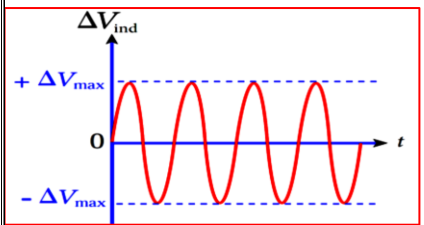
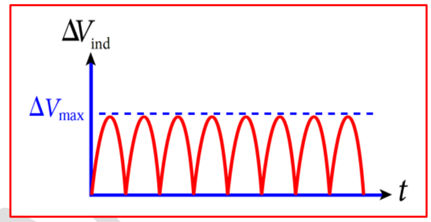
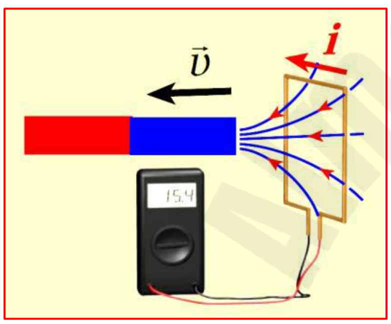
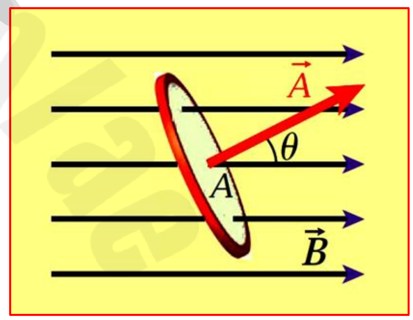
KHALIFA BIN ZAYED SECONDERY SCHOOL





PHYSICS





GRADE 12 ADVANCE

2024-2025

TERM 3 , <u>UNIT 9</u>

Electromagnetic induction

Student Name ……………………………………

DONE BY: HABIS ALADWAN

0503224514

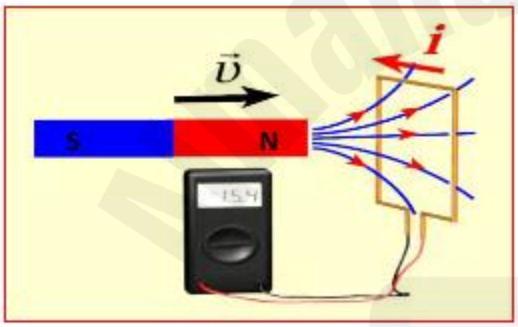
1.1 Faraday's experiments

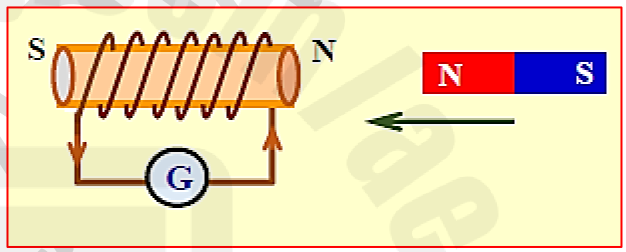
(Faraday and Henry) *Their experiments demonstrated that*
[changing magnetic field inside a conducting loop induces a current in the loop]

Lenz's Law: *the current induced in a loop by a changing magnetic flux produces a* magnetic *field that* **opposes the change** *in magnetic flux.*

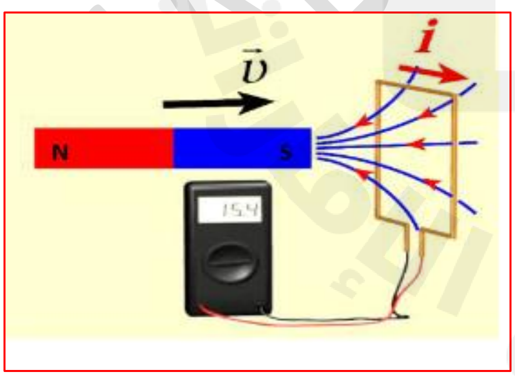
1. **A magnet moving toward a wire loop it will induce a current flowing in the loop :**

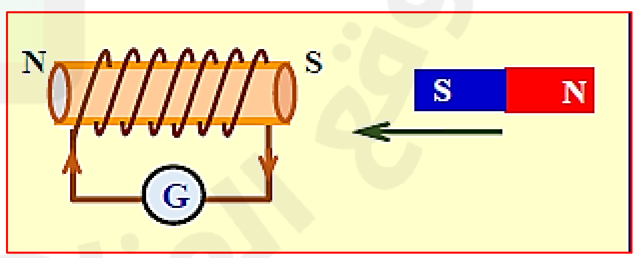
A- *With the north pole of the magnet* **pointing toward the** loop *(a positive current) results [a counterclockwise current flow in the loop]*





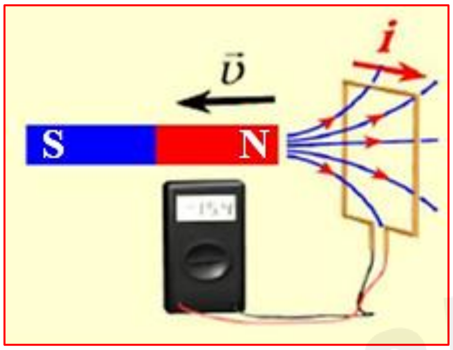
B- *With the south pole of the magnet* ***pointing toward the loop*** *(a negative current) results.[a clockwise current flow in the loop]*

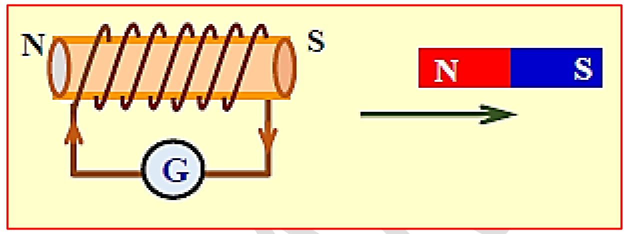




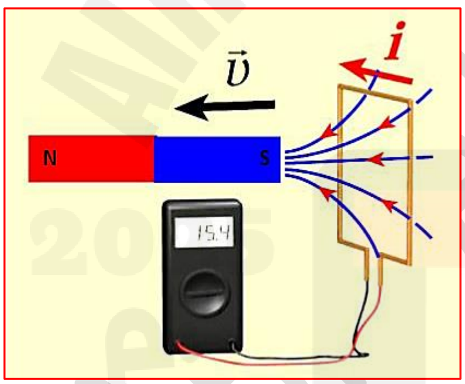
2- Magnet moving away from a wire loop induce a current flowing in the Loop:

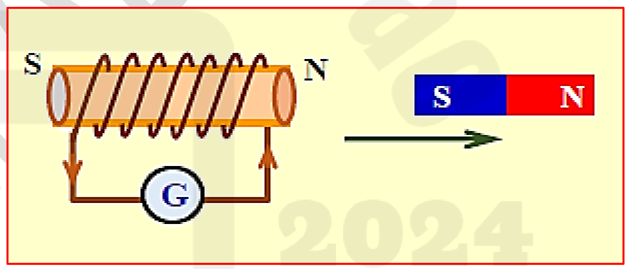
A- With the north pole of the magnet ***pointing away from the loop****, a negative current Results.   [a clockwise current flow in the loop]*





B-*With the south pole of the magnet* **pointing away from the loop** *(a positive current) results. [a counterclockwise current flows in the loop]*





******* For all of the above cases***

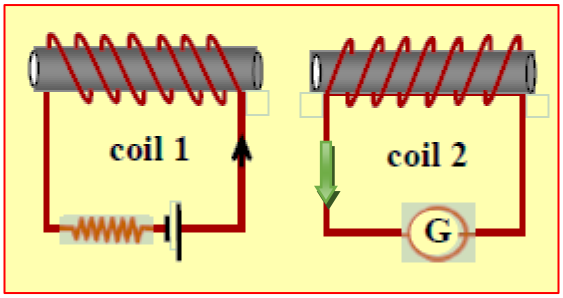
- We get the same result if the magnet stationary and the loop is moving

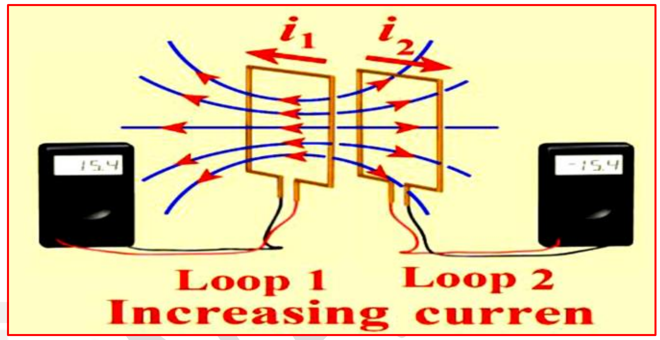
- No current is generated when there is no relative Movement

or movement at the same speed and in the same direction.

