إجابات هيكل الرياضيات 2023- 2024

الصف الثاني عشر متقدم (الفصل الثانى)

أولاً :الالكترونى

إعداد :

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إيجاد الاعداد الحرجة لدالة معطاة

ص 258 من 3-6

3) $(a)\ f(x) = x^2 + 5x - 1$ $(b)\ f(x) = -x^2 + 4x + 2$

3. (a) $f(x) = x^2 + 5x - 1$
$f'(x) = 2x + 5$
$2x + 5 = 0$
$x = -5/2$ is a critical number.
This is a parabola opening upward, so we have a minimum at $x = -5/2$.

(b) $f(x) = -x^2 + 4x + 2$
$f'(x) = -2x + 4 = 0$ when $x = 2$.
This is a parabola opening downward, so we have a maximum at $x = 2$.

4) $(a)\ f(x) = x^3 - 3x + 1$ $(b)\ f(x) = -x^3 + 6x^2 + 2$

(a) $f(x) = x^3 - 3x + 1$
$f'(x) = 3x^2 - 3$
$= 3(x^2 - 1)$
$= 3(x + 1)(x - 1) = 0$
$x = \pm 1$ are critical numbers and $f(1) = -1$, $f(-1) = 3$.
This is a cubic with a positive leading coefficient so $x = -1$ is a local max, $x = 1$ is a local min.

(b) $f(x) = -x^3 + 6x^2 + 2$
$f'(x) = -3x^2 + 12x = -3x(x + 4) = 0$ when $x = 0$ and $x = -4$.
$f(0) = 2$, $f(-4) = 162$.
This is a cubic with a negative leading coefficient so $x = 0$ is a local min and $x = -4$ is a local max.

5) $(a)\ f(x) = x^3 - 3x^2 + 6x$ $\qquad$ $(b)\ f(x) = -x^3 + 3x^2 - 3x$

(a) $f(x) = x^3 - 3x^2 + 6x$

$f'(x) = 3x^2 - 6x + 6$

$3x^2 - 6x + 6 = 3(x^2 - 2x + 2) = 0$

We can use the quadratic formula to find the roots, which are $x = 1 \pm \sqrt{-1}$. These are imaginary so there are no real critical points.

(b) $f(x) = -x^3 + 3x^2 - 3x$

$$\begin{aligned} f'(x) &= -3x^2 + 6x - 3 \\ &= 3(-x^2 + 2x - 1) \\ &= -3(x^2 - 2x + 1) \\ &= -3(x-1)^2 \end{aligned}$$

$f'(x) = 3(x-1)^2 = 0$ when $x = 1$.

Since $f(x)$ is a cubic with only one critical number it is neither local min nor max.

6) $(a)\ f(x) = x^4 - 2x^3 + 6x$ $\qquad$ $(b)\ f(x) = x^4 - 3x^3 + 2$

(a) $f(x) = x^4 - 2x^2 + 1$

$$\begin{aligned} f'(x) &= 4x^3 - 4x \\ &= 4x(x^2 - 1) \\ &= 4x(x-1)(x+1) \end{aligned}$$

$f'(x) = 0$ when $x = 0, \pm 1$.

$x = 0, \pm 1$ are critical numbers. $x = 0$ is local maximum and $x = \pm 1$ are local minimum.

(b) $f(x) = x^4 - 3x^3 + 2$

$$\begin{aligned} f'(x) &= 4x^3 - 9x^2 \\ &= x^2(4x - 9) \end{aligned}$$

$f'(x) = 0$ when $x = 0, \frac{9}{4}$.

$x = 0, \frac{9}{4}$ are critical points. $x = \frac{9}{4}$ is local minimum and $x = 0$ is neither max nor min.

في التمارين 34-25، جـــد القيم القصوى المطلقة لدالة

ص 258

25. $f(x) = x^3 - 3x + 1$
$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$
$f'(x) = 0$ for $x = \pm 1$.

(a) On $[0, 2]$, 1 is the only critical number.
We calculate:
$f(0) = 1$
$f(1) = -1$ is the abs min.
$f(2) = 3$ is the abs max.

(b) On the interval $[-3, 2]$, we have both 1 and -1 as critical numbers.
We calculate:
$f(-3) = -17$ is the abs min.
$f(-1) = 3$ is the abs max.
$f(1) = -1$
$f(2) = 3$ is also the abs max.

27. $f(x) = x^{2/3}$
$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$
$f'(x) \neq 0$ for any x, but $f'(x)$ undefined for $x = 0$, so $x = 0$ is critical number.

(a) On $[-4, -2]$:
$0 \notin [-4, -2]$ so we only look at endpoints.
$f(-4) = \sqrt[3]{16} \approx 2.52$
$f(-2) = \sqrt[3]{4} \approx 1.59$
So $f(-4) = \sqrt[3]{16}$ is the abs max and $f(-2) = \sqrt[3]{4}$ is the abs min.

(b) On $[-1, 3]$, we have 0 as a critical number.
$f(-1) = 1$
$f(0) = 0$ is the abs min.
$f(3) = 3^{2/3}$ is the abs max.

28. $f(x) = \sin x + \cos x$
$f'(x) = \cos x - \sin x = 0$ when $x = \frac{\pi}{4} + k\pi$ for integers k.

(a) On $[0, 2\pi]$:
$f(0) = 1$, $f(\pi/4) = \sqrt{2}$, $f(5\pi/4) = -\sqrt{2}$, and $f(2\pi) = 1$.
The abs min on this interval is $f(5\pi/4) = -\sqrt{2}$ and the abs max is $f(\pi/4) = \sqrt{2}$.

(b) On $[\pi/2, \pi]$:
$f(\pi/2) = 1$, $f(\pi) = -1$.
The abs min on this interval is $f(\pi) = -1$ and the abs max is $f(\pi/2) = 1$.

29. $f(x) = e^{-x^2}$
$f'(x) = -2xe^{-x^2}$
Hence $x = 0$ is the only critical number.

(a) On $[0, 2]$:
$f(0) = 1$ is the abs max.
$f(2) = e^{-4}$ is the abs min.

(b) On $[-3, 2]$:
$f(-3) = e^{-9}$ is the abs min.
$f(0) = 1$ is the abs max.
$f(2) = e^{-4}$

30. $f(x) = x^2e^{-4x}$
$f'(x) = 2xe^{-4x} - 4x^2e^{-4x} = 0$ when $x = 0$ and $x = 1/2$.

(a) On $[-2, 0]$:
$f(-2) = 4e^8$, $f(0) = 0$.
The abs min is $f(0) = 0$ and the abs max is $f(-2) = 4e^8$.

(b) On $[0, 4]$:
$f(1/2) = e^{-2}/4$, $f(4) = 16e^{-16}$.
The abs min is $f(0) = 0$ and the abs max is $f(1/2) = e^{-2}/4$.

31. $f(x) = \dfrac{3x^2}{x-3}$
Note that $x = 3$ is not in the domain of f.

$$f'(x) = \frac{6x(x-3) - 3x^2(1)}{(x-3)^2} = \frac{6x^2 - 18x - 3x^2}{(x-3)^2} = \frac{3x^2 - 18x}{(x-3)^2} = \frac{3x(x-6)}{(x-3)^2}$$

The critical points are $x = 0$, $x = 6$.

(a) On $[-2, 2]$:
$f(-2) = -12/5$
$f(2) = -12$
$f(0) = 0$
Hence abs max is $f(0) = 0$ and abs m
is $f(2) = -12$.

(b) On $[2, 8]$, the function is not continuo
and in fact has no absolute max or mi

32. $f(x) = \tan^{-1}(x^2)$

$f'(x) = \dfrac{2x}{1+x^4} = 0$ when $x = 0$.

(a) On $[0, 1]$:
$f(0) = 0$ and $f(1) = \pi/4$.
The abs min is $f(0) = 0$ and the abs max is $f(1) = \pi/4$.

(b) On $[-3, 4]$:
$f(-3) \approx 1.46$, $f(0) = 0$, and $f(4) \approx 1.51$.
The abs min is $f(0) = 0$ and the abs max is $f(4) = \tan^{-1} 16$.

33. $f(x) = \dfrac{x}{x^2+1}$

$$f'(x) = \frac{(x^2+1)\cdot 1 - x\cdot(2x)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)\cdot 1 - x\cdot(2x)}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2} = 0$$

when $x = \pm 1$.
$x = \pm 1$ are critical numbers.

(a) On $[0,\ 2]$:
$f(0) = \dfrac{0}{0^2+1} = 0$ is the abs minimum.
$f(2) = \dfrac{2}{2^2+1} = \dfrac{2}{5}$
$f(1) = \dfrac{1}{2}$ is the abs maximum.

(b) On $[-3,\ 3]$:
$f(3) = -\dfrac{3}{10}$
$f(-1) = -\dfrac{1}{2}$ is the abs minimum.
$f(1) = \dfrac{1}{2}$ is the abs maximum.
$f(3) = \dfrac{3}{10}$

34. $f(x) = \dfrac{3x}{x^2+16}$

$$f'(x) = \frac{(x^2+16)\cdot 3 - 3x\cdot(2x)}{(x^2+16)^2}$$

$$= \frac{(x^2+16)\cdot 3 - 3x\cdot(2x)}{(x^2+16)^2} = 0$$

$$= \frac{-3x^2+48}{(x^2+16)^2} = 0 \text{ when } x = \pm 4.$$

$x = \pm 4$ are critical numbers.

(a) On $[0, 2]$:

$f(0) = \dfrac{0}{0^2+16} = 0$ is the abs minimum.

$f(2) = \dfrac{2}{2^2+1} = \dfrac{3}{10}$ is the abs maximum.

(b) on $[0, 6]$:

$f(0) = 0$ is abs minimum.

$f(4) = \dfrac{3}{8}$ is abs maximum.

$f(6) = \dfrac{9}{26}$

التعرف على مفهومي الدالة المتناقصة والدالة المتزايدة

ص 267 من 1 – 10

في التمارين 1-10، جـد (يدويًا) الفترات التي تكون فيها الدالة متزايدة والفترات التي تكون فيها متناقصة. استخدم هذه المعلومات في تحديد جميع القيم القصوى المحلية وارسم تمثيلًا بيانيًا.

1. $y = x^3 - 3x + 2$

2. $y = x^3 + 2x^2 + 1$

3. $y = x^4 - 8x^2 + 1$

4. $y = x^3 - 3x^2 - 9x + 1$

5. $y = (x+1)^{2/3}$

6. $y = (x-1)^{1/3}$

7. $y = \sin x + \cos x$

8. $y = \sin^2 x$

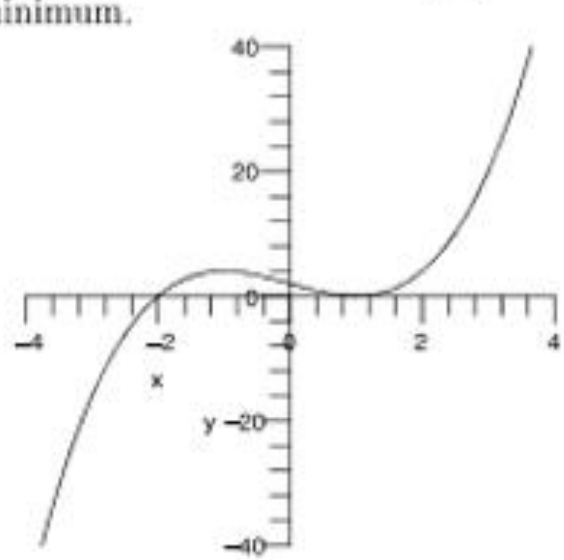
9. $y = e^{x^2-1}$

10. $y = \ln(x^2 - 1)$

3.4 Increasing and Decreasing Functions

1. $y = x^3 - 3x + 2$
   $y' = 3x^2 - 3 = 3(x^2 - 1)$
   $= 3(x + 1)(x - 1)$
   $x = \pm 1$ are critical numbers.
   $(x + 1) > 0$ on $(-1, \infty)$, $(x + 1) < 0$ on $(-\infty, -1)$
   $(x - 1) > 0$ on $(1, \infty)$, $(x - 1) < 0$ on $(-\infty, -1)$
   $3(x + 1)(x - 1) > 0$ on $(1, \infty) \cup (-\infty, -1)$ so

   y is increasing on $(1, \infty)$ and on $(-\infty, -1)$
   $3(x + 1)(x - 1) < 0$ on $(-1, 1)$, so y is decreasing on $(-1, 1)$.
   $y'' = 6x$
   $y'' = -6 < 0$ at $x = -1$
   Hence the function is a local maximum at $x = -1$.
   $y'' = 6 > 0$ at $x = 1$. Hence $y(1) = 0$ is a local minimum.



