

End of Term 3 Revision



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ⇨ المناهج الإماراتية ⇨ الصف الثاني عشر المتقدم ⇨ رياضيات ⇨ الفصل الثالث ⇨ ملفات متنوعة ⇨ الملف

تاريخ إضافة الملف على موقع المناهج: 19:03:55 2026-06-15

ملفات اكتب للمعلم اكتب للطالب | اختبارات الكترونية | اختبارات | حلول | عروض بوربوينت | أوراق عمل
منهج انجليزي | ملخصات وتقارير | مذكرات وبنوك | الامتحان النهائي | للمدرس

المزيد من مادة
رياضيات:

إعداد: Nair Prakash

التواصل الاجتماعي بحسب الصف الثاني عشر المتقدم



صفحة المناهج
الإماراتية على
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

المواد على تلغرام

المزيد من الملفات بحسب الصف الثاني عشر المتقدم والمادة رياضيات في الفصل الثالث

مراجعة هيكل الامتحان كراسة تدريبية

1

ملزمة الهيكل لقوانين التكامل الأساسية

2

هيكل المشتقة العكسية

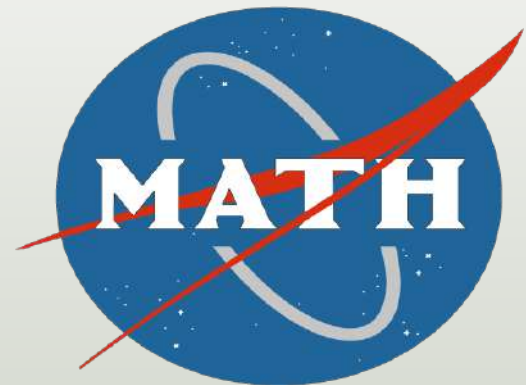
3

حلول تجميع أسئلة وتدريبات هيكل الامتحان

4

تجميع أسئلة وتدريبات هيكل الامتحان غير محلول

5

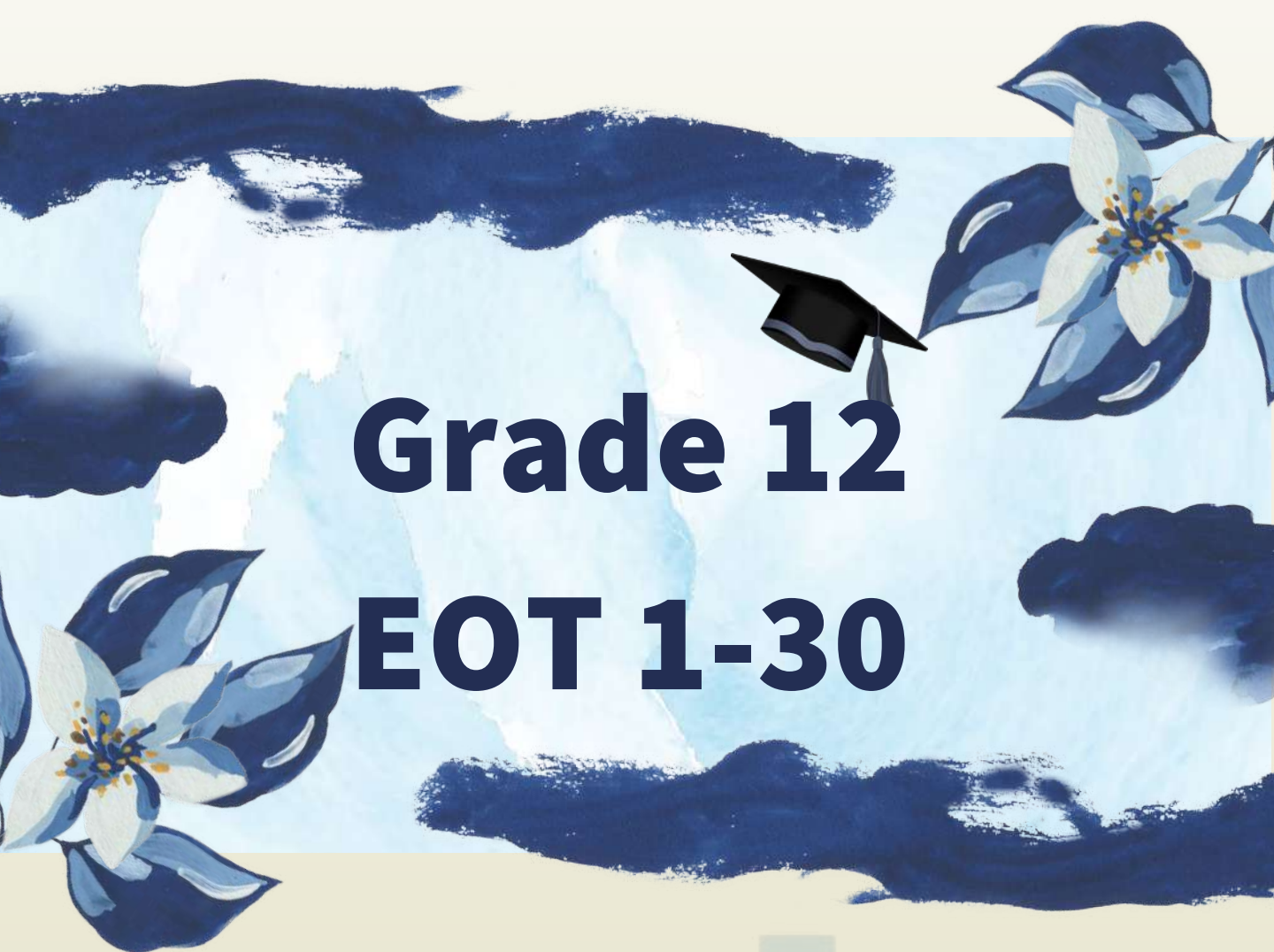


BE READY END OF TERM 3 REVISION MATHS GRADE 12 ADV

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حمدة العامري



Grade 12

EOT 1-30



مجمع زايد التعليمي
الخوانيج

1	Find the antiderivative of a given function إيجاد عكس المشتقة لدالة معطاة	Exercises (11-20)	P329
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LESSON 5-1 Antiderivatives

11. $\int (2 \sin x + \cos x) dx$

12. $\int (3 \cos x - \sin x) dx$

EOT 1

Question 11 ☞

$$\int (2 \sin x + \cos x) dx$$

• Step-by-step solution: Integrate each term individually:

$$\int 2 \sin x dx = -2 \cos x$$
$$\int \cos x dx = \sin x$$

• Final Answer:

$$-2 \cos x + \sin x + C$$

Question 12 ☞

$$\int (3 \cos x - \sin x) dx$$

• Step-by-step solution:

$$\int 3 \cos x dx = 3 \sin x$$
$$\int -\sin x dx = \cos x$$

• Final Answer:

$$3 \sin x + \cos x + C$$

1	Find the antiderivative of a given function إيجاد عكس المشتقة لدالة معطاة	Exercises (11-20)	P329
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LESSON 5-1 Antiderivatives

13. $\int 2 \sec x \tan x dx$

14. $\int \frac{4}{\sqrt{1-x^2}} dx$

Question 13 ☞

$$\int 2 \sec x \tan x dx$$

• Step-by-step solution: Using the standard trigonometric integration identity

$$\int \sec x \tan x dx = \sec x$$
$$2 \int \sec x \tan x dx = 2 \sec x$$

• Final Answer:

$$2 \sec x + C$$

Question 14 ☞

$$\int \frac{4}{\sqrt{1-x^2}} dx$$

• Step-by-step solution: Pull out the constant factor: $4 \int \frac{1}{\sqrt{1-x^2}} dx$. Recall that $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$.

• Final Answer:

$$4 \sin^{-1} x + C$$

1	Find the antiderivative of a given function إيجاد عكس المشتقة لدالة معطاة.	Exercises (11-20)	P329
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LESSON 5-1 Antiderivatives

15. $\int 5 \sec^2 x \, dx$

16. $\int 4 \frac{\cos x}{\sin^2 x} \, dx$

Question 15 1

$\int 5 \sec^2 x \, dx$

- Step-by-step solution: Since $\frac{d}{dx}(\tan x) = \sec^2 x$:

$5 \int \sec^2 x \, dx = 5 \tan x$

- Final Answer: $5 \tan x + C$

Question 16 2

$\int 4 \frac{\cos x}{\sin^2 x} \, dx$

- Step-by-step solution: Rewrite the integrand: $\frac{\cos x}{\sin^2 x} = \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = \csc x \cot x$.

$4 \int \csc x \cot x \, dx$

Since $\frac{d}{dx}(\csc x) = -\csc x \cot x$:

- Final Answer: $-4 \csc x + C$

1	Find the antiderivative of a given function إيجاد عكس المشتقة لدالة معطاة.	Exercises (11-20)	P329
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LESSON 5-1 Antiderivatives

17. $\int (3e^x - 2) \, dx$

18. $\int (4x - 2e^x) \, dx$

Question 17 1

$\int (3e^x - 2) \, dx$

- Step-by-step solution:

$\int 3e^x \, dx = 3e^x$

$\int -2 \, dx = -2x$

- Final Answer: $3e^x - 2x + C$

Question 18

$\int (4x - 2e^x) \, dx$

- Step-by-step solution:

$\int 4x \, dx = 4 \cdot \frac{x^2}{2} = 2x^2$

$\int -2e^x \, dx = -2e^x$

- Final Answer: $2x^2 - 2e^x + C$

1	Find the antiderivative of a given function إيجاد عكس المشتقة لدالة معطاة.	Exercises (11-20)	P329
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LESSON 5-1
Antiderivatives

19. $\int (3 \cos x - 1/x) dx$

20. $\int (2x^{-1} + \sin x) dx$

Question 19

$$\int \left(3 \cos x - \frac{1}{x} \right) dx$$

• Step-by-step solution:

$$\int 3 \cos x dx = 3 \sin x$$

$$\int -\frac{1}{x} dx = -\ln|x|$$

• Final Answer:

$$3 \sin x - \ln|x| + C$$

Question 20

$$\int (2x^{-1} + \sin x) dx$$

• Step-by-step solution: Rewrite $2x^{-1}$ as $\frac{2}{x}$:

$$\int \frac{2}{x} dx = 2 \ln|x|$$

$$\int \sin x dx = -\cos x$$

• Final Answer:

$$2 \ln|x| - \cos x + C$$

2	Compute straightforward indefinite integrals إجراء التكامل غير المحدود بشكل مباشر	Exercises (35-40)	P330
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LESSON 5-1
Antiderivatives

35. $f'(x) = 3e^x + x, f(0) = 4$

36. $f'(x) = 4 \cos x, f(0) = 3$

Find $f(x)$ given $f'(x) = 3e^x + x$ and $f(0) = 4$.

• Step-by-step solution:

1. Integrate $f'(x)$:

$$f(x) = \int (3e^x + x) dx = 3e^x + \frac{x^2}{2} + C$$

2. Substitute the initial condition $f(0) = 4$:

$$4 = 3e^0 + \frac{0^2}{2} + C \Rightarrow 4 = 3(1) + 0 + C \Rightarrow C = 1$$

• Final Answer:

$$f(x) = 3e^x + \frac{x^2}{2} + 1$$

Question 36

Find $f(x)$ given $f'(x) = 4 \cos x$ and $f(0) = 3$.

• Step-by-step solution: 1. Integrate $f'(x)$:

$$f(x) = \int 4 \cos x dx = 4 \sin x + C$$

2. Substitute the initial condition $f(0) = 3$:

$$3 = 4 \sin(0) + C \Rightarrow 3 = 0 + C \Rightarrow C = 3$$

• Final Answer:

$$f(x) = 4 \sin x + 3$$

2	Compute straightforward indefinite integrals إجراء التكامل غير المحدود بشكل مباشر	Exercises (35-40)	P330
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LESSON 5-1 Antiderivatives

37. $f''(x) = 12x^2 + 2e^x, f'(0) = 2, f(0) = 3$

37. $f''(x) = 12x^2 + 2e^x, f'(0) = 2, f(0) = 3$

Find $f(x)$ given $f''(x) = 12x^2 + 2e^x, f'(0) = 2$, and $f(0) = 3$.

- Step-by-step solution: 1. Integrate $f''(x)$ to find $f'(x)$:
 $f'(x) = \int (12x^2 + 2e^x) dx = 4x^3 + 2e^x + C_1$
- 2. Apply condition $f'(0) = 2$:
 $2 = 4(0)^3 + 2e^0 + C_1 \Rightarrow 2 = 2 + C_1 \Rightarrow C_1 = 0$
 So, $f'(x) = 4x^3 + 2e^x$.
- 3. Integrate $f'(x)$ to find $f(x)$:
 $f(x) = \int (4x^3 + 2e^x) dx = x^4 + 2e^x + C_2$
- 4. Apply condition $f(0) = 3$:
 $3 = 0^4 + 2e^0 + C_2 \Rightarrow 3 = 2 + C_2 \Rightarrow C_2 = 1$
- Final Answer: $f(x) = x^4 + 2e^x + 1$

38. $f'''(x) = 20x^3 + 2e^{2x}, f'(0) = -3, f(0) = 2$

Math HW - Unit 4: Integration

38. Given $f'''(x) = 20x^3 + 2e^{2x}, f'(0) = -3$, and $f(0) = 2$. Find $f(x)$.

- Step 1: Integrate $f'''(x)$ to find $f''(x)$.
 $f''(x) = \int (20x^3 + 2e^{2x}) dx = 5x^4 + e^{2x} + C_1$
- Step 2: Apply condition $f'(0) = -3$.
 $-3 = 5(0)^4 + e^{2(0)} + C_1$ Subtract 1
 $-3 = 1 + C_1$
 $-4 = C_1$
 So, $f''(x) = 5x^4 + e^{2x} - 4$.
- Step 3: Integrate $f''(x)$ to find $f'(x)$.
 $f'(x) = \int (5x^4 + e^{2x} - 4) dx = x^5 + \frac{1}{2}e^{2x} - 4x + C_2$
- Step 4: Apply condition $f'(0) = -3$.
 $-3 = 0^5 + \frac{1}{2}e^{2(0)} - 4(0) + C_2$
 $-3 = \frac{1}{2} + C_2 \rightarrow C_2 = -3 - \frac{1}{2} = -\frac{7}{2}$
 Therefore, $C_2 = -\frac{7}{2}$.
 Final Answer: $f'(x) = x^5 + \frac{1}{2}e^{2x} - 4x - \frac{7}{2}$

2	Compute straightforward indefinite integrals إجراء التكامل غير المحدود بشكل مباشر	Exercises (35-40)	P330
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LESSON 5-1 Antiderivatives

39. $f''(t) = 2 + 2t, f(0) = 2, f(3) = 2$

39. Find $f(t)$ given $f''(t) = 2 + 2t, f(0) = 2$, and $f(3) = 2$.

- Step-by-step solution: 1. Integrate $f''(t)$ to find $f'(t)$:
 $f'(t) = \int (2 + 2t) dt = 2t + t^2 + C_1$
- 2. Integrate $f'(t)$ to find $f(t)$:
 $f(t) = \int (2t + t^2 + C_1) dt = t^2 + \frac{t^3}{3} + C_1t + C_2$
- 3. Apply condition $f(0) = 2$:
 $2 = 0^2 + \frac{0^3}{3} + C_1(0) + C_2 \Rightarrow C_2 = 2$
 So, $f(t) = t^2 + \frac{t^3}{3} + C_1t + 2$.
- 4. Apply condition $f(3) = 2$:
 $2 = 3^2 + \frac{3^3}{3} + C_1(3) + 2$
 $2 = 9 + 9 + 3C_1 + 2$
 $2 = 20 + 3C_1$
 $3C_1 = -18 \Rightarrow C_1 = -6$
 So, $C_1 = -6$.
- Final Answer: $f(t) = t^2 + \frac{t^3}{3} - 6t + 2$

40. $f''(t) = 4 + 6t, f(1) = 3, f(-1) = -2$

40. Find $f(t)$ given $f''(t) = 4 + 6t, f(1) = 3$, and $f(-1) = -2$.

- Step-by-step solution: 1. Integrate $f''(t)$ to find $f'(t)$:
 $f'(t) = \int (4 + 6t) dt = 4t + 3t^2 + C_1$
- 2. Integrate $f'(t)$ to find $f(t)$:
 $f(t) = \int (4t + 3t^2 + C_1) dt = 2t^2 + t^3 + C_1t + C_2$
- 3. Apply boundary conditions to build a linear system:
 - For $f(1) = 3$:
 $3 = 2(1)^2 + (1)^3 + C_1(1) + C_2$
 $\Rightarrow 3 = 1 + 1 + C_1 + C_2$
 $\Rightarrow 3 = 2 + C_1 + C_2 \Rightarrow C_1 + C_2 = 1$
 - For $f(-1) = -2$:
 $-2 = 2(-1)^2 + (-1)^3 + C_1(-1) + C_2$
 $\Rightarrow -2 = 2 - 1 - C_1 + C_2$
 $\Rightarrow -2 = 1 - C_1 + C_2$
 $\Rightarrow -2 = 1 - C_1 + C_2 \Rightarrow -C_1 + C_2 = -3$
- 4. Solve the system:
 - Add Equation 1 and Equation 2:
 $(C_1 + C_2) + (-C_1 + C_2) = 1 + (-3)$
 $2C_2 = -2 \Rightarrow C_2 = -1$
 - From $C_1 + C_2 = 1$:
 $C_1 + (-1) = 1 \Rightarrow C_1 = 2$
- Final Answer: $f(t) = t^3 + 2t^2 - t - 1$

