حل أسئلة امتحانات سابقة الدرس الخامس Motion Projectile من الوحدة السادسة

1. A diver drops from a height of 64 feet. Which of the following gives the initial conditions?

A. $h(0) = 64\ ft,\ h'(0) = 0$
B. $h(0) = 64\ ft,\ h'(0) = 32\ ft/s$
C. $h(0) = 64\ ft,\ h''(t) = 9.8\ ft/s$
D. $h(0) = 0,\ h''(t) = -32\ ft/s^2$

2. A diver drops from a height of 64 feet. What is the velocity at impact?

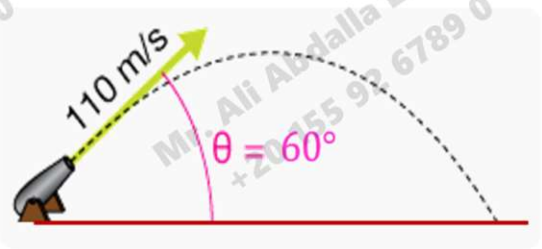
A. $32\ ft/s$
B. $23\ ft/s$
C. $64\ ft/s$
D. $46\ ft/s$

3. Identify the initial condition $y(0)$ and $y'(0)$ for the vertical motion, if the object is thrown at a velocity of $6\ m/s$ from a height of $30\ m$. (Take the origin to be the ground)

A. $y(0) = 30,\ y'(0) = -6$
B. $y(0) = 0,\ y'(0) = -6$
C. $y(0) = 30,\ y'(0) = 6$
D. $y(0) = 0,\ y'(0) = 6$

4. For an object moving in two dimensions shown in figure below. What is the initial value of the vertical component of the velocity?

A. $y'(0) = 110\ sin(60) = 55\sqrt{3}\ m/s$
B. $y'(0) = 110\ cos(60) = 55\ m/s$
C. $y'(0) = 110\ tan(60) = 110\sqrt{3}\ m/s$
D. $y'(0) = \frac{110}{sin(60)} = \frac{55}{\sqrt{3}}\ m/s$



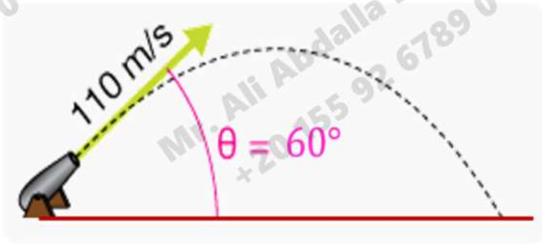
5. For an object moving in two dimensions shown in figure below. What is the vertical acceleration?

A. $y''(t) = -32\ ft/s^2$

B. $y''(t) = 32\ ft/s^2$

C. $y''(t) = -9.8\ m/s^2$

D. $y''(t) = 0$



6. If an object dropped from a height of H feet, at what time t it will hit the ground?

A. $t = \frac{1}{4}\sqrt{H}$

B. $t = \frac{1}{2}\sqrt{H}$

C. $t = 8\sqrt{H}$

D. $t = 4\sqrt{H}$

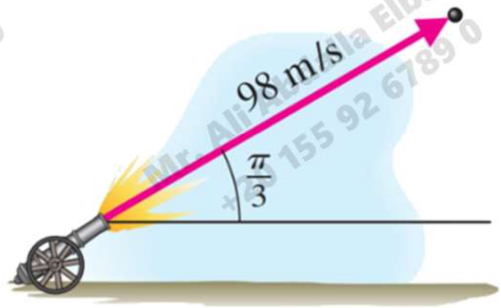
7. An object is launched at angle $\theta = \frac{\pi}{3}$ radians from the horizontal with an initial speed of $98\ m/s$. Determine the time of flight.