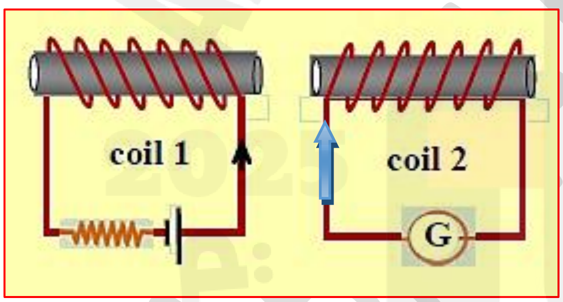
<u>Using two loops of conductive material</u>

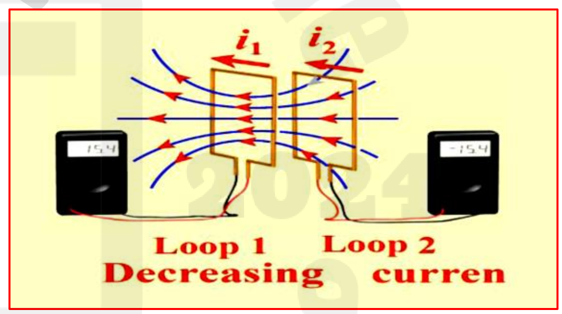
- If a constant current is flowing through **loop 1,** no current is induced in **loop 2.**
- If the current in **loop 1** is increased, a current is induced in **loop 2** in the **opposite direction.**





- if current is flowing in **loop 1** in the same direction as before and is then **<u>decreased the current</u>** induced in **loop 2** flows in **the same direction** as the current in **loop 1.**

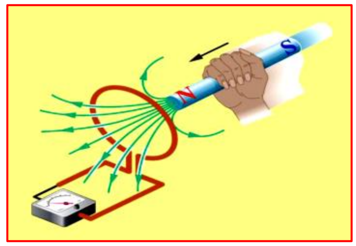




No current generated in the loop (2) if the current in the loop (1) is **a constant current of** intensity.

- If the current in loop (1) **is alternating (AC)** , it generates in the loop (2) **(AC)** current with the same frequency as the loop)6(.

Q1: The Figure shows a conducting loop connected to a sensitive ammeter:



1- Determine the direction of the induced current if we move a bar magnet toward the loop.

……………………………………………………………………………………

2- What happens to the induced Current When we increase the speed of a bar magnet.

………

3- What happens to the induced current When the a bar magnet stope moving .

……………………………………………………………………………………………

* Clockwise
* Faster motion produces a greater current.
* The current disappears (Equal zero)

1.2 Faraday's law of induction

Faraday's Law of Induction states that:

- a potential difference is induced in a loop when there is **a change in the magnetic flux** through the loop .
- potential difference is induced in a loop when **the number of magnetic field lines passing through the loop changes** with time

The rate of change of the magnetic field lines determines the induced potential difference.

$$\Delta V_{\text{ind}} = -\frac{d\Phi_B}{dt}$$

magnetic flux (Φ) : is defined as the surface integral of the magnetic field passing through a differential element of area:

$$\Phi_B = \iint \vec{B} \cdot d\vec{A}$$

where $\vec{B}$ is the magnetic field at each differential area element, $d\vec{A}$, of a closed surface.

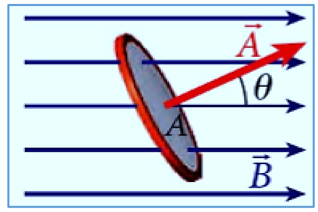
- Integration of the magnetic flux over a closed surface yields zero:

$$\oiint \vec{B} \cdot d\vec{A} = 0.$$

dA, always points out of the enclosed volume and is perpendicular to the surface everywhere.

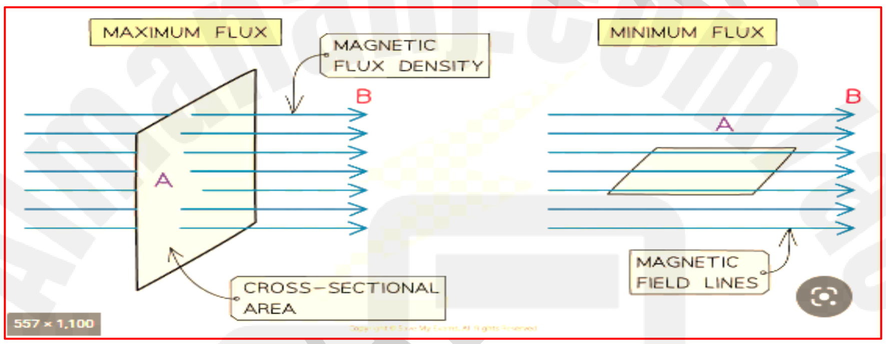
- **Consider the special case of a flat loop of area A in a constant magnetic field, as illustrated in For this case, we can**

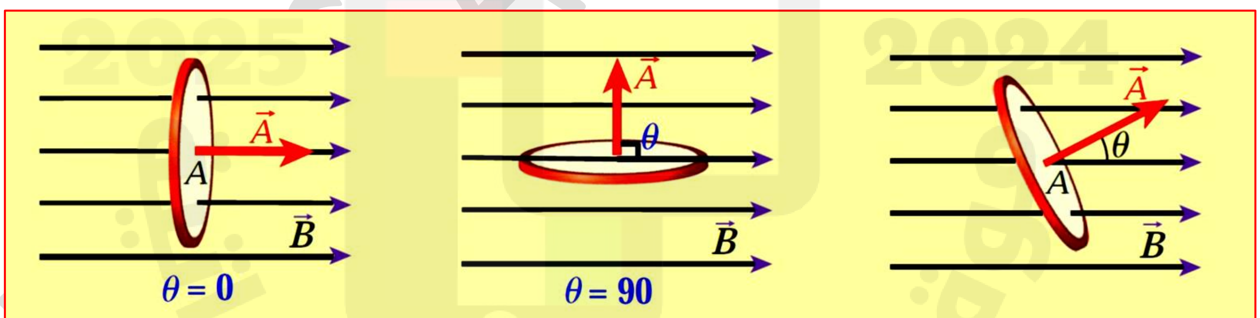
$$\Phi_B = BA\cos\theta$$



B is the magnitude of the constant magnetic field
A is the area of the loop,
Θ is the angle between the surface normal vector to the plane of the loop and the magnetic field lines.

- **The unit of magnetic flux is [ϕ_B] = [B][A] = T m^2.**
- **This unit has received a special name, the weber (Wb):**
- $1\ \mathrm{Wb} = 1\ \mathrm{T\ m^2}$.





the magnetic field is perpendicular to the plane of the loop θ = 0° Φ = A B cos 0 = A B maximum magnetic flux	magnetic field is parallel to the plane of the loop θ = 90° Φ = A B cos 90 = 0 magnetic flux = 0	the angle between the surface and the magnetic field line = 60° θ = 30° Φ = A B cos 30

Faraday's Law of Induction is thus expressed by the equation

$$\Delta V_{\text{ind}} = -\frac{d\Phi_B}{dt}.$$

- The negative sign in equation is necessary because the induced potential difference establishes an induced current whose magnetic field tends to oppose the flux change---- **Lenz's Law**

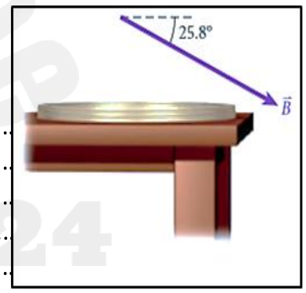
The magnetic flux can be changed in several ways

$$\Phi_B = BA\cos\theta$$

- including changing the magnitude of the magnetic field **(B)**
- changing the area of the loop **(A)**
- changing the angle, the loop makes with respect to the magnetic field **(θ)**

In all situations that involve some form of motion of a conductor relative to the source of a magnetic field, the induced potential difference is called a motional **emf.**

Q2 :A circular coil of wire with (20 turns) and a radius of (r= 40.0 cm) is laying flat on a horizontal table as shown in the figure. There is a uniform magnetic field extending over the entire table with a magnitude of (B= 5.00 T) and directed to the north and downward, making an (θ= 25.8°) with the horizontal).



What is the magnitude of the magnetic flux through the coil?:

………
………
………
………
………
……………………………………………………………

21.9 T m²

induction in a Flat loop inside a Magnetic Field

where uniform means that the field has the same value (same magnitude and same direction)

$$\Delta V_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA\cos\theta).$$

We can use the product rule from calculus to expand this derivative:

$$\Delta V_{\text{ind}} = -A\cos\theta\frac{dB}{dt} - B\cos\theta\frac{dA}{dt} + AB\sin\theta\frac{d\theta}{dt}.$$

Because the time derivative of the angular displacement is the angular velocity, dϴ/dt = ω,

$$\Delta V_{\text{ind}} = -A\cos\theta\frac{dB}{dt} - B\cos\theta\frac{dA}{dt} + \omega AB\sin\theta.$$

Holding two of the three variables in equation **(A, B, and Θ)** constant results in the following three special cases:

1- Holding the area of the loop and its orientation relative to the magnetic field constant but varying the magnetic field in time yields

A and θ constant: $\Delta V_{ind} = -A\cos\theta \frac{dB}{dt}$.

2- Holding the magnetic field as well as the orientation of the loop relative to the magnetic field constant but changing the area of the loop that is exposed to the magnetic field yields

B and θ constant: $\Delta V_{ind} = -B\cos\theta \frac{dA}{dt}$.

3- Holding the magnetic field constant and keeping the area of the loop fixed but allowing the angle between the two to change as a function of time yields

A and B constant: $\Delta V_{ind} = \omega AB\sin\theta$.