EOT
3

3	Use the sigma notation to compute basic summation. استخدام رمز المجموع سيجما لإيجاد المجاميع البسيطة.	Exercises (9-12)	P337
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LESSON 5-2 Sums and Sigma Notation

Calculator

9. $\sum_{i=1}^{70} (3i - 1)$

9. $\sum_{i=1}^{70} (3i - 1)$

• Step-by-step solution: Split into separate sums:

$$\sum_{i=1}^{70} (3i - 1) = \sum_{i=1}^{70} 3i - \sum_{i=1}^{70} 1$$

Apply formulas with $n = 70$:

$$= 3 \cdot \sum_{i=1}^{70} i - \sum_{i=1}^{70} 1$$

$$= 3 \cdot \left(\frac{70 \cdot 71}{2} \right) - (1 \cdot 70)$$

$$= 3 \cdot \left(\frac{4970}{2} \right) - 70$$

$$= 3 \cdot (2485) - 70$$

$$= 7455 - 70$$

$$= 7385$$

• Final Answer: 7385

10. $\sum_{i=1}^{45} (3i - 4)$

Question 10

$$\sum_{i=1}^{45} (3i - 4)$$

• Step-by-step solution: Split into separate sums:

$$3 \sum_{i=1}^{45} i - \sum_{i=1}^{45} 4$$

Apply formulas with $n = 45$:

$$3 \cdot \left(\frac{45 \cdot 46}{2} \right) - (4 \cdot 45)$$

$$3 \cdot (1035) - 180 = 3105 - 180 = 2925$$

• Final Answer: 2925

3	Use the sigma notation to compute basic summation. استخدام رمز المجموع سيجما لإيجاد المجاميع البسيطة.	Exercises (9-12)	P337
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LESSON 5-2 Sums and Sigma Notation

Calculator

11. $\sum_{i=1}^{40} (4 - i^2)$

11. $\sum_{i=1}^{40} (4 - i^2)$

Useful standard summation rules:

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

• Step-by-step solution:

$$\sum_{i=1}^{40} (4 - i^2) = \sum_{i=1}^{40} 4 - \sum_{i=1}^{40} i^2$$

Apply formulas with $n = 40$:

$$= (4 \cdot 40) - \left(\frac{40 \cdot 41 \cdot 81}{6} \right)$$

$$= 160 - \frac{40 \cdot 41 \cdot 81}{6}$$

$$= 160 - \frac{22140}{6}$$

$$= 160 - 22140$$

$$= -21980$$

• Final Answer: -21980

12. $\sum_{i=1}^{50} (8 - i)$

12. $\sum_{i=1}^{50} (8 - i)$

• Step-by-step solution:

$$\sum_{i=1}^{50} (8 - i) = \sum_{i=1}^{50} 8 - \sum_{i=1}^{50} i$$

Apply formulas with $n = 50$:

$$= (8 \cdot 50) - \left(\frac{50 \cdot 51}{2} \right)$$

$$= 400 - \frac{50 \cdot 51}{2}$$

$$= 400 - \frac{2550}{2}$$

$$= 400 - 1275$$

$$= -875$$

• Final Answer: -875

4	Understand the notion of a definite integral التعرف على مفهوم التكامل المحدود.	Definition 4.1	P347
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LESSON 5-4 The Definite Integral

☆ IMPORTANT:

DEFINITION 4.1

For any function f defined on $[a, b]$, the definite integral of f from a to b is

$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x,$$

whenever the limit exists and is the same for every choice of evaluation points, $c_1, c_2, \dots, c_n$. When the limit exists, we say that f is integrable on $[a, b]$.

Notes:

- $\Delta x = \frac{b-a}{n}$ (width of each subinterval)
- c_i = sample point in the i -th subinterval $[x_{i-1}, x_i]$
- The value of the limit does not depend on the choice of points c_i .
- If the limit exists, f is integrable on $[a, b]$.

EOT
4

5	Learn the properties of definite integrals. التعرف على خصائص التكامل المحدود.	Exercises (37-38)	P356
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LESSON 5-4 The Definite Integral

37. (a) $\int_1^3 [f(x) + g(x)] \, dx$

(b) $\int_1^3 [2f(x) - g(x)] \, dx$

37. (a)
$$\begin{aligned} \int_1^3 (f(x) + g(x)) \, dx &= \int_1^3 f(x) \, dx + \int_1^3 g(x) \, dx \\ &= 3 + (-2) = \boxed{1} \end{aligned}$$

(b)
$$\begin{aligned} \int_1^3 (2f(x) - g(x)) \, dx &= 2 \int_1^3 f(x) \, dx - \int_1^3 g(x) \, dx \\ &= 2(3) - (-2) = \boxed{8} \end{aligned}$$

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T 5

5	Learn the properties of definite integrals. التعرف على خصائص التكامل المحدود.	Exercises (37-38)	P356
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LESSON 5-4 The Definite Integral

38. (a) $\int_1^3 [f(x) - g(x)] dx$

(b) $\int_1^3 [4g(x) - 3f(x)] dx$

38. (a) $\int_1^3 (f(x) - g(x)) dx$
 $= \int_1^3 f(x) dx - \int_1^3 g(x) dx$
 $= 3 - (-2) = \boxed{5}$

(b) $\int_1^3 (4g(x) - 3f(x)) dx$
 $= 4 \int_1^3 g(x) dx - 3 \int_1^3 f(x) dx$
 $= 4(-2) - 3(3) = \boxed{-17}$

6	Find the area under a curve on a given interval using Riemann sums إيجاد المساحة تحت المنحنى الدالة على فترة مغلقة باستخدام مجاميع ريمان.	Exercises (49-52)	P357
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LESSON 5-4 The Definite Integral

Calculator

49. $\int_0^2 3x dx$

50. $\int_1^4 2x dx$

49. $\int_0^2 3x dx$
 $= 3 \int_0^2 x dx$
 $= 3 \left[\frac{x^2}{2} \right]_0^2$
 $= 3 \left(\frac{2^2}{2} - \frac{0^2}{2} \right)$
 $= 3 \left(\frac{4}{2} - 0 \right)$
 $= 3(2)$
 $= \boxed{6}$

50. $\int_1^4 2x dx$
 $= 2 \int_1^4 x dx$
 $= 2 \left[\frac{x^2}{2} \right]_1^4$
 $= 2 \left(\frac{4^2}{2} - \frac{1^2}{2} \right)$
 $= 2 \left(\frac{16}{2} - \frac{1}{2} \right)$
 $= 2 \left(8 - \frac{1}{2} \right)$
 $= 2 \left(\frac{16}{2} - \frac{1}{2} \right)$
 $= 2 \left(\frac{15}{2} \right)$
 $= \boxed{15}$

6	Find the area under a curve on a given interval using Riemann sums إيجاد المساحة تحت المنحنى لدالة على فترة مغلقة باستخدام مجاميع ريمان.	Exercises (49-52)	P357
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LESSON 5-4 The Definite Integral

Calculator

51. $\int_0^2 \sqrt{4-x^2} dx$

51. $\int_0^2 \sqrt{4-x^2} dx$

= Area of upper half of a circle with radius 2

$= \frac{1}{2} \times \pi r^2$ (area of full circle = πr^2)

$= \frac{1}{2} \times \pi (2)^2$ ($r = 2$)

$= \frac{1}{2} \times \pi \times 4$

$= \frac{4\pi}{2}$

$= \pi$

52. $\int_{-3}^0 \sqrt{9-x^2} dx$

52. $\int_{-3}^0 \sqrt{9-x^2} dx$

= Area of a semicircle centered at the origin with radius 3 (upper-left quadrant)

$= \frac{1}{4} \times \pi r^2$ (area of full circle = πr^2)

$= \frac{1}{4} \times \pi (3)^2$ ($r = 3$)

$= \frac{1}{4} \times \pi \times 9$

$= \frac{9\pi}{4}$

$= \frac{9\pi}{4}$

7	Apply the Integral Mean Value Theorem. تطبيق نظرية القيمة المتوسطة في التكامل.	Exercises (33-34)	P356
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LESSON 5-4 The Definite Integral

Calculator

33. $\int_0^2 3x^2 dx (= 8)$

33. $\int_0^2 3x^2 dx (= 8)$

$\int_0^2 3x^2 dx = 8$, find $c \in [0, 2]$

• Step-by-step solution:

$f(c) = \frac{1}{2-0}(8) \Rightarrow 3c^2 = 4 \Rightarrow c^2 = \frac{4}{3} \Rightarrow c = \frac{2}{\sqrt{3}}$

• Final Answer:

$c = \frac{2}{\sqrt{3}} \approx 1.155$

34. $\int_{-1}^1 (x^2 - 2x) dx (= \frac{2}{3})$

34. $\int_{-1}^1 (x^2 - 2x) dx = \frac{2}{3}$, find $c \in [-1, 1]$

• Step-by-step solution:

Let $f(x) = x^2 - 2x$. Then $\int_{-1}^1 f(x) dx = \frac{2}{3}$.

By the Mean Value Theorem for Integrals,

$f(c) = \frac{1}{1-(-1)} \left(\frac{2}{3} \right)$

$\Rightarrow f(c) = \frac{1}{2} \left(\frac{2}{3} \right) = \frac{1}{3}$

So, $c^2 - 2c = \frac{1}{3} \Rightarrow c^2 - 2c - \frac{1}{3} = 0$

Using the quadratic formula $c = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-\frac{1}{3})}}{2}$

$= \frac{2 \pm \sqrt{4 + \frac{4}{3}}}{2} = \frac{2 \pm \sqrt{\frac{16}{3}}}{2} = \frac{2 \pm \frac{4}{\sqrt{3}}}{2} = 1 \pm \frac{2}{\sqrt{3}}$

Only $c = 1 - \frac{2}{\sqrt{3}} \approx -0.155$ falls inside the interval $[-1, 1]$.

• Final Answer:

$c = 1 - \frac{2}{\sqrt{3}}$

EOT
7

8	Learn the Fundamental Theorem of Calculus (Part I) and use it to compute various definite integrals	Exercises (1-8)	P366
	التعرف على النظرية الأساسية الأولى للتفاضل والتكامل وتطبيقها على دوال متنوعة لإيجاد تكاملات محدودة.		

Calculator

1. $\int_0^2 (2x - 3) dx$

1. $\int_0^2 (2x - 3) dx$

• Step-by-step solution:

$$\int_0^2 (2x - 3) dx$$
$$= \left[x^2 - 3x \right]_0^2$$
$$= (2^2 - 3(2)) - (0^2 - 3(0))$$
$$= (4 - 6) - 0$$
$$= -2$$

• Final Answer:

-2

2. $\int_0^3 (x^2 - 2) dx$

2. $\int_0^3 (x^2 - 2) dx$

• Step-by-step solution:

$$\int_0^3 (x^2 - 2) dx$$
$$= \left[\frac{x^3}{3} - 2x \right]_0^3$$
$$= \left(\frac{3^3}{3} - 2(3) \right) - \left(\frac{0^3}{3} - 2(0) \right)$$
$$= (27/3 - 6) - (0 - 0)$$
$$= (9 - 6) - 0$$
$$= 3 - 0$$
$$= 3$$

• Final Answer:

3

LESSON 5-5 The Fundamental Theorem of Calculus

8	Learn the Fundamental Theorem of Calculus (Part I) and use it to compute various definite integrals	Exercises (1-8)	P366
	التعرف على النظرية الأساسية الأولى للتفاضل والتكامل وتطبيقها على دوال متنوعة لإيجاد تكاملات محدودة.		

LESSON 5-5 The Fundamental Theorem of Calculus

Calculator

3. $\int_{-1}^1 (x^3 + 2x) dx$

3. $\int_{-1}^1 (x^3 + 2x) dx$

• Step-by-step solution:

Since $x^3 + 2x$ is an odd function evaluated over symmetric boundaries $[-1, 1]$, the integral simplifies directly to zero.

• Final Answer:

0

4. $\int_0^2 (x^3 + 3x - 1) dx$

4. $\int_0^2 (x^3 + 3x - 1) dx$

$$= \left(\frac{x^4}{4} - \frac{3x^2}{2} - x \right) \bigg|_0^2 = -4$$

8	Learn the Fundamental Theorem of Calculus (Part I) and use it to compute various definite integrals	Exercises (1-8)	P366
	التعرف على النظرية الأساسية الأولى للتفاضل والتكامل وتطبيقها على دوال متنوعة لإيجاد تكاملات محدودة.		

LESSON 5-5 The Fundamental Theorem of Calculus

Calculator

5. $\int_1^4 \left(x\sqrt{x} + \frac{3}{x} \right) dx$

6. $\int_1^2 \left(4x - \frac{2}{x^2} \right) dx$

5. $\int_1^4 \left(x\sqrt{x} + \frac{3}{x} \right) dx$

1. Rewrite the integrand using exponents:

$$x\sqrt{x} = x^1 \cdot x^{1/2} = x^{3/2}$$
$$\int_1^4 \left(x^{3/2} + \frac{3}{x} \right) dx$$

2. Find the antiderivative:

$$F(x) = \frac{x^{5/2}}{5/2} + 3 \ln|x| = \frac{2}{5} x^{5/2} + 3 \ln|x|$$

3. Evaluate from 1 to 4:

$$F(4) = \frac{2}{5} (4)^{5/2} + 3 \ln(4) = \frac{2}{5} (32) + 3 \ln(4) = \frac{64}{5} + 3 \ln(4)$$
$$F(1) = \frac{2}{5} (1)^{5/2} + 3 \ln(1) = \frac{2}{5} (1) + 0 = \frac{2}{5}$$

4. Subtract:

$$\left(\frac{64}{5} + 3 \ln(4) \right) - \frac{2}{5} = \frac{62}{5} + 3 \ln(4)$$

6. $\int_1^2 \left(4x - \frac{2}{x^2} \right) dx$

1. Rewrite the integrand:

$$\int_1^2 \left(4x - \frac{2}{x^2} \right) dx$$
$$= \int_1^2 (4x - 2x^{-2}) dx$$

2. Find the antiderivative:

$$F(x) = 2x^2 - \frac{2x^{-1}}{-1} = 2x^2 + \frac{2}{x}$$

3. Evaluate from 1 to 2:

$$F(2) = 2(2)^2 + \frac{2}{2} = 8 + 1 = 9$$
$$F(1) = 2(1)^2 + \frac{2}{1} = 2 + 2 = 4$$

4. Subtract:

$$9 - 4 = \boxed{5}$$

8	Learn the Fundamental Theorem of Calculus (Part I) and use it to compute various definite integrals	Exercises (1-8)	P366
	التعرف على النظرية الأساسية الأولى للتفاضل والتكامل وتطبيقها على دوال متنوعة لإيجاد تكاملات محدودة.		

LESSON 5-5 The Fundamental Theorem of Calculus

Calculator

7. $\int_0^1 (6e^{-3x} + 4) dx$

8. $\int_0^2 \left(\frac{e^{2x} - 2e^{3x}}{e^{3x}} \right) dx$

7. $\int_0^1 (6e^{-3x} + 4) dx$

Exercise 7

$$\int_0^1 (6e^{-3x} + 4) dx$$

1. Find the antiderivative:

$$F(x) = \frac{6e^{-3x}}{-3} + 4x = -2e^{-3x} + 4x$$

2. Evaluate from 0 to 1:

$$F(1) = -2e^{-3(1)} + 4(1) = -2e^{-3} + 4$$
$$F(0) = -2e^0 + 4(0) = -2(1) + 0 = -2$$

3. Subtract:

$$(-2e^{-3} + 4) - (-2) = -2e^{-3} + 4 + 2 = 6 - \frac{2}{e^3}$$

8. $\int_0^2 \left(\frac{e^{2x} - 2e^{3x}}{e^{3x}} \right) dx$

Exercise 8

$$\int_0^2 \left(\frac{e^{2x} - 2e^{3x}}{e^{3x}} \right) dx$$

1. Simplify the fraction before integrating:

$$\frac{e^{2x}}{e^{3x}} - \frac{2e^{3x}}{e^{3x}} = e^{-x} - 2$$
$$\int_0^2 (e^{-x} - 2) dx$$

2. Find the antiderivative:

$$F(x) = -e^{-x} - 2x$$

3. Evaluate from 0 to 2:

$$F(2) = -e^{-2} - 2(2) = -e^{-2} - 4$$
$$F(0) = -e^0 - 2(0) = -1$$

4. Subtract:

$$(-e^{-2} - 4) - (-1) = -e^{-2} - 4 + 1 = -3 - \frac{1}{e^2}$$

9	Learn the Fundamental Theorem of Calculus (Part II) and use it to compute derivatives of functions defined as definite integrals	Exercises (13,14)	P366
	التعرف على النظرية الأساسية الثانية للتفاضل والتكامل وتطبيقها على دوال معرفة كتكاملات محدودة لإيجاد مشتقاتها.		