A. 18.32

B. 17.32

C. 19.32

D. 16.32



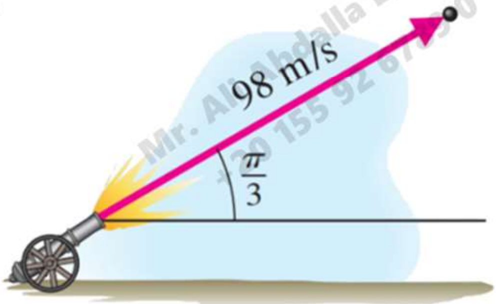
8. An object is launched at angle $\theta = \frac{\pi}{3}$ radians from the horizontal with an initial speed of $98\ m/s$. Which of the following give the horizontal range?

A. $x(t) = 98t$

B. $x(t) = 49\sqrt{3}\,t$

C. $x(t) = \sqrt{3}\,t$

D. $x(t) = 49t$



9. A diver drops from $30\ ft$ above the water. Which of the following give the height of the diver at time t?

A. $h(t) = -16t^2 + 30$

B. $h(t) = 16t^2 + 30$

C. $h(t) = -4.9t^2 + 30$

D. $h(t) = 4.9t^2 + 30$

10. A ball is propelled straight upward from the ground with initial velocity $64\ ft/s$. Ignoring air resistance, determine the amount of time the ball spends in the air.

A. 2

B. 3

C. 4

D. 5

11. A certain not-so-wily coyote discovers that he just stepped off the edge of a cliff. Four seconds later, he hits the ground in a puff of dust. How high in meters was the cliff?

A. Height $= 78.4\ m$

B. Height $= 88.4\ m$

C. Height $= 98.2\ m$

D. Height $= 49.2\ m$

12. The coyote's next scheme involves launching himself into the air with an Acme catapult. If the coyote is propelled vertically from the ground with initial velocity $19.6\ m/s$, find an equation for the height of the coyote at any time t.

A. $h(t) = -16t^2 + 19.6t$　　**B.** $h(t) = -16t^2 + 19.6$

C. $h(t) = -4.9t^2 + 19.6$　　**D.** $h(t) = -4.9t^2 + 19.6t$

13. The coyote's next scheme involves launching himself into the air with an Acme catapult. If the coyote is propelled vertically from the ground with initial velocity $19.6\ m/s$, Find his maximum height.

A. $h(t) = 19.6\ m$　　**B.** $h(t) = 13.4\ m$

C. $h(t) = 29.6\ m$　　**D.** $h(t) = 9.8\ m$

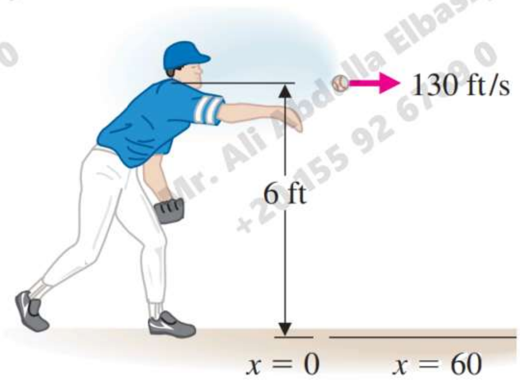
14. A baseball pitcher releases the ball horizontally from a height of $6\ ft$ with an initial speed of $130\ ft/s$. Find the height of the ball when it reaches home plate 60 feet away.

A. $y(\frac{6}{13}) \approx 1.59\ ft$

B. $y(\frac{6}{13}) = 2.95\ ft$

C. $y(\frac{6}{13}) = 2.59\ ft$

D. $y(\frac{6}{13}) = 2.89\ ft$



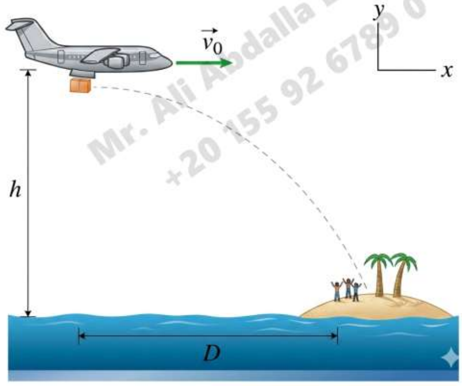
15. A plane at an altitude of 256 feet want to drop supplies to a specific location on the ground. If the plane has a horizontal velocity of $100\ ft/s$, how far away from the target should the plane release the supplies in order to hit the target location?

