ملخص الدرس الثامن Rates Related من الوحدة الرابعة تطبيقات التفاضل منهج ريفيل

Math lessons

Simplification

اللهم إني أسألك فَهْمَ النَّبِيِّينَ وَحِفْظَ الْمُرْسَلِينَ، وَالْمَلَائِكَةَ الْمُقَرَّبِينَ، اللهم اجعل لِسَانِي عَامِرًا بِذِكْرِكَ، وَقَلْبِي بِخَشْيَتِكَ، وَسِرِّي بِطَاعَتِكَ، إِنَّكَ عَلَىٰ كُلِّ شَيْءٍ قَدِيرٌ، وَحَسْبِيَ اللَّهُ وَنِعْمَ الْوَكِيلُ

By: Fa

Chpt 4: Applications of differentiation

Lesson 4.8: Related Rates

1. Make a simple sketch, if appropriate.
2. Set up an equation relating all of the relevant quantities.
3. Differentiate (implicitly) both sides of the equation with respect to time (t).
4. Substitute in values for all known quantities and derivatives.
5. Solve for the remaining rate.

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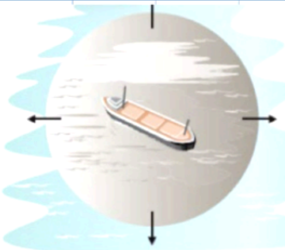
CHECK YOUR UNDERSTANDING

An oil tanker has an accident and oil pours out at the rate of 150 liters per minute.



Suppose that the oil spreads onto the water in a circle at a thickness of 1/120 m.

Given that 1 liter equals 0.001 m^3, determine the rate at which the radius of the spill is increasing when the radius reaches 500 meters.

step 1: Write the given's and the unknown

** write the rule you will use to find the unknown value

Given's:-

$v = 150L$

$h = \frac{1}{120}$

Unknown:-

$r' = ??$

$v = \pi r^2 h$

step 2: Convert L/min ----> m^3/min

Given that 1 liter equals 0.001 m^3

$v = 150L/min \times 0.001 = 0.15 m^3/min$

step 3: Differentiating both sides of the equation with respect to (t), we get

$v = \pi r^2 h$

$v' = 2\pi r \cdot r' \cdot h$

step 4: Substituting in $V'(t) = 0.15$ and $r = 500$, we have

$v' = 2\pi r \cdot r' \cdot h$

$0.15 = 2\pi(500) r' \times \frac{1}{120}$

$r' = 5.73 \times 10^{-3}$ m/min

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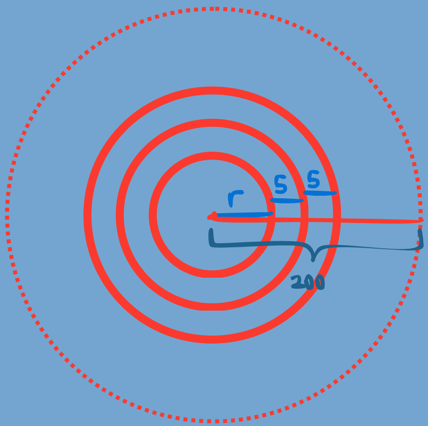
CHECK YOUR UNDERSTANDING

Suppose a forest fire spreads in a circle with radius changing at a rate of 5 ft/min. When the radius reaches 200 feet, at what rate is the area of the burning region increasing?

Simple sketch:



** You can see that as the radius increases by 5 feet, the fire spreads over a larger area of the forest. The question is, how can we determine the new radius when the burned area of the forest reaches 200?

step 1: Write the given's and the unknown

** write the rule you will use to find the unknown value

Given's:-

$r'(t) = 5$

$r = 200ft$

Unknown:-

$A'(t) = ??$

$A = \pi r^2$

step 2: Differentiating both sides of the equation with respect to (t), we get

$A = \pi r^2$

$A'(t) = 2\pi \cdot r \cdot r'$

step 3: Substituting in $r' = 5$ and $r = 200$, we have

$A'(t) = 2\pi \cdot r \cdot r'$

$= 2\pi(200)(5)$

$2,000\pi = 6283.2ft^2/min$

READ THE STEPS CAREFULLY AND ATTENTIVELY. ONCE YOU ARE SURE YOU UNDERSTAND, GO TO THE NEXT SLIDE TO SOLVE ANOTHER QUESTION ON YOUR OWN.