2. $y = x^3 + 2x^2 + 1$

$y' = 3x^2 + 4x = x(3x + 4)$

The function is increasing when $x < -\frac{4}{3}$, decreasing when $-\frac{4}{3} < x < 0$, and increasing when $x > 0$.

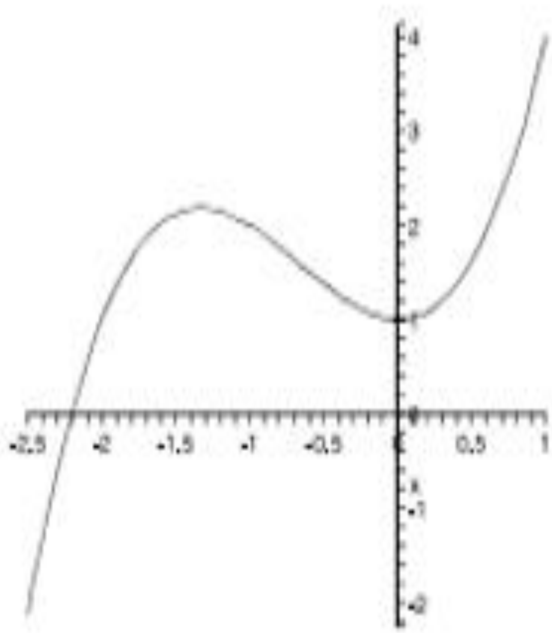
$y'' = 6x + 4$

$y'' = -12 < 0$ at $x = -\frac{4}{3}$

Hence $f(-\frac{4}{3})$ is a local maximum at $x = -\frac{4}{3}$.

$y'' = 4 > 0$ at $x = 0$

Hence $y(0)$ is a local minimum at $x = 0$.



3. $y = x^4 - 8x^2 + 1$

$y' = 4x^3 - 16x = 4x(x^2 - 4)$

$\quad = 4x(x-2)(x+2)$

$x = 0, 2, -2$ are critical numbers.

$4x > 0$ on $(0, \infty)$, $4x < 0$ on $(-\infty, 0)$

$(x-2) > 0$ on $(2, \infty)$, $(x-2) < 0$ on $(-\infty, 2)$

$(x+2) > 0$ on $(-2, \infty)$, $(x+2) < 0$ on $(-\infty, -2)$

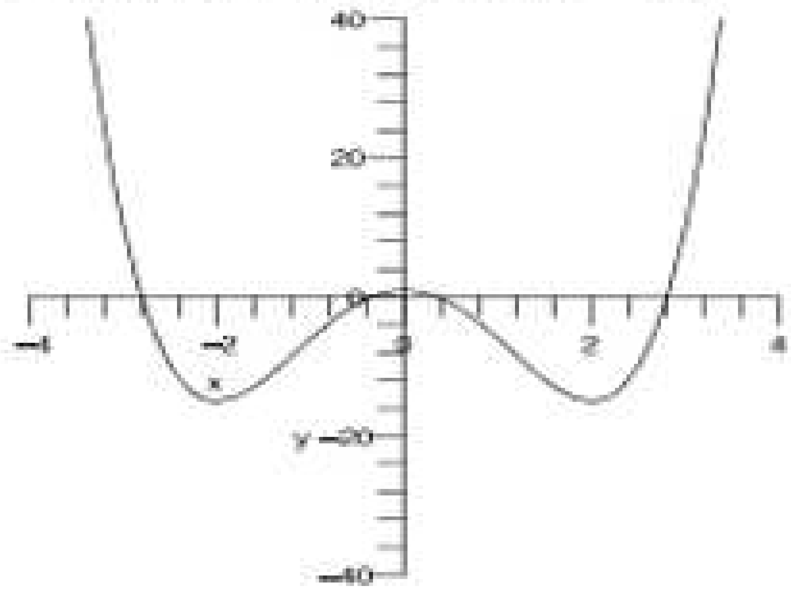
$4(x-2)(x+2) > 0$ on $(-2, 0) \cup (2, \infty)$, so the function is increasing on $(-2, 0)$ and on $(2, \infty)$.

$4(x-2)(x+2) < 0$ on $(-\infty, -2) \cup (0, 2)$, so y is decreasing on $(-\infty, -2)$ and on $(0, 2)$.

$y'' = 12x^2 - 16$

At $x = 0$, $y'' < 0$. Hence $y(0)$ is a local maximum at $x = 0$.

$y'' = 12(\pm 2)^2 - 16 > 0$ at $x = \pm 2$. Hence $y(\pm 2)$ are local minima at $x = \pm 2$.



4. $y = x^3 - 3x^2 - 9x + 1$

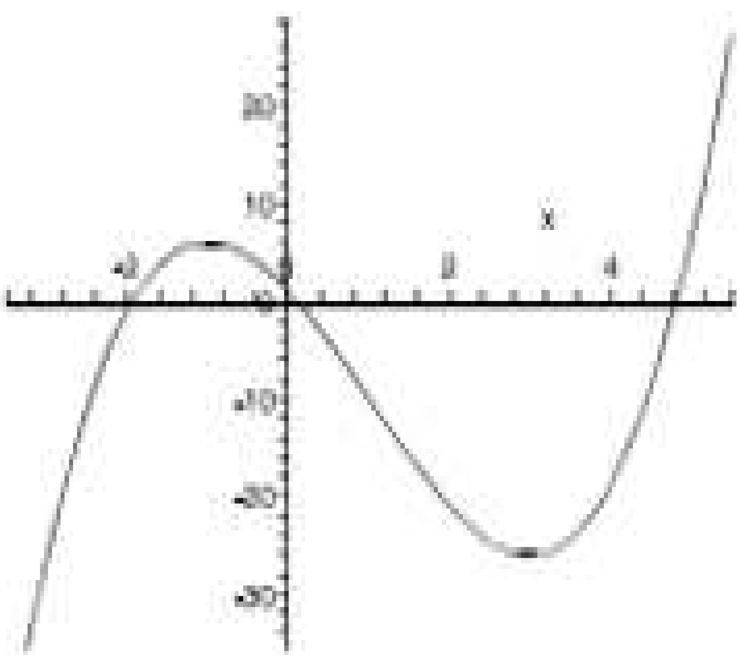
$y' = 3x^2 - 6x - 9 = 3(x-3)(x+1)$.

The function is increasing when $x < -1$, decreasing when $-1 < x < 3$, and increasing when $x > 3$.

$y'' = 6x - 6$

$y'' = -12 < 0$ at $x = -1$. Hence the function is a local maximum at $x = -1$.

$y'' = 12 > 0$ at $x = 3$. Hence the function is a local minimum at $x = 3$.



5. $y = (x+1)^{2/3}$

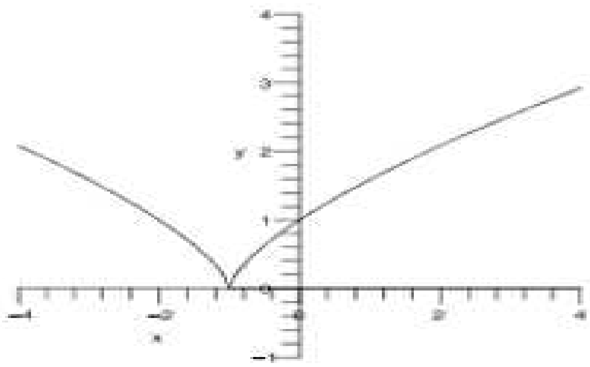
$y' = \frac{2}{3}(x+1)^{-1/3} = \frac{2}{3\sqrt[3]{x+1}}$

y' is not defined for $x = -1$

$\frac{2}{3\sqrt[3]{x+1}} > 0$ on $(-1, \infty)$, y is increasing

$\frac{2}{3\sqrt[3]{x+1}} < 0$ on $(-\infty, -1)$, y is decreasing

The graph has minimum at $x = -1$.

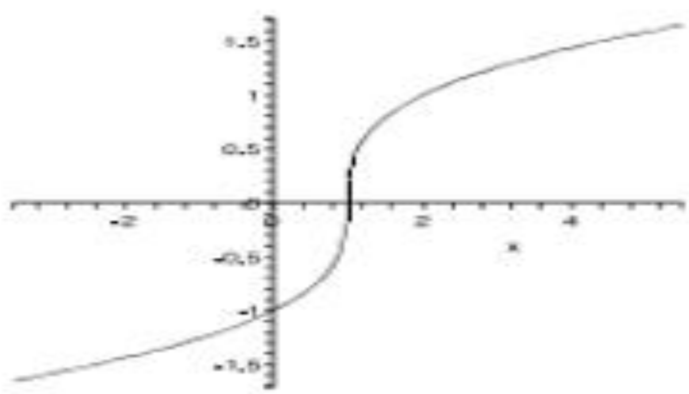


6. $y = (x-1)^{1/3}$

$y' = \frac{1}{3}(x-1)^{-2/3}$.

The function is increasing for all x. The slope approaches vertical as x approaches 1.

The graph has no extrema.



7. $y = \sin x + \cos x$

$y' = \cos x - \sin x = 0$

$\cos x = \sin x$

$x = \pi/4,\ 5\pi/4,\ 9\pi/4$, etc. $\cos x - \sin x > 0$ on $(-3\pi/4, \pi/4) \cup (5\pi/4, 9\pi/4) \cup \ldots$

$\cos x - \sin x < 0$ on $(\pi/4, 5\pi/4) \cup (9\pi/4, 13\pi/4) \cup \ldots$

So $y = \sin x + \cos x$ is decreasing on $(\pi/4, 5\pi/4)\,,(9\pi/4, 13\pi/4)$, etc., and is increasing on $(-3\pi/4, \pi/4)\,,(5\pi/4, 9\pi/4)$, etc.

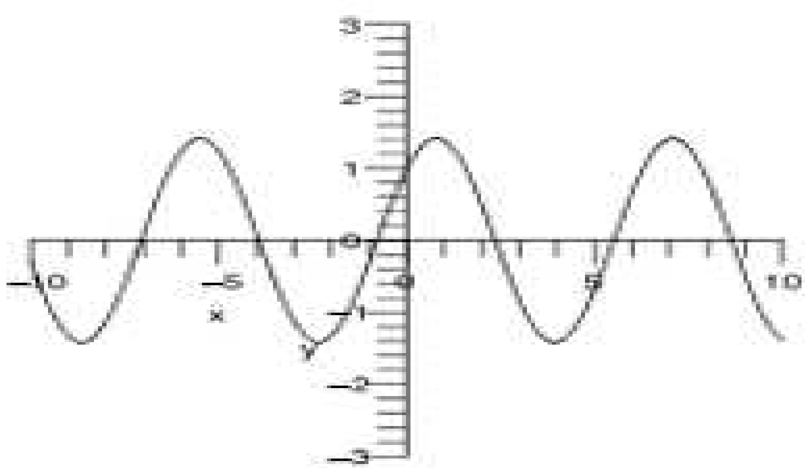
$y'' = -\sin x - \cos x$

$y'' = -\dfrac{2}{\sqrt{2}} < 0$ at $x = \pi/4,\ x = 9\pi/4$, etc.

Hence the function is local maximum at $x = \pi/4,\ x = 9\pi/4$, etc.

$y'' = \sqrt{2} > 0$ at $x = 5\pi/4,\ x = 13\pi/4$ etc.

Hence the function is local minimum at $x = 5\pi/4,\ x = 13\pi/4$ etc.