A. $D = 300\ ft$

B. $D = 400\ ft$

C. $D = 200\ ft$

D. $D = 350\ ft$



16. An object is launched from the ground at an angle $\theta = 20^\circ$ with an initial speed of $48\ m/s$. Find the time of the flight? (ignore the air resistance)

A. $t = 1.026\ s$ **B.** $t = 16.4\ s$

C. $t = 2.03\ s$ **D.** $t = 45.1\ s$

17. One of the authors has a vertical "jump" of $20\ in$. What is the initial velocity required to jump this high?

A. $v_0 = 1.033\ ft/s$ **B.** $v_0 = 10.33\ ft/s$

C. $v_0 = 103.3\ ft/s$ **D.** $v_0 = 8\sqrt{5}\ ft/s$

18. A diver drops from $120\ ft$ above the water (about the height of divers at the Acapulco Cliff Diving competition). What is the diver's velocity at impact?

A. $v = 16\sqrt{30}\ ft/s$ **B.** $v = 16\sqrt{3}\ ft/s$

C. $v = -16\sqrt{30}\ ft/s$ **D.** $v = -16\sqrt{3}\ ft/s$

QUIZ BUBBLE SHEET

1. (A) (B) (C) (D)	7. (A) (B) (C) (D)	13. (A) (B) (C) (D)
2. (A) (B) (C) (D)	8. (A) (B) (C) (D)	14. (A) (B) (C) (D)
3. (A) (B) (C) (D)	9. (A) (B) (C) (D)	15. (A) (B) (C) (D)
4. (A) (B) (C) (D)	10. (A) (B) (C) (D)	16. (A) (B) (C) (D)
5. (A) (B) (C) (D)	11. (A) (B) (C) (D)	17. (A) (B) (C) (D)
6. (A) (B) (C) (D)	12. (A) (B) (C) (D)	18. (A) (B) (C) (D)

Solution to Q1:

Let $h(t)$ be the height of the diver at time t.

Since the diver drops from a height of 64 feet, the initial height is:

$h(0) = 64$

Because the diver "drops" (starts from rest), the initial velocity is zero:

$h'(0) = 0$

Answer: A

Solution to Q2:

Using Newton's second law for vertical motion (in feet), the acceleration is:

$h''(t) = -32$

The initial conditions are $h(0) = 64$ and $h'(0) = 0$. Integrating the acceleration gives the velocity:

$$\int h''(t)\,dt = \int -32\,dt$$

$h'(t) = -32t + c_1$

Applying the initial velocity condition $h'(0) = 0$:

$0 = -32(0) + c_1 \implies c_1 = 0$

So, the velocity function is $h'(t) = -32t$. Integrating again gives the position:

$$\int h'(t)\,dt = \int -32t\,dt$$

$h(t) = -16t^2 + c_2$

Applying the initial height condition $h(0) = 64$:

$64 = -16(0)^2 + c_2 \implies c_2 = 64 \implies h(t) = -16t^2 + 64$

The diver hits the water when $h(t) = 0$:

$0 = -16t^2 + 64 \implies 16t^2 = 64 \implies t = 2$ s.

The velocity at impact ($t = 2$) is:

$h'(2) = -32(2) = -64$ ft/s. Its magnitude is 64 ft/s.

