ملخص الدرس الرابع Increasing and decreasing functions من الوحدة الرابعة تطبيقات التفاضل منهج ريفيل

Math lessons

Simplification

اللهم إني أسألك فَهْمَ النَّبِيِّينَ وَحِفْظَ الْمُرْسَلِينَ، وَالْمَلَائِكَةَ الْمُقَرَّبِينَ، اللهم اجعل لِسَانِي عَامِرًا بِذِكْرِكَ، وَقَلْبِي بِخَشْيَتِكَ، وَسِرِّي بِطَاعَتِكَ، إِنَّكَ عَلَىٰ كُلِّ شَيْءٍ قَدِيرٌ، وَحَسْبِيَ اللَّهُ وَنِعْمَ الْوَكِيلُ

By: 'Fa

Some Rules from Term 1 You Should Know::

Chain Rule

$$\frac{d}{dx}f\big(g(x)\big) = f'\big(g(x)\big)\cdot g'(x)$$

Power Rule

$$\frac{d}{dx}x^n = nx^{n-1}$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Function	
Polynomials	Continuous Everywhere
$\sin x, \cos x, \tan^{-1} x$	Continuous Everywhere
$\sqrt[n]{x}$	Continuous Everywhere when n is odd
$\sqrt[n]{x}$	Continuous for $x > 0$ when n is even
$\ln x$	Continuous for $x > 0$
$\sin^{-1} x, \cos^{-1} x$	Continuous for $-1 < x < 1$

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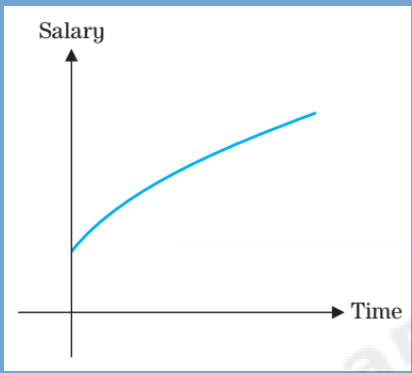
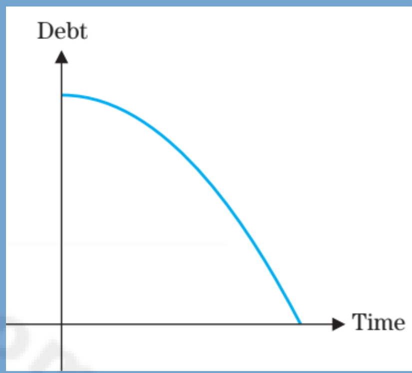
Chpt 4: Applications of differentiation

Lesson 4.4: Increasing and Decreasing Functions

The general idea:



Increasing function

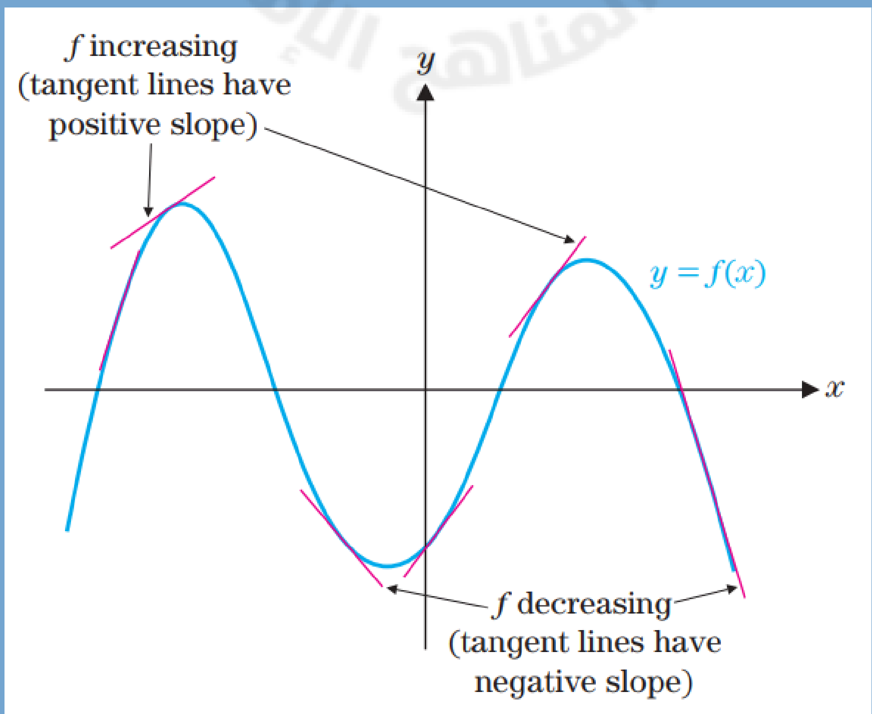


Decreasing function

THEOREM 4.1

Suppose that f is differentiable on an interval I.