8. $y = \sin^2 x$

$y' = 2\sin x \cos x.$

The function is increasing for $0 < x < \frac{\pi}{2}$, and decreasing for $\frac{\pi}{2} < x < \pi$, and this pattern repeats with period π.

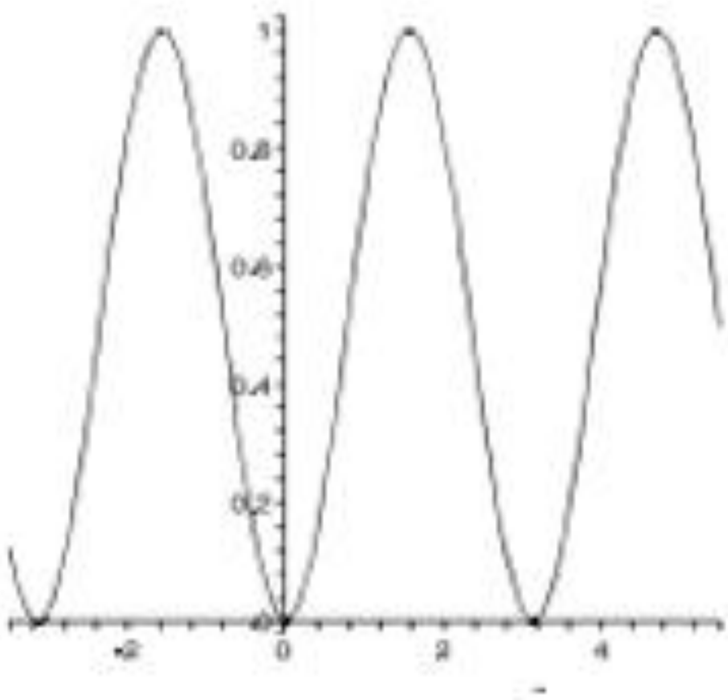
$y'' = 2\cos 2x$

$y'' = -2 < 0$ at $x = \pi/2$, $x = 3\pi/2$, etc.

Hence the function is local maximum at $x = \pi/2$, $x = 3\pi/2$, etc.

$y'' = \sqrt{2} > 0$ at $x = 0$, $x = \pi$, etc.

Hence the function is a local minimum $x = 0$, $x = \pi$, etc.



9. $y = e^{x^2-1}$

$y' = e^{x^2-1} \cdot 2x = 2xe^{x^2-1}$

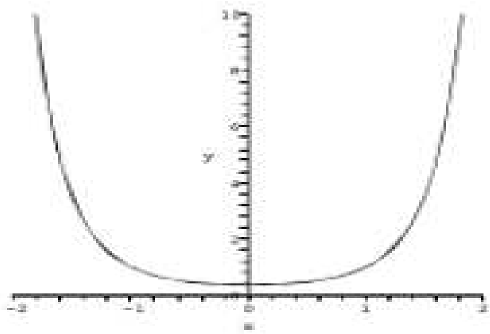
$x = 0$

$2xe^{x^2-1} > 0$ on $(0, \infty)$, y is increasing

$2xe^{x^2-1} < 0$ on $(-\infty, 0)$, y is decreasing

$y'' = 2e^{x^2-1}\left[2x^2+1\right]$

$y'' = 0.736 > 0$ at $x = 0$. Hence the function is a local minimum at $x = 0$.

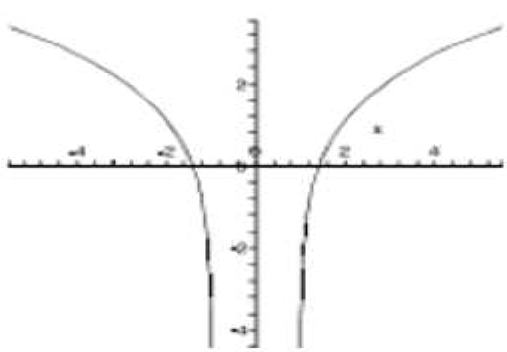


10. $y = \ln(x^2 - 1)$

$y' = \dfrac{2x}{x^2-1}$.

The function is defined for $|x| > 1$. The function is decreasing for $x < -1$ and increasing for $x > 1$.

The graph has no extrema.



إيجاد القيم القصوى المحلية لدالة معينة

باستخدام اختبار المشتقة الأولى

ص 267 من 11 – 20

في التمارين 11-20، جـد (يدويًا) جميع الأعداد الحرجة واستخدم اختبار المشتقة الأولى لتصنيف كل واحدة على أنها قيمة عظمى محلية أو قيمة صغرى محلية أو غير ذلك.

11. $y = x^4 + 4x^3 - 2$

12. $y = x^5 - 5x^2 + 1$

13. $y = xe^{-2x}$

14. $y = x^2e^{-x}$

15. $y = \tan^{-1}(x^2)$

16. $y = \sin^{-1}\left(1 - \frac{1}{x^2}\right)$

17. $y = \frac{x}{1+x^3}$

18. $y = \frac{x}{1+x^4}$

19. $y = \sqrt{x^3 + 3x^2}$

20. $y = x^{4/3} + 4x^{1/3}$

11. $y = x^4 + 4x^3 - 2$

$y' = 4x^3 + 12x^2 = 4x^2(x+3)$

Critical numbers are $x = 0$, $x = -3$.

$4x^2(x+3) > 0$ on $(-3, 0) \cup (0, \infty)$

$4x^2(x+3) < 0$ on $(-\infty, -3)$

Hence $x = -3$ is a local minimum and $x = 0$ is not an extremum.

12. $y = x^5 - 5x^2 + 1$

$y' = 5x^4 - 10x = 5x(x^3 - 2)$.

At $x = 0$ the slope changes from positive to negative indicating a local maximum. At $x = \sqrt[3]{2}$ the slope changes from negative to positive indicating a local minimum.

13. $y = xe^{-2x}$

$$\begin{aligned} y' &= 1 \cdot e^{-2x} + x \cdot e^{-2x}(-2) \\ &= e^{-2x} - 2xe^{-2x} \\ &= e^{-2x}(1 - 2x) \end{aligned}$$

$x = \frac{1}{2}$

$e^{-2x}(1 - 2x) > 0$ on $(-\infty, 1/2)$

$e^{-2x}(1 - 2x) < 0$ on $(1/2, \infty)$

So $y = xe^{-2x}$ has a local maximum at $x = 1/2$.

14. $y = x^2 e^{-x}$
$y' = 2xe^{-x} - x^2 e^{-x} = xe^{-x}(2 - x)$.
At $x = 0$ the slope changes from negative to positive indicating a local minimum. At $x = 2$ the slope changes from positive to negative indicating a local maximum.

15. $y = \tan^{-1}(x^2)$
$y' = \dfrac{2x}{1 + x^4}$
Critical number is $x = 0$.
$\dfrac{2x}{1 + x^4} > 0$ for $x > 0$
$\dfrac{2x}{1 + x^4} < 0$ for $x < 0$. Hence $x = 0$ is a local minimum.

16. $y = \sin^{-1}\left(1 - \frac{1}{x^2}\right)$
$y' = \dfrac{2}{x^3} \cdot \dfrac{1}{\sqrt{1 - (1 - \frac{1}{x^2})^2}}$.
The derivative is never 0 and is defined where the function is defined, so there are no critical points.

17. $y = \dfrac{x}{1+x^3}$ Note that the function is not defined for $x = -1$.

$$y' = \frac{1(1+x^3) - x(3x^2)}{(1+x^3)}$$

$$= \frac{1+x^3-3x^3}{(1+x^3)^2}$$

$$= \frac{1-2x^3}{(1+x^3)^2}$$

Critical number is $x = \sqrt[3]{1/2}$

$y' > 0$ on $(-\infty,-1) \cup (-1, -\sqrt[3]{1/2})$

$y' < 0$ on $(\sqrt[3]{1/2}, \infty)$

Hence $x = \sqrt[3]{1/2}$ is a local max.

18. $y = \dfrac{x}{1+x^4}$

$$y' = \frac{(1+x^4) - 4x^4}{(1+x^4)^2} = \frac{1-3x^4}{(1+x^4)^2}.$$

At $x = -\sqrt[4]{1/3}$ the slope changes from negative to positive indicating a local minimum. At $x = \sqrt[4]{1/3}$ the slope changes from positive to negative incicating a local maximum.

19. $y = \sqrt{x^3 + 3x^2} = (x^3 + 3x^2)^{1/2}$

Domain is all $x \geq -3$.

$$y' = \frac{1}{2}(x^3 + 3x^2)^{-1/2}(3x^2 + 6x)$$

$$= \frac{3x^2 + 6x}{2\sqrt{x^3 + 3x^2}}$$

$$= \frac{3x(x+2)}{2\sqrt{x^3 + 3x^2}}$$

$x = 0, -2, -3$ are critical numbers.

y' undefined at $x = 0, -3$

$y' > 0$ on $(-3, -2) \cup (0, \infty)$

$y' < 0$ on $(-2, 0)$

So $y = \sqrt{x^3 + 3x^2}$ has local max at $x = -2$, local min at $x = 0, -3$.

20. $y = x^{4/3} + 4x^{1/3}$

$$y' = \frac{4}{3}x^{1/3} + \frac{4}{3x^{2/3}} = \frac{4}{3} \cdot \frac{x+1}{x^{2/3}}.$$

At $x = -1$ the slope changes from negative to positive indicating a local minimum. At $x = 0$ the slope is vertical and is positive on positive side and negative on negative side, so this is neither a minimum nor a maximum.

التعريف على مفهوم نقطة الانعطاف وإيجادها

ص 276

في التمارين 8–1، حدد الفترات التي يكون فيها التمثيل البياني لدالة معطاة مقعّرًا إلى الأعلى والفترات التي يكون فيها مقعّرًا إلى الأسفل، وحدد نقاط الانعطاف.

1. $f(x) = x^3 - 3x^2 + 4x - 1$
2. $f(x) = x^4 - 6x^2 + 2x + 3$
3. $f(x) = x + 1/x$
4. $f(x) = x + 3(1 - x)^{1/3}$
5. $f(x) = \sin x - \cos x$
6. $f(x) = \tan^{-1}(x^2)$
7. $f(x) = x^{4/3} + 4x^{1/3}$
8. $f(x) = xe^{-4x}$

3.5 Concavity and the Second Derivative Test

1. $f'(x) = 3x^2 - 6x + 4$
$f''(x) = 6x - 6 = 6(x - 1)$
$f''(x) > 0$ on $(1, \infty)$
$f''(x) < 0$ on $(-\infty, 1)$
So f is concave down on $(-\infty, 1)$ and concave up on $(1, \infty)$.
$x = 1$ is a point of inflection.

2. $f'(x) = 4x^3 - 12x + 2$ and $f''(x) = 12x^2 - 12$.
The graph is concave up where $f''(x)$ is positive, and concave down where $f''(x)$ is negative. Concave up for $x < -1$ and $x > 1$, and concave down for $-1 < x < 1$.
$x = -1, 1$ are points of inflection.

3. $f(x) = x + \frac{1}{x} = x + x^{-1}$
$f'(x) = 1 - x^{-2}$
$f''(x) = 2x^{-3}$
$f''(x) > 0$ on $(0, \infty)$
$f''(x) < 0$ on $(-\infty, 0)$
So f is concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$.
$x = 0$ is a point of inflection.

4. $y' = 1 - (1 - x)^{-2/3}$ and $y'' = \frac{-2}{3}(1 - x)^{-5/3}$.
Concave up for $x > 1$ and concave down for $x < 1$.
$x = 1$ is a point of inflection.

5. $f'(x) = \cos x + \sin x$

$f''(x) = -\sin x + \cos x$

$f''(x) < 0$ on $\ldots \left(\frac{\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{9\pi}{4}, \frac{13\pi}{4}\right) \ldots$

$f''(x) > 0$ on $\ldots \left(\frac{3\pi}{4}, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{9\pi}{4}\right) \ldots$

f is concave down on $\ldots \left(\frac{\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{9\pi}{4}, \frac{13\pi}{4}\right) \ldots$,
concave up on $\ldots \left(\frac{3\pi}{4}, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{9\pi}{4}\right) \ldots$

$x = k\pi + \frac{\pi}{4}$ are the points of inflection for any interger k.