$d\Phi = \Phi_f - \Phi_i$ $dA = A_f - A_i$ $dB = B_f - B_i$

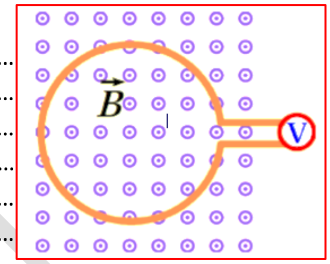
$\Delta V_{ind} = -A\cos\theta \frac{dB}{dt}$	$\Delta V_{ind} = \omega AB\sin\theta$	$\Delta V_{ind} = -B\cos\theta \frac{dA}{dt}$
تغيير المجال المغناطيسي	تغيير الزاوية	تغير المساحة
changing the magnetic field	changing the angle	changing the area
(A , θ constant ثوابت)	(B و A constant ثوابت)	(B و θ constant ثوابت)

For loop (N) turns . we multiply each equation by (N)

$d\Phi = \Phi_f - \Phi_i$ $dA = A_f - A_i$ $dB = B_f - B_i$

Q3: The plane of the circular loop shown in the figure is perpendicular to a magnetic field with magnitude *($B = 0.500$ T)*. The magnetic field goes to zero at a constant rate in *(0.250 s)*. The induced voltage in the loop is *($\Delta V_{ind} = 1.24$ V)* during that time.

What is the radius of the loop?

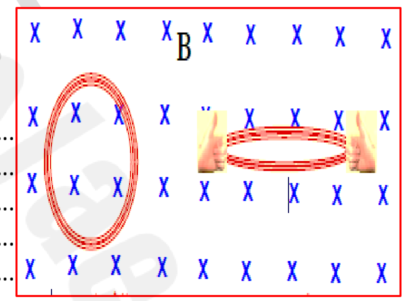


r = 0.44m

Q4: A circular coil of *($N = 10$)* turns and area *($0.5\ m^2$)* is located perpendicular inside a uniform magnetic field *($B = 0.04$ T)* (figure 1) Over a period of *(0.2 s)* the loop is decrease to *($0.125\ m^2$)* (figure) .

1- what is the direction of the induced current?

2- Calculate the induced potential difference in the coil .

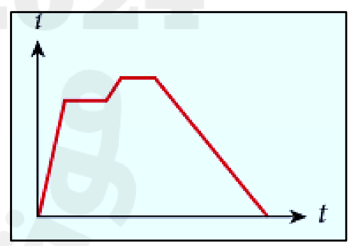


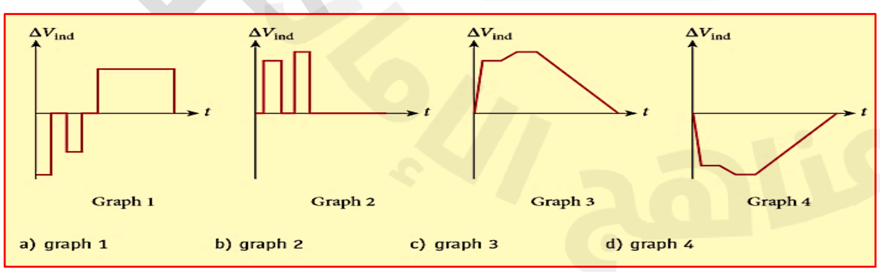
Clock wise

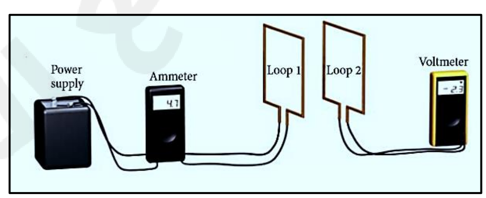
V=0.75 v

Q5: A power supply is connected to loop 1 and an ammeter as shown in the figure. Loop 2 is close to loop 1 and is connected to a voltmeter. A graph of the current i through loop 1 as a function of time, t, is also shown in the figure.

Which graph best describes the induced potential difference, ΔV_{ind}, in loop 2 as a function of time, t?

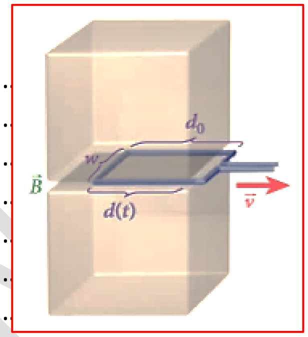






Q6: A rectangular wire loop of width *($w = 3.1$ cm)* and depth*($d_0 = 4.8$ cm)* is pulled out of the gap between two permanent magnets. A magnetic field of magnitude *($B = 0.073$ T)* is present throughout the gap (Figure). If the loop is removed at a constant speed of *($v = 1.6$ cm/s)* ,

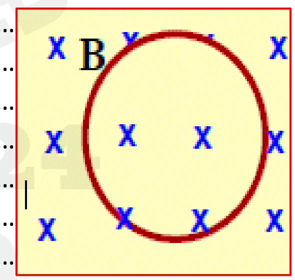
what is the induced voltage in the loop as a function of time?



$= 3.6 \cdot 10^{-5}$ V.

Q7: The plane of the circular loop with radius (0.01 m) is perpendicular to a magnetic field with magnitude (B = 1.2 T) within a time of (2 s) the magnetic field is decreases to (B = 0.6 T) .

calculate the induced potential difference in the loop.

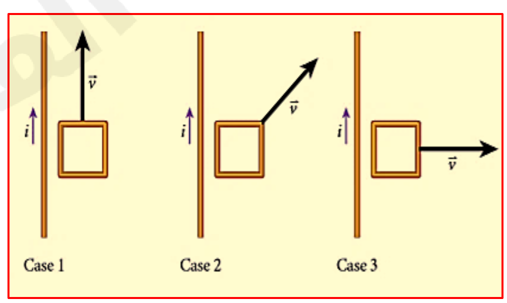


$\Delta v = 9.42 \times 10^{-5}$ v

Q8: A long wire carries a current, i, as shown in the figure. A square loop moves in the same plane as the wire as indicated.

In which cases will the loop have an induced current?

a) cases 1 and 2
b) cases 1 and 3
c) cases 2 and 3
d) None of the loops will have an induced current.

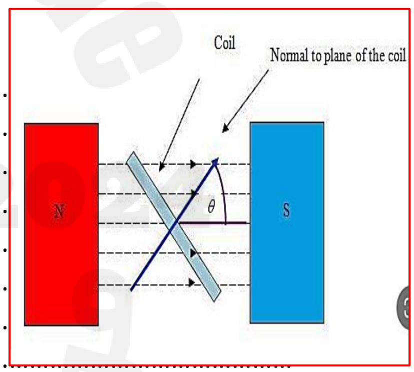


Q9: When a magnet in an MRI is abruptly shut down, the magnet is said to be quenched. Quenching can occur in as little as (Δt = 20.0 s). Suppose a magnet with an initial field of(B_I 1.20 T) is quenched in (Δt = 20.0 s) , and the final field(B_2=0.0) .Under these conditions, what is the average induced potential difference around a conducting loop of radius (1.00 cm) (about the size of a wedding ring) oriented perpendicular to the field?

$1.89 \cdot 10^{-5}$ V.

Q10: An*(8-turn)* coil has square loops measuring *(r= 0.200 m*) along a side and a resistance of *(R= 3.00 Ω)* . It is placed in a magnetic field that makes an angle of (θ =40.0°*)* with the plane of each loop. The magnitude of this field varies with time according to **$B(t) = 1.50\,t^3$** , where t is measured in seconds and B in teslas.

1- calculate the induced potential difference in the loop at t = 2.00 s?
2- What is the induced current in the coil at ***t = 2.00 s****?*

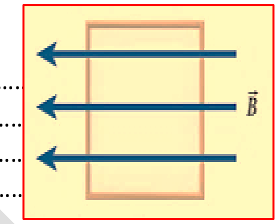


ΔV=4.41 V

I= 1.47 A

Q11: A metal loop has an area of(0.100 m^2) and is placed flat on the ground. There is a uniform magnetic field pointing west, as shown in the figure. This magnetic field initially has a magnitude of (0.123 T), which decreases steadily to(0.075 T) during a period of(0.579 s).

Find the potential difference induced in the loop during this time.



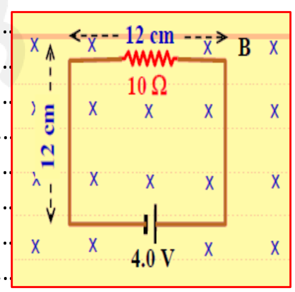
0 v

Q12: A respiration monitor has a flexible loop of copper wire, which wraps about the chest. As the wearer breathes, the radius of the loop of wire increases and decreases. When a person in the Earth's magnetic field ***(0.426×10^{-4} T)*** inhales, what is the average current in the loop, assuming that it has a resistance of***(R= 30.0 Ω)*** and increases in radius from***(r_1= 20.0 cm to r_2= 25.0 cm)*** over ***(Δt= 1.00 s)*** ? Assume that the magnetic field is perpendicular to the plane of the loop.

$-1.00374 \cdot 10^{-7}$ A

Q13: In the adjacent figure, the magnetic field passing through the electric circuit decreases at a rate ***(150 T/S).***

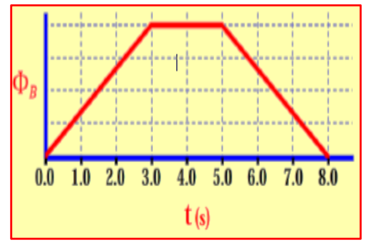
calculate the current passing through resistor during the decreasing of the magnetic field .