(i) If $f'(x) > 0$ for all $x \in I$, then f is increasing on I.
(ii) If $f'(x) < 0$ for all $x \in I$, then f is decreasing on I.



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THEOREM 4.2 (First Derivative Test)

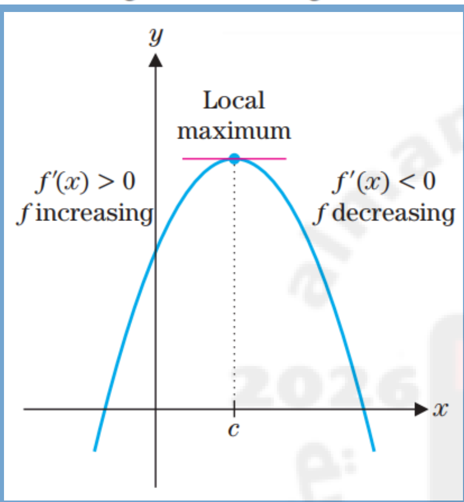
Suppose that f is continuous on the interval $[a, b]$ and $c \in (a, b)$ is a critical number.

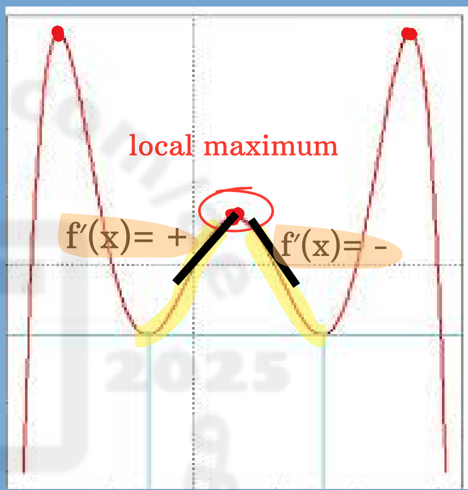
(i) If $f'(x) > 0$ for all $x \in (a, c)$ and $f'(x) < 0$ for all $x \in (c, b)$ (i.e., f changes from increasing to decreasing at c), then $f(c)$ is a local maximum.

(ii) If $f'(x) < 0$ for all $x \in (a, c)$ and $f'(x) > 0$ for all $x \in (c, b)$ (i.e., f changes from decreasing to increasing at c), then $f(c)$ is a local minimum.

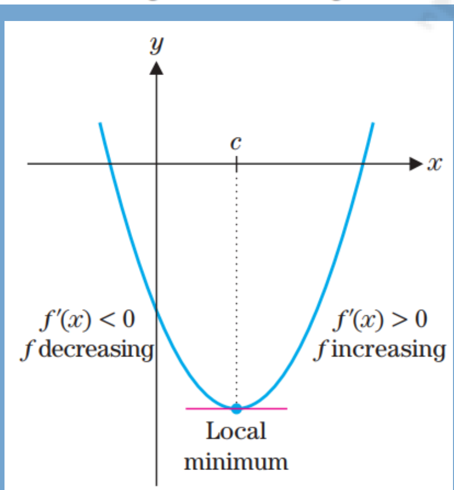
(iii) If $f'(x)$ has the *same* sign on (a, c) and (c, b), then $f(c)$ is *not* a local extremum.