6. $f'(x) = \dfrac{2x}{1+x^4}$ and $f''(x) = \dfrac{2-6x^4}{(1+x^4)^2}$.

Concave up for $-\sqrt[4]{\dfrac{1}{3}} < x < \sqrt[4]{\dfrac{1}{3}}$, and concave down for $x < -\sqrt[4]{\dfrac{1}{3}}$ and $x > \sqrt[4]{\dfrac{1}{3}}$.

$x = -\sqrt[4]{\dfrac{1}{3}}, \sqrt[4]{\dfrac{1}{3}}$ are the points of inflection.

7. $f'(x) = \dfrac{4}{3}x^{1/3} + \dfrac{4}{3}x^{-2/3}$

$$f''(x) = \frac{4}{9}x^{-2/3} + \frac{8}{9}x^{-5/3} = \frac{4}{9x^{2/3}}\left(1 - \frac{2}{x}\right)$$

The quantity $\dfrac{4}{9x^{2/3}}$ is never negative, so the sign of the second derivative is the same as the sign of $1 - \dfrac{2}{x}$. Hence the function is concave up for $x > 2$ and $x < 0$, and is concave down for $0 < x < 2$.

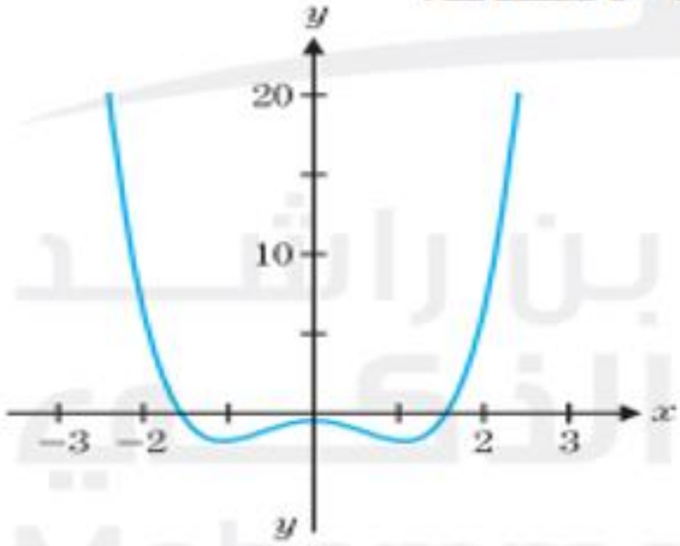
$x = 0\,2$ are the points of inflection.

تحديد فترات التقعر إلى أعلى وإلى أسفل لدالة معينة

باستخدام المشتقتين الأولى والثانية ص 276 سؤال 45 و 46

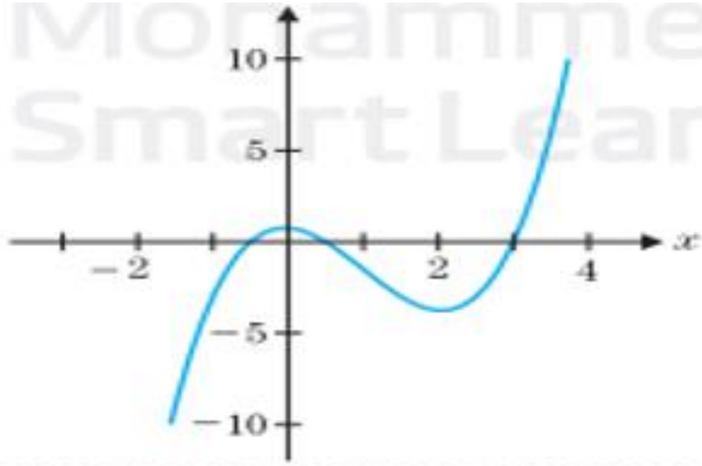
في التمرينين 45 و46، قدّر الفترات المتزايدة والمتناقصة ومواقع القيم القصوى المحلية، وفترات التقعر، ومواقع نقاط الانعطاف.

.45



45. $f(x)$ is concave up on $(-\infty, -0.5)$ and $(0.5, \infty)$; $f(x)$ is concave down on $(-0.5, 0.5)$. $f(x)$ is decreasing on the intervals $(-\infty, 1)$ and $(0, 1)$; increasing on the intervals $(-1, 0)$ and $(1, \infty)$. $f(x)$ has local maxima at 0 and minima at -1 and 1. Inflection points of $f(x)$ are -0.5 and 0.5.

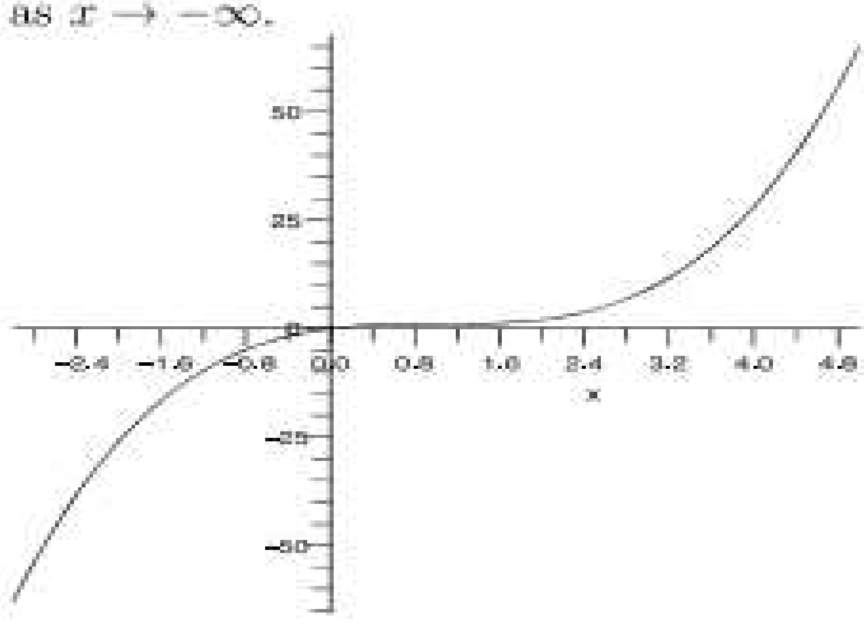
.46



46. $f(x)$ is concave up on $(1, \infty)$; $f(x)$ is concave down on $(-\infty, 1)$. $f(x)$ is increasing on the intervals $(-\infty, 0)$ and $(2, \infty)$; decreasing on the intervals $(0, 2)$. Inflection point of $f(x)$ is 1.

3.6 Overview of Curve Sketching

1. $f(x) = x^3 - 3x^2 + 3x$
$= x(x^2 - 3x + 3)$
The only x-intercept is $x = 0$; the y-intercept is $(0, 0)$.
$f'(x) = 3x^2 - 6x + 3$
$= 3(x^2 - 2x + 1) = 3(x-1)^2$
$f'(x) > 0$ for all x, so $f(x)$ is increasing for all x and has no local extrema.
$f''(x) = 6x - 6 = 6(x-1)$
There is an inflection point at $x = 1$; $f(x)$ is concave down on $(-\infty, 1)$ and concave up on $(1, \infty)$.
Finally, $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$.



2. $f(x) = x^4 - 3x^2 + 2x$
$= x(x^3 - 3x + 2)$

The x-intercepts are $x = -2$, $x = 1$ and $x = 0$; the y-intercept is $(0, 0)$.

$f'(x) = 4x^3 - 6x + 2$
$= 2(2x^3 - 3x + 1)$

The critical numbers are $x = -1.366$, 0.366 and 1.

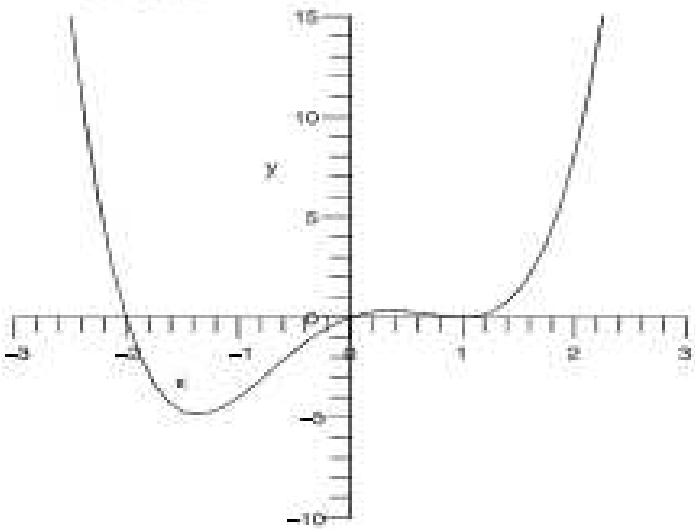
$f'(x) > 0$ on $(-1.366, 0.366)$ and $(1, \infty)$, so $f(x)$ is increasing on these intervals. $f'(x) < 0$ on $(-\infty, -1.366)$ and $(0.366, 1)$, so $f(x)$ is decreasing on these intervals. Thus $f(x)$ has local minima at $x = -1.366$ and $x = 1$ and a local maximum at $x = 0.366$.

$f''(x) = 12x^2 - 6 = 6(2x^2 - 1)$

The critical numbers here are $x = \pm 1/\sqrt{2}$.

$f''(x) > 0$ on $(-\infty, -1/\sqrt{2})$ and $(1/\sqrt{2}, \infty)$ so $f(x)$ is concave up on these intervals. $f''(x) < 0$ on $(-1/\sqrt{2}, 1/\sqrt{2})$ so $f(x)$ is concave down on this interval. Thus $f(x)$ has inflection points at $x = \pm 1/\sqrt{2}$.

Finally, $f(x) \to \infty$ as $x \to \pm\infty$.



3. $f(x) = x^5 - 2x^3 + 1$

The x-intercepts are $x = 1$ and $x \approx -1.5129$; the y-intercept is $(0, 1)$.

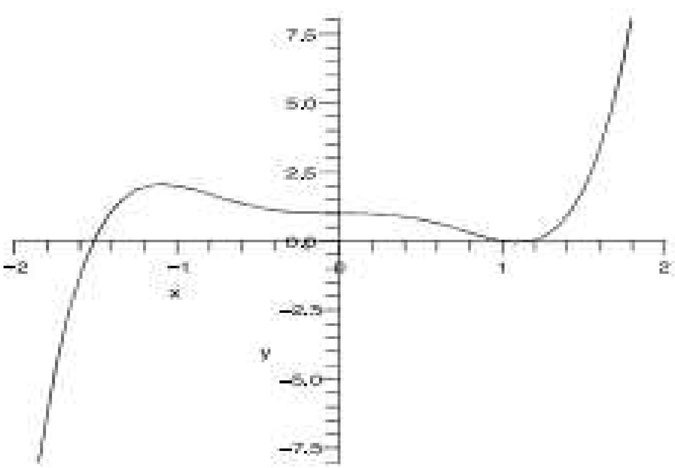
$f'(x) = 5x^4 - 6x^2 = x^2(5x^2 - 6)$

The critical numbers are $x = 0$ and $x = \pm\sqrt{6/5}$. Plugging values from each of the intervals into $f'(x)$, we find that $f'(x) > 0$ on $(-\infty, -\sqrt{6/5})$ and $(\sqrt{6/5}, \infty)$ so $f(x)$ is increasing on these intervals. $f'(x) < 0$ on $(-\sqrt{6/5}, 0)$ and $(0, \sqrt{6/5})$ so $f(x)$ is decreasing on these intervals. Thus $f(x)$ has a local maximum at $-\sqrt{6/5}$ and a local minimum at $\sqrt{6/5}$.

$f''(x) = 20x^3 - 12x = 4x(5x^2 - 3)$

The critical numbers are $x = 0$ and $x = \pm\sqrt{3/5}$. Plugging values from each of the intervals into $f''(x)$, we find that $f''(x) > 0$ on $(-\sqrt{3/5}, 0)$ and $(\sqrt{3/5}, \infty)$ so $f(x)$ is concave up on these intervals. $f''(x) < 0$ on $(-\infty, -\sqrt{3/5})$ and $(0, \sqrt{3/5})$ so $f(x)$ is concave down on these intervals. Thus $f(x)$ has inflection points at all three of these critical numbers.