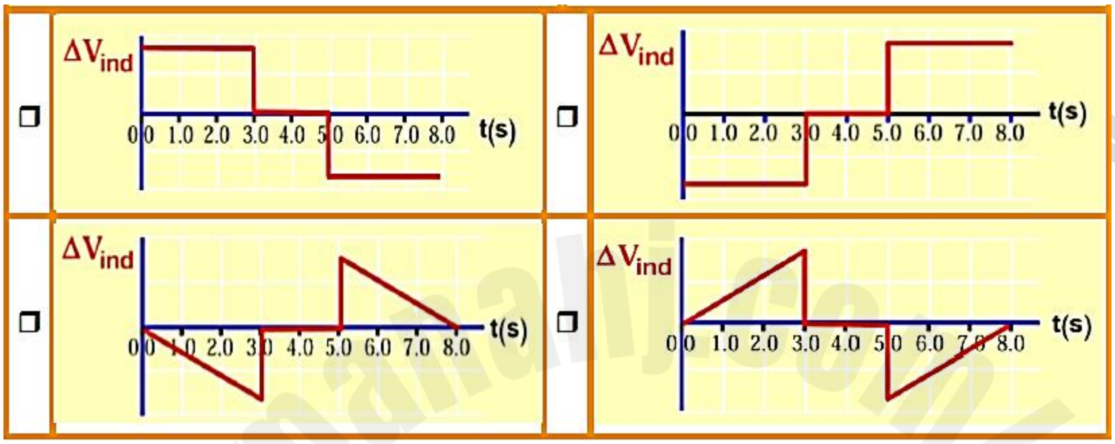


I_{ind}=0.216 A
I_{pass}=I-I_{ind}
I_{pass}=0.4- 0.216=0.184 A

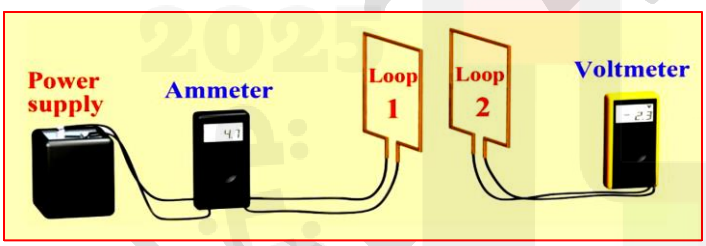
CHOSE THE BEST ANSWER

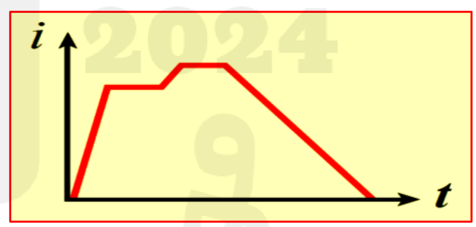
Q14: The diagram shows the Changes of the magnetic flux. that passes through a closed circuit as a function of Time. which of the following figures represents the potential? difference changes as a function of Time.

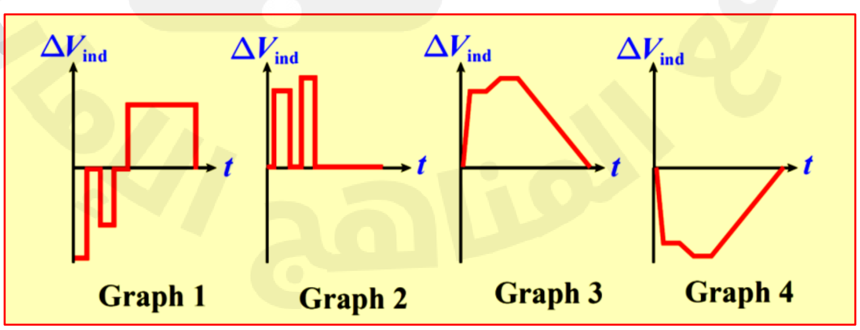




Q15: A power supply is connected to loop (**1**) and an ammeter as shown in the figure. Loop (**2**) is close to loop (**1**) and is connected to a voltmeter.
A graph of the current *i* through loop (1) asa function of time, *t,* is also shown in the figure.
Which **graph best describes** the induced potential difference,(ΔV_{ind}) in loop (**2**) as a function of **time *t*** ?

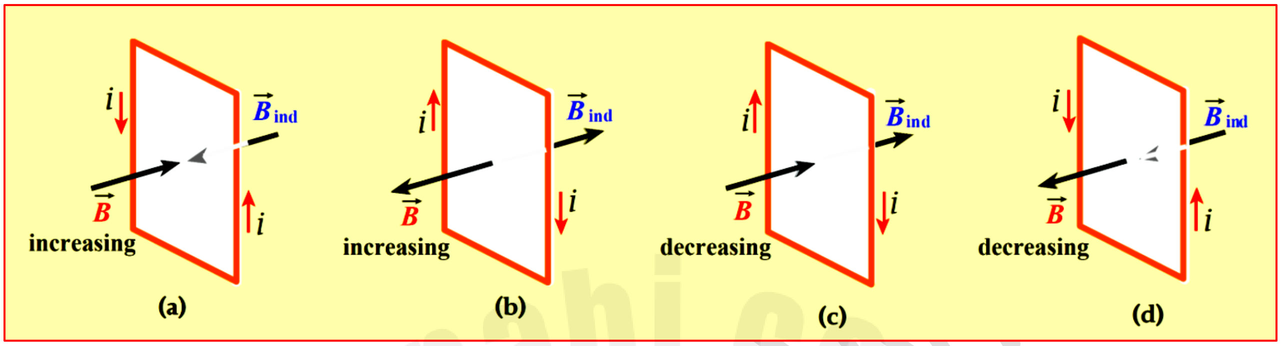






9.3 Lenz's law

Lenz's Law provides a rule for determining the direction of an induced current in a loop.
" An induced current will have a direction such that the magnetic field due to the induced current opposes the change in the magnetic flux that induces the current"



The relationship between the external magnetic field (B) and the induced current (i) and the magnetic field (Bind) caused by that induced current :

A- An increasing magnetic field pointing to the right induces a current that creates a magnetic field pointing to the left. direction of current is counterclockwise.

B- An increasing magnetic field pointing to the left induces a current that creates a magnetic field pointing to the right. direction of current is clockwise

C- A decreasing magnetic field pointing to the right induces a current that creates a magnetic field pointing to the right. direction of current is clockwise

D- A decreasing magnetic field pointing to the left induces a current that creates a magnetic field pointing to the left. direction of current is counterclockwise

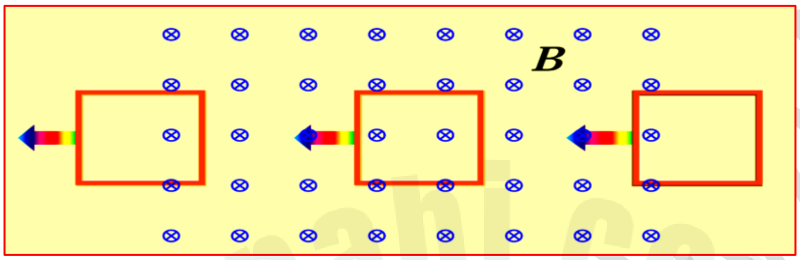
Q16: A square conducting loop with very small resistance is moved at constant speed from a region with no magnetic field through a region of constant magnetic field and then into a region with no magnetic field, as shown in the figure.

As the loop enters the magnetic field, what is the direction of the induced current?

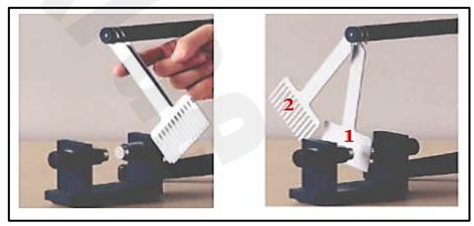
………..

As the loop leaves the magnetic field, what is the direction of the induced current?

…….



Eddy Currents



- One metal plate is solid, and the other has slots cut in it.
- The pendulums are pulled to one side and released.
- The pendulum with the solid metal plate stops in the gap.
- while the slotted plate passes through the magnetic field, only slowing slightly.
- This demonstration illustrates the very important phenomenon of **induced eddy currents.**

Why the solid metal plate is stopping ?

As the pendulum with the solid plate (1) enters the magnetic field between the magnets, these currents produce induced magnetic fields opposing the external field that created the currents.
These induced magnetic fields interact with the external magnetic field (via their spatial gradients) to stop the pendulum.

Why the slotted plate passes?

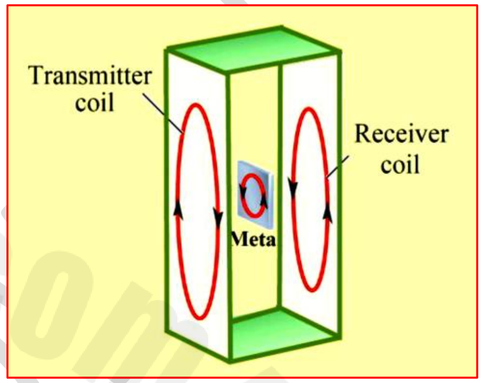
- Larger induced currents produce larger induced magnetic fields and thus lead to more rapid deceleration of the pendulum. In the slotted plate (2), the induced eddy currents are broken up by the slots, and the slotted plate passes through the magnetic field, only slowing slightly.