LESSON 5-5 The Fundamental Theorem of Calculus

13. $\int_0^{1/2} \frac{3}{\sqrt{1-x^2}} dx$

14. $\int_{-1}^1 \frac{4}{1+x^2} dx$

Calculator

13. $\int_0^{1/2} \frac{3}{\sqrt{1-x^2}} dx$

Exercise 13

$$\int_0^{1/2} \frac{3}{\sqrt{1-x^2}} dx$$

1. Identify the inverse trig derivative rule ($\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x)$):

$$F(x) = 3 \arcsin(x)$$

2. Evaluate from 0 to 1/2:

$$F(1/2) = 3 \arcsin\left(\frac{1}{2}\right) = 3\left(\frac{\pi}{6}\right) = \frac{\pi}{2}$$
$$F(0) = 3 \arcsin(0) = 3(0) = 0$$

3. Subtract:

$$\frac{\pi}{2} - 0 = \frac{\pi}{2}$$

14. $\int_{-1}^1 \frac{4}{1+x^2} dx$

Exercise 14

$$\int_{-1}^1 \frac{4}{1+x^2} dx$$

1. Identify the inverse trig derivative rule ($\int \frac{1}{1+x^2} dx = \arctan(x)$):

$$F(x) = 4 \arctan(x)$$

2. Evaluate from -1 to 1:

$$F(1) = 4 \arctan(1) = 4\left(\frac{\pi}{4}\right) = \pi$$
$$F(-1) = 4 \arctan(-1) = 4\left(-\frac{\pi}{4}\right) = -\pi$$

3. Subtract:

$$\pi - (-\pi) = \pi + \pi = 2\pi$$

EOT
9

10	Learn the Fundamental Theorem of Calculus (Part II) and use it to compute derivatives of functions defined as definite integrals	Exercises (25-28)	P366
	التعرف على النظرية الأساسية الثانية للتفاضل والتكامل وتطبيقها على دوال معرفة كتكاملات محدودة لإيجاد مشتقاتها.		

LESSON 5-5 The Fundamental Theorem of Calculus

25. $f(x) = \int_0^x (t^2 - 3t + 2) dt$

Replace t with x

25. $f(x) = \int_0^x (t^2 - 3t + 2) dt$

Exercise 25

$$f(x) = \int_0^x (t^2 - 3t + 2) dt$$

• Since the upper limit is simply x, just substitute t with x:

$$f'(x) = x^2 - 3x + 2$$
$$f(x) = \int_0^x (t^2 - 3t + 2) dt$$
$$= \int_0^x (t^2) dt - \int_0^x (3t) dt + \int_0^x (2) dt$$
$$= \left[\frac{t^3}{3} \right]_0^x - \left[\frac{3t^2}{2} \right]_0^x + \left[2t \right]_0^x$$
$$= \left(\frac{x^3}{3} - 0 \right) - \left(\frac{3x^2}{2} - 0 \right) + (2x - 0)$$
$$= \frac{x^3}{3} - \frac{3x^2}{2} + 2x$$
$$f'(x) = x^2 - 3x + 2$$

10	Learn the Fundamental Theorem of Calculus (Part II) and use it to compute derivatives of functions defined as definite integrals	Exercises (25-28)	P366
	التعرف على النظرية الأساسية الثانية للتفاضل والتكامل وتطبيقها على دوال معرفة كتكاملات محدودة لإيجاد مشتقاتها.		

LESSON 5-5 The Fundamental Theorem of Calculus

Replace t with x

26. $f(x) = \int_2^x (t^2 - 3t - 4) dt$

26. $f(x) = \int_2^x (t^2 - 3t - 4) dt$

Exercise 26

$f(x) = \int_2^x (t^2 - 3t - 4) dt$

• Standard application where the lower limit is a constant and the upper limit is x:

$f'(x) = x^2 - 3x - 4$

10	Learn the Fundamental Theorem of Calculus (Part II) and use it to compute derivatives of functions defined as definite integrals	Exercises (25-28)	P366
	التعرف على النظرية الأساسية الثانية للتفاضل والتكامل وتطبيقها على دوال معرفة كتكاملات محدودة لإيجاد مشتقاتها.		

LESSON 5-5 The Fundamental Theorem of Calculus

Replace t with x

27. $f(x) = \int_0^{x^2} (e^{-t^2} + 1) dt$

27. $f(x) = \int_0^{x^2} (e^{-t^2} + 1) dt$

Exercise 27

$f(x) = \int_0^{x^2} (e^{-t^2} + 1) dt$

1. Substitute t with the upper bound x^2 : $e^{-(x^2)^2} + 1 = e^{-x^4} + 1$

2. Multiply by the derivative of the upper bound $\left(\frac{d}{dx}(x^2)\right) = 2x$:

$f'(x) = (e^{-x^4} + 1) \cdot 2x = 2x(e^{-x^4} + 1)$

10	Learn the Fundamental Theorem of Calculus (Part II) and use it to compute derivatives of functions defined as definite integrals	Exercises (25-28)	P366
	التعرف على النظرية الأساسية الثانية للتفاضل والتكامل وتطبيقها على دوال معرفة كتكاملات محدودة لإيجاد مشتقاتها.		

LESSON 5-5 The Fundamental Theorem of Calculus

Replace t with x

28. $f(x) = \int_x^2 \sec t \, dt$

28. $f(x) = \int_x^2 \sec t \, dt$

Exercise 28

$f(x) = \int_x^2 \sec t \, dt$

1. Flip the limits of integration to bring the variable to the top (which introduces a negative sign):

$f(x) = - \int_2^x \sec t \, dt$

2. Take the derivative directly:

$f'(x) = - \sec x$

10	Learn the Fundamental Theorem of Calculus (Part II) and use it to compute derivatives of functions defined as definite integrals	Exercises (25-28)	P366
	التعرف على النظرية الأساسية الثانية للتفاضل والتكامل وتطبيقها على دوال معرفة كتكاملات محدودة لإيجاد مشتقاتها.		

LESSON 5-5 The Fundamental Theorem of Calculus

28. $f(x) = \int_x^2 \sec t \, dt$

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Exercise 28

$f(x) = \int_x^2 \sec t \, dt$

1. Flip the limits of integration to bring the variable to the top (which introduces a negative sign):

$f(x) = - \int_2^x \sec t \, dt$

2. Take the derivative directly:

$f'(x) = - \sec x$

11	Write the equation of a tangent line at a given point to a function defined as definite integral كتابة معادلة المماس لدالة معرفة كتكامل محدد عند نقطة معطاة.	Exercises (39-42)	P366,367
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LESSON 5-5 The Fundamental Theorem of Calculus

Calculator

39. $y = \int_0^x \sin \sqrt{t^2 + \pi^2} dt, x = 0$

39. $y = \int_0^x \sin \sqrt{t^2 + \pi^2} dt, x = 0$
 Given function: $y = \int_0^x \sin \sqrt{t^2 + \pi^2} dt$
 At $x = 0$:
 $y(0) = \int_0^0 \sin \sqrt{t^2 + \pi^2} dt$
 By the definite integral property $\int_a^a f(t) dt = 0$, $\therefore y = 0$
 Also, $y' = \frac{dy}{dx} = \sin \sqrt{x^2 + \pi^2}$ (by FTC)
 At $x = 0$, $y' = \sin \sqrt{0 + \pi^2} = \sin(\pi) = 0$
 $y' = \sin \sqrt{0 + \pi^2} = \sin(\pi) = 0$

11	Write the equation of a tangent line at a given point to a function defined as definite integral كتابة معادلة المماس لدالة معرفة كتكامل محدد عند نقطة معطاة.	Exercises (39-42)	P366,367
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LESSON 5-5 The Fundamental Theorem of Calculus

Calculator

40. $y = \int_{-1}^x \ln(t^2 + 2t + 2) dt, x = -1$

40. $y = \int_{-1}^x \ln(t^2 + 2t + 2) dt, x = -1$
 Given function: $y = \int_{-1}^x \ln(t^2 + 2t + 2) dt$
 At $x = -1$:
 $y(-1) = \int_{-1}^{-1} \ln(t^2 + 2t + 2) dt$
 By the definite integral property $\int_a^a f(t) dt = 0$, At $x = -1$, $y' = \ln(-(-1)^2 + 2(-1) + 2) = 0 \therefore y' = 0$
 Now, $y' = \frac{dy}{dx} = \ln(x^2 + 2x + 2)$ (by FTC)

11	Write the equation of a tangent line at a given point to a function defined as definite integral كتابة معادلة المماس لدالة معرفة كتكامل محدود عند نقطة معطاة.	Exercises (39-42)	P366,367
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LESSON 5-5 The Fundamental Theorem of Calculus

Calculator

42. $y = \int_0^x e^{-t^2+1} dt, x = 0$

42. $y = \int_0^x e^{-t^2+1} dt, x = 0$
 Given function: $y = \int_0^x e^{-t^2+1} dt$ Find the derivative at $x = 0$:
 At $x = 0$: $y(0) = \int_0^0 e^{-t^2+1} dt$ Also, $y' = \frac{dy}{dx} = e^{-x^2+1}$ (by FTC)
 By the definite integral property $\int_a^a f(t) dt = 0$, At $x = 0$, $y' = e^{-0^2+1} = e^1 \therefore y' = e$

12	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات.	Exercises (5-14)	P376
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LESSON 5-6 Integration by Substitution

5. $\int x^3 \sqrt{x^4 + 3} dx$

5. $\int x^3 \sqrt{x^4 + 3} dx$
 Use u-substitution:
 Let $u = x^4 + 3$ $du = 4x^3 dx \therefore dx = \frac{du}{4x^3}$
 Substitute into the integral: $\int x^3 \sqrt{u} \frac{du}{4x^3} = \int \frac{1}{4} \sqrt{u} du$
 Integrate $\frac{1}{4} \sqrt{u}$: $\frac{1}{4} \cdot \left(\frac{u^{3/2}}{3/2} \right) + C = \frac{1}{4} \cdot \frac{2}{3} \cdot u^{3/2} + C = \frac{1}{6} u^{3/2} + C$
 Back substitute $u = x^4 + 3$: $\therefore \frac{1}{6} (x^4 + 3)^{3/2} + C$

12	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات.	Exercises (5-14)	P376
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LESSON 5-6 Integration by Substitution

6. $\int \sqrt{1 + 10x} \, dx$

6. $\int \sqrt{1 + 10x} \, dx$
 Use u-substitution:
 Let $u = 1 + 10x$ $du = 10 \, dx \therefore dx = \frac{du}{10}$
 Substitute into the integral: $\int \sqrt{u} \frac{du}{10} = \int \frac{1}{10} u^{1/2} \, du$
 Factor out the constant: $\frac{1}{10} \int u^{1/2} \, du$
 Integrate using the power rule: $\frac{1}{10} \cdot \left(\frac{u^{1/2+1}}{(1/2)+1} \right) + C = \left(\frac{u^{3/2}}{3/2} \right) = \frac{1}{10} \left(\frac{2}{3} \right) + C = \frac{1}{15} u^{3/2} + C$
 Back-substitute $u = 1 + 10x$: $\therefore \frac{1}{15} (1 + 10x)^{3/2} + C$

12	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات.	Exercises (5-14)	P376
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LESSON 5-6 Integration by Substitution

7. $\int \frac{\sin x}{\sqrt{\cos x}} \, dx$

7. $\int \frac{\sin x}{\sqrt{\cos x}} \, dx$
 Use u-substitution:
 Let $u = \cos x$
 Then $du = -\sin x \, dx$, so $\sin x \, dx = -du$
 Substitute into the integral: $\int \frac{\sin x \, dx}{\sqrt{\cos x}} = \int \frac{-du}{\sqrt{u}} \, du$
 Rewrite using exponents: $= - \int u^{-1/2} \, du$
 Integrate using the power rule: $- \left[\frac{u^{-1/2+1}}{-1/2+1} \right] + C = -2u^{1/2} + C$
 Back-substitute $u = \cos x$: $= -2(\cos x)^{1/2} + C$
 $\therefore$ Final Answer: $-2\sqrt{\cos x} + C$

12	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات.	Exercises (5-14)	P376
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LESSON 5-6 Integration by Substitution

8. $\int \sin^3 x \cos x \, dx$

8. $\int \sin^3 x \cos x \, dx$
 Use u-substitution:
 Let $u = \sin x$ Then $du = \cos x \, dx$, so $\cos x \, dx = du$
 Substitute into the integral: $\int (\sin x)^3 \cos x \, dx = \int u^3 \, du$
 Integrate using the power rule: $= \left[\frac{u^4}{4} \right] + C$
 $\therefore$ Final Answer: $\frac{1}{4} \sin^4 x + C$ $\therefore$ Final Answer: $\frac{1}{4} \sin^4 x + C$

12	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات.	Exercises (5-14)	P376
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LESSON 5-6 Integration by Substitution

9. $\int t^2 \cos t^3 \, dt$

9. $\int t^2 \cos t^3 \, dt$
 Use u-substitution:
 Let $u = t^3$ Then $du = 3t^2 \, dt$, so $t^2 \, dt = \frac{du}{3}$
 Substitute into the integral: $\int t^2 \cos t^3 \, dt = \int (\cos u) \frac{du}{3}$
 Factor out the constant: $= \frac{1}{3} \int \cos u \, du$
 Integrate $\cos u$: $= \frac{1}{3} \sin u + C$
 Back-substitute $u = t^3$: $= \frac{1}{3} \sin(t^3) + C$
 $\therefore$ Final Answer: $\frac{1}{3} \sin(t^3) + C$

12	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات.	Exercises (5-14)	P376
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LESSON 5-6 Integration by Substitution

11. $\int x e^{x^2+1} dx$

10. $\int \sin t (\cos t + 3)^{3/4} dt$

11. Find the integral: $\int x e^{x^2+1} dx$
 Use u-substitution:
 Let $u = x^2 + 1$ $\frac{du}{dx} = 2x$, so $du = 2x dx$, and $dx = \frac{du}{2x}$
 Substitute into the integral:
 $\int x \cdot e^u \cdot \frac{du}{2x} = \int \left(\frac{1}{2}\right) e^u du$
 Factor out the constant: $= \frac{1}{2} \int e^u du$
 Integrate: $= \frac{1}{2} e^u + C$
 Back-substitute u : $= \frac{1}{2} e^{x^2+1} + C$

10. Find the integral: $\int \sin t (\cos t + 3)^{3/4} dt$
 Use u-substitution:
 Let $u = \cos t + 3$ $\frac{du}{dt} = -\sin t$, so $du = -\sin t dt$, and $dt = \frac{du}{-\sin t}$
 Substitute into the integral:
 $\int \sin t \cdot u^{3/4} \cdot \frac{du}{-\sin t} = \int -u^{3/4} du$
 Integrate:
 $= -\int u^{3/4} du = -\frac{u^{(3/4)+1}}{(3/4)+1} + C = -\frac{u^{7/4}}{7/4} + C$
 Back-substitute u : $= -\frac{4}{7} (\cos t + 3)^{7/4} + C$

12	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات.	Exercises (5-14)	P376
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LESSON 5-6 Integration by Substitution

12. $\int e^x \sqrt{e^x + 4} dx$

13. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

12. Find the integral: $\int e^x \sqrt{e^x + 4} dx$
 Use u-substitution:
 Let $u = e^x + 4$ $\frac{du}{dx} = e^x$, so $du = e^x dx$, and $dx = \frac{du}{e^x}$
 Substitute into the integral:
 $\int e^x \cdot \sqrt{u} \cdot \frac{du}{e^x} = \int \sqrt{u} du$
 Rewrite using exponents: $= \int u^{1/2} du$
 Integrate: $= \frac{u^{3/2}}{3/2} + C = \frac{2}{3} u^{3/2} + C$
 Back-substitute u : $= \frac{2}{3} (e^x + 4)^{3/2} + C$

13. Find the integral: $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$
 Use u-substitution:
 Let $u = \sqrt{x}$ $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, so $du = \frac{dx}{2\sqrt{x}}$, and $dx = 2\sqrt{x} du$
 Substitute into the integral:
 $\int \frac{e^u}{\sqrt{x}} \cdot (2\sqrt{x} du) = \int 2e^u du$
 Factor out the constant: $= 2 \int e^u du$
 Integrate: $= 2e^u + C$
 Back-substitute u : $= 2e^{\sqrt{x}} + C$