Finally, $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$.



4. $f(x) = x^4 + 4x^3 - 1$

The x-intercepts are $x \approx -4.01541$ and $x \approx 0.6012$; the y-intercept is $(0, -1)$.

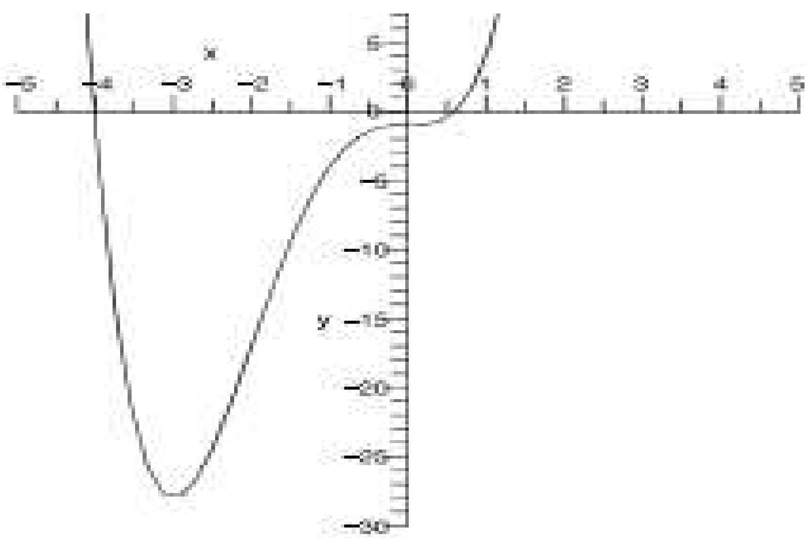
$f'(x) = 4x^3 + 12x^2 = 4x^2(x + 3)$

The critical numbers are $x = 0$ and $x = -3$. Plugging values from each of the intervals into $f'(x)$, we find that $f'(x) > 0$ on $(-3, 0)$ and $(0, \infty)$ so $f(x)$ is increasing on these intervals. $f'(x) < 0$ on $(-\infty, -3)$ so $f(x)$ is decreasing on these intervals. Thus $f(x)$ has a local minimum at -3.

$f''(x) = 12x^2 + 24x = 12x(x + 2)$

The critical numbers are $x = 0$ and $x = -2$. Plugging values from each of the intervals into $f''(x)$, we find that $f''(x) > 0$ on $(-\infty, -2)$ and $(0, \infty)$ so $f(x)$ is concave up on $(-\infty, -2)$ and $(0, \infty)$. $f''(x) < 0$ on $(-2, 0)$ so $f(x)$ is concave down on $(-2, 0)$. The graph has inflection points at -2 and 0.

Finally, $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to \infty$ as $x \to -\infty$.



5. $f(x) = x + \dfrac{4}{x} = \dfrac{x^2+4}{x}$

This function has no x- or y-intercepts. The domain is $\{x | x \neq 0\}$.

$f(x)$ has a vertical asymptote at $x = 0$ such that $f(x) \to -\infty$ as $x \to 0^-$ and $f(x) \to \infty$ as $x \to 0^+$.

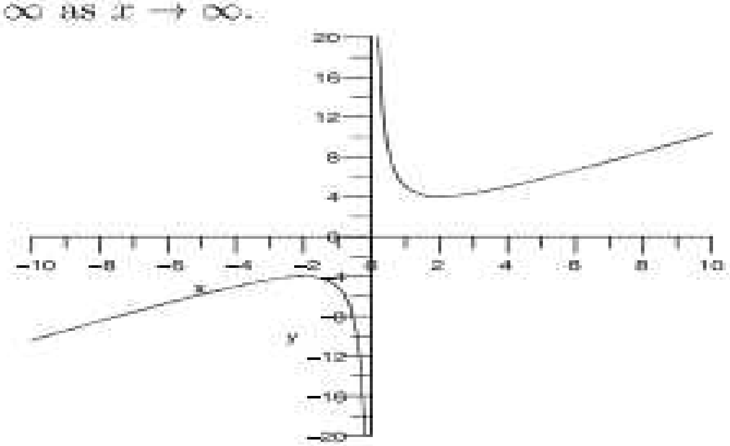
$f'(x) = 1 - 4x^{-2} = \dfrac{x^2-4}{x^2}$

The critical numbers are $x = \pm 2$. We find that $f'(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$ so $f(x)$ is increasing on these intervals. $f'(x) < 0$ on $(-2, 0)$ and $(0, 2)$, so $f(x)$ is decreasing on these intervals. Thus $f(x)$ has a local maximum at $x = -2$ and a local minimum at $x = 2$.

$f''(x) = 8x^{-3}$

$f''(x) < 0$ on $(-\infty, 0)$ so $f(x)$ is concave down on this interval and $f''(x) > 0$ on $(0, \infty)$ so $f(x)$ is concave up on this interval, but $f(x)$ has an asymptote (not an inflection point) at $x = 0$.

Finally, $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.



6. $f(x) = \dfrac{x^2-1}{x} = x - \dfrac{1}{x}$

There are x-intercepts at $x = \pm 1$, but no y-intercepts. The domain is $\{x | x \neq 0\}$.

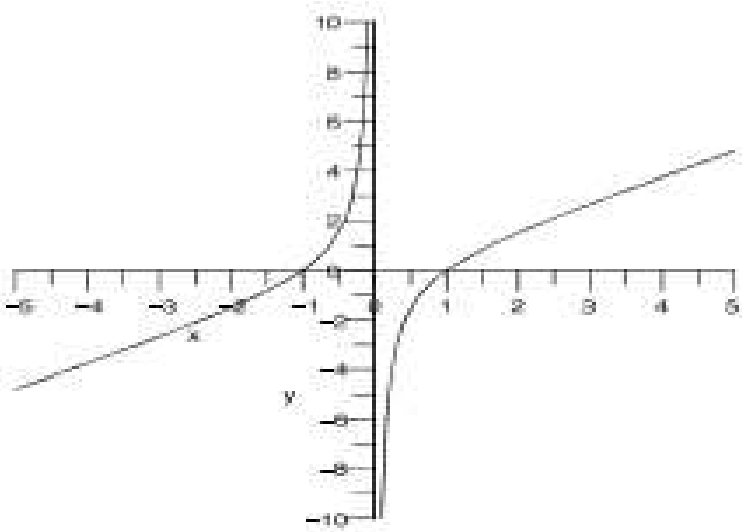
$f(x)$ has a vertical asymptote at $x = 0$ such that $f(x) \to \infty$ as $x \to 0^-$ and $f(x) \to -\infty$ as $x \to 0^+$.

$f'(x) = 1 + x^{-2} > 0$, So there is no critical numbers. $f(x)$ is increasing function.

$f''(x) = -2x^{-3}$

$f''(x) > 0$ on $(-\infty, 0)$ so $f(x)$ is concave up on this interval and $f''(x) < 0$ on $(0, \infty)$ so $f(x)$ is concave down on this interval, but $f(x)$ has an vertical asymptote (not an inflection point) at $x = 0$.

Finally, $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.



7. $f(x) = \dfrac{x^2+4}{x^3}$ has no x-intercept and no y-intercept. The domain of f includes all real numbers $x \neq 0$. $f(x)$ has a vertical asymptote at $x = 0$

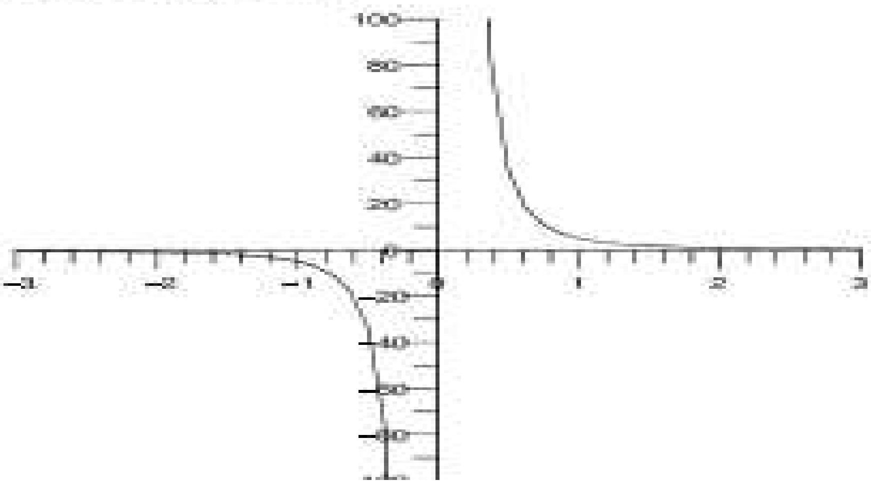
$$f'(x) = \frac{2x(x^3) - (x^2+4)(3x^2)}{(x^3)^2}$$
$$= \frac{-(x^2+12)}{x^4}$$

Since $f'(x) = 0$ has no real roots, the graph has no extrema. $f'(x) < 0$ on $(-\infty, 0)$ and $(0, \infty)$ so $f(x)$ is decreasing on these intervals. $f''(x) = -\left[\dfrac{x^4(2x) - (x^2+12)(4x^3)}{(x^4)^2}\right]$

$$= \frac{2[x^2+24]}{x^5}$$

$f''(x) < 0$ on $(-\infty, 0)$ so $f(x)$ is concave down on this interval and $f''(x) > 0$ on $(0, \infty)$ so $f(x)$ is concave up on this interval, but $f(x)$ has an asymptote (not an inflection point) at $x = 0$.

Finally, $f(x) \to 0$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to \infty$. Therefore, the graph has horizontal asymptot $y = 0$.



8. $f(x) = \dfrac{x-4}{x^3}$

The graph has x-intercepts at $x = 4$, but no y-intercepts. The domain of f includes all real numbers $x \neq 0$. $f(x)$ has a vertical asymptote at $x = 0$

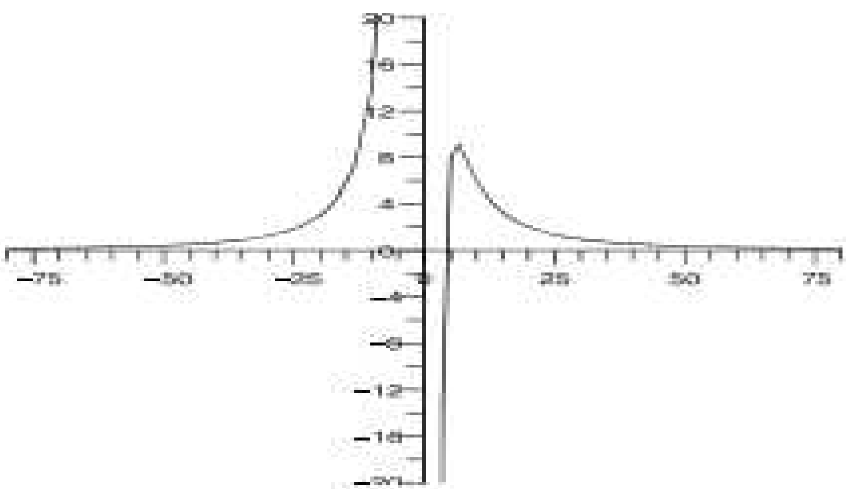
$$f'(x) = \frac{x^3 - (x-4)(3x^2)}{(x^3)^2}$$
$$= \frac{-2x+12}{x^4}$$

The critical numbers is $x = 6$. We find that $f'(x) > 0$ on $(-\infty, 0)$ and $(0,6)$ so $f(x)$ is increasing on these intervals. $f'(x) < 0$ on $(6, \infty)$, so $f(x)$ is decreasing on these intervals. Therefore, the graph has a local maximum at $x = 6$.

$$f''(x) = \frac{(x^4)(-2) - (-2x+12)(4x^3)}{(x^4)^2}$$
$$= \frac{6x-48}{x^5}$$

$f''(x) > 0$ on $(-\infty, 0)$ and $(8, \infty)$ so $f(x)$ is concave up on this interval and $f''(x) < 0$ on $(0, 8)$ so $f(x)$ is concave down on this interval, but $f(x)$ has an inflection point at $x = 8$.