- Eddy currents are often undesirable, forcing equipment designers to minimize them by segmenting or laminating electrical devices that must operate in an environment of changing magnetic fields. However, eddy currents can also be useful and are employed in certain practical applications, such as the brakes of train cars.

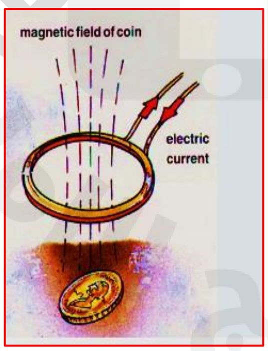
Metal Detector

A metal detector works by using electromagnetic induction, often called pulse induction. A metal detector has **a transmitter coil and a receiver coil**

. A transmitter coil and a receiver coil are located on opposite sides of an entry door.
The person or object to be scanned passes through the door between the two coils.
Suppose that the current in the transmitter coil is flowing in the direction shown and increasing.

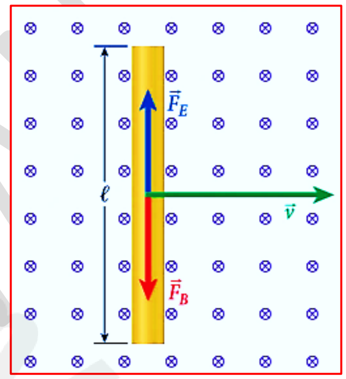


A current will be induced in the metal plate in the opposite direction and will tend to oppose the increase in the current in the transmitter coil. The increasing current in the metal plate will induce a current in the receiver coil that is in the opposite direction and tends to oppose the increase in the current in the metal plate (not shown in the diagram). Thus, the metal plate induces a current in the receiver coil that flows in the same direction as the current in the transmitter coil.
Without the metal plate, the increasing current in the transmitter coil induces a current in the opposite direction in the receiver coil that tends to oppose the increase in the current in the transmitter coil (as shown in the diagram). Thus, the overall effect of the metal plate in the metal detector is to decrease the observed current in the receiver coil. The metal object does not have to be a flat plate; any piece of metal, provided it is large enough, will have currents induced in it that can be detected by measuring the induced current in the receiver coil
Metal detectors are also used to control traffic lights.



Induced Potential Difference on a Wire Moving in a Magnetic Field

Consider a conducting wire of **length L** moving with constant **velocity v** perpendicular to a constant **magnetic field, B**, directed into the page.
The wire is oriented so that it is perpendicular to the velocity and to the magnetic field. The magnetic field exerts a force, F_B, on the conduction electrons in the wire, causing them to move downward.
This motion of the electrons produces a net negative charge at the bottom end of the wire and a net positive charge at the top end of the wire.
This charge separation produces an electric field, E, which exerts a force, F_e, on the conduction electrons that tends to cancel the magnetic force.
After some time, the two forces become equal in magnitude (but opposite in direction) producing a zero net force:



$$F_B = evB = F_E = eE.$$

$$E = vB.$$

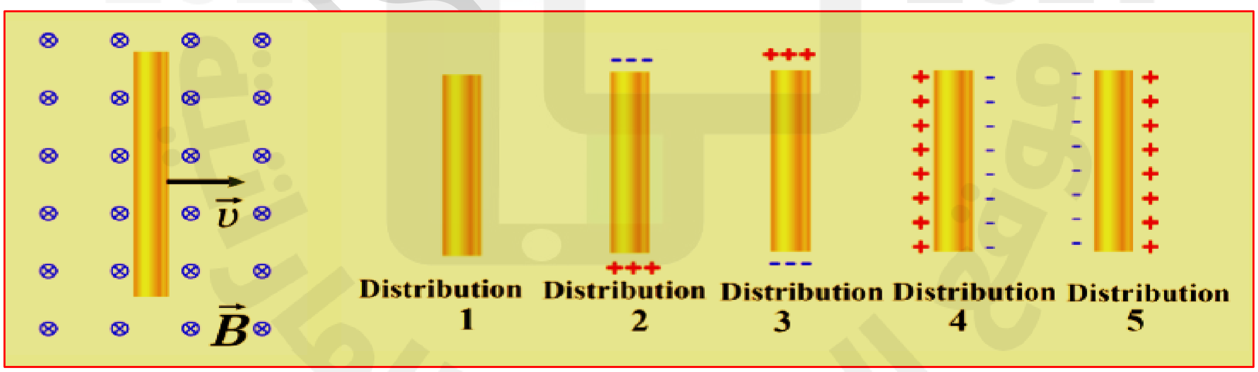
Because the electric field is constant in the wire, it produces a potential difference between th... ire given by

$$E = \frac{\Delta V_{\text{ind}}}{\ell} = vB.$$

The induced potential difference between the ends of the wire is then

$$\Delta V_{\text{ind}} = v\ell B$$

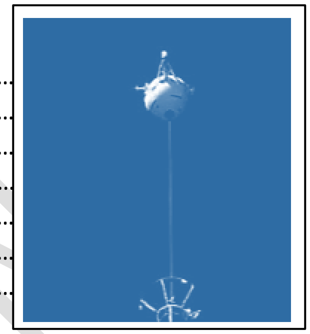
Q17: A metal bar is moving with constant velocity v through a uniform magnetic field pointing into the page, as shown in the figure.
Which of the following most accurately represents the charge distributionon the surface of the metal bar?



A- 1 **B- 2** **C- 3** **D- 4** **E- 5**

Q18: In 1996, the Space Shuttle Columbia deployed a tethered satellite on a wire out to a distance of ***(20 km),*** The wire was oriented perpendicular to the Earth's magnetic field at that point, and the magnitude of the field was ***($B = 5.1 \times 10^{-5}$ T) .*** Columbia was traveling at a speed of ***(7.6 km/s).***

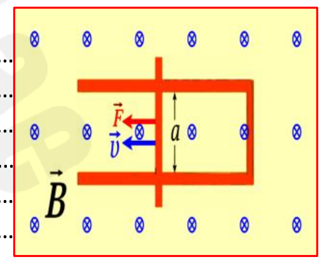
What was the potential difference induced between the ends of the wire?



7800 V.

Q19: A conducting rod is pulled horizontally by a constant force, along a set of conducting rails separated by a distance ***(a = 0.500 m)*** (Figure). The two rails are connected, and no friction occurs between the rod and the rails. A uniform magnetic field with magnitude ***(B = 0.500 T)*** is directed into the page. The rod moves at constant speed***, (v =5.00 m/s).***

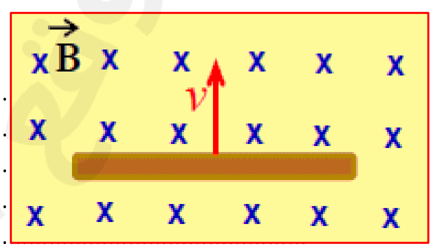
What is the magnitude of the induced potential difference in the loop created by the connected rails and the moving rod?



1.25 V.

Q20: Metal bar its length *(ℓ = 0.2 m)*is moving with constant velocity *(v)* perpendicular to a uniform magnetic field *(B = 0.45 T)*pointing into the page as shown in the figure .If the induced potential difference generated in the wire equal *(V_{ind} = 1.35 V)* .

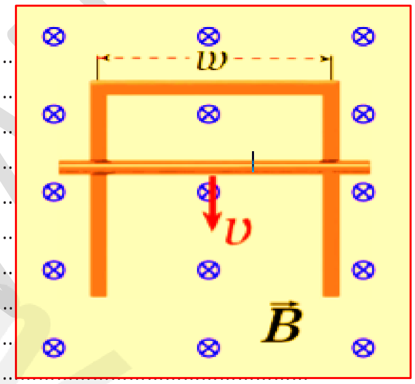
Calculate the velocity of metal bar (v) .



V=15.0 m/s

Challenge Question

Q21: A rectangular frame of conducting wire has Negligible resistance and width ($W = 0.2\ m$) and is held vertically in a magnetic field of magnitude ($B = 0.1\ T$) as shown in the figure
A metal bar with weight ($F_g = 0.05\ N$) and Resistance ($R = 0.4\ \Omega$) is placed across the frame, maintaining contact with the frame.
Calculate the terminal velocity of the bar if itis allowed to fall freely along this frame starting from rest. [Neglect friction between the wires and the metal bar]



....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

9.4 Generators and Motors

Electric generator: A device that converts electrical energy into kinetic energy.
Electric motor: A device that converts kinetic energy into electrical energy.

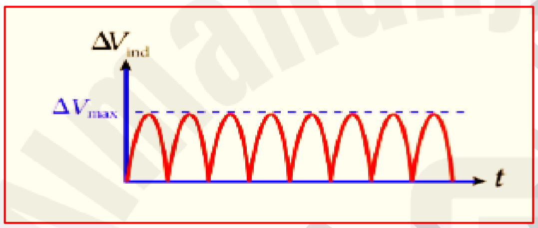
The force that causes the loop to rotate can be supplied by:

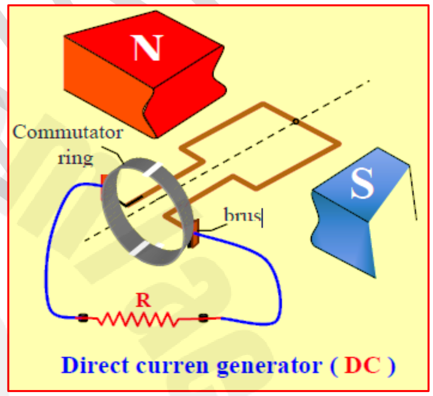
1- hot steam running over a turbine, as occurs in nuclear and coal-fired power plants.
2- flowing water or wind to generate electricity in a pollution-free way.