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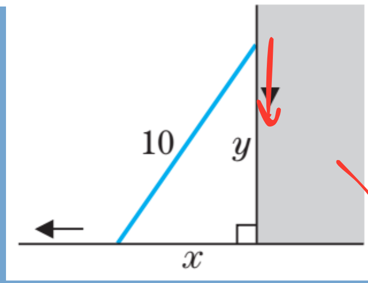
A 10-feet ladder leans against the side of a building. If the top of the ladder begins to slide down the wall at the rate of 2 ft/sec, how fast is the bottom of the ladder sliding away from the wall when the top of the ladder is 8 ft off the ground?

Because the length of the ladder will not change and will remain 10, whether it stays standing or horizontal, as y increases, x decreases, and as x decreases, y increases, because the length of the ladder is constant.



since the ladder is sliding down the wall at rate of 2ft/sec, we must have $y' = -2$

Solution First, we make a sketch of the problem, as seen in Figure 4.94. We have denoted the height of the top of the ladder as y and the distance from the wall to the bottom of the ladder as x. Since the ladder is sliding *down* the wall at the rate of 2 ft/sec, we must have that $\frac{dy}{dt} = -2$. (Note the minus sign here.) Observe that both x and y are functions of time, t. By the Pythagorean Theorem, we have

$$[x(t)]^2 + [y(t)]^2 = 100.$$

DON'T forget to find X first

$10^2 = x^2 + y^2$

$100 = x^2 + 8^2$

$x = 6$

step 1: Write the given's and the unknown

** write the rule you will use to find the unknown value

Given's:-

$y = 8$

$y' = -2$

$x = 6$

Unknown:-

$x'(t) = ??$

$10 = x^2 + y^2$

step 2: Differentiating both sides of the equation with respect to (t), we get

$10 = x^2 + y^2$

$0 = 2x \cdot x' + 2y \cdot y'$

step 4: Substituting in y=8, y' (-2) = 0.15 and x = 6, we have

$0 = 2(6) \cdot x' + 2(8) \cdot (-2)$

$x' = \frac{8}{3}$ ft/sec

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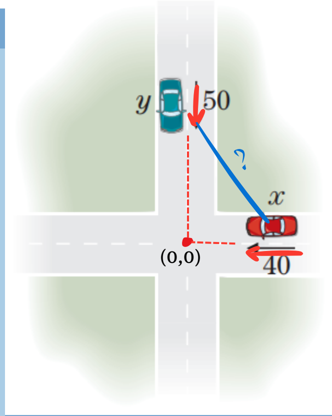
OTHER IDEA

A car is traveling at 50 mph due south at a point $\frac{1}{2}$ mile north of an intersection. A police car is traveling at 40 mph due west at a point $\frac{1}{4}$ mile east of the same intersection. At that instant, the radar in the police car measures the rate at which the distance between the two cars is changing. What does the radar gun register?

Solution First, we draw a sketch and denote the vertical distance of the first car from the center of the intersection y and the horizontal distance of the police car x. (See Figure 4.95.) Notice that at the moment in question (call it $t = t_0$), $\frac{dx}{dt} = -40$, since the police car is moving in the direction of the negative x-axis and $\frac{dy}{dt} = -50$, since the other car is moving in the direction of the negative y-axis. From the Pythagorean Theorem, the distance between the two cars is $d = \sqrt{x^2 + y^2}$. Since all quantities are changing with time, we have

$$d(t) = \sqrt{[x(t)]^2 + [y(t)]^2} = \{[x(t)]^2 + [y(t)]^2\}^{1/2}.$$



step 1: Write the given's and the unknown

** write the rule you will use to find the unknown value

Given's:-	Unknown:-
$y = \frac{1}{2}$	$d'(t) = ??$
$y' = -50$	
$x = \frac{1}{4}$	
$x' = -40$	

step 2: Differentiating both sides of the equation with respect to (t), we get

$$d^2 = x^2 + y^2$$

$$2dd' = 2x \cdot x' + 2y \cdot y'$$

$$dd' = x \cdot x' + y \cdot y'$$

step 3: Find d:

Distance Formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(x,y) ---> (0,0)

$$d = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^2} = 0.559$$

step 4: Substituting in y, y′, x, and x′ we have

$$0.559d' = \frac{1}{4} \cdot (-40) + \frac{1}{2} \cdot (-50)$$

$$d' = -62.61$$

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A Related Rates Problem

Exercise **Q19 Page: 304**

Suppose a $6ft$-tall **person** is moving **away** from $18ft$-**tall lamppost** if the person is **moving away from lamppost** at a **rate of** $2ft/sec$ at what **rate is the length of the shadow changing**?

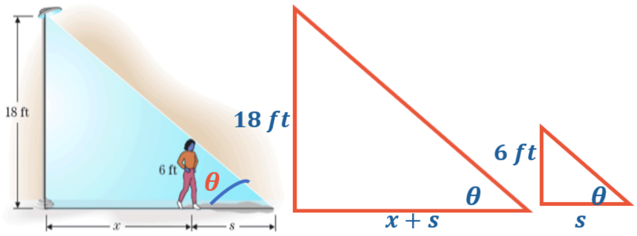
Solution

Given:

Rate of Change of x: $\frac{dx}{dt} = x'(t) = 2ft/sec$

Required:

Rate of Change of shadow Length(s): $\frac{ds}{dt} = s'(t) = ?$



Equation Relating Variables:

Using similarity of triangles

Or Using tangent of angle θ

$\tan\theta = \frac{18}{x+s}$ $\qquad$ $\tan\theta = \frac{6}{s}$

⇨ $\frac{18}{x+s} = \frac{6}{s}$

⇨ $\frac{x+s}{18} = \frac{s}{6}$ $\qquad$ ⇨ $\frac{x+s}{3} = s$

$x(t) + s(t) = 3s(t)$

x and s are functions of time

$x(t) = 2s(t)$

Differentiate implicitly with respect to time:

$x'(t) = 2s'(t)$

$2 = 2s'(t)$

⇨ $s'(t) = 1\ ft/sec$

The length of the shadow is increasing by rate of $1ft/sec$ *as this person walking with a rate of* $2\ ft/sec$ *away from lamppost.*

Estimating a Rate of Change in Economics

Exercise **Q14 Page: 304**

Suppose that the **average yearly cost** per item **for producing** x **items** of a business product is $\overline{C}(x) = 12 + \frac{94}{x}$. The three most recent yearly production figures are given in the table below. Estimate the **value of** $x'(2)$ and the **current (year 2) rate of change of the average cost**.

Solution

Given:

Production: $x(t)$

Current Production: $x(2) = 9.4$

Year (t)	0	1	2
Production $(x(t))$	8.2	8.8	9.4

Production is increasing by 0.6 per year $\quad 0.6 \quad 0.6$

Required:

Current Rate of Change of Production: $x'(2) = ?$

Current Rate of Change of average cost : $\overline{C}'(2) = ?$

From the tabble:

$x'(2) \approx 0.6$ ***Per year***

Equation Relating Variables:

$\overline{C}(t) = 12 + \frac{94}{x(t)}$

Differentiate implicitly with respect to time:

$\overline{C}'(t) = -\frac{94}{x^2(t)}x'(t)$

Substitute and Solve for $s'(4)$ **:**

$\overline{C}'(2) = -\frac{94}{x^2(2)}x'(2) = -\frac{94}{(9.4)^2}(0.6) \approx -0.6383$

Per year

For a more detailed explanation:-

- Math 12-Adv | 4.8 | Related Rates
- Related Rates | Applications of Differentiation |
- المعدلات المرتبطة Related Rates 4-8
- المعدلات المرتبطة (الجزء 1) صف 12 متقدم الفصل الثاني

By: Fa

Done with lesson 4.8..
Hope you understand!

اللهم إني أستودعُكَ ما قرأتُ وما حفظتُ وما تعلمتُ، فردّه إليَّ عند حاجتي إليه، إنك على كل شيءٍ قدير.

اللهم ذكرني منه ما نسيتُ، وعلمّني منه ما جهلتُ

By: 𝔉a