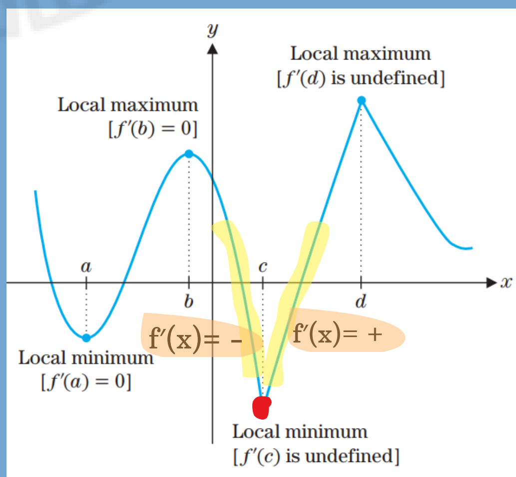
(i) If $f'(x) > 0$ for all $x \in (a, c)$ and $f'(x) < 0$ for all $x \in (c, b)$ (i.e., f changes from increasing to decreasing at c), then $f(c)$ is a local maximum.





(ii) If $f'(x) < 0$ for all $x \in (a, c)$ and $f'(x) > 0$ for all $x \in (c, b)$ (i.e., f changes from decreasing to increasing at c), then $f(c)$ is a local minimum.





(iii) If $f'(x)$ has the *same* sign on (a, c) and (c, b), then $f(c)$ is *not* a local extremum.

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LET'S APPLY WHAT WE'VE LEARNED

FIND THE INTERVALS WHERE THE FUNCTION IS INCREASING AND DECREASING AND DETERMINE THE LOCAL EXTREMA:

$y = x^3 - 3x + 2$

step 1: Determine the domain of the function.

since it's a polynomial function in this qn the domain will be $(-\infty,\infty)$

step 2: Find the derivative of the original function.

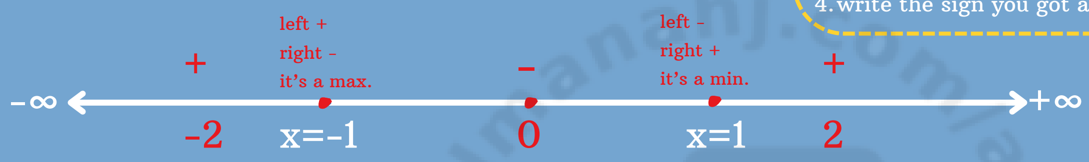
$f'(x) = 3x^2 - 3$

step 3: equate the resulting function to 0 and find the value of x.

$f'(x)=0$ ---> $3x^2 - 3 = 0$

$x=1$ $x=-1$

1. Draw a number line and locate both values of x you found.
2. take numbers close to both values
3. substitute the numbers you took in the f′(x) function.
4. write the sign you got above each number separately.



Then, you can see that the function is increasing between the intervals: $(-\infty,-1)\cup(1,\infty)$
and decreasing between the interval: $(-1,1)$

Now we have finished finding the intervals...
LET'S DETERMINE THE LOCAL MAX. AND MIN :)

We can see that we have a local maximum at x=1
substitute 1 in the original function to find the y coordinate ---> $y = (1)^3 - 3(1) + 2 = 0$

local max. at (1,0)

then, we have also a local minimum at x=-1
substitute -1 in the original function to find the y coordinate ---> $y = (-1)^3 - 3(-1) + 2 = 4$

local min. at (-1,4)

Summary of the steps::

step 1: Determine the domain of the function.
step 2: Find the derivative of the original function.
step 3: equate the resulting function to 0 and find the value of x.
step 4: Find the intervals where the function is increasing and decreasing by using a number line.
step 5: Find both local max. and min. by substituting x values in the original function

READ THE STEPS CAREFULLY AND ATTENTIVELY. ONCE YOU ARE SURE YOU UNDERSTAND, GO TO THE NEXT SLIDE TO SOLVE ANOTHER QUESTION ON YOUR OWN.

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CHECK YOUR UNDERSTANDING

FIND THE INTERVALS WHERE THE FUNCTION IS INCREASING AND DECREASING AND DETERMINE THE LOCAL EXTREMA:

$$y = x^4 - 8x^2 + 1$$

step 1: Determine the domain of the function.
(-3)
since it's a polynomial function in this qn the domain will be (-∞,∞)

step 2: Find the derivative of the original function.