Finally, $f(x) \to 0$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to \infty$. Therefore, the graph has horizontal asymptote $y = 0$.



9. $f(x) = \dfrac{2x}{x^2 - 1}$

The graph has x-intercept and y-intercept at $(0, 0)$. The domain of f includes all real numbers $x = \pm 1$. $f(x)$ has vertical asymptotes at $x = \pm 1$.

$$f'(x) = \frac{2\left(x^2 - 1\right) - (2x)(2x)}{\left(x^2 - 1\right)^2}$$

$$= \frac{-2\left(x^2 + 1\right)}{\left(x^2 - 1\right)^2}$$

Since $f'(x) = 0$ has no real roots, the graph has no extrema. $f'(x) < 0$ on $(-\infty, -1)$, $(-1, 0)$, $(0, 1)$ and $(1, \infty)$ so $f(x)$ is decreasing on these intervals.

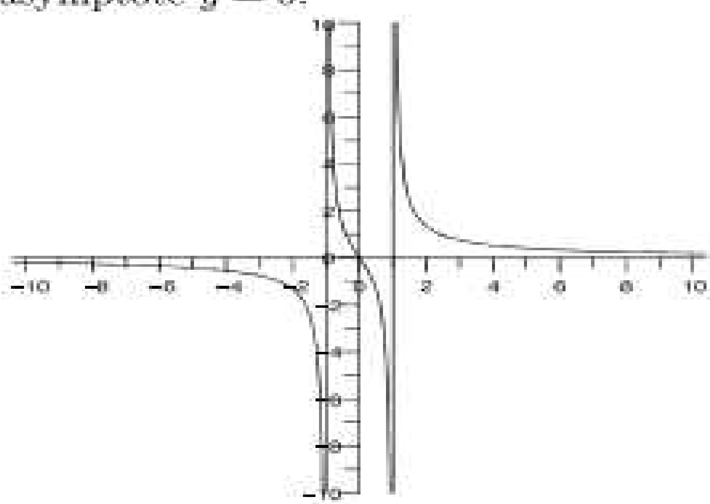
$$f''(x) = -2\left[\frac{2x\left(x^2 - 1\right)\left[x^2 - 1 - 2x^2 - 2\right]}{\left(x^2 - 1\right)^4}\right]$$

$$= \frac{4x\left[x^2 + 3\right]}{\left(x^2 - 1\right)^3}$$

$f''(x) > 0$ on $(-1, 0)$ and $(1, \infty)$ so $f(x)$ is concave up on this interval and $f''(x) < 0$ on

$(-\infty, -1)$ and $(0, 1)$ so $f(x)$ is concave down on this interval, but $f(x)$ has an inflection point at $x = 0$.

Finally, $f(x) \to 0$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to \infty$. Therefore, the graph has horizontal asymptote $y = 0$.



10. $f(x) = \dfrac{3x^2}{x^2+1}$

The graph has x-intercept and y-intercept at $(0, 0)$. The domain of f includes all real numbers.

$$f'(x) = \frac{(x^2+1)(6x) - (3x^2)(2x)}{(x^2+1)^2}$$

$$= \frac{6x}{(x^2+1)^2}$$

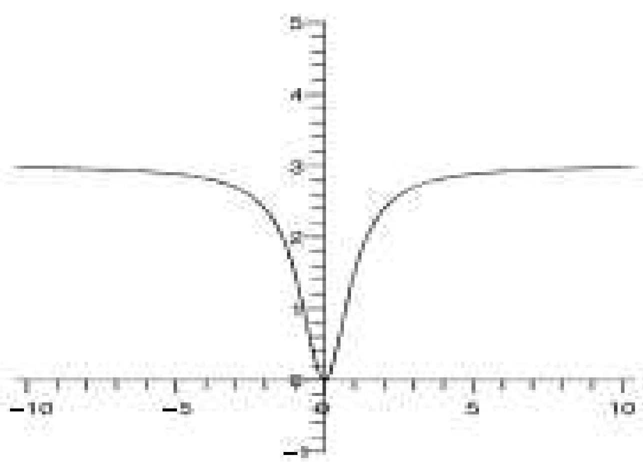
$f'(x) < 0$ on $(-\infty, 0)$ so $f(x)$ is decreasing on these intervals and $f'(x) > 0$ on $(0, \infty)$ so $f(x)$ is increasing on these interval.

$$f''(x) = \frac{(x^2+1)\left[6(x^2+1) - 24x^2\right]}{(x^2+1)^4}$$

$$= \frac{6 - 18x^2}{(x^2+1)^3}$$

The critical numbers are $x = \pm\sqrt{\frac{1}{3}}$. We find that $f''(x) > 0$ on $\left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right)$ so $f(x)$ is concave up on this interval and we find that $f''(x) < 0$ on $\left(-\infty, -\sqrt{\frac{1}{3}}\right)$ and $\left(\sqrt{\frac{1}{3}}, \infty\right)$ so $f(x)$ is concave down on this interval, but the graph has inflection points at $x = \pm\sqrt{\frac{1}{3}}$.

Finally, $f(x) \to 3$ as $x \to -\infty$ and $f(x) \to 3$ as $x \to \infty$. Therefore, the graph has horizontal asymptote at $y = 3$.



11. $f(x) = (x + \sin x)$

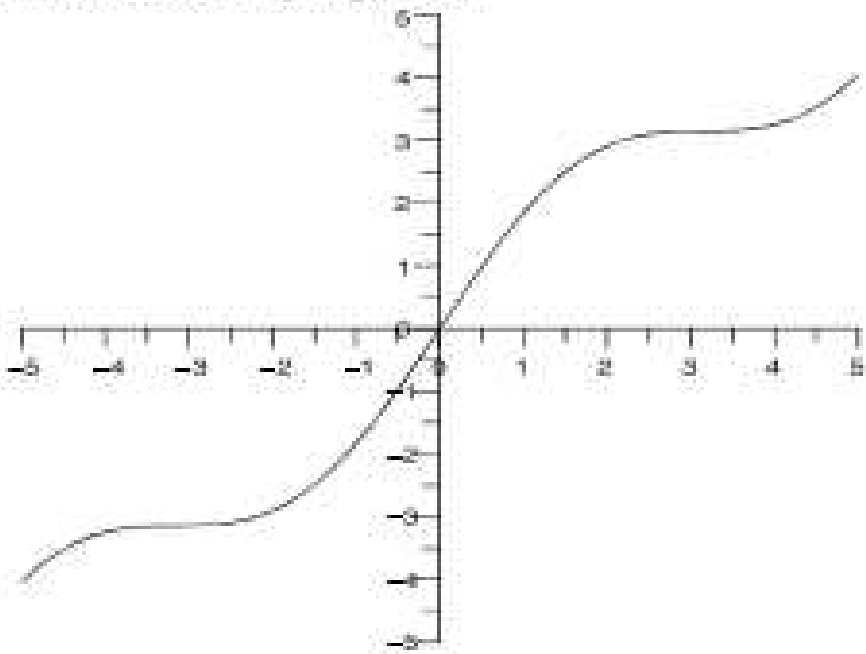
The graph has x-intercepts and y-intercepts at $(0, 0)$. The domain of f includes all real numbers.

$f'(x) = 1 + \cos x \geq 0$,therefore the graph has no extrema and $f(x)$ is a increasing function.

$f''(x) = -\sin x$

$f''(x) < 0$ on $(2n\pi, (2n+1)\pi)$ so $f(x)$ is concave down on this interval and we find that $f''(x) > 0$ on $((2n+1)\pi, 2(n+1)\pi)$ so $f(x)$ is concave up on this interval, but the graph has inflection points at $x = n\pi$.

Finally, $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$. Therefore, the graph has no horizontal asymptote.



12. $f(x) = \sin x - \cos x$

$f'(x) = \cos x + \sin x$ is zero for $x = n\pi - \frac{\pi}{4}$.

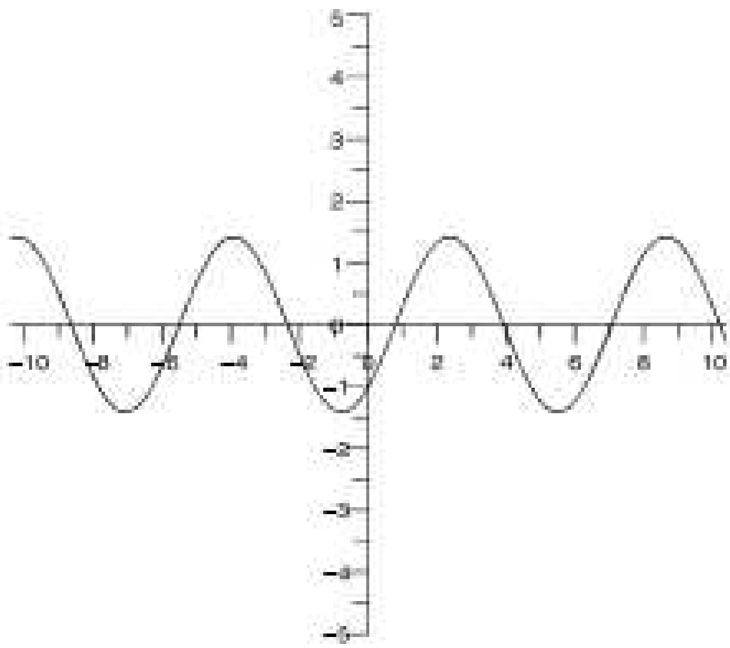
$f''(x) = -\sin x + \cos x$

When n is even, $f''(x) > 0$ and so f is minimum at $x = n\pi - \frac{\pi}{4}$.

When n is odd, $f''(x) < 0$ and so f is maximum at $x = n\pi - \frac{\pi}{4}$.

$f''(x) = 0$ for $x = n\pi + \frac{\pi}{4}$. So inflection points are $n\pi + \frac{\pi}{4}$.

$f''(x) < 0$ on $\left(\frac{\pi}{4} + n\pi, \frac{5\pi}{4} + n\pi\right)$ so $f(x)$ is concave down on this interval and we find that $f''(x) > 0$ on $\left(\frac{5\pi}{4} + n\pi, \frac{9\pi}{4} + n\pi\right)$ so $f(x)$ is concave up on this interval.



13. $f(x) = x \ln x$

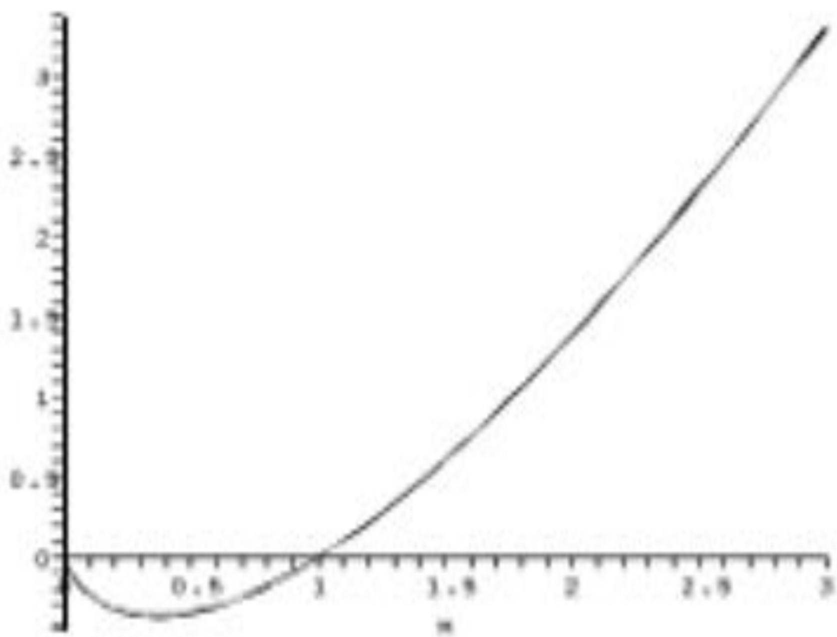
The domain is $\{x | x > 0\}$. There is an x-intercept at $x = 1$ and no y-intercept.