Answer: C

Solution to Q3:

The object is thrown from a height of 30 m, giving the initial position:

$$y(0) = 30$$

It is thrown with a velocity of 6 m/s. According to the choices provided, the object is thrown downward, meaning the initial velocity is negative:

$$y'(0) = -6$$

Answer: A

Solution to Q4:

From basic trigonometry, the vertical component of the initial velocity is given by $v_y = v_0 \sin(\theta)$.

Given an initial speed of $v_0 = 110$ m/s and launch angle of $\theta = 60^\circ$:

$$y'(0) = 110 \sin(60^\circ) = 110 \left(\frac{\sqrt{3}}{2}\right) = 55\sqrt{3} \text{ m/s.}$$

Answer: A

Solution to Q5:

Ignoring air resistance, the only force acting on the object is gravity.

In the SI system (meters), the vertical acceleration is:

$y''(t) = -9.8$ m/s^2.

Answer: C

Solution to Q6:

Acceleration: $h''(t) = -32$. Initial conditions: $h'(0) = 0$ (dropped), $h(0) = H$.

Integrating to find velocity:

$$\int h''(t)\,dt = \int -32\,dt \implies h'(t) = -32t + c_1$$

Using $h'(0) = 0$, we get $c_1 = 0 \implies h'(t) = -32t$.

Integrating to find position:

$$\int h'(t)\,dt = \int -32t\,dt \implies h(t) = -16t^2 + c_2$$

Using $h(0) = H$, we get $c_2 = H \implies h(t) = -16t^2 + H$.

The object hits the ground when $h(t) = 0$:

$$0 = -16t^2 + H \implies 16t^2 = H \implies t^2 = \frac{H}{16} \implies t = \frac{1}{4}\sqrt{H}$$

Answer: A

Solution to Q7:

Vertical acceleration: $y''(t) = -9.8$. Initial height: $y(0) = 0$.

Initial vertical velocity: $y'(0) = 98\sin\left(\frac{\pi}{3}\right) = 98\left(\frac{\sqrt{3}}{2}\right) = 49\sqrt{3}$.

$$\int y''(t)\,dt = \int -9.8\,dt \implies y'(t) = -9.8t + c_1$$

Using $y'(0) = 49\sqrt{3} \implies y'(t) = -9.8t + 49\sqrt{3}$.

$$\int y'(t)\,dt = \int (-9.8t + 49\sqrt{3})\,dt \implies y(t) = -4.9t^2 + 49\sqrt{3}t + c_2$$

Using $y(0) = 0 \implies y(t) = -4.9t^2 + 49\sqrt{3}t$.

Time of flight ($y(t) = 0$): $0 = t(-4.9t + 49\sqrt{3})$.

Ignoring $t = 0$: $4.9t = 49\sqrt{3} \implies t = 10\sqrt{3} \approx 17.32$ s.

Answer: B

Solution to Q8:

Horizontal acceleration: $x''(t) = 0$. Initial position: $x(0) = 0$.

Initial horizontal velocity: $x'(0) = 98\cos\left(\frac{\pi}{3}\right) = 98\left(\frac{1}{2}\right) = 49$.

Integrating to find velocity:

$$\int x''(t)\,dt = \int 0\,dt \implies x'(t) = c_1$$

Using $x'(0) = 49 \implies c_1 = 49 \implies x'(t) = 49$.

Integrating to find position:

$$\int x'(t)\,dt = \int 49\,dt \implies x(t) = 49t + c_2$$

Using $x(0) = 0 \implies c_2 = 0 \implies x(t) = 49t$.

Answer: D

Solution to Q9:

Acceleration: $h''(t) = -32$. Initial conditions: $h'(0) = 0$, $h(0) = 30$.

Integrating for velocity:

$$\int h''(t)\,dt = \int -32\,dt \implies h'(t) = -32t + c_1$$

With $h'(0) = 0$, $c_1 = 0 \implies h'(t) = -32t$.

Integrating for position:

$$\int h'(t)\,dt = \int -32t\,dt \implies h(t) = -16t^2 + c_2$$

With $h(0) = 30$, $c_2 = 30 \implies h(t) = -16t^2 + 30$.