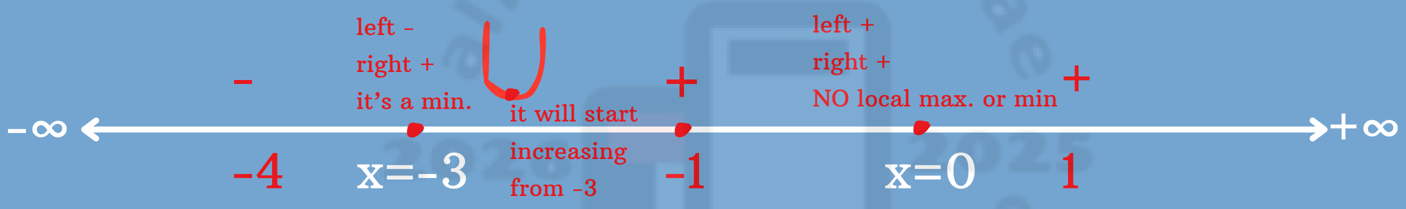
$f'(x) = 4x^3 - 16x$

step 3: equate the resulting function to 0 and find the value of x.

$f'(x)=0$ ---> $4x^3 - 16x = 0$

x=-3 , x=0

1. Draw a number line and locate both values of x you found.
2. take numbers close to both values
3. substitute the numbers you took in the f′(x) function.
4. write the sign you got above each number separately.



Then, you can see that the function is increasing between the intervals: (-3,∞)
and decreasing between the interval: (-∞,-3)

Now we have finished finding the intervals...
LET'S DETERMINE THE LOCAL MAX. AND MIN :)

We can see that we don't have any local maximum..

here, we have only a local minimum at x=-3
substitute -3 in the original function to find the y coordinate ---> $y = x^4 - 8x^2 + 1$

local min. at (-3,10)

READ THE STEPS CAREFULLY AND ATTENTIVELY. ONCE YOU ARE SURE YOU UNDERSTAND, GO TO THE NEXT SLIDE TO SOLVE ANOTHER QUESTION ON YOUR OWN.

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LAST QN

FIND THE INTERVALS WHERE THE FUNCTION IS INCREASING AND DECREASING AND DETERMINE THE LOCAL EXTREMA:

Given $f(x) = \frac{x^2}{x+2}$ → $x+2=0$, $x \neq -2$

step 1: Determine the domain of the function.

Here, after we equate the denominator to zero, we see that the value of x cannot be 2, so the domain of the function will be $(-\infty,2)\cup(2,\infty)$

step 2: Find the derivative of the original function by using quotient rule

derivative

X^2	$X + 2$
$2x$	1

$$f'(x) = \frac{2x(x+2) - x^2(1)}{(x+2)^2} = \frac{2x^2+4x - x^2}{(x+2)^2} = \frac{x^2+4x}{(x+2)^2}$$

step 3: equate the resulting function to 0 and find the value of x.

$f'(x)=0$ ---> $x^2 + 4 = 0$

$x=0$ $x=-4$

$f'(x)$ is not defined at $x=-2$

1. Draw a number line and locate both values of x you found.
2. take numbers close to both values
3. substitute the numbers you took in the f'(x) function.
4. write the sign you got above each number separately.

$-\infty$ ← -5 (+) | x=-4: left +, right −, it's a max. | -3 (−) | x=-2: left +, right +, NO extrema | -1 (−) | x=0: left −, right +, it's a min. | 1 (+) → $+\infty$

Then, you can see that the function is increasing between the intervals: $(-\infty,-4)\cup(0,\infty)$
and decreasing between the interval: $(-4,-2)\cup(-2,0)$

Now we have finished finding the intervals...
LET'S DETERMINE THE LOCAL MAX. AND MIN :)

We can see that we have a local maximum at x=-4
substitute -4 in the original function to find the y coordinate ---> $f(-4) = \frac{x^2}{x+2}$

local max. at (-4,-2)

then, we have also a local minimum at x=0
substitute 0 in the original function to find the y coordinate ---> $f(0) = \frac{x^2}{x+2}$

local min. at (0,0)

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Done with lesson 4.4..
Hope you understand!

اللهم إني أستودعُكَ ما قرأتُ وما حفظتُ وما تعلمتُ، فردّه إليَّ عند حاجتي إليه، إنك على كل شيءٍ قدير.

اللهم ذكرني منه ما نسيتُ، وعلمّني منه ما جهلتُ

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