12	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات.	Exercises (5-14)	P376
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LESSON 5-6 Integration by Substitution

14. $\int \frac{\cos(1/x)}{x^2} dx$

14. $\int \frac{\cos(\frac{1}{x})}{x^2} dx$ Let $u = \frac{1}{x} = x^{-1}$
 Use u-substitution: Find du : $du = -x^{-2}(-\frac{1}{x^2})dx \therefore dx = -x^2 du$
 Substitute into the integral: $\int \frac{\cos u}{x^2} \cdot (-x^2 du) = \int -\cos u du$
 Factor out the constant: $-\int \cos u du$ Integrate: $-\sin u + C$
 Back-substitute $u = \frac{1}{x}$: $-\sin(\frac{1}{x}) + C$
 $\therefore \int \frac{\cos(1/x)}{x^2} dx = -\sin(\frac{1}{x}) + C$

13	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات.	Exercises (24,26,35)	P376
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LESSON 5-6 Integration by Substitution

24. (a) $\int \frac{x^2}{1+x^6} dx$
 (b) $\int \frac{x^3}{1+x^6} dx$

24. (a)
 Evaluate $\int \frac{x^2}{1+x^6} dx$.
 Let $u = x^3$, then $du = 3x^2 dx$
 Solution: $= \frac{1}{3} \int \frac{du}{1+u^2} = \frac{1}{3} \arctan(u) + C$
 Substituting back $= \frac{1}{3} \arctan(x^3) + C$

24. (b)
 Evaluate $\int \frac{x^5}{1+x^6} dx$.
 Let $v = x^6$, then $dv = 6x^5 dx$
 The $\frac{1}{6} \int \frac{dv}{1+v} = \frac{1}{6} \ln|1+v| + C$
 Solution $= \frac{1}{6} \ln(1+x^6) + C$

EOT

13	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات	Exercises (24,26,35)	P376
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LESSON 5-6 Integration by Substitution

26. (a) $\int \frac{3\sqrt{x}}{1+x^3} dx$ (b) $\int \frac{x\sqrt{x}}{1+x^5} dx$

26. (a) $\int \frac{3\sqrt{x}}{1+x^3} dx$

Solution (a): Let $u = x^{3/2}$
 $du = \frac{3}{2}x^{1/2}dx$
 $2du = 3\sqrt{x}dx$

$\int \frac{2du}{1+u^2} = 2\int \frac{du}{1+u^2} = 2\arctan(u) + C$
 $= 2\arctan(x^{3/2}) + C$

26. (b) $\int \frac{x\sqrt{x}}{1+x^5} dx$

Solution (b): Let $u = x^{5/2}$, $du = \frac{5}{2}x^{3/2}dx$, $\frac{2}{5}du = x^{3/2}dx = x\sqrt{x}dx$

$\int \frac{\frac{2}{5}du}{1+u^2} = \frac{2}{5} \int \frac{du}{1+u^2} = \frac{2}{5} \arctan(u) + C = \frac{2}{5} \arctan(x^{5/2}) + C$

13	Compute integrals using substitution استخدام طريقة التكامل بالتعويض لإيجاد تكاملات	Exercises (24,26,35)	P376
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LESSON 5-6 Integration by Substitution

Calculator

35. $\int_0^2 \frac{e^x}{1+e^{2x}} dx$

35. $\int_0^2 \frac{e^x}{1+e^{2x}} dx$

Solution:
 Rewriting the denominator: $\int_0^2 \frac{e^x}{1+(e^x)^2} dx$

Step 1: substitution (u-sub)
 Let $u = e^x$
 $du = e^x dx$

Step 2: Change the limits of integration
 Lower limit: When $x=0$, $u=e^0=1$
 Upper limit: When $x=2$, $u=e^2$

Step 3: Integrate with new variables
 $\int_1^{e^2} \frac{1}{1+u^2} du$
 The antiderivative of $\frac{1}{1+u^2}$ is $\arctan(u)$
 $[\arctan(u)]_1^{e^2}$

Step 4: Evaluate the limits
 $\arctan(e^2) - \arctan(1)$
 We know $\arctan(1) = \frac{\pi}{4}$

Final Answer: $\arctan(e^2) - \frac{\pi}{4}$

14	Find the area between two curves using definite integrations إيجاد مساحة المنطقة المحصورة بين منحنيين باستخدام التكامل المحدود	Exercises (5-8)	P414
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LESSON 6-1 Area Between Curves

5. $y = x^2 - 1, y = 7 - x^2$

6. $y = x^2 - 1, y = \frac{1}{2}x^2$

MATH PROBLEM SET

5. Find the area enclosed by the curves: $y = x^2 - 1, y = 7 - x^2$

Steps:

1. Find intersection points: $x^2 - 1 = 7 - x^2, 2x^2 = 8$
 $x^2 = 4, x = \pm 2$

2. Find Upper Curve: Test $x = 0: y = 7, y = -1$. Upper: $7 - x^2$

3. Set up Integral: $\int_{-2}^2 (x - 1 - 7 - x^2) = \int_{-2}^2 (x^2 + 4) = \frac{17 - 41}{3} = \text{Area} = \frac{64}{3}$ square units

6. Find the area enclosed by the curves: $y = x^2 - 1, y = \frac{1}{2}x^2$

Steps:

1. Find intersection points: $x^2 - 1 = \frac{1}{2}x^2, x = \pm\sqrt{2}$

2. Find Upper Curve: Test $x = 0: y = 0, y = -1$. Upper: $\frac{1}{2}x^2$

3. Set up Integral: $\int_{-\sqrt{2}}^{\sqrt{2}} (x^2 - \frac{1}{2})x^2 = \int_{-\sqrt{2}}^{\sqrt{2}} ((x + -1)) - \frac{4}{3} = \frac{(4\sqrt{2} - 1)}{3} = \text{Area} = \frac{4\sqrt{2}}{3}$ square units

14	Find the area between two curves using definite integrations إيجاد مساحة المنطقة المحصورة بين منحنيين باستخدام التكامل المحدود	Exercises (5-8)	P414
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LESSON 6-1 Area Between Curves

7. $y = x^3, y = 3x + 2$

8. $y = \sqrt{x}, y = x^2$

MATH PROBLEM SET 3

7. Find the area enclosed by the curves: $y = x^3, y = 3x + 2$

Steps:

1. Find intersection points: $x^3 - 1 = 3x + 2, 3x = 2$
 $x^2 = 4, x = -1, 2$

2. Find Upper/Lower curves: $x = -1: y = 0, y = 2$. Upper: $3x + 2$

3. Set up Integral: $\int_{-1}^2 (x - 1 - 7 - x^2) = \int_{-1}^2 (x^2 + 4) = \frac{17 - 41}{3} = \text{Area} = \frac{27}{4}$ square units

8. Find the area enclosed by the curves: $y = \sqrt{x}, y = x^2$

Steps:

1. Find intersection points: $x^2 - 1 = 7 - x + x^2, x = 0 = 1$
 $x = 0, 1, x = 0, 1$

2. Find Upper/Lower curves: $x = \frac{1}{4}: y = \frac{1}{2}, y = \frac{1}{16}$. Upper: $\sqrt{x}$

3. Set up Integral: $\int_0^1 (x^2 \sqrt{x} - \frac{1}{4} \sqrt{x}) = \int_0^1 (\frac{1}{4} \sqrt{x} + \frac{1}{3}) = \frac{(12 + \sqrt{2} - 1)}{3} = \text{Area} = \frac{1}{3}$ square units

15	Compute the area of a region using definite integration with y as a variable x إيجاد مساحة منطقة كتمثل محدود بمعلومية y عوضاً عن x.	Exercises (19-24)	P414
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LESSON 6-1 Area Between Curves

19. $y = x, y = 2 - x, y = 0$
20. $y = x, y = 2, y = 6 - x, y = 0$

MATH PROBLEM SET 4

19. Find the area enclosed by: $y = x, y = 2 - x, y = 0$

Steps:

- Find intersection points: $x^2 - 1 = 2 - x, x = 0$
 $(0,0), (1,1), (2,0)$
- Set up Integral: $\int_0^1 [(2-y)-y] dy = \frac{17-41}{3} = \text{Area} = 1$

Answer for 19: 1 square unit

20. Find the area enclosed by: $y = x, y = 2, y = 6 - x, y = 0$

Steps:

- Find intersection points: $x^2 - 1 = 7 - x + x, x = 0, x = 1$
 $(0,0), (2,2), (4,2), (6,0)$
- Set up Integral (using symmetry with resp to y):
 $\int_0^2 (6-y)-y dy = \int_0^2 (6-y)-y dy = \frac{1}{3} || = \frac{12-4}{8} = \text{Area} = 8$

Answer for 20: 8 square units

15	Compute the area of a region using definite integration with y as a variable x إيجاد مساحة منطقة كتمثل محدود بمعلومية y عوضاً عن x.	Exercises (19-24)	P414
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LESSON 6-1 Area Between Curves

21. $x = y, x = -y, x = 1$
22. $x = 3y, x = 2 + y^2$

MATH PROBLEM SET 5

21. Find the area enclosed by the curves: $x = y, x = -y, x = 1$

Steps:

- Find intersection points: $x - y = 1$, Points $(0,0), (1,1), (1,-1)$
- Set up Integral (using symmetry with resp to x):
 $\int_{y=0}^{y=1} (x-7-x^2) dy = \int_{y=0}^{y=1} (x^2+4) = \frac{17-41}{3} = \frac{17-41}{3} = \text{Area} = 1$

Answer for 21: 1 square unit

22. Find the area enclosed by the curves: $x = 3y, x = 2 + y^2$

Steps:

- Find intersection points: $x^3 - 3y = x + 2$, Points at $y = 1$ and $y = 2$
- Set up Integral:
 $\int_{y=1}^{y=2} (3y-(2+y^2)) dy = \int_{y=1}^{y=2} 3y-(2+y^2) dy = \left(\frac{3}{2} - \frac{1}{11} \right) ||_1^2 = \text{Area} = \frac{1}{6}$

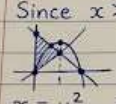
Answer for 22: $\frac{1}{6}$ square units

15	Compute the area of a region using definite integration with y as a variable x إيجاد مساحة منطقة ككامل محدود بمعلومية y عوضاً عن x .	Exercises (19-24)	P414
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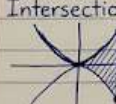
LESSON 6-1 Area Between Curves

23. $y = 2x (x > 0), y = 3 - x^2, x = 0$

24. $x = y^2, x = 4$

23. $y = 2x (x > 0), y = 3 - x^2, x = 0$
 23. $y = 2x$ (for $x > 0$), $y = 3 - x^2$, $x = 0$
 Find intersection points: $2x = 3 - x^2 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x+3)(x-1) = 0$
 Since $x > 0$, $x = 1$. Points are $(0,0), (1,2), (0,3)$


$$\text{Area} = \int_0^1 (3 - x^2 - 2x) dx = \left[3x - \frac{x^3}{3} - x^2 \right]_0^1 = \left(3 - \frac{1}{3} - 1 \right) - 0 = 2 - \frac{1}{3} = \frac{5}{3}$$

 24. $x = y^2, x = 4$
 Intersection: $y^2 = 4 \Rightarrow y = \pm 2$


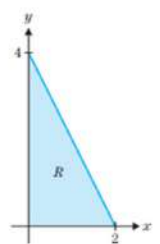
$$\text{Area} = 2 \int_0^2 (4 - y^2) dy = 2 \left[4y - \frac{y^3}{3} \right]_0^2 = 2 \left[\frac{16}{3} \right] = \frac{32}{3}$$

16	Find the volume of a solid of revolution by using the method of washers إيجاد حجم مجسم مخروطي ناتج عن دوران منطقة حول مستقيم معلوم باستخدام طريقة الحلقات.	Exercises (25-26)	P430
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LESSON 6-2 Volume: Slicing, Disks and Washers

25. Let R be the region bounded by $y = 4 - 2x$, the x -axis and the y -axis. Compute the volume of the solid formed by revolving R about the given line.

- (a) the y -axis (b) the x -axis (c) $y = 4$
 (d) $y = -4$ (e) $x = 2$ (f) $x = -2$



25. Let R be the region bounded by $y = 4 - 2x$, the x -axis and the y -axis. Compute the volume of the solid formed by revolving R about the given line.
 (a) the y -axis, (b) the x -axis, (c) $y = 4$
 (d) $y = -4$, (e) $x = 2$, (f) $x = -2$
 Solutions:
 For region R : x -intercept at $x = 2$ (when $y = 0$), y -intercept at $y = 4$ (when $x = 0$).
 Also, $y = 4 - 2x \Rightarrow x = 2 - \frac{y}{2}$ (for integration w.r.t. y)
 (a) Revolving about y -axis:
 Solid is a right circular cone. Radius $r = 2$ (max x), Height $h = 4$ (max y).
 Alternatively, use washers (or discs w.r.t. y).

$$V_a = \pi \int_0^4 x^2 dy = \pi \int_0^4 \left(2 - \frac{y}{2} \right)^2 dy = \pi \int_0^4 \left(4 - 2y + \frac{y^2}{4} \right) dy = \pi \left[4y - y^2 + \frac{y^3}{12} \right]_0^4 = \frac{16\pi}{3}$$

 (b) Revolving about x -axis:
 Solid is a right circular cone. Radius $r = 4$, Height $h = 2$.
 Alternatively, use discs w.r.t. x .

$$V_b = \pi \int_0^2 y^2 dx = \pi \int_0^2 (4 - 2x)^2 dx = \pi \left[\frac{(4-2x)^3}{3} \right]_0^2 = \frac{64\pi}{3}$$

 (c) Revolving about $y = 4$:
 Solid is an inverted cone with a cylindrical core removed.
 Outer radius $R = 4$, Inner radius $r(x) = y = 4 - 2x$.

$$V_c = \pi \int_0^2 [R^2 - r(x)^2] dx = \pi \int_0^2 [4^2 - (4 - 2x)^2] dx = \pi \int_0^2 [8x^2 - 4x^2] dx = \pi \left[\frac{8x^3}{3} - \frac{4x^3}{3} \right]_0^2 = \frac{64\pi}{3}$$

 (d) 

$$V_d = \pi \int_0^4 \left[\left(2 - \frac{y}{2} \right)^2 \right] dy = \pi \int_0^4 \left(4 - 2y + \frac{y^2}{4} \right) dy = \frac{16\pi}{3}$$

 (e) 

$$V_e = \pi \int_0^2 (4 - 2x)^2 dx = \frac{64\pi}{3}$$

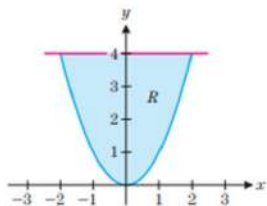
 (f) 

$$V_f = \pi \int_0^2 (4 - 2x)^2 dx = \frac{64\pi}{3}$$

16	Find the volume of a solid of revolution by using the method of washers إيجاد حجم مجسم مجوف ناتج عن دوران منطقة حول مستقيم معلوم باستخدام طريقة الحلقات.	Exercises (25-26)	P430
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6-2 Volume: Slicing, Disks and Washers

26. Let R be the region bounded by $y = x^2$ and $y = 4$. Compute the volume of the solid formed by revolving R about the given line.
- (a) $y = 4$ (b) the y -axis (c) $y = 6$
 (d) $y = -2$ (e) $x = 2$ (f) $x = -4$



26. Let R be the region bounded by $y = x^2$ and $y = 4$. Compute the volume of the solid formed by revolving R about the given line.