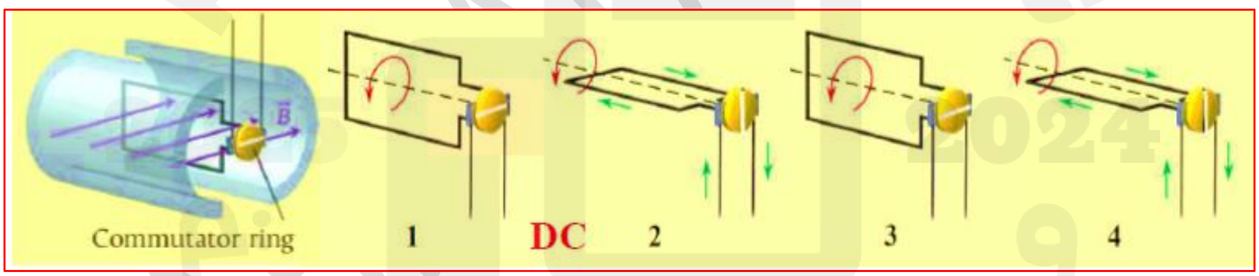
types of simple generators.

1- ***Direct current generator (DC)***

the rotating loop is connected to an external circuit through a split commutator ring, as illustrated.
As the loop turns, the connection is reversed twice per revolution,
so the induced potential difference always has the same sign.





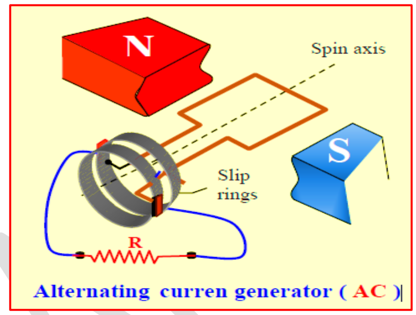


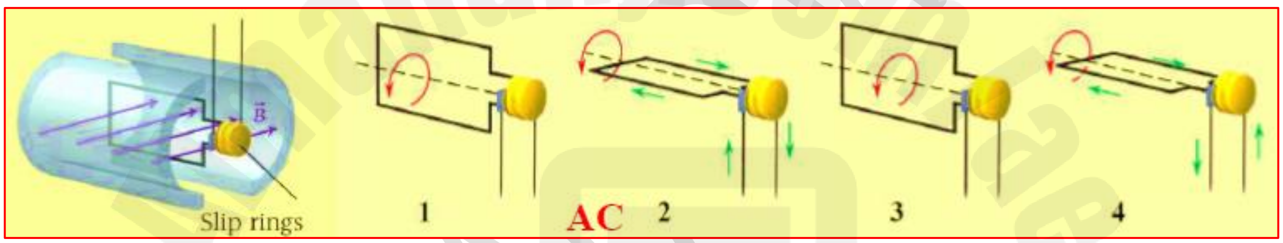
(1) & **(3)** The loop perpendicular to the magnetic field : [Φ_{max}] , [$\Delta V = 0$] , [$i = 0$]

(2) & **(4)** The loop parallel to the magnetic field : [$\Phi = 0$] , [ΔV_{max}] , [i_{max}]

2- Alternating current generator (AC)

An alternating current is a current that varies in time between positive and negative values, with the variation often showing a sinusoidal form. Each end of the loop is connected to the external circuit through its own solid slip ring.





(1) & **(3)** The loop perpendicular to the magnetic field : [Φ_{max}] , [$\Delta V = 0$] , [$i = 0$]
(2) & **(4)** The loop parallel to the magnetic field : [$\Phi = 0$] , [ΔV_{max}] , [i_{max}]

In this case, the angle between the conducting loop and the magnetic field is varied over time, while keeping the area of the loop as well as the magnetic field strength constant.

In this situation.

A and B constant:	$\Delta V_{ind} = \omega AB\sin\theta.$

$\Delta V_{ind} = NAB\omega \sin\theta$	$\Delta V_{ind} = NAB\omega \sin\omega t$
$\Delta V_{max} = NAB\omega$	$\Delta V_{ind} = \Delta V_{max} \sin\omega t$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Q22: A simple generator consists of a loop rotating inside a constant magnetic field If the loop is rotating with frequency (f) , the magnetic flux is given by *ϕ(t) = BA cos (2Пft)*.

If ***(B = 1.00 T and A = 1.00 m^2,)***

what must the value of f be for **the maximum induced potential difference** to be **(110. V)** ?

....

....

....

....

....

....

....

....

.... 17.5 Hz.

Q23: A motor has a single loop inside a magnetic field of magnitude *(B= 0.87 T).* If the area of the loop is (*A= 300. cm^2),*

find ***the maximum angular speed*** possible for this motor when connected to a source of emf providing (170 V) .

....

....

....

....

....

....

....

....

....

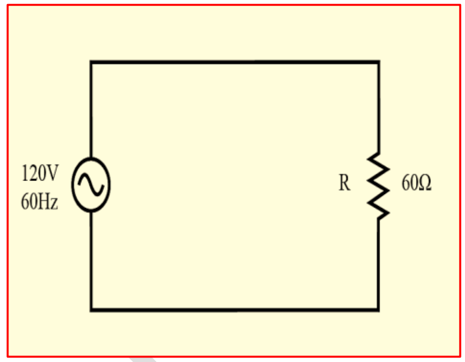
....

....

$\omega = 6500$ Hz

Home work

An AC power supply the out put voltage give as function.
$V_{emf} = 120 \sin(120\pi t)$,where t is measured in s and v in volt .
The power supply is connected to a resistor of(15.0 Ω) .



1- What is maximum voltage?
2- Calculate the frequency for this power supply?
3- Calculate the maximum current.
4- What is the power dissipated in resistor .

....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....

Q24: Your friend decides to produce electrical power by turning a coil of *(1.00×10^5 circular)* loops of wire around an axis parallel to a diameter in the Earth's magnetic field, which has a local magnitude of *($B= 0.300$ G)*. The loops have a radius of *($r= 25.0$ cm.)*

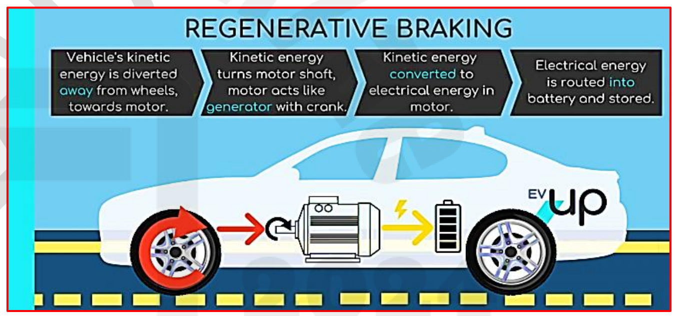
a) If your friend turns the coil at a frequency of ($f= 150.0$ Hz), what peak current will flow in a resistor, ($R = 1500.\ \Omega$), connected to the coil?

b) The average current flowing in the coil will be 0.7071 times the peak current. What will be the average power obtained from this device?

………
………
………
………
………
………
………
………

(a) $i_{ind,peak} = 0.370$ A

(b) $i_{avg} = 0.262$ A, $P_{avg} = 103$ W



Regenerative braking

Hybrid cars are propelled by a combination of gasoline power and electrical power. One attractive feature of a hybrid vehicle is that it is capable of regenerative braking. When the brakes are used to slow or stop a nonhybrid vehicle, the kinetic energy of the vehicle is turned into heat in the brake pads. This heat dissipates into the environment, and energy is lost. In a hybrid car, the brakes are connected to the electric motor, which functions as a generator, charging the car's battery.
Thus, the kinetic energy of the car is partially recovered during braking, and this energy can later be used to propel the car, contributing to its efficiency and greatly increasing its gas mileage in stop-and-go driving.

9.5 induced electric Field

Consider a positive charge q moving in a circular path with radius r in an electric field, ⃗ E. The work done on the charge is equal to the integral of the scalar product of the force and the differential displacement vector.

$$W = \oint \vec{F}\cdot d\vec{s} = q\oint \vec{E}\cdot d\vec{s}.$$

$$\oint \vec{F}\cdot d\vec{s} = \oint q\vec{E}\cdot d\vec{s} = \oint q\cos 0° E\, ds = qE\oint ds = qE(2\pi r).$$

Since the work done by a constant electric field is ΔV_{ind}, we get

$$\Delta V_{ind} = \oint \vec{E}\cdot d\vec{s}.$$

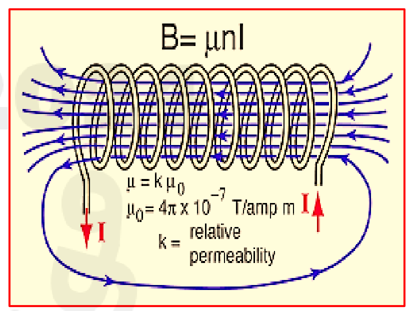
Now we can express the induced potential difference in a different way

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}.$$

9.6 inductance of a Solenoid

Consider a long solenoid with N turns carrying a current, i. This current creates a magnetic field in the center of the solenoid, resulting in a magnetic flux, ϕ_B.
The same magnetic flux goes through each of the N windings of the solenoid.

$$N\Phi_B = L\, i$$



L, called the inductance.