Answer: A

Solution to Q10:

Acceleration: $h''(t) = -32$. Initial conditions: $h'(0) = 64$, $h(0) = 0$.

Integrating for velocity:

$$\int h''(t)\,dt = \int -32\,dt \implies h'(t) = -32t + c_1$$

With $h'(0) = 64$, $c_1 = 64 \implies h'(t) = -32t + 64$.

Integrating for position:

$$\int h'(t)\,dt = \int (-32t + 64)\,dt \implies h(t) = -16t^2 + 64t + c_2$$

With $h(0) = 0$, $c_2 = 0 \implies h(t) = -16t^2 + 64t$.

Lands when $h(t) = 0$: $0 = -16t^2 + 64t \implies 16t(4 - t) = 0$.

Ignoring launch time, $t = 4$ seconds.

Answer: C

Solution to Q11:

Working in meters: $y''(t) = -9.8$. Initial conditions: $y'(0) = 0$ (steps off), $y(0) = H$.

Integrating for velocity:

$$\int y''(t)\,dt = \int -9.8\,dt \implies y'(t) = -9.8t + c_1$$

With $y'(0) = 0 \implies c_1 = 0 \implies y'(t) = -9.8t$.

Integrating for position:

$$\int y'(t)\,dt = \int -9.8t\,dt \implies y(t) = -4.9t^2 + c_2$$

With $y(0) = H \implies c_2 = H \implies y(t) = -4.9t^2 + H$.

At $t = 4$ seconds, he hits the ground ($y(4) = 0$):

$0 = -4.9(4)^2 + H \implies H = 4.9(16) = 78.4$ m.

Answer: A

Solution to Q12:

Acceleration: $h''(t) = -9.8$. Initial conditions: $h'(0) = 19.6$, $h(0) = 0$.

Integrating for velocity:

$$\int h''(t)\,dt = \int -9.8\,dt \implies h'(t) = -9.8t + c_1$$

With $h'(0) = 19.6 \implies c_1 = 19.6 \implies h'(t) = -9.8t + 19.6$.

Integrating for position:

$$\int h'(t)\,dt = \int (-9.8t + 19.6)\,dt \implies h(t) = -4.9t^2 + 19.6t + c_2$$

With $h(0) = 0 \implies c_2 = 0 \implies h(t) = -4.9t^2 + 19.6t$.

Answer: D

Solution to Q13:

Using the functions derived in Q12:

$h'(t) = -9.8t + 19.6$ and $h(t) = -4.9t^2 + 19.6t$.

Maximum height occurs when the velocity is zero:

$0 = -9.8t + 19.6 \implies 9.8t = 19.6 \implies t = 2$ s.

Substitute $t = 2$ into the height function:

$h(2) = -4.9(2)^2 + 19.6(2) = -4.9(4) + 39.2 = -19.6 + 39.2 = 19.6$ m.

Answer: A

Solution to Q14:

Vertical Motion: $y''(t) = -32$. Initial conditions: $y'(0) = 0$ (horizontal release), $y(0) = 6$.

$$\int y''(t)\,dt = \int -32\,dt \implies y'(t) = -32t + c_1 \implies y'(t) = -32t.$$

$$\int y'(t)\,dt = \int -32t\,dt \implies y(t) = -16t^2 + c_2 \implies y(t) = -16t^2 + 6.$$

Horizontal Motion: $x''(t) = 0$. Initial conditions: $x'(0) = 130$, $x(0) = 0$.

$$\int x''(t)\,dt = \int 0\,dt \implies x'(t) = c_3 \implies x'(t) = 130.$$

$$\int x'(t)\,dt = \int 130\,dt \implies x(t) = 130t + c_4 \implies x(t) = 130t.$$

Find time when it reaches home plate $x(t) = 60$:

$130t = 60 \implies t = \frac{6}{13}$ s.

