac{\cos x}{x^2} = \lim_{x\to 0}\frac{-\sin x}{2x} = \lim_{x\to 0}\frac{-\cos x}{2} = -\frac{1}{2}$$

Q42) $$\lim_{x\to 0}\frac{e^x - 1}{x^2} = \lim_{x\to 0}\frac{e^x}{2x} = \lim_{x\to 0}\frac{e^x}{2} = \frac{1}{2}$$

If we get $(0.\infty)$ $\boldsymbol{or}$ $(\infty - \infty)$

Q23) $\lim_{x\to \infty} te^{-t}$

Q24) $\lim_{t\to \infty} t\sin(\frac{1}{t})$

Q18) $\lim_{x\to 0}(\cot x - \frac{1}{x})$

Q20) $\lim_{x\to \frac{\pi}{2}}(\tan x + \frac{1}{x - \frac{\pi}{2}})$

If we get ∞^0 , 0^0 , *or* 1^∞

Q33) $\lim_{x\to\infty}(1+\frac{1}{x})^x$

Note:

$$\lim_{x\to\infty}(1+\frac{a}{x})^x = e^a$$

Q40) $\lim_{t\to\infty}(\frac{t-3}{2t+1})^t$

Q37) $\lim_{x\to 0^+}(\frac{1}{x})^x$

Q38) $\lim_{x\to 0^+}(\cos x)^{1/x}$

Example 8: evaluate $\lim_{x\to 1^+} x^{\frac{1}{x-1}}$

Example 9: evaluate $\lim_{x\to 0^+}(\sin x)^x$