The unit of inductance is the Henry (H)

The flux linkage for this solenoid is

$$N\Phi_B = (n\ell)(BA),$$

$$L = \frac{\mu_o N^2 A}{\ell} = \mu_o n^2 A\ell$$

$L = \frac{\mu N^2 A}{l}$
Where:
L = Inductance in henries (H)
μ = permeability $(Wb/A\cdot m)$
N = number of turns in coil
A = area encircled by coil (m^2)
l = lenth of coil (m)

Inductance: is the tendency of an electrical conductor to oppose a change in the electric current flowing through it.

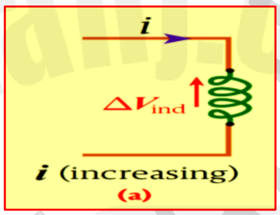
- You can see from equation that the inductance of a solenoid depends only on the geometry (length(l), area(A), and number of turns (N) of the device.
- Any solenoid has an inductance, and when a solenoid is used in an electric circuit, it is called an inductor, simply because its inductance is its most important property as far as the current flow is concerned.

9.7 Self-inductance and Mutual induction

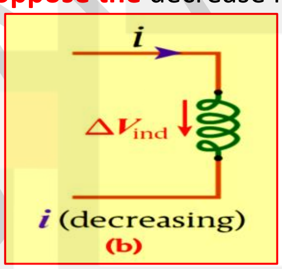
When the current changes in a coil. then. the magnetic field is change, this change in the magnetic field and flux generate an induced potential difference in the same coil, it is called self-induction .

$$\boxed{\Delta V_{\text{ind},L} = -N\frac{d\Phi_B}{dt} = -L\frac{di}{dt}}$$

- *The negative sign in equation provides the clue that the induced potential difference always opposes any change in current*
- Figure a shows current flowing through an inductor **and increasing with time**. Thus, the self-induced potential difference **will oppose** the increase in current**.**



- In Figure b, the current flowing through an inductor **is decreasing with time**. Thus, a self-induced potential difference will **oppose the** decrease in current.

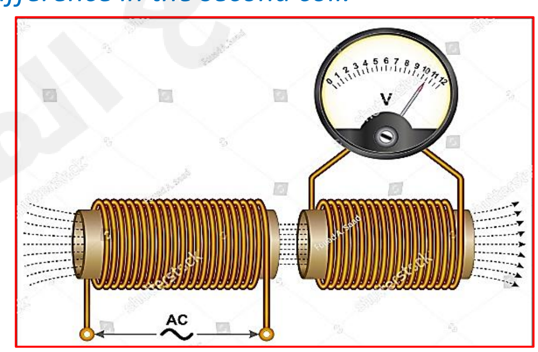


- We have assumed that these inductors are ideal inductors; that is, **they have no resistance**.

mutual induction

Changing the current in the first coil also induces a potential difference in the second coil.

Now let's consider two coils with their central axes aligned as shown.
Coil 1 has N1 turns, and coil 2 has N2 turns.
The current in coil 1 produces a magnetic field, B1.
The flux linkage in coil 2 resulting from the magnetic field in coil 1 is $N_2\Phi_{1\to 2}$. The mutual inductance, $M_{1\to 2}$, of coil 2 due to coil 1



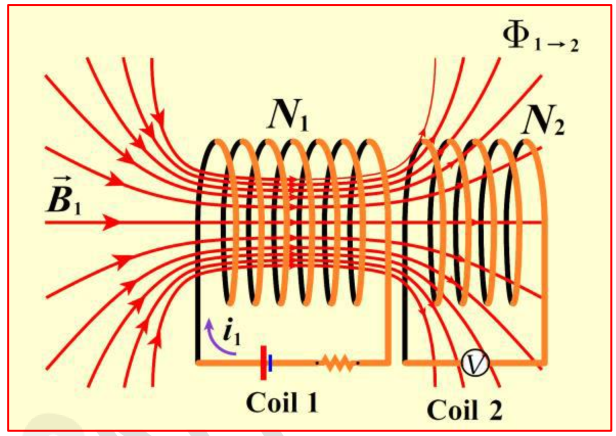
Coil 1 coil 2

It occurs between two coils close to each other Changing the current in the first coil generates induces a potential difference in the second coil:

$$\Delta V_{\text{ind},2} = -M\frac{di_1}{dt} = N_2\frac{d\Phi_{1\to2}}{dt}$$

$$\Delta V_{\text{ind},1} = -M\frac{di_2}{dt} = N_1\frac{d\Phi_{2\to1}}{dt}$$

$$M = \frac{NBA}{i} = \frac{N(\mu_0 n i)(\pi r_1^2)}{i} = N\pi\mu_0 n r_1^2.$$



We see that the potential difference induced in one coil is proportional to the change of current in the other coil

where M is the mutual inductance between the two coils.

The SI unit of mutual inductance is the Henry.

Q25: a solenoid with length ***(l= 20cm)***, it has ***(N= 1000 laps)*** with radius ***(r= 10cm),*** flowing an electrical current of ***(i= 4A)*** through it.

1- **calculate the solenoid inductance(L)?**
2- **What is the induced potential difference in the solenoid if the current change its direction during t = 0.2 s?**

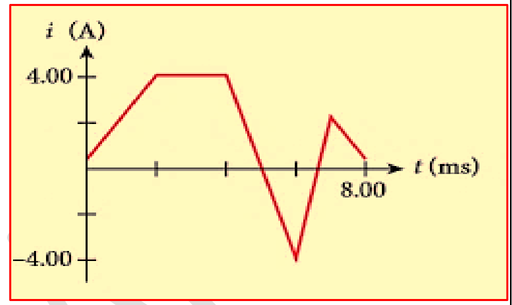
L= 0.2 H

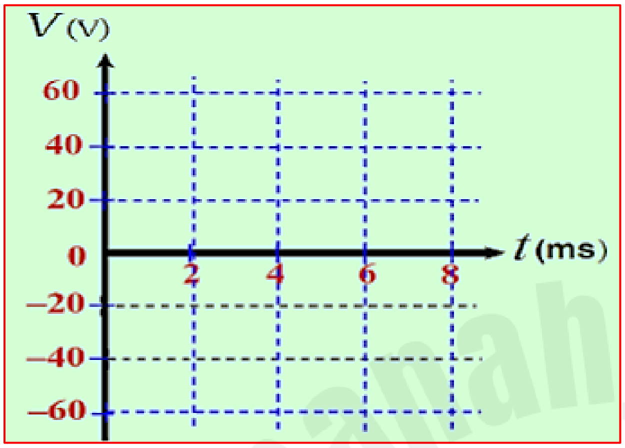
ΔV = 7.9 V

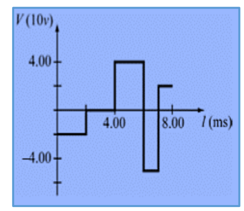
Q26: The figure shows the current through an inductor with **(L=10.0-mH)** over a time interval of ***(Δt= 8.00 ms)*** .

1- Draw a graph showing the self-induced potential difference, **ΔV_{ind},** for the inductor over the same interval.

2- What is **the maximum potential difference** induced in the conductor **ΔV_L**?

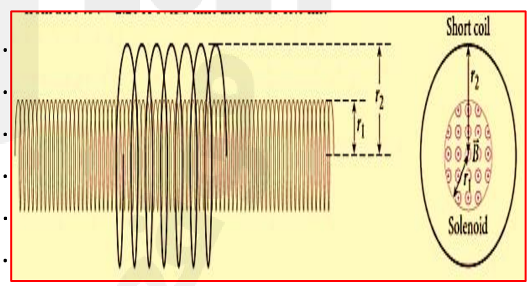






Q27: A long solenoid with circular cross section of radius **(r_1 = 2.80 cm)** and **(n = 290 turns/cm)** is inside and coaxial with a short coil with circular cross section of radius **(r_2 = 4.90 cm)** and **(N = 31 turns),** The current in the solenoid is increased at a constant rate from zero to **(i = 2.20 A)** over a time interval of **(Δt= 48.0 ms) .**

What is the potential difference induced in the short coil while the current is changing?



M=2.779x10^{-3} Henry

ΔV= -0.127 v

Q28: A constant current passing through a coil whose intensity changes according to the equation : [i (t) = 5 + 7 t – 2 t2] at the moment (t = 3.0 s) the induced potential
difference generated in a coil ($\Delta V_L = 0.036$ V)
Calculate **the inductance** (**L**) of the coil .