(a) $y = 4$ (b) the y -axis (c) $y = 6$
 (d) $y = -2$ (e) $x = 2$ (f) $x = -4$

(a) Disk/Washer Method. $R_{\text{outer}} = 4 - y_{\text{lower}} = 4 - x^2$, $R_{\text{inner}} = 4 - y_{\text{upper}} = 4 - 4 = 0$
 $V = \int_{-2}^2 \pi (4 - x^2)^2 dx = 2\pi \int_0^2 (16 - 8x^2 + x^4) dx = 2\pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right]_0^2 = \frac{256\pi}{5}$

(b) Disk/Washer Method w.r.t y . $R = \sqrt{y}$
 $V = \int_0^4 \pi (\sqrt{y})^2 dy = \pi \left[\frac{1}{2}y^2 \right]_0^4 = 8\pi$

(c) $V = \int_{-2}^2 \pi (6 - 6)^2 dx = 2\pi \int_0^2 (16 - 8x^2 + x^4) dx = 2\pi \left[\frac{8}{3}x^3 \right]_0^2 = 8\pi$

(d) $V = \int_{-2}^2 \pi (y - 2)^2 dx = 2\pi \int_0^2 (16 - 8x^2 + x^4) dx = 2\pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right]_0^2 = \frac{256\pi}{5}$

(e) $V = \int_0^4 \pi (\sqrt{y})^2 dy = \pi \left[\frac{1}{2}y^2 \right]_0^4 = 8\pi$

(f) $V = \int_{-4}^4 \pi (x - 4)^2 dx = 2\pi \int_0^4 \pi (17)^2 dy = \pi \left[\frac{1}{2}y^2 \right]_0^4 = \frac{256\pi}{5}$

17	Find the volume of a solid of revolution by using the method of washers إيجاد حجم مجسم مجوف ناتج عن دوران منطقة حول مستقيم معلوم باستخدام طريقة الحلقات.	Exercises (17-20)	P430
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6-2 Volume: Slicing, Disks and Washers

17. Region bounded by $y = 2 - x$, $y = 0$ and $x = 0$ about (a) the x -axis; (b) $y = 3$

17. Region bounded by $y = 2 - x$, $y = 0$ and $x = 0$ about (a) the x -axis; (b) $y = 3$

(a) $(0,2)$, $(2,0)$, $(0,0)$
 $V_a = \pi \int_0^2 (2-x)^2 dx$
 $(a) = \pi \left[-\frac{(2-x)^3}{3} \right]_0^2 = \frac{8\pi}{3}$

(b) $y = 3$
 $V_b = \pi \int_0^2 [(3-0)^2 - (3-(2-x))^2] dx$
 $(b) = \pi \int_0^2 [9 - (1+x)^2] dx = \pi \left[9x - \frac{1}{3}x^3 \right]_0^2 = \frac{28\pi}{3}$

17	Find the volume of a solid of revolution by using he method of washers إيجاد حجم مجسم مجوف ناتج عن دوران منطقة حول مستقيم معلوم باستخدام طريقة الحلقات.	Exercises (17-20)	P430
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6-2 Volume: Slicing, Disks and Washers

18. Region bounded by $y = x^2$, $y = 4 - x^2$ about (a) the x -axis;
 (b) $y = 4$

18. Region bounded by $y = x^2$, $y = 4 - x^2$ about (a) the x -axis;
 (b) $y = 4$

(a) $V = \pi \int_{-a}^a [(R_{outer})^2 - (R_{inner})^2] dx = \pi \int_{x=-\sqrt{2}}^{x=\sqrt{2}} dx - \int_a^{\sqrt{2}} dx = 23.764\pi$

(b) $V = \pi \int_{-a}^a [(R_{outer})^2 - (R_{inner})^2] dx = \pi \int_a^{\sqrt{2}} (x^2 - x^4) dx = 24.7519\pi$

17	Find the volume of a solid of revolution by using he method of washers إيجاد حجم مجسم مجوف ناتج عن دوران منطقة حول مستقيم معلوم باستخدام طريقة الحلقات.	Exercises (17-20)	P430
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6-2 Volume: Slicing, Disks and Washers

19. Region bounded by $y = \sqrt{x}$, $y = 2$ and $x = 0$ about (a) the y -axis; (b) $x = 4$

1. Radius $R(y) = y^2$

$$V = \pi \int_0^2 (y^2)^2 dy = \pi \int_0^2 y^4 dy$$

$$V = \pi \left[\frac{1}{5} y^5 \right]_0^2 = \frac{32\pi}{5}$$

2. Outer Radius $R(y) = 4 - 0 = 4$
 Inner Radius $r(y) = 4 - y^2$

$$V = \pi \int_0^2 [4^2 - (4 - y^2)^2] dy$$

$$V = \pi \int_0^2 [16 - (16 - 8y^2 + y^4)] dy$$

$$V = \pi \int_0^2 (8y^2 - y^4) dy$$

$$V = \pi \left[\frac{8}{3} y^3 - \frac{1}{5} y^5 \right]_0^2$$

$$V = \pi \left(\frac{64}{3} - \frac{32}{5} \right) = \pi \left(\frac{320 - 96}{15} \right)$$

$$V = \frac{224\pi}{15}$$

17	Find the volume of a solid of revolution by using the method of washers إيجاد حجم مجسم مجوف ناتج عن دوران منطقة حول مستقيم معلوم باستخدام طريقة الحلقات.	Exercises (17-20)	P430
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LESSON 6-2 Volume: Slicing, Disks and Washers

20. Region bounded by $y = x^2$ and $x = y^2$ about (a) the y -axis;
 (b) $x = 1$

Outer Radius $R(y) = \sqrt{y}$
 Inner Radius $r(y) = y^2$

$$V = \pi \int_0^1 [(\sqrt{y})^2 - (y^2)^2] dy = \pi \int_0^1 (y - y^4) dy$$

$$V = \pi \left[\frac{1}{2}y^2 - \frac{1}{5}y^5 \right]_0^1$$

$$V = \pi \left(\frac{1}{2} - \frac{1}{5} \right) = \frac{3\pi}{10}$$

Outer Radius $R(y) = 1 - y^2$
 Inner Radius $r(y) = 1 - \sqrt{y}$

$$V = \pi \int_0^1 [(1 - y^2)^2 - (1 - \sqrt{y})^2] dy$$

$$V = \pi \int_0^1 [(1 - 2y^2 + y^4) - (1 - 2\sqrt{y} + y)] dy$$

$$V = \pi \int_0^1 (2y^{1/2} - y - 2y^2 + y^4) dy$$

$$V = \pi \left[\frac{4}{3}y^{3/2} - \frac{1}{2}y^2 - \frac{2}{3}y^3 + \frac{1}{5}y^5 \right]_0^1$$

$$V = \pi \left(\frac{4}{3} - \frac{1}{2} - \frac{2}{3} + \frac{1}{5} \right) = \pi \left(\frac{2}{3} - \frac{1}{2} + \frac{1}{5} \right)$$

$$V = \pi \left(\frac{20 - 15 + 6}{30} \right) = \frac{11\pi}{30}$$

18	Find arc length in a given interval using definite integration إيجاد طول قوس من منحنى دالة معطاة في فترة معينة باستخدام التكامل المحدود.	Exercises (17-20)	P447
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LESSON 6-4 Arc Length and Surface Area

17. $y = 2x - x^2, 0 \leq x \leq 2$

17. $y = 2x - x^2, 0 \leq x \leq 2$
 Find area: $\int_0^2 (2x - x^2) dx$
 Evaluate: $\left[x^2 - \frac{x^3}{3} \right]_0^2$

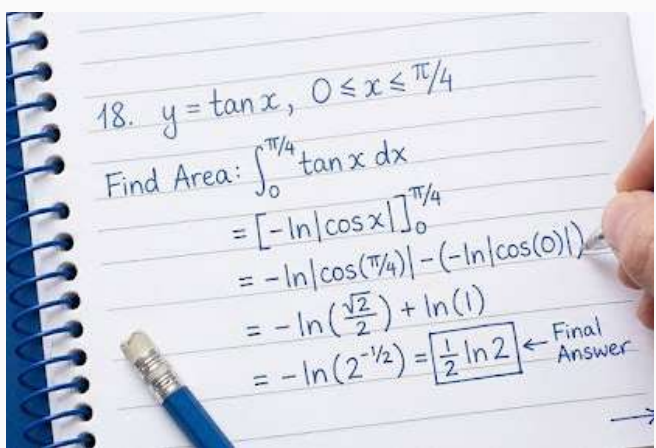
EOT

18

18	Find arc length in a given interval using definite integration إيجاد طول قوس من منحنى دالة معطاة في فترة معطاة باستخدام التكامل المحدود.	Exercises (17-20)	P447
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LESSON 6-4 Arc Length and Surface Area

18. $y = \tan x, 0 \leq x \leq \pi/4$

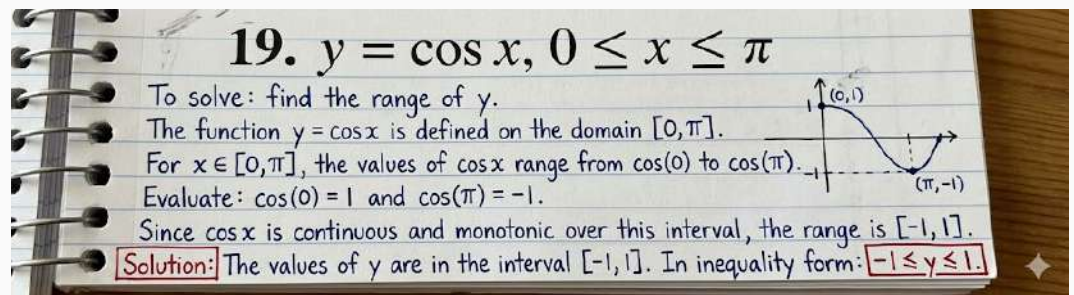


18. $y = \tan x, 0 \leq x \leq \pi/4$
 Find Area: $\int_0^{\pi/4} \tan x \, dx$
 $= [-\ln|\cos x|]_0^{\pi/4}$
 $= -\ln|\cos(\pi/4)| - (-\ln|\cos(0)|)$
 $= -\ln(\frac{\sqrt{2}}{2}) + \ln(1)$
 $= -\ln(2^{-1/2}) = \frac{1}{2} \ln 2 \leftarrow \text{Final Answer}$

18	Find arc length in a given interval using definite integration إيجاد طول قوس من منحنى دالة معطاة في فترة معطاة باستخدام التكامل المحدود.	Exercises (17-20)	P447
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LESSON 6-4 Arc Length and Surface Area

19. $y = \cos x, 0 \leq x \leq \pi$



19. $y = \cos x, 0 \leq x \leq \pi$
 To solve: find the range of y .
 The function $y = \cos x$ is defined on the domain $[0, \pi]$.
 For $x \in [0, \pi]$, the values of $\cos x$ range from $\cos(0)$ to $\cos(\pi)$.
 Evaluate: $\cos(0) = 1$ and $\cos(\pi) = -1$.
 Since $\cos x$ is continuous and monotonic over this interval, the range is $[-1, 1]$.
Solution: The values of y are in the interval $[-1, 1]$. In inequality form: $-1 \leq y \leq 1$.

18	Find arc length in a given interval using definite integration إيجاد طول قوس من منحنى دالة معطاة في فترة معطاة باستخدام التكامل المحدود.	Exercises (17-20)	P447
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LESSON 6-4 Arc Length and Surface Area

20. $y = \ln x, 1 \leq x \leq 3$

20. $y = \ln x, 1 \leq x \leq 3$

Solution:
 We need to find the length of the arc of the curve given by
 We find the length of an arc of a curve $y = f(x)$ on $[a, b]$:

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$L = \int_1^3 \sqrt{1 + \left(\frac{1}{x}\right)^2} dx = \int_1^3 \sqrt{1 + \frac{1}{x^2}} dx = \int_1^3 \frac{\sqrt{x^2 + 1}}{x} dx$$

$$L = \int_1^3 \frac{\sqrt{x^2 + 1}}{x} dx$$

$$L = \int_{\pi/4}^{\arctan(3)} \frac{\sec \theta}{\tan \theta} d\theta = \int_{\pi/4}^{\arctan(3)} \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} d\theta = \int_{\pi/4}^{\arctan(3)} \frac{1}{\sin \theta \cos^2 \theta} d\theta$$

$$L = \left[\sqrt{10} - \ln \left| \frac{3(1+\sqrt{2})}{1+\sqrt{10}} \right| \right]_1^3$$

19	Find surface area of a solid of revolution using definite integration حساب مساحة السطح الناتج عن دوران منطقة معينة حول مستقيم معلوم في فترة معطاة باستخدام التكامل المحدود.	Exercises (32-36)	P447
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LESSON 6-4 Arc Length and Surface Area

32. $y = x^3 - 4x, -2 \leq x \leq 0$, revolved about the x-axis

$$y = x^3 - 4x, -2 \leq x \leq 0$$

$$V = \pi \int_{-2}^0 (x^3 - 4x)^2 dx$$

$$V = \pi \int_{-2}^0 (x^6 - 8x^4 + 16x^2) dx$$

$$V = \pi \left[\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3} \right]_{-2}^0$$

$$V = \pi(0) - \pi \left(\frac{(-2)^7}{7} - \frac{8(-2)^5}{5} + \frac{16(-2)^3}{3} \right)$$

$$V = -\pi \left(-\frac{128}{7} + \frac{256}{5} - \frac{128}{3} \right)$$

$$V = -\pi \left(\frac{-1920 + 5376 - 4480}{105} \right) = -\pi \left(\frac{-1024}{105} \right) = \frac{1024\pi}{105}$$

19	Find surface area of a solid of revolution using definite integration	Exercises (32-36)	P447
حساب مساحة السطح الناتج عن دوران منطقة معينة حول مستقيم معلوم في فترة معينة باستخدام التكامل المحدود.			

LESSON 6-4 Arc Length and Surface Area

33. $y = e^x, 0 \leq x \leq 1$, revolved about the x-axis

$$y = e^x, \quad 0 \leq x \leq 1$$

$$V = \pi \int_0^1 (e^x)^2 dx = \pi \int_0^1 e^{2x} dx$$

$$V = \pi \left[\frac{1}{2} e^{2x} \right]_0^1$$

$$V = \frac{\pi}{2} (e^2 - e^0) = \frac{\pi}{2} (e^2 - 1)$$

19	Find surface area of a solid of revolution using definite integration	Exercises (32-36)	P447
حساب مساحة السطح الناتج عن دوران منطقة معينة حول مستقيم معلوم في فترة معينة باستخدام التكامل المحدود.			

LESSON 6-4 Arc Length and Surface Area

34. $y = \ln x, 1 \leq x \leq 2$, revolved about the x-axis

$$(y = \ln x, \quad ; \quad 1 \leq x \leq 2)$$

Surface area formula: $S = \int_1^2 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

Derivative: $y' = \frac{1}{x}$

Substitute into the formula: $S = 2\pi \int_1^2 \ln x \sqrt{1 + \frac{1}{x^2}} dx$

Simplify the radical: $\sqrt{1 + \frac{1}{x^2}} = \frac{\sqrt{x^2 + 1}}{x}$

Rewrite: $S = 2\pi \int_1^2 \ln x \cdot \frac{\sqrt{x^2 + 1}}{x} dx$

19	Find surface area of a solid of revolution using definite integration حساب مساحة السطح الناتج عن دوران منطقة معينة حول مستقيم معلوم في فترة معينة باستخدام التكامل المحدود.	Exercises (32-36)	P447
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LESSON 6-4 Arc Length and Surface Area

35. $y = \cos x, 0 \leq x \leq \pi/2$, revolved about the x-axis

$$y = \cos x, \quad 0 \leq x \leq \frac{\pi}{2}$$

$$V = \pi \int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \pi \int_0^{\frac{\pi}{2}} \frac{1 + \cos(2x)}{2} \, dx$$

$$V = \frac{\pi}{2} \left[x + \frac{\sin(2x)}{2} \right]_0^{\frac{\pi}{2}}$$

$$V = \frac{\pi}{2} \left(\left(\frac{\pi}{2} + 0 \right) - (0 + 0) \right) = \frac{\pi^2}{4}$$

19	Find surface area of a solid of revolution using definite integration حساب مساحة السطح الناتج عن دوران منطقة معينة حول مستقيم معلوم في فترة معينة باستخدام التكامل المحدود.	Exercises (32-36)	P447
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LESSON 6-4 Arc Length and Surface Area

36. $y = \sqrt{x}, 1 \leq x \leq 2$, revolved about the x-axis

$$y = \sqrt{x}, \quad 1 \leq x \leq 2$$

$$V = \pi \int_1^2 (\sqrt{x})^2 \, dx = \pi \int_1^2 x \, dx$$

$$V = \pi \left[\frac{x^2}{2} \right]_1^2$$

$$V = \pi \left(\frac{4}{2} - \frac{1}{2} \right) = \frac{3\pi}{2}$$

20	Solve physical problems involving velocity. حل مسائل تطبيقات فيزيائية على السرعة المتجهة.	Exercises (5-7)	P455
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6-5 Projectile Motion

5. A diver drops from 30 ft above the water (about the height of an Olympic platform dive). What is the diver's velocity at impact?

$$\begin{aligned}d &= 30 \text{ ft} \\v &= -\sqrt{2(32)(30)} \\v &= -\sqrt{1920} \\v &\approx -43.82 \text{ ft/s} \quad (\approx -29.9 \text{ mph})\end{aligned}$$

20	Solve physical problems involving velocity. حل مسائل تطبيقات فيزيائية على السرعة المتجهة.	Exercises (5-7)	P455
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6-5 Projectile Motion

6. A diver drops from 120 ft above the water (about the height of divers at the Acapulco Cliff Diving competition). What is the diver's velocity at impact?

6. A diver drops from 120 ft above the water (about the height of divers at the Acapulco Cliff Diving competition). What is the diver's velocity at impact?

Solution to Problem 6
Objective: Find velocity at impact (v_f).
Given Info:

- Initial position: $h_i = 120 \text{ ft}$ above water.
- Final position (at impact): $h_f = 0 \text{ ft}$ (at water level)
- Initial velocity: $v_i = 0 \text{ ft/s}$ (since the diver "drops")
- Acceleration: $g \approx 32.2 \text{ ft/s}^2$ (downward)

* $KE_{\text{final}} = PE_{\text{initial}} \quad \frac{1}{2}mv_f^2 = mgh \quad v_f = \sqrt{2(32.2)(120)} \approx 87.9 \text{ ft/s}$

Diagram: A diver is shown at a height of 120 ft above the ground level. The initial velocity $v_i = 0$ and the final velocity $v_f = ?$ is indicated at the water level.