$f'(x) = \ln x + 1$

The only critical number is $x = e^{-1}$. $f'(x) < 0$ on $(0, e^{-1})$ and $f'(x) > 0$ on (e^{-1}, ∞) so $f(x)$ is decreasing on $(0, e^{-1})$ and increasing on (e^{-1}, ∞). Thus $f(x)$ has a local minimum at $x = e^{-1}$.

$f''(x) = 1/x$, which is positive for all x in the domain of f, so $f(x)$ is always concave up.

$f(x) \to \infty$ as $x \to \infty$.



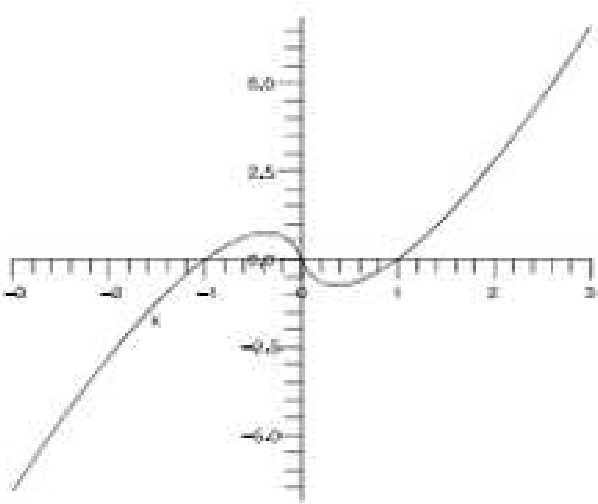
14. $f(x) = x \ln x^2$

The domain is $\{x | x \neq 0\}$. There are x-intercepts at $x = \pm 1$ but no y-intercept.

$f'(x) = \ln x^2 + 2$

The critical numbers are $x = \pm e^{-1}$. $f''(x) = 2/x$, so $x = -e^{-1}$ is a local maximum and $x = e^{-1}$ is a local minimum. $f(x)$ is increasing on $(-\infty, -e^{-1})$ and (e^{-1}, ∞); $f(x)$ is decreasing on $(-e^{-1}, 0)$ and $(0, e^{-1})$. $f(x)$ is concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$.

$f(x) \to -\infty$ as $x \to \infty$ and $f(x) \to \infty$ as $x \to \infty$.



15. $f(x) = \sqrt{x^2+1}$

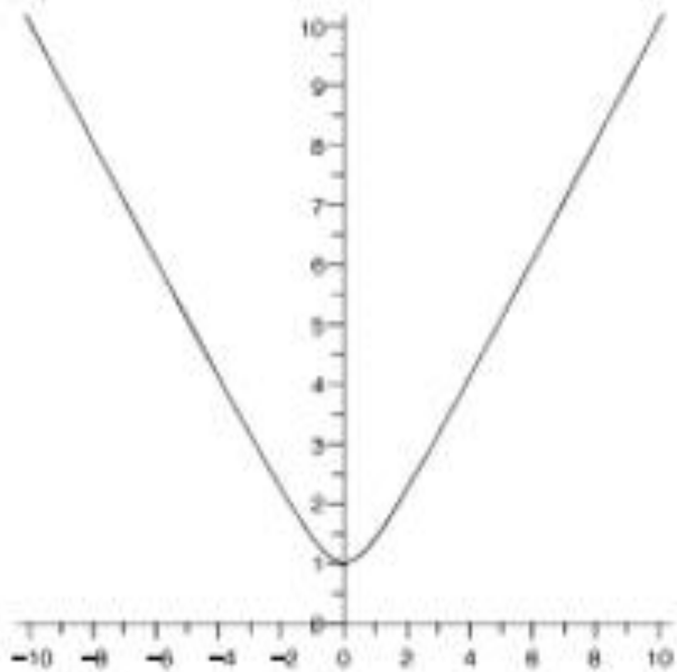
The y-intercept is $(0, 1)$. There are no x-intercepts.

$f'(x) = \frac{1}{2}(x^2+1)^{-1/2}2x = \frac{x}{\sqrt{x^2+1}}$ The only critical number is $x = 0$. $f'(x) < 0$ when $x < 0$ and $f'(x) > 0$ when $x > 0$ so $f(x)$ is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$. Thus $f(x)$ has a local minimum at $x = 0$.

$$f''(x) = \frac{\sqrt{x^2+1} - x\frac{1}{2}(x^2+1)^{-1/2}2x}{x^2+1}$$
$$= \frac{1}{(x^2+1)^{3/2}}$$

Since $f''(x) > 0$ for all x, we see that $f(x)$ is concave up for all x.

$f(x) \to \infty$ as $x \to \pm\infty$.



16. $f(x) = \sqrt{2x-1}$

The domain is $\{x | x \geq 1/2\}$. There is an x-intercept at $x = 1/2$.

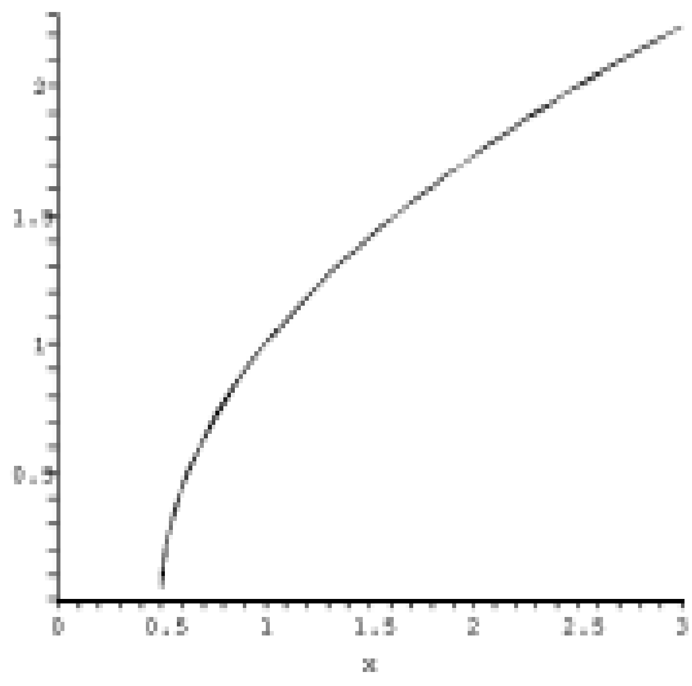
$f'(x) = \frac{1}{2}(2x-1)^{-1/2}2 = \frac{1}{\sqrt{2x-1}}$

$f'(x)$ is undefined at $x = 1/2$, but this is an endpoint of $f(x)$ and there are no other critical points. Since $f'(x)$ is positive for all x in the domain of f, we see that $f(x)$ is increasing for all x in the domain.

$f''(x) = -\frac{1}{2}(2x-1)^{-3/2}2 = \frac{-1}{(2x-1)^{3/2}}$

$f''(x) < 0$ for all x in the domain of f, so f is concave down for all x for which it is defined.

$f(x) \to \infty$ as $x \to \infty$.



17. $f(x) = \sqrt[3]{x^3 - 3x^2 + 2x}$

17. $f(x) = (x^3 - 3x^2 + 2x)^{1/3}$

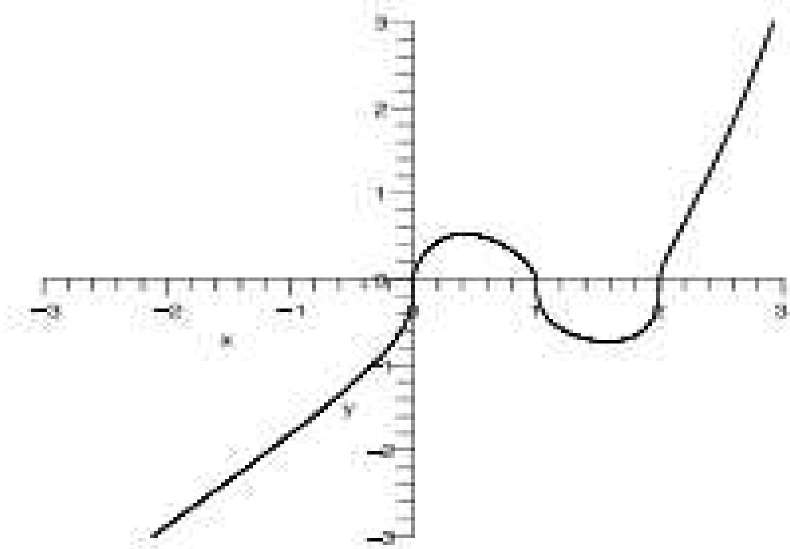
$f'(x) = \dfrac{3x^2 - 6x + 2}{3(x^3 - 3x^2 + 2x)^{2/3}}$

There are critical numbers at $x = \dfrac{3 \pm \sqrt{3}}{3}$, 0, 1 and 2.

$f''(x) = \dfrac{-6x^2 + 12x - 8}{9(x^3 - 3x^2 + 2x)^{5/3}}$

with critical numbers $x = 0$, 1 and 2. $f''(x)$ changes sign at these values, so these are inflection points. The Second Derivative test shows that $x = \dfrac{3 + \sqrt{3}}{3}$ is a minimum, and $x = \dfrac{3 - \sqrt{3}}{3}$ is a maximum.

$f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.



18. $f(x) = \sqrt{x^3 - 3x^2 + 2x}$

18. $f(x) = (x^3 - 3x^2 + 2x)^{1/2}$
$f(x)$ is defined for $0 \le x \le 1$ and $x \ge 2$.
$f(x) \to \infty$ as $x \to \infty$.

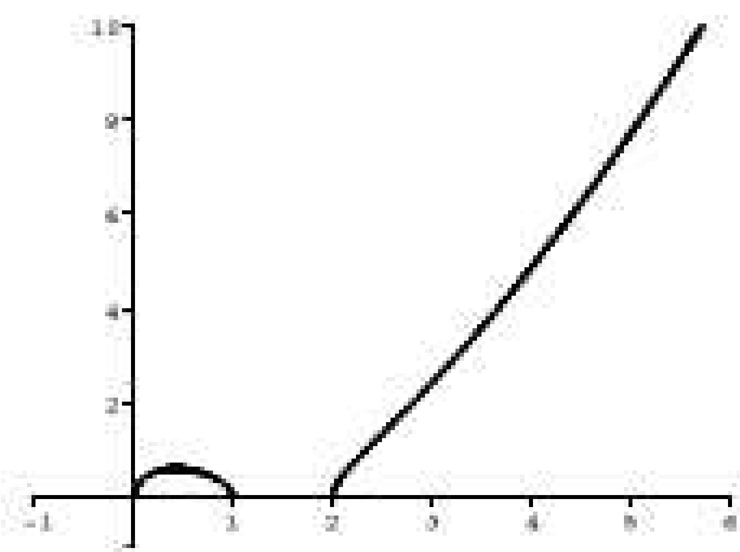
$$f'(x) = \frac{3x^2 - 6x + 2}{2(x^3 - 3x^2 + 2x)^{1/2}}$$

There are critical numbers at $x = \dfrac{3 \pm \sqrt{3}}{3}$, 0, 1 and 2.

$$f''(x) = \frac{3x^4 - 12x^3 + 12x^2 - 4}{4(x^3 - 3x^2 + 2x)^{3/2}}$$

with critical numbers $x = 0$, 1 and 2 and $x \approx -0.4679$ and 2.4679. $f(x)$ is undefined at $x = -0.4679$, so we do not consider this point.

$f''(x)$ changes sign at $x = 2.4679$, so this is an inflection point. The Second Derivative test shows that $x = \dfrac{3 - \sqrt{3}}{3}$ is a maximum.
At $x = 0, 1, 2$, $f(x)$ is minimum.



19. $f(x) = x^{5/3} - 5x^{2/3}$

19. $f(x) = x^{5/3} - 5x^{2/3}$

The domain of f includes all real numbers.

$$f'(x) = \frac{5}{3}x^{\frac{2}{3}} - \frac{10}{3}x^{-\frac{1}{3}}$$
$$= \frac{5}{3}\left(x^{\frac{2}{3}} - 2x^{-\frac{1}{3}}\right)$$
$$= \frac{5}{3}\left(\frac{x-2}{x^{1/3}}\right)$$

Critical number is $x = 2$.