Height at this time:

$y\left(\frac{6}{13}\right) = -16\left(\frac{6}{13}\right)^2 + 6 \approx -3.408 + 6 = 2.59$ ft.

Answer: C

Solution to Q15:

Vertical Motion: $y''(t) = -32$. Initial conditions: $y'(0) = 0$ (dropped), $y(0) = 256$.

$$\int y''(t)\,dt = \int -32\,dt \implies y'(t) = -32t.$$

$$\int y'(t)\,dt = \int -32t\,dt \implies y(t) = -16t^2 + c \implies y(t) = -16t^2 + 256.$$

Time to hit the ground ($y(t) = 0$):

$0 = -16t^2 + 256 \implies 16t^2 = 256 \implies t^2 = 16 \implies t = 4$ s.

Horizontal Motion: $x''(t) = 0$. Initial conditions: $x'(0) = 100$, $x(0) = 0$.

$$\int x''(t)\,dt = \int 0\,dt \implies x'(t) = 100.$$

$$\int x'(t)\,dt = \int 100\,dt \implies x(t) = 100t.$$

Horizontal distance D traveled in 4 seconds:

$D = x(4) = 100(4) = 400$ ft.

Answer: B

Solution to Q16:

(Note: Based on the answer choices, initial speed unit is $48\ ft/s$, using $g = 32\ ft/s^2$.)

Vertical acceleration: $y''(t) = -32$. Initial height: $y(0) = 0$, $y'(0) = 48\sin(20^\circ)$.

$$\int y''(t)\,dt = \int -32\,dt \implies y'(t) = -32t + 48\sin(20^\circ).$$

$$\int y'(t)\,dt = \int (-32t + 48\sin(20^\circ))\,dt \implies y(t) = -16t^2 + 48\sin(20^\circ)t.$$

Flight time is when $y(t) = 0$:

$t(-16t + 48\sin(20^\circ)) = 0 \implies 16t = 48\sin(20^\circ) \implies t = 3\sin(20^\circ) \approx 1.026\ s.$

Answer: A

Solution to Q17:

Convert height to feet: $h = 20\ in = \frac{20}{12}\ ft = \frac{5}{3}\ ft.$

Acceleration: $y''(t) = -32$. Initial conditions: $y'(0) = v_0$, $y(0) = 0$.

$$\int y''(t)\,dt = \int -32\,dt \implies y'(t) = -32t + v_0.$$

$$\int y'(t)\,dt = \int (-32t + v_0)\,dt \implies y(t) = -16t^2 + v_0 t.$$

Max height at $y'(t) = 0 \implies t = \frac{v_0}{32}$.

$$y\left(\frac{v_0}{32}\right) = -16\left(\frac{v_0}{32}\right)^2 + v_0\left(\frac{v_0}{32}\right) = \frac{v_0^2}{64}.$$

Set equal to $\frac{5}{3}$: $\frac{v_0^2}{64} = \frac{5}{3} \implies v_0^2 = \frac{320}{3} \approx 106.67$

$\implies v_0 = \sqrt{106.67} \approx 10.33\ ft/s.$

Answer: B

Solution to Q18:

Acceleration: $h''(t) = -32$. Initial conditions: $h'(0) = 0$, $h(0) = 120$.

$$\int h''(t)\,dt = \int -32\,dt \implies h'(t) = -32t.$$

$$\int h'(t)\,dt = \int -32t\,dt \implies h(t) = -16t^2 + 120.$$

Impact occurs when $h(t) = 0$: $16t^2 = 120 \implies t^2 = 7.5 \implies t = \frac{\sqrt{30}}{2}$.

Velocity at impact: $h'\left(\frac{\sqrt{30}}{2}\right) = -32\left(\frac{\sqrt{30}}{2}\right) = -16\sqrt{30}$ ft/s.

Answer: C