Equations to Consider: Since acceleration is constant, we can use kinematics. A convenient equation relating velocity and displacement without needing time (t) is:
 $v_f^2 = v_i^2 + 2g \cdot d$

Step 1: $v_f^2 = (0 \text{ ft/s})^2 + 2(32.2 \text{ ft/s}^2)(120 \text{ ft})$
Step 2: $v_f^2 = 0 + (64.4)(120) \text{ ft}^2/\text{s}^2$
Step 3: Solve for $v_f = \sqrt{7728} \text{ ft/s}$
Step 4: $v_f \approx 87.909... \text{ ft/s}$

The diver's velocity at a reasonable impact is approximately 87.9 ft/s.
(Direction is downwards, toward the water).

20	Solve physical problems involving velocity. حل مسائل تطبيقات فيزيائية على السرعة المتجهة.	Exercises (5-7)	P455
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6-5 Projectile Motion

7. Compare the impact velocities of objects falling from 120 ft (exercise 5), 120 ft (exercise 6) and 3000 ft (example 5.6). If height is increased by a factor of h , by what factor does the impact velocity increase?

7. Compare impact velocities for objects falling from 120 ft & 3000 ft. If height is increased by factor 'h', by what factor does impact velocity increase?

General formula for impact velocity.
 Formula: $v = \sqrt{2gd}$, where $g \approx 32.2 \text{ ft/s}^2$

120 ft fall: $v_1 = \sqrt{2 \times 32.2 \times 120} \approx 87.9 \text{ ft/s}$

3000 ft fall: $v_2 = \sqrt{2 \times 32.2 \times 3000} \approx 439.5 \text{ ft/s}$

Comparing: $v_2 = \sqrt{2g \times 3000} = \sqrt{2g \times (25 \times 120)} = 5 \times \sqrt{2g \times 120} = 5 \cdot v_1$
 So velocity from 3000 ft is 5 times greater than from 120 ft.
 Relationship for factor of increase:
 Let new height be $h \cdot d_1$.
 $v_{\text{new}} = \sqrt{2g(h \cdot d_1)} = \sqrt{h} \cdot \sqrt{2gd_1} = \sqrt{h} \cdot v_{\text{old}}$
 If height is increased by a factor of 'h', the impact velocity increases by a factor of $\sqrt{h}$.

21	Solve problems on projectiles حل مسائل لتطبيقات على حركة المقذوفات	Exercises (17-20)	P456
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6-5 Projectile Motion

17. An object is launched at angle $\theta = \pi/3$ radians from the horizontal with an initial speed of 98 m/s. Determine the time of flight and the horizontal range. Compare to example 5.4.



HEET 21

Question 17 – Projectile Motion

Starting point:
 $y'' = -9.8, y'(0) = 98 \sin(\frac{\pi}{3}) = 49\sqrt{3}$.

We get:
 $y(t) = -4.9t^2 + 49\sqrt{3}t$
 $= -4.9t(t - 10\sqrt{3})$

Time of flight: $t = 10\sqrt{3} \text{ s}$

Horizontal Range:
 $x'(t) = 98 \cos(\frac{\pi}{3}) = 49$
 $x(t) = 49t$
 Range (R) = $49(10\sqrt{3})$
 $\rightarrow R = 490\sqrt{3} \text{ meters}$

18. Find the time of flight and horizontal range of an object launched at angle 30° with initial speed 40 m/s. Repeat with an angle of 60° .

18. Solution

Here $y'(0) = 40 \sin(\frac{\pi}{6}) = 20$
 Therefore $y(t) = -4.9t^2 + 20t$
 $\rightarrow$ the time of flight = $\frac{20}{4.9} = 4.082 \text{ s}$

Now, for the horizontal range $x(t)$
 $x'(t) = 40 \cos(\frac{\pi}{6}) = 20\sqrt{3}$
 Therefore $x(t) = 20\sqrt{3}t$
 $\rightarrow (4.082) = 20(1.7321)(4.082) = 141.3919 \text{ m}$

Repeating the same for the angle 60°
 Here $y'(0) = 40 \sin(\frac{\pi}{3}) = 34.6410$
 Therefore $y(t) = -4.9t^2 + 34.6410t$
 $\rightarrow$ the time of flight = $\frac{34.6410}{4.9} = 7.0696 \text{ s}$

Now, for the horizontal range $x(t)$
 $x'(t) = 40 \cos(\frac{\pi}{3}) = 20$
 Therefore $x(t) = 20t$
 $\rightarrow x(7.0696) = 20(7.0696) = 141.3919 \text{ m}$

21	Solve problems on projectiles حل مسائل تطبيقية على حركة المقذوفات	Exercises (17-20)	P456
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6-5 Projectile Motion

19. Repeat example 5.5 with an initial angle of 6° . By trial and error, find the smallest and largest angles for which the serve will be in.

Step 1: Convert Initial Speed & Set Up Velocity

Convert speed:
 $120 \text{ mph} = 120 \cdot \left(\frac{5280}{3600}\right) = 176 \text{ ft/s}$

For an angle of 6° below horizontal ($\theta = -6^\circ$):
 $x'(0) = 176 \cos(-6^\circ) \approx 175.03 \text{ ft/s}$
 $y'(0) = 176 \sin(-6^\circ) \approx -18.39 \text{ ft/s}$

Step 2: Write Position Equations (by Integration)

Horizontal position:
 $x(t) = 175.03t$

Vertical position (using acceleration $g = -32 \text{ ft/s}^2$ and initial height $y(0) = 10 \text{ ft}$):
 $y(t) = -16t^2 - 18.39t + 10$

Step 3: Check Net Clearance (at $x = 39 \text{ ft}$)

Find time to reach net:
 $175.03t = 39 \Rightarrow t \approx 0.2228 \text{ s}$

Find ball height at this time:
 $y(0.2228) = -16(0.2228)^2 - 18.39(0.2228) + 10 \approx 5.11 \text{ ft}$

Find time to reach net:
 $175.03t = 39 \Rightarrow t \approx 0.2228 \text{ s}$

Find ball height at this time:
 $y(0.2228) = -16(0.2228)^2 - 18.39(0.2228) + 10 \approx 5.11 \text{ ft}$

Step 4: Check Landing Distance (where $y = 0$)

Find time to reach net:
 $175.03t = 39 \Rightarrow t \approx 0.2228 \text{ s}$

Step 5: Find Minimum Angle Limit (Lands exactly at $x = 60 \text{ ft}$)
($t_x = 60 \text{ ft}$, $x = 60 \text{ ft}$)
 $x(t) = 60^\circ = 120^\circ \Rightarrow t \approx 1.222 \text{ ft}$
 $y(t) = 60^\circ = -16^\circ \Rightarrow t \approx 12.50 \text{ ft}$

Step 6: Find Maximum Angle Limit (Clears net exactly at $x = 39 \text{ ft}$, $y = 3 \text{ ft}$)
Angle: $\theta = -16^\circ \Rightarrow \theta = -16^\circ$
Angler: $-16t^2 - 1.39t + 10$
 $\pi = -6x - 39$, $y = 0$

Conclusion

For an initial angle of 6° , the serve travels 70.50 ft and is OUT. The serve will be IN if the angle is between 7.69° and 9.01° below the horizontal.

21	Solve problems on projectiles حل مسائل تطبيقية على حركة المقذوفات	Exercises (17-20)	P456
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6-5 Projectile Motion

20. Repeat example 5.5 with an initial speed of 170 ft/s. By trial and error, find the smallest and largest initial speeds for which the serve will be in.

Exercise 20. Solution Notes

Reference: Example 5.5, Fig 6.46

Problem Parameters:

- Launch height (y_0) = 10 ft
- Launch angle (θ) = -7° (down)
- Net: 3 ft high at $x = 39 \text{ ft}$
- Service line: $x = 60 \text{ ft}$
- $g = -32 \text{ ft/s}^2$

Part 1: Initial Speed (v_0) = 170 ft/s

Find Velocity Components:

- $x'(0) = 170 \cos(-7^\circ) \approx 168.73 \text{ ft/s}$
- $y'(0) = 170 \sin(-7^\circ) \approx -20.72 \text{ ft/s}$

Integrate to find Position Equations:

- $x(t) = 168.73t$
- $y(t) = -16t^2 - 20.72t + 10$

Check Net Clearance:

Ball reaches net at $x = 39$.
 $168.73t = 39 \Rightarrow t \approx 0.2311 \text{ s}$
 $y(0.2311) = -16(0.2311)^2 - 20.72(0.2311) + 10$
 $y(0.2311) \approx 4.35 \text{ ft}$
Since $4.35 > 3$, it clears the net.

Check Landing Constraint:

Ball lands at $y = 0$.
 $-16t^2 - 20.72t + 10 = 0$
Positive solution for $t \approx 0.3807 \text{ s}$.
 $x(0.3807) = 168.73 \cdot 0.3807 \approx 64.24 \text{ ft}$
Since $64.24 > 60$, the serve is OUT (too long).

Part 2: Find Boundary v_0 by Trial & Error

General path equation $y(x)$:

$$y(x) = x \tan(-7^\circ) - \frac{32x^2}{2(v_0 \cos(-7^\circ))^2} + 10$$
$$y(x) = -0.1228x - \frac{16.196x^2}{v_0^2} + 10$$

Constraint A: Clears the Net

Try $v_0 = 176$ (Example): IN (too long)
Try $v_0 = 170$ (Our test): IN (too long)
Try $v_0 = 145$:
 $y(39) = 10 - 0.1228(39) - \frac{16.196(39)^2}{145^2} \approx 4.04 \text{ ft}$
It clears.
Try $v_0 = 135$:
 $y(39) = 10 - 4.79 - 1.35 \approx 3.86 \text{ ft}$
It clears.
Solving precisely for boundary ($y=3$), $v_0 \approx 113.6 \text{ ft/s}$.

Constraint B: Lands before 60ft

Find speed landing exactly on line ($y=0$ at $x=60$):
 $0 = 10 - 0.1228(60) - \frac{16.196(60)^2}{v_0^2}$
 $0 = 10 - 7.368 - (58305.6) / v_0^2$
 $58305.6 / v_0^2 = 2.632$
 $v_0^2 = 22152.6 \Rightarrow v_0 \approx 148.84 \text{ ft/s}$

Final Conclusion:

For the serve to land IN,
Smallest $v_0 \approx 113.6 \text{ ft/s}$
Largest $v_0 \approx 148.8 \text{ ft/s}$
With $v_0 = 170$, the ball travels $\sim 64 \text{ ft}$ (OUT).

22	Learn the notion of integration by parts. التعرف على التكامل بطريقة التكامل بالأجزاء.	Exercises (1-4)	P496
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7-2 Integration by Parts

1. $\int x \cos x \, dx$

1. Let $u = x$, $dv = \cos x \, dx$
 $du = dx$, $v = \sin x$.

$$\int x \cos x \, dx = x \sin x - \int \sin x \, dx$$

$$\int x \cos x \, dx = x \sin x - \int$$

$$= x \sin x + \cos x + c$$

2. $\int x \sin 4x \, dx$

2. Let $u = x$, $dv = \sin 4x \, dx$
 $du = dx$, $v = -\frac{1}{4} \cos 4x$

$$\int x \sin 4x \, dx$$

$$= -\frac{1}{4} x \cos 4x - \int -\frac{1}{4} \cos 4x \, dx$$

$$= -\frac{1}{4} x \cos 4x + \frac{1}{16} \sin 4x + c.$$

22	Learn the notion of integration by parts. التعرف على التكامل بطريقة التكامل بالأجزاء.	Exercises (1-4)	P496
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7-2 Integration by Parts

3. $\int x e^{2x} \, dx$

3. Let $u = x$, $dv = e^{2x} \, dx$
 $du = dx$, $v = \frac{1}{2} e^{2x}$.

$$\int x e^{2x} \, dx = \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} \, dx$$

$$\int x e^{2x} \, dx = \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} \, dx$$

$$= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + c.$$

4. $\int x \ln x \, dx$

4. Let $u = \ln x$, $dv = x \, dx$
 $du = \frac{1}{x} \, dx$ and $v = \frac{x^2}{2}$

$$\int x \ln x \, dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x \, dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + c.$$

23	Use integration by parts to compute definite and indefinite integrals إيجاد تكاملات محدودة وغير محدودة متنوعة باستخدام طريقة التكامل بالأجزاء.	Exercises (19-24)	P496
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7-2 Integration by Parts

Calculator

$$19. \int_0^1 x \sin 2x \, dx$$

$$20. \int_0^{\pi} 2x \cos x \, dx$$

19. Let $u = x$, $dv = \sin 2x \, dx$
 $du = dx$, $v = -\frac{1}{2} \cos 2x$

$$\begin{aligned} \int_0^1 x \sin 2x \, dx &= \left[-\frac{1}{2} x \cos 2x \right]_0^1 - \int_0^1 \left(-\frac{1}{2} \cos 2x \right) dx \\ &= -\frac{1}{2} (1 \cos 2 - 0 \cos 0) + \frac{1}{2} \int_0^1 \cos 2x \, dx \\ &= -\frac{1}{2} \cos 2 + \frac{1}{2} \left[\frac{1}{2} \sin 2x \right]_0^1 \\ &= -\frac{1}{2} \cos 2 + \frac{1}{4} (\sin 2 - \sin 0) \\ &= -\frac{1}{2} \cos 2 + \frac{1}{4} \sin 2 \end{aligned}$$

20. Let $u = 2x$, $dv = \cos x \, dx$
 $du = 2 \, dx$ and $v = \sin x$.

$$\begin{aligned} \int_0^{\pi} 2x \cos x \, dx &= 2x \sin x \Big|_0^{\pi} - 2 \int_0^{\pi} \sin x \, dx \\ &= (2x \sin x + 2 \cos x) \Big|_0^{\pi} = -4. \end{aligned}$$

EOT
23

23	Use integration by parts to compute definite and indefinite integrals إيجاد تكاملات محدودة وغير محدودة متنوعة باستخدام طريقة التكامل بالأجزاء.	Exercises (19-24)	P496
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7-2 Integration by Parts

Calculator

$$21. \int_0^1 x^2 \cos \pi x \, dx$$

$$22. \int_0^1 x^2 e^{3x} \, dx$$

21. $\int_0^1 x^2 \cos \pi x \, dx$
Let $u = x^2$, $dv = \cos \pi x \, dx$,
 $du = 2x \, dx$, $v = \frac{\sin \pi x}{\pi}$.

$$\begin{aligned} \int_0^1 x^2 \cos \pi x \, dx &= x^2 \frac{\sin \pi x}{\pi} \Big|_0^1 - \int_0^1 \frac{\sin \pi x}{\pi} 2x \, dx \\ &= (0 - 0) - \frac{2}{\pi} \int_0^1 x \sin(\pi x) \, dx \\ &= -\frac{2}{\pi} \int_0^1 x \sin(\pi x) \, dx \\ \text{Let } u &= x, dv = \sin(\pi x) \, dx, \\ du &= dx, v = -\frac{\cos(\pi x)}{\pi}. \\ &= -\frac{2}{\pi} \int_0^1 x \sin(\pi x) \, dx \\ &= -\frac{2}{\pi} \left\{ -\frac{x \cos(\pi x)}{\pi} \Big|_0^1 - \int_0^1 -\frac{\cos(\pi x)}{\pi} \, dx \right\} \\ &= -\frac{2}{\pi} \left\{ \left[-\frac{\cos \pi}{\pi} - 0 \right] + \frac{1}{\pi} \left[\frac{\sin(\pi x)}{\pi} \right]_0^1 \right\} \\ &= -\frac{2}{\pi} \left\{ \frac{1}{\pi} + \frac{1}{\pi} (0 - 0) \right\} = -\frac{2}{\pi^2} \end{aligned}$$

22. $\int_0^1 x^2 e^{3x} \, dx$ Let $u = x^2$, $dv = e^{3x} \, dx$,
 $du = 2x \, dx$, $v = \frac{e^{3x}}{3}$.