………
………
………
………
………
………
………
………
………
………
……… L=7.2mH
………

HOM WORK

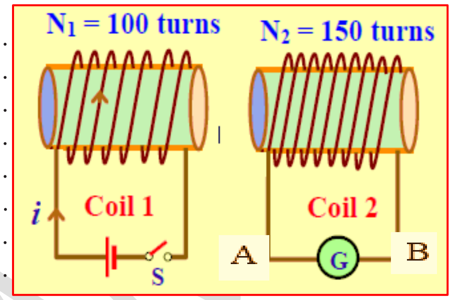
Q29: inductor with cross section area (***A*** **= 7 cm2**) and (***N*** **= 1000 turns**) wrapped on an iron core [μ **= 2 × 10^{-3} T.m/**] and its length (**ℓ = 10 cm**)
1- Calculate the **the inductance (L)** .
2- If the iron is pulled out from the coil and the current changes from (**9 A**) to (**3 A**) within(**3 ms**) .
Calculate the **induced potential.**
difference generated in a Coil.

………
………
………
………
………
………
………
………
………
………
………
………
………
………
………
………
………
………
……………………………………………………………………………………………

Q30: Two identical inductor are shown in the figure inductor 1 has a current (i = 2 A) flowing in the direction as shown in the figure. When the switch in the circuit containing inductor 1 is opened within *(0.5 S)*.[The mutual inductance between the two coils (M = 0.03 H)]

1- Determine the direction of the current passing through coil 2 .

2- Calculate the induced potential difference generated in a circuit 2

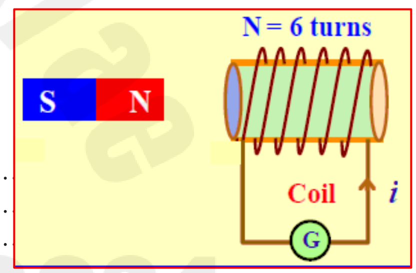


1- A to B
2- $\Delta V = 0.12$ v

Q31: inductor with a circular cross section of ($A = 1 \times 10^{-4}\ m^2$) and ($N = 6$ turns) and its length ($\ell = 0.08$ m) .the magnetic permeability of the medium inside the inductor is : [$\mu = 2 \times 10^{-3}$ T.m/A]

1- Determine the direction of movement of the magnet relative to the coil

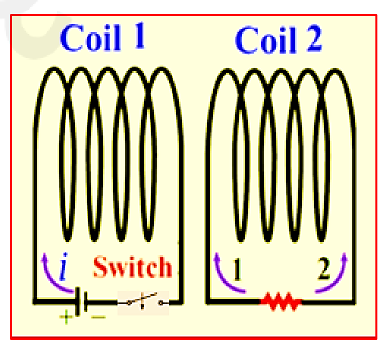
2- Calculate the inductance (L) of the coil .



1- Away from coil (to the left)
2- $L = 9.0 \times 10^{-5}$ H

Q32: Two identical coils are shown in the figure. Coil 1 has a current *I* flowing in the direction shown. When the switch in the circuit containing coil 1 is opened,

What happens in coil 2 ? Chose the best answer



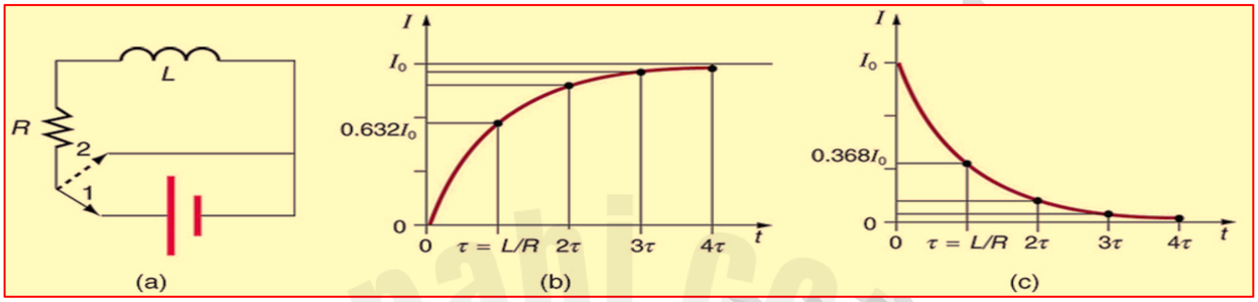
A- current is induced in coil 2 that flows in direction 1

B- A current is induced in coil 2 that flows in direction 2

C- No current is induced in coil 2

9.8 RL Circuits

RL circuit : a single-loop circuit containing a source of emf is connected to a resistor with resistance **(*R*)** and an inductor with inductance **(*L*)** in series .
- the increasing current flowing through the inductor creates a self-induced potential difference that tends to oppose the increase in current .
- As time passes, the change in current decreases, and the opposing self-induced potential difference also decreases .
- After a long time, the current becomes steady at the value $i = V_{emf}/R$.



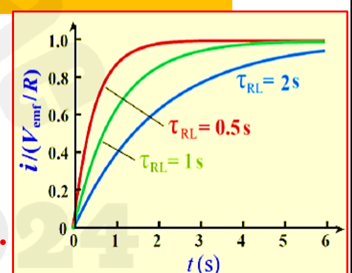
Time constant

$$\tau_{RL} = \frac{L}{R}$$

with a source of emf V_{emf}

$$i_{(t)} = \frac{V_{emf}}{R}\left(1 - e^{-t/(L/R)}\right)$$

$$L\frac{di}{dt} + iR = V_{emf}$$



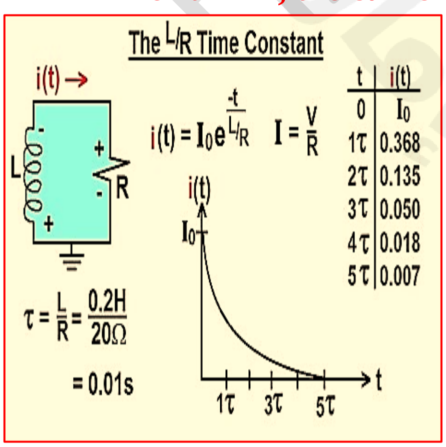
- **you can see that for $t = 0$, the current is zero.**
- **For $t \to \infty$, the current is given by $i = V_{emf}/R$, which is as expected.**

source of emf had been connected and is suddenly removed. We can use equation.

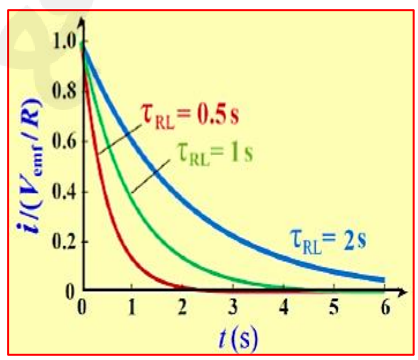
$$i_{(t)} = i_{\circ}\, e^{-(t/\tau_{RL})}$$

$$L\frac{di}{dt} + iR = 0$$

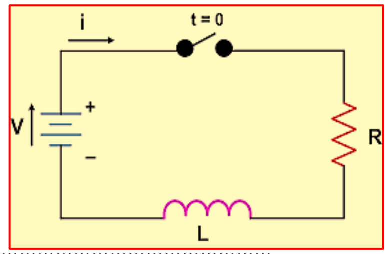
- **you can see that for $t = 0$, $i = V_{emf}/R$.**
- **For $t \to \infty$, the current is zero**



When t= time constant I= 0.63 I_O



Q33: Consider an RL circuit with resistance (R = 2.0 MΩ) and inductance (L = 1.0 H) which is powered by a (10.0 V) battery.



1- What is the time constant of the circuit ?

2- If the switch is closed at time (t = 0) .

a) what is the current just after that time?

b) After 2.0 μs ?

c) When a long time has passed ? .

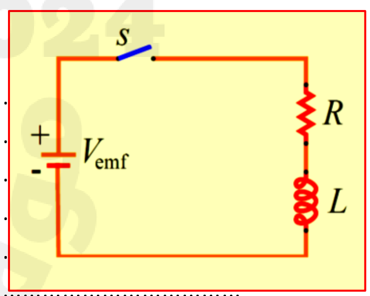
1- T=0.5μs

2- a- 0.0A

b-4.9μA

c- 5μA

Q34: In the circuit in the figure (R = 210 Ω) , (L = 3.0 H) and (V_{emf} = 40.0 V). After the switch is closed, How long will it take the current in the inductor to reach (3.0mA) ?



2.28×10^{-4} s

Q35: The current is increasing at a rate of (3.6 A/s) in an RL circuit with (R =3.15 Ω) and (L = 440 mH). What is the potential difference across the circuit at the moment?
when the current in the circuit is (3.0 A) ?

……
……
……
……
……
……
……
………

$\boxed{V=11.0\ V}$

9.10 Energy and energy Density of a Magnetic Field

We can think of an inductor as a device that can store energy in a magnetic field.

$$\boxed{P = V_{\text{emf}}\, i = \left(L\frac{di}{dt}\right) i.}$$

Now let's consider an ideal solenoid with length ℓ, cross-sectional area A, and n turns per unit length, carrying current i. The energy stored in the magnetic field of the solenoid using is

$$\boxed{U_B = \tfrac{1}{2} L i^2 = \tfrac{1}{2}\mu_0 n^2 \ell A i^2.}$$

Q36) Consider a long solenoid with a circular cross section of radius (r = 8.10 cm) and (n = 2.0× 10^4 turns/m). The solenoid has length (ℓ = 0.540 m) and is carrying a current of magnitude ($i = 4.04 \times 10^{-3}$ A).
How much <u>energy is stored</u> in the magnetic field of the solenoid?

……
……
……
……
……
……
……
………………………

$\boxed{4.55 \cdot 10^{-5}\ J}$