$$\begin{aligned} \int_0^1 x^2 e^{3x} \, dx &= \left[\frac{x^2 e^{3x}}{3} \right]_0^1 - \int_0^1 \left(\frac{e^{3x}}{3} \right) 2x \, dx \\ &= \frac{1}{3} (e^3 - 0) - \frac{2}{3} \int_0^1 x e^{3x} \, dx \\ \text{Let } u &= x, dv = e^{3x} \, dx, \\ du &= dx, v = \frac{e^{3x}}{3}. \\ &= \frac{e^3}{3} - \frac{2}{3} \int_0^1 \left\{ x \left(\frac{e^{3x}}{3} \right) \Big|_0^1 - \int_0^1 \frac{e^{3x}}{3} \, dx \right\} \\ &= \frac{e^3}{3} - \frac{2}{3} \left\{ \left(\frac{e^3}{3} \right) - \int_0^1 \left(\frac{e^{3x}}{3} \right) dx \right\} \\ &= \frac{e^3}{3} - \frac{2}{3} \left\{ \left(\frac{e^3}{3} \right) - \left[\frac{e^{3x}}{9} \right]_0^1 \right\} \\ &= \frac{e^3}{3} - \frac{2e^3}{9} + \frac{2}{27} (e^3 - 1) \\ &= \frac{e^3}{3} - \frac{2e^3}{9} + \frac{2e^3}{27} - \frac{2}{27} = \frac{5e^3}{27} - \frac{2}{27} \end{aligned}$$

23	Use integration by parts to compute definite and indefinite integrals إيجاد تكاملات محدودة وغير محدودة متتوية باستخدام طريقة التكامل بالأجزاء.	Exercises (19-24)	P496
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7-2 Integration by Parts

Calculator

23. $\int_1^{10} \ln 2x \, dx$

24. $\int_1^2 x \ln x \, dx$

23. $\int_1^{10} \ln(2x) \, dx$
 Let $u = \ln(2x)$, $dv = dx$
 $du = \frac{1}{x} dx$, $v = x$
 $\int_1^{10} \ln(2x) \, dx = x \ln(2x) \Big|_1^{10} - \int_1^{10} x \left(\frac{1}{x}\right) dx$
 $= (10 \ln(20) - \ln(2)) - \int_1^{10} dx$
 $= (10 \ln(20) - \ln(2)) - [x]_1^{10}$
 $= (10 \ln(20) - \ln(2)) - (10 - 1)$
 $= (10 \ln(20) - \ln(2) - 9)$

24. Let $u = \ln x$, $dv = x \, dx$
 $du = \frac{1}{x} dx$, $v = \frac{x^2}{2}$
 $\int_1^2 x \ln x \, dx = \frac{1}{2} x^2 \ln x \Big|_1^2 - \int_1^2 \frac{1}{2} x \, dx$
 $= \left(\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 \right) \Big|_1^2 = 2 \ln 2 - \frac{3}{4}$

EOT
 24

24	Integrate functions of the form $\sin^n(x) \cos^m(x)$ إيجاد تكاملات دوال بصيغة $\sin^n(x) \cos^m(x)$	Exercises (3-6)	P507
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7-3 Trigonometric Techniques of Integration

Calculator

3. $\int_0^{\pi/4} \cos 2x \sin^3 2x \, dx$

4. $\int_{\pi/4}^{\pi/3} \cos^3 3x \sin^3 3x \, dx$

3. Let $u = \sin 2x$, $du = 2 \cos 2x \, dx$
 $\int_0^{\pi/4} \cos 2x \sin^3 2x \, dx$
 $= \frac{1}{2} \int_0^1 u^3 \, du = \frac{1}{2} \left[\frac{u^4}{4} \right]_0^1 = \frac{1}{8}$
 $\int \cot x (1 + \cot^2 x) \cdot \csc^2 x \, dx$
 $= - \int \cot x (1 + \cot^2 x) \cdot \csc^2 x \, dx$
 $= - \int (u + u^3) \, du = - \frac{u^2}{2} - \frac{u^4}{4} + C$
 $= - \frac{\cot^2 x}{2} - \frac{\cot^4 x}{4} + C$

4. Let $u = \cos 3x$, $du = -3 \sin 3x \, dx$
 $\int_{\pi/4}^{\pi/3} (\cos^3 3x) (\sin^3 3x) \, dx$
 $= - \frac{1}{3} \int_{-1/\sqrt{2}}^{-1} u^3 (1 - u^2) \, du$
 $= - \frac{1}{3} \left[\frac{u^4}{4} - \frac{u^6}{6} \right]_{-1/\sqrt{2}}^{-1}$
 $= - \frac{1}{3} \left(\frac{3}{16} - \frac{7}{48} \right) = - \frac{1}{72}$



24	Integrate functions of the form $\sin^n(x) \cos^m(x)$ إيجاد تكاملات دوال بصيغة $\sin^m(x) \cos^n(x)$	Exercises (3-6)	P507
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7-3 Trigonometric Techniques of Integration

Calculator

5. $\int_0^{\pi/2} \cos^2 x \sin x \, dx$

6. $\int_{-\pi/2}^0 \cos^3 x \sin x \, dx$

5. Let $u = \cos x$, $du = -\sin x \, dx$

$$\int_0^{\pi/2} \cos^2 x \sin x \, dx = \int_1^0 u^2 (-du)$$

$$= \left(-\frac{1}{3} u^3 \right) \Big|_1^0 = \frac{1}{3}$$

6. Let $u = \cos x$, $du = -\sin x \, dx$

$$\int_{-\pi/2}^0 \cos^3 x \sin x \, dx = -\int_0^1 u^3 \, du = -1$$



25	Integrate trigonometric functions using the substitution $x=a \sin(y)$ إيجاد تكاملات دوال مثلثية باستخدام التبديل $x=a \sin(y)$	Exercises (23-26)	P507
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7-3 Trigonometric Techniques of Integration

23. $\int \frac{x^2}{\sqrt{16-x^2}} \, dx$

24. $\int \frac{x^3}{\sqrt{9-x^2}} \, dx$

Problem 23

$$23. \int \frac{x^2}{\sqrt{16-x^2}} \, dx = \int \frac{(16 \sin^2 \theta) 4 \cos \theta}{\sqrt{16-(4 \sin \theta)^2}} \, d\theta$$

$$= 64 \int \frac{(\sin^2 \theta) \cos \theta}{\sqrt{16-16 \sin^2 \theta}} \, d\theta$$

$$= 64 \int \frac{(\sin^2 \theta) \cos \theta}{4 \sqrt{(1-\sin^2 \theta)}} \, d\theta$$

$$= 16 \int \frac{\sin^2 \theta \cos \theta}{\cos \theta} \, d\theta = 16 \int \sin^2 \theta \, d\theta$$

$$= 16 \int \left(\frac{1-\cos 2\theta}{2} \right) \, d\theta$$

$$= 8 \left[\int d\theta - \int (\cos 2\theta) \, d\theta \right]$$

$$= 8 \left[\theta - \frac{\sin 2\theta}{2} \right] + c$$

$$= 8 \sin^{-1} \left(\frac{x}{4} \right) - 4 \sin \left[2 \sin^{-1} \left(\frac{x}{4} \right) \right] + c.$$

$$= 8 \sin^{-1} \left(\frac{x}{4} \right) - \frac{x \sqrt{16-x^2}}{2} + c$$

24. Let $x = 3 \sin \theta$, so that $dx = 3 \cos \theta \, d\theta$.

$$\int \frac{x^3}{\sqrt{9-x^2}} \, dx = \int \frac{27 (\sin^3 \theta) (3 \cos \theta) \, d\theta}{\sqrt{9-(3 \sin \theta)^2}}$$

$$= 81 \int \frac{(\sin^3 \theta) (\cos \theta)}{\sqrt{9-9 \sin^2 \theta}} \, d\theta$$

$$= 81 \int \left(\frac{\sin^3 \theta}{3 \cos \theta} \right) \cos \theta \, d\theta = 27 \int \sin^3 \theta \, d\theta$$

$$= 27 \int \left(\frac{3 \sin \theta - \sin 3\theta}{4} \right) \, d\theta$$

$$= \frac{27}{4} \left[3 \int \sin \theta \, d\theta - \int \sin 3\theta \, d\theta \right]$$

$$= \frac{27}{4} \left[-3 \cos \theta + \frac{\cos 3\theta}{3} \right] + c$$

$$= \frac{27}{4} \left\{ -3 \cos \left[\sin^{-1} \left(\frac{x}{3} \right) \right] + \frac{\cos \left[3 \sin^{-1} \left(\frac{x}{3} \right) \right]}{3} \right\} + c.$$

$$= \frac{27}{4} \left\{ -3 \cos \left[\sin^{-1} \left(\frac{x}{3} \right) \right] + \frac{\cos \left[\sin^{-1} \left(\frac{x}{3} \right) \right]}{3} \right\} + c.$$

**EOT
25**

25	Integrate trigonometric functions using the substitution $x=a \sin(y)$ إيجاد تكاملات دوال مثلثية باستخدام التبدل $x=a \sin(y)$	Exercises (23-26)	P507
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7-3 Trigonometric Techniques of Integration

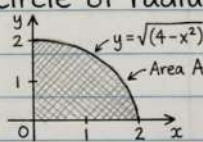
25. $\int_0^2 \sqrt{4-x^2} dx$

26. $\int_0^1 \frac{x}{\sqrt{4-x^2}} dx$

Calculator

25. This is the area of a quarter of a circle of radius 2,

$$\int_0^2 \sqrt{(4-x^2)} dx = \pi$$



Check: Quarter-circle area $A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2^2) = \pi$, as derived above.

26. Let $u = 4 - x^2$, $du = -2x dx$

$$\int_0^1 \frac{x}{\sqrt{4-x^2}} dx = - \int_4^3 \frac{du}{2\sqrt{u}}$$

$$= -u^{1/2} \Big|_4^3 = 2 - \sqrt{3}$$

26	Integrate trigonometric functions using the substitution $x=a \sec(y)$ إيجاد تكاملات دوال مثلثية باستخدام التبدل $x=a \sec(y)$	Exercises (25-30)	P507
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7-3 Trigonometric Techniques of Integration

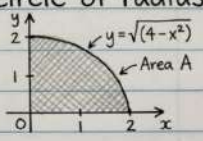
25. $\int_0^2 \sqrt{4-x^2} dx$

26. $\int_0^1 \frac{x}{\sqrt{4-x^2}} dx$

Calculator

25. This is the area of a quarter of a circle of radius 2,

$$\int_0^2 \sqrt{(4-x^2)} dx = \pi$$



Check: Quarter-circle area $A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2^2) = \pi$, as derived above.

26. Let $u = 4 - x^2$, $du = -2x dx$

$$\int_0^1 \frac{x}{\sqrt{4-x^2}} dx = - \int_4^3 \frac{du}{2\sqrt{u}}$$

$$= -u^{1/2} \Big|_4^3 = 2 - \sqrt{3}$$

EOT
26

25	Integrate trigonometric functions using the substitution $x = a \sec(y)$ إيجاد تكاملات دوال مثلثية باستخدام التبديل $x = a \sec(y)$	Exercises (25-30)	P507
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7-3 Trigonometric Techniques of Integration

27. $\int \frac{x^2}{\sqrt{x^2-9}} dx$

27. $I = \int \frac{x^2}{\sqrt{x^2-9}} dx$

Let $x = 3 \sec \theta$, $dx = 3 \sec \theta \tan \theta d\theta$
 Then $\sqrt{x^2-9} = \sqrt{9 \sec^2 \theta - 9} = 3 \tan \theta$
 So,

$$I = \int \frac{x^2}{\sqrt{x^2-9}} dx = \int \frac{(3 \sec \theta)^2}{3 \tan \theta} (3 \sec \theta \tan \theta d\theta)$$

$$= \int \frac{9 \sec^2 \theta}{3 \tan \theta} \cdot 3 \sec \theta \tan \theta d\theta$$

$$= \int 9 \sec^3 \theta d\theta$$

$$= 9 \int \sec^3 \theta d\theta$$

Use integration by parts.

Let $u = \sec \theta$, $dv = \sec^2 \theta d\theta \Rightarrow du = \sec \theta \tan \theta d\theta$, $v = \tan \theta$

$$\int \sec^3 \theta d\theta = \int u dv$$

$$= u v - \int v du$$

$$= \sec \theta \tan \theta - \int \tan \theta (\sec \theta \tan \theta) d\theta$$

$$= \sec \theta \tan \theta - \int \sec \theta \tan^2 \theta d\theta$$

$$= \sec \theta \tan \theta - \int \sec \theta (\sec^2 \theta - 1) d\theta$$

$$= \sec \theta \tan \theta - \int \sec^3 \theta d\theta + \int \sec \theta d\theta$$

$$2 \int \sec^3 \theta d\theta = \sec \theta \tan \theta + \int \sec \theta d\theta$$

$$\int \sec^3 \theta d\theta = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \int \sec \theta d\theta$$

Compute $\int \sec \theta d\theta$.

Let $u = \sec \theta + \tan \theta \Rightarrow du = \sec \theta \tan \theta + \sec^2 \theta d\theta = \sec \theta (\tan \theta + \sec \theta) d\theta = u d\theta$
 $d\theta = \frac{du}{u}$

$$\int \sec \theta d\theta = \int \frac{\sec \theta (\sec \theta + \tan \theta)}{\sec \theta + \tan \theta} d\theta = \int \frac{1}{u} du = \ln |u| + C$$

$$= \ln |\sec \theta + \tan \theta| + C$$

Therefore, $\int \sec^3 \theta d\theta = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| + C$
 So, $I = 9 \int \sec^3 \theta d\theta = \frac{9}{2} \sec \theta \tan \theta + \frac{9}{2} \ln |\sec \theta + \tan \theta| + C$

Back-substitute in terms of x .

Since $x = 3 \sec \theta \Rightarrow \sec \theta = \frac{x}{3}$

$$\tan \theta = \sqrt{\sec^2 \theta - 1} = \sqrt{\frac{x^2}{9} - 1} = \frac{\sqrt{x^2-9}}{3}$$

Then $\sec \theta \tan \theta = \frac{x}{3} \cdot \frac{\sqrt{x^2-9}}{3} = \frac{x \sqrt{x^2-9}}{9}$
 and $|\sec \theta + \tan \theta| = \left| \frac{x}{3} + \frac{\sqrt{x^2-9}}{3} \right| = \frac{1}{3} |x + \sqrt{x^2-9}|$

Substitute: $I = \frac{9}{2} \left(\frac{x \sqrt{x^2-9}}{9} \right) + \frac{9}{2} \ln \left| \frac{1}{3} (x + \sqrt{x^2-9}) \right| + C$

$$I = \frac{1}{2} x \sqrt{x^2-9} + \frac{9}{2} (\ln |x + \sqrt{x^2-9}| - \ln 3) + C$$

$$I = \frac{1}{2} x \sqrt{x^2-9} + \frac{9}{2} \ln |x + \sqrt{x^2-9}| + C \quad (\text{since } -\frac{9}{2} \ln 3 \text{ is absorbed in } C)$$

$$I = \frac{1}{2} x \sqrt{x^2-9} + \frac{9}{2} \ln |x + \sqrt{x^2-9}| + C$$

26	Integrate trigonometric functions using the substitution $x = a \sec(y)$ إيجاد تكاملات دوال مثلثية باستخدام التبديل $x = a \sec(y)$	Exercises (25-30)	P507
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7-3 Trigonometric Techniques of Integration

28. $\int x^3 \sqrt{x^2-1} dx$

28. Let $u = x^2 - 1$, $du = 2x dx$

$$\int x^3 \sqrt{x^2-1} dx$$

$$= \frac{1}{2} \int x^2 \sqrt{x^2-1} (2x) dx$$

$$= \frac{1}{2} \int (u+1) \sqrt{u} du$$

$$= \frac{1}{2} \int u^{3/2} + u^{1/2} du$$

$$= \frac{1}{2} \left(\frac{2u^{5/2}}{5} + \frac{2u^{3/2}}{3} \right) + c$$

$$= \frac{1}{5} (x^2-1)^{5/2} + \frac{1}{3} (x^2-1)^{3/2} + c$$

26	Integrate trigonometric functions using the substitution $x = a \sec(y)$ إيجاد تكاملات دوال مثلثية باستخدام التبدل $x = a \sec(y)$	Exercises (25-30)	P507
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7-3 Trigonometric Techniques of Integration

29. $\int \frac{2}{\sqrt{x^2 - 4}} dx$

30. $\int \frac{x}{\sqrt{x^2 - 4}} dx$

29. Let $x = 2 \sec \theta$, $dx = 2 \sec \theta \tan \theta d\theta$
 $\int \frac{2}{\sqrt{x^2 - 4}} dx = \int \frac{4 \sec \theta \tan \theta}{2 \tan \theta} d\theta$
 $= 2 \int \sec \theta d\theta$
 $= 2 \ln |2 \sec \theta + 2 \tan \theta| + c$
 $= 2 \ln |x + \sqrt{x^2 - 4}| + c$

30. Let $x = 2 \sec \theta$, $dx = 2 \sec \theta \tan \theta d\theta$
 $\int \frac{x}{\sqrt{x^2 - 4}} dx = \int \frac{4 \sec^2 \theta \tan \theta}{2 \tan \theta} d\theta$
 $= 2 \int \sec^2 \theta d\theta$
 $= 2 \tan \theta + C$
 $= \sqrt{x^2 - 4} + c$

27	Integrate rational functions using partial fractions in different cases إيجاد تكاملات دوال نسبية باستخدام طريقة الكسور الجزئية في حالات مختلفة	Exercises (1-6)	P516
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7-4 Integration of Rational Functions Using Partial Fractions

1. $\frac{x-5}{x^2-1}$

2. $\frac{5x-2}{x^2-4}$

1. $\frac{x-5}{x^2-1} = \frac{x-5}{(x+1)(x-1)}$
 $= \frac{A}{x+1} + \frac{B}{x-1}$
 $x-5 = A(x-1) + B(x+1)$
 $x = -1 : -6 = -2A; A = 3$
 $x = 1 : -4 = 2B; B = -2$
 $\frac{x-5}{x^2-1} = \frac{3}{x+1} - \frac{2}{x-1}$
 $\int \frac{x-5}{x^2-1} dx = \int \left(\frac{3}{x+1} - \frac{2}{x-1} \right) dx$
 $= 3 \ln |x+1| - 2 \ln |x-1| + c$

2. $\frac{5x-2}{x^2-4} = \frac{5x-2}{(x+2)(x-2)}$
 $= \frac{A}{x+2} + \frac{B}{x-2}$
 $5x-2 = A(x-2) + B(x+2)$
 $x = -2 : -12 = -4A; A = 3$
 $x = 2 : 8 = 4B; B = 2$
 $\frac{5x-2}{x^2-4} = \frac{3}{x+2} + \frac{2}{x-2}$
 $\int \frac{5x-2}{x^2-4} dx = \int \left(\frac{3}{x+2} + \frac{2}{x-2} \right) dx$
 $= 3 \ln |x+2| + 2 \ln |x-2| + c$

EOT
 27

27	Integrate rational functions using partial fractions in different cases إيجاد تكاملات دوال نسبية باستخدام طريقة الكسور الجزئية في حالات مختلفة.	Exercises (1-6)	P516
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7-4 Integration of Rational Functions Using Partial Fractions

3. $\frac{6x}{x^2 - x - 2}$

$$\frac{6x}{x^2 - x - 2} = \frac{6x}{(x-2)(x+1)}$$

$$= \frac{A}{x-2} + \frac{B}{x+1}$$

$$6x = A(x+1) + B(x-2)$$

$$x = 2 : 12 = 3A; A = 4$$

$$x = -1 : -6 = -3B; B = 2$$

$$\frac{6x}{x^2 - x - 2} = \frac{4}{x-2} + \frac{2}{x+1}$$

$$\int \frac{6x}{x^2 - x - 2} dx = \int \left(\frac{4}{x-2} + \frac{2}{x+1} \right) dx$$

$$= 4 \ln|x-2| + 2 \ln|x+1| + c$$

4. $\frac{3x}{x^2 - 3x - 4}$

$$\frac{3x}{x^2 - 3x - 4} = \frac{3x}{(x+1)(x-4)} = \frac{A}{x+1} + \frac{B}{x-4}$$

$$3x = A(x-4) + B(x+1)$$

$$\frac{3x}{x^2 - 3x - 4} = \frac{3/5}{x+1} + \frac{12/5}{x-4}$$

$$x = -1 : -3 = -5A; A = \frac{3}{5}$$

$$x = 3 : 12 = 5B; B = \frac{12}{5}$$

$$\int \frac{3x}{x^2 - 3x - 4} dx = \int \left(\frac{3/5}{x+1} + \frac{12/5}{x-4} \right) dx = \frac{3}{5} \ln|x+1| + \frac{12}{5} \ln|x-4| + c$$

27	Integrate rational functions using partial fractions in different cases إيجاد تكاملات دوال نسبية باستخدام طريقة الكسور الجزئية في حالات مختلفة.	Exercises (1-6)	P516
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7-4 Integration of Rational Functions Using Partial Fractions

5. $\frac{-x+5}{x^3 - x^2 - 2x}$

$$\frac{-x+5}{x^3 - x^2 - 2x} = \frac{-x+5}{x(x-2)(x+1)}$$

$$= \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$-x+5 = A(x-2)(x+1) + Bx(x+1) + Cx(x-2)$$

$$x = 0 : 5 = -2A : A = -\frac{5}{2}$$

$$x = 2 : 3 = 6B : B = \frac{1}{2}$$

$$x = -1 : 6 = 3C : C = 2$$

$$\frac{-x+5}{x^3 - x^2 - 2x} = -\frac{5/2}{x} + \frac{1/2}{x-2} + \frac{2}{x+1}$$

$$\int \frac{-x+5}{x^3 - x^2 - 2x} dx = \int \left(-\frac{5/2}{x} + \frac{1/2}{x-2} + \frac{2}{x+1} \right) dx$$

$$= -\frac{5}{2} \ln|x| + \frac{1}{2} \ln|x-2| + 2 \ln|x+1| + c$$

6. $\frac{3x+8}{x^3 + 5x^2 + 6x}$

$$\frac{3x+8}{x^3 + 5x^2 + 6x} = \frac{3x+8}{x(x+2)(x+3)}$$

$$= \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x+3}$$

$$3x+8 = A(x+2)(x+3) + Bx(x+3) + Cx(x+2)$$

$$x = 0 \text{ عند} : 8 = 6A \Rightarrow A = \frac{4}{3}$$

$$x = -2 \text{ عند} : 2 = -2B \Rightarrow B = -1$$

$$x = -3 \text{ عند} : -1 = 3C \Rightarrow C = -\frac{1}{3}$$

$$\frac{3x+8}{x^3 + 5x^2 + 6x} = \frac{4/3}{x} - \frac{1}{x+2} - \frac{1/3}{x+3}$$

$$\int \frac{3x+8}{x^3 + 5x^2 + 6x} dx = \int \left(\frac{4/3}{x} - \frac{1}{x+2} - \frac{1/3}{x+3} \right) dx$$

$$= \left[\frac{4}{3} \ln|x| - \ln|x+2| - \frac{1}{3} \ln|x+3| + c \right]$$

EOT
28

28	Integrate rational functions using partial fractions in different cases إيجاد تكاملات دوال نسبية باستخدام طريقة الكسور الجزئية في حالات مختلفة.	Exercises (37-40)	P516
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7-4 Integration of Rational Functions Using Partial Fractions

37. In this exercise, we find the partial fractions decomposition of $\frac{4x^2 + 2}{(x^2 + 1)^2}$. The form for the decomposition is

$$\frac{4x^2 + 2}{(x^2 + 1)^2} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{(x^2 + 1)^2}$$

Multiplying through by $(x^2 + 1)^2$, we get

$$\begin{aligned} 4x^2 + 2 &= (Ax + B)(x^2 + 1) + Cx + D \\ &= Ax^3 + Bx^2 + Ax + B + Cx + D \end{aligned}$$

As in example 4.5, we match up coefficients of like powers of x . For x^3 , we have $0 = A$. For x^2 , we have $4 = B$. Match the coefficients of x and the constants to finish the decomposition.

28	Integrate rational functions using partial fractions in different cases إيجاد تكاملات دوال نسبية باستخدام طريقة الكسور الجزئية في حالات مختلفة.	Exercises (37-40)	P516
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7-4 Integration of Rational Functions Using Partial Fractions

37.
$$\frac{4x^2 + 2}{(x^2 + 1)^2} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{(x^2 + 1)^2}$$

$$\begin{aligned} 4x^2 + 2 &= (Ax + B)(x^2 + 1) + (Cx + D) \\ &= Ax^3 + Bx^2 + (A + C)x + (B + D) \\ A &= 0; B = 4; C = 0; D = -2 \end{aligned}$$
$$\frac{4x^2 + 2}{(x^2 + 1)^2} = \frac{4}{x^2 + 1} + \frac{-2}{(x^2 + 1)^2}$$

28	Integrate rational functions using partial fractions in different cases إيجاد تكاملات دوال نسبية باستخدام طريقة الكسور الجزئية في حالات مختلفة.	Exercises (37-40)	PS16
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7-4 Integration of Rational Functions Using Partial Fractions

38. $\frac{x^3 + 2}{(x^2 + 1)^2}$

38. $\frac{x^3 + 2}{(x^2 + 1)^2} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{(x^2 + 1)^2}$

Multiply both sides by $(x^2 + 1)^2$:

$$x^3 + 2 = (Ax + B)(x^2 + 1) + (Cx + D)$$

$$x^3 + 2 = Ax^3 + Bx^2 + Ax + B + Cx + D$$

$$x^3 + 2 = Ax^3 + Bx^2 + (A + C)x + (B + D)$$

Compare coefficients:

x^3 coefficients: $A = 1$

x^2 coefficients: $B = 0$

x coefficients: $A + C = 0 \Rightarrow 1 + C = 0 \Rightarrow C = -1$

Constants: $B + D = 2 \Rightarrow 0 + D = 2 \Rightarrow D = 2$

Therefore:

$$\frac{x^3 + 2}{(x^2 + 1)^2} = \frac{x}{x^2 + 1} + \frac{-x + 2}{(x^2 + 1)^2}$$

39. $\frac{4x^2 + 3}{(x^2 + x + 1)^2}$

39. $\frac{4x^2 + 3}{(x^2 + x + 1)^2} = \frac{Ax + B}{x^2 + x + 1} + \frac{Cx + D}{(x^2 + x + 1)^2}$

$$4x^2 + 3 = (Ax + B)(x^2 + x + 1) + Cx + D$$

$$= Ax^3 + Ax^2 + Ax + Bx^2 + Bx + B + Cx + D$$

$$A = 0$$

$$A + B = 4 : B = 4$$

$$A + B + C = 0 : C = -4$$

$$B + D = 3 : D = -1$$

$$\frac{4x^2 + 3}{(x^2 + x + 1)^2} = \frac{4}{x^2 + x + 1} - \frac{4x + 1}{(x^2 + x + 1)^2}$$

28	Integrate rational functions using partial fractions in different cases إيجاد تكاملات دوال نسبية باستخدام طريقة الكسور الجزئية في حالات مختلفة.	Exercises (37-40)	PS16
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7-4 Integration of Rational Functions Using Partial Fractions

40. $\frac{x^4 + x^3}{(x^2 + 4)^2}$

40. $\frac{x^4 + x^3}{(x^2 + 4)^2} = 1 + \frac{x^3 - 8x^2 - 8}{(x^2 + 4)^2}$

$$= 1 + \frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2 + 4)^2}$$

$$x^3 - 8x^2 - 8 = (Ax + B)(x^2 + 4) + Cx + D$$

$$= Ax^3 + Bx^2 + (4A + C)x + (4B + D)$$

$$A = 1 ; B = -8 ; C = -4 ; D = 24$$

$$\frac{x^4 + x^3}{(x^2 + 4)^2} = 1 + \frac{x - 8}{x^2 + 4} + \frac{-4x + 24}{(x^2 + 4)^2}$$

EOT
 29

29	Learn differential equations of the form $y'=ky$ and their general solution. التعرف على مفهوم المعادلة التفاضلية $y'=ky$ وحلها العام.	Exercises (1-6)	P533
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LESSON 7-6 Modeling with Differential Equations

1. $y' = 4y, y(0) = 2$

initial condition: $t=0$
 A) $y' = 4y \quad y(0) = 2 \rightarrow y=2$

$$\frac{y'}{y} = 4 \Rightarrow \int \frac{y'}{y} dt = \int 4 dt$$

$$\ln|y| = 4t + C$$

$$y = e^{4t+C} = Ae^{4t}$$

$$2 = Ae^{4(0)} = Ae^0 \Rightarrow A=2$$

$y = 2e^{4t}$

2. $y' = 3y, y(0) = -2$

B) $y' = 3y, y(0) = -2$

$$\frac{y'}{y} = 3 \Rightarrow \int \frac{y'}{y} dt = \int 3 dt$$

$$\ln|y| = 3t + C$$

$$e^{\ln|y|} = e^{3t+C} = e^{3t} \cdot e^C = Ae^{3t}$$

$y = Ae^{3t}$

29	Learn differential equations of the form $y'=ky$ and their general solution. التعرف على مفهوم المعادلة التفاضلية $y'=ky$ وحلها العام.	Exercises (1-6)	P533
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LESSON 7-6 Modeling with Differential Equations

3. $y' = -3y, y(0) = 5$

c) $y' = -3y, y(0) = 5$

$$\frac{y'}{y} = -3 \Rightarrow \int \frac{y'}{y} dt = \int -3 dt$$

$$\ln|y| = -3t + C$$

$$\Rightarrow y = e^{-3t+C}, A=e^C$$

$$5 = Ae^0 = A$$

$y = 5e^{-3t}$

4. $y' = -2y, y(0) = -6$

$y' = -2y, y(0) = -6$

$$y' = ky \Rightarrow y = Ce^{kt}$$

Substitute ($k=-2$)

$$y = Ce^{-2t}$$

Apply the initial condition

$$y(0) = Ce^0 = C = -6$$

Write the final solution

$$y = -6e^{-2t}$$

29	Learn differential equations of the form $y' = ky$ and their general solution. التعرف على مفهوم المعادلة التفاضلية $y' = ky$ وحلها العام.	Exercises (1-6)	P533
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7-6 Modeling with Differential Equations

5. $y' = 2y, y(1) = 2$

$$y' = 2y, y(1) = 2$$

$$y = Ce^{2t}$$

$$2 = Ce^2$$

$$C = 2e^{-2}$$

$$y = 2e^{-2}e^{2t}$$

$$y = 2e^{2(t-1)}$$

6. $y' = -y, y(1) = 2$

$$y' = -y, y(1) = 2$$

$$\frac{y'}{y} = -1 \Rightarrow \int \frac{y'}{y} dt = \int -1 dt \quad y = Ae^{-t}$$

$$= 2e^1 \cdot e^{-t}$$

$$\ln|y| = -t + c$$

$$\Rightarrow y = e^{-t+c} = Ae^{-t}, (e^c = A)$$

$$2 = Ae^{-1} \Rightarrow A = 2e$$

$$y = 2e^{1-t}$$

30	Find the general solution of separable differential equations of first order. إيجاد الحل العام للمعادلات تفاضلية من الدرجة الأولى قابلة للفصل.	Exercises (11-16)	P544
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7-7 Separable Differential Equations

11. $y' = \frac{2x}{y}e^{y-x}$

$$\int ye^{-y} dy = \int 2xe^{-x} dx$$

$$-ye^{-y} - e^{-y} = -2xe^{-x} - 2e^{-x} + C$$

$$e^{-y}(y+1) = e^{-x}(2x+2) + C$$

$$\frac{y+1}{e^y} = \frac{2x+2}{e^x} + C$$

12. $y' = \frac{\sqrt{1-y^2}}{x \ln x}$

$$y' = \frac{\sqrt{1-y^2}}{x \ln x}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{1}{x \ln x} dx$$

$$\sin^{-1}y = \ln|\ln|x|| + C$$

$$y = \sin(\ln|\ln|x|| + C)$$

EOT
30

30	Find the general solution of separable differential equations of first order. إيجاد الحل العام لمعادلات تفاضلية من الدرجة الأولى قابلة للفصل.	Exercises (11-16)	P544
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7-7 Separable Differential Equations

13. $y' = \frac{\cos x}{\sin y}$

$$y' = \frac{\cos x}{\sin y}$$

$$\int \sin y \, dy = \int \cos x \, dx$$

$$-\cos y = \sin x + C$$

$$\cos y + \sin x = K \quad (K = -C)$$

$$\cos y + \sin x = K$$

14. $y' = x \cos^2 y$

$$y' = x \cos^2 y$$

$$\int \frac{dy}{\cos^2 y} = \int x \, dx$$

$$\int \sec^2 y \, dy = \int x \, dx$$

$$\tan y = \frac{1}{2}x^2 + C$$

$$y = \tan^{-1}\left(\frac{1}{2}x^2 + C\right)$$

30	Find the general solution of separable differential equations of first order. إيجاد الحل العام لمعادلات تفاضلية من الدرجة الأولى قابلة للفصل.	Exercises (11-16)	P544
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7-7 Separable Differential Equations

15. $y' = \frac{xy}{1+x^2}$

$$y' = \frac{xy}{1+x^2}$$

$$\frac{dy}{y} = \frac{x}{1+x^2} dx$$

$$\int \frac{dy}{y} = \int \frac{x}{1+x^2} dx$$

$$\ln|y| = \frac{1}{2} \ln|1+x^2| + C$$

$$\ln|y| = \ln|(1+x^2)^{1/2}| + C$$

$$y = e^C \sqrt{1+x^2}$$

$$y = A\sqrt{1+x^2}$$

16. $y' = \frac{2}{xy+y}$

$$y' = \frac{2}{y(x+1)} \Rightarrow \frac{dy}{dx} = \frac{2}{y(x+1)}$$

$$y \, dy = \frac{2}{x+1} dx$$

$$\int y \, dy = \int \frac{2}{x+1} dx$$

$$\frac{1}{2}y^2 = 2 \ln|x+1| + C_1$$

$$y^2 = 4 \ln|x+1| + C$$