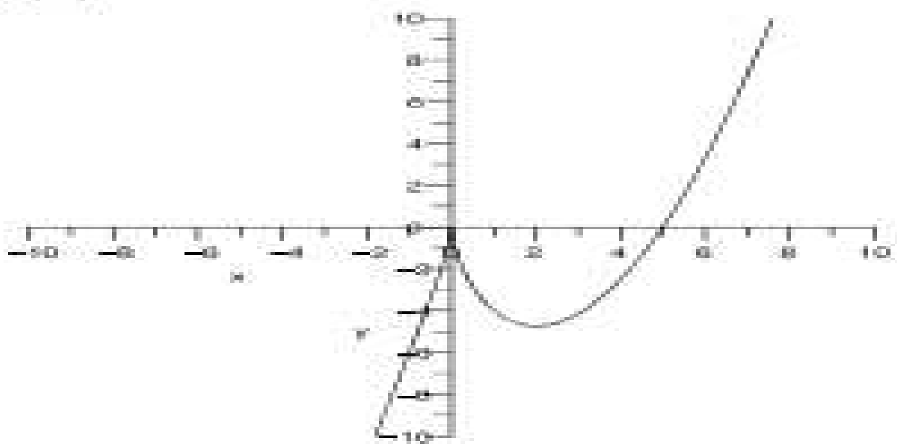
$f'(x) > 0$ on $(-\infty, 0)$ and $(2, \infty)$. So $f(x)$ is increasing on these intervals.

$f'(x) < 0$ on $(0, 2)$ and so $f(x)$ is decreasing on this interval.

Therefore $f(x)$ is maximum at $x = 0$ and minimum at $x = 2$.

$$f''(x) = \frac{5}{3}\left(\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}x^{-\frac{4}{3}}\right)$$
$$= \frac{10}{9}\left(x^{-\frac{1}{3}} + x^{-\frac{4}{3}}\right)$$
$$= \frac{10}{9}\left(\frac{x+1}{x^{4/3}}\right)$$

The critical number is at $x = 0, -1$. $f''(x)$ changes sign at these values, so these are inflection points. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.



20. $f(x) = x^3 - \frac{3}{400}x$

20. $f(x) = x^3 - \frac{3}{400}x = x(x^2 - \frac{3}{400})$

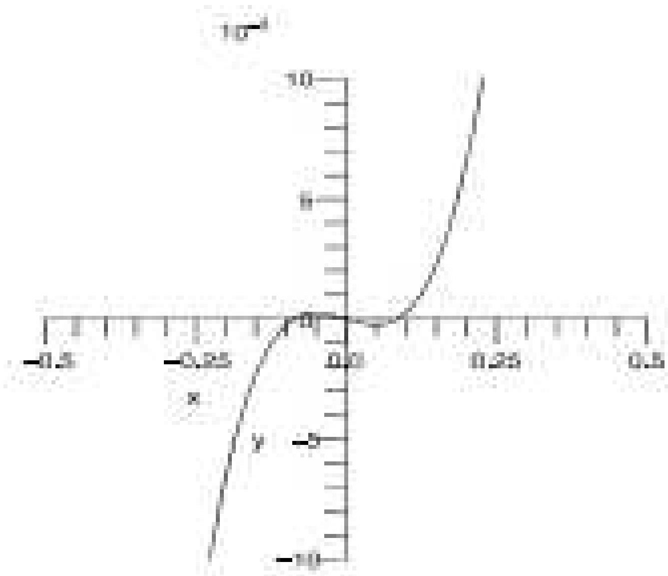
The y-intercept (also an x-intercept) is $(0, 0)$ and there are also x-intercepts at $x = \pm\sqrt{3}/20$.

$f'(x) = 3x^2 - \frac{3}{400}$

The critical numbers are $x = \pm 1/20$.

$f''(x) = 6x$, so $x = -1/20$ is a local maximum and $x = 1/20$ is a local minimum. $f(x)$ is increasing on $(-\infty, -1/20)$ and $(1/20, \infty)$ and decreasing on $(-1/20, 1/20)$. It is concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$, with an inflection point at $x = 0$.

$f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.



21. $f(x) = e^{-2/x}$

$$f'(x) = e^{-2/x}\left(\frac{2}{x^2}\right) = \frac{2}{x^2}e^{-2/x}$$

$$f''(x) = \frac{-4}{x^3}e^{-2/x} + \frac{2}{x^2}e^{-2/x}\left(\frac{2}{x^2}\right)$$

$$= \frac{4}{x^4}e^{-2/x} - \frac{4}{x^3}e^{-2/x}$$

$f'(x) > 0$ on $(-\infty, 0) \cup (0, \infty)$

$f''(x) > 0$ on $(-\infty, 0) \cup (0, 1)$

$f''(x) < 0$ on $(1, \infty)$

f increasing on $(-\infty, 0)$ and on $(0, \infty)$, concave up on $(-\infty, 0) \cup (0, 1)$, concave down on $(1, \infty)$, inflection point at $x = 1$. f is undefined at $x = 0$.

$\lim_{x\to 0^+} e^{-2/x} = \lim_{x\to 0^+} \frac{1}{e^{2/x}} = 0$ and

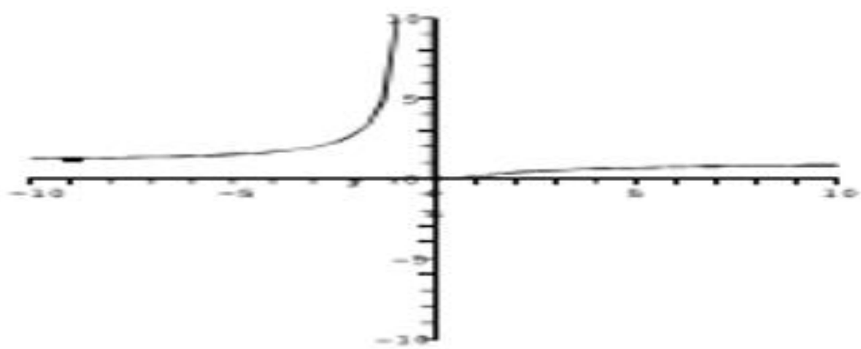
$\lim_{x\to 0^-} e^{-2/x} = \infty$

So f has a vertical asymptote at $x = 0$.

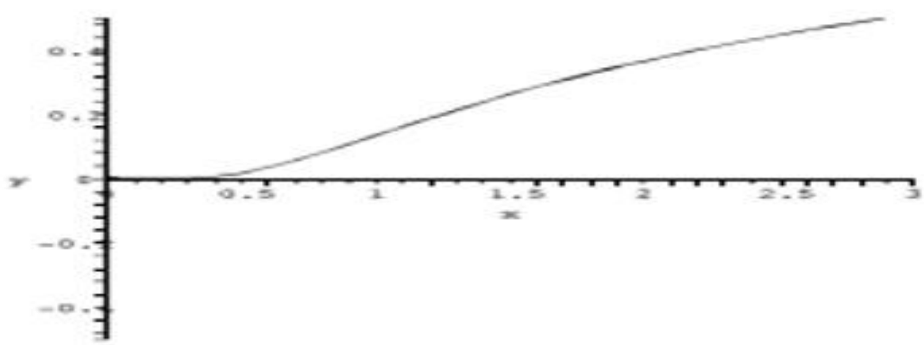
$\lim_{x\to\infty} e^{-2/x} = \lim_{x\to-\infty} e^{-2/x} = 1$

So f has a horizontal asymptote at $y = 1$.

Global graph of $f(x)$:



Local graph of $f(x)$:



22. $f(x) = e^{1/x^2}$

22. $f(x) = e^{1/x^2}$

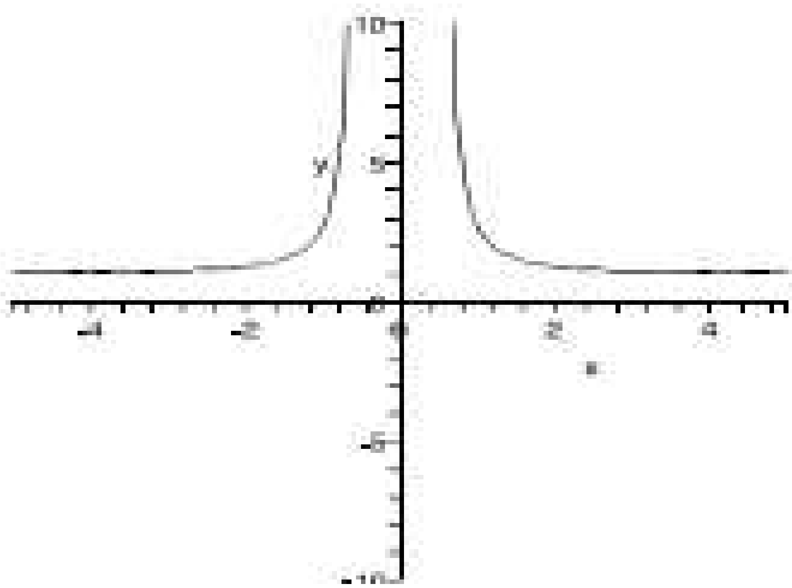
The function has a vertical asymptote at $x = 0$ such that $f(x) \to \infty$ as x approaches 0 from the right or left. There is a horizontal asymptote of $y = 1$ as $x \to \pm\infty$.

$f'(x) = \dfrac{-2}{x^3} \cdot e^{1/x^2}$

$f'(x) > 0$ for $x < 0$, so $f(x)$ is increasing on $(-\infty, 0)$ and $f'(x) < 0$ for $x > 0$, so $f(x)$ is decreasing on $(-\infty, 0)$.

$$f''(x) = \frac{2e^{1/x^2}(3x^2 + 2)}{x^6}$$

is positive for all $x \neq 0$, so $f(x)$ is concave up for all $x \neq 0$.



في التمارين 23–36، حددِ جميع المميزات المهمة (تقريبيًا إذاَ لزم الأمر) وارسم تمثيلًا بيانيًا.

23. $f(x) = \dfrac{1}{x^3 - 3x^2 - 9x + 1}$

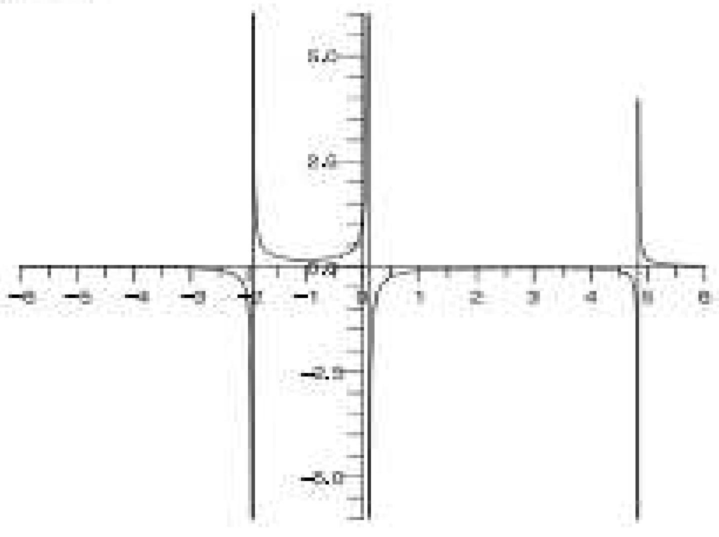
$$f'(x) = -\frac{3x^2 - 6x - 9}{(x^3 - 3x^2 - 9x + 1)^2}$$

The critical numbers are $x = -1, 3$.

$$f''(x) = \frac{6(6x^4 - 4x^3 - 7x^2 + 12x + 2)}{(x^3 - 3x^2 - 9x + 1)^3}$$

The Second Derivative test shows that the graph has a local minimum at $x = -1$ and a local maximum at $x = 3$. The graph has a vertical asymptote at $x = -1.9304$. Similarly, the graph has vertical asymptotes at $x = 0.1074$ and 4.8231.

$f(x) \to 0$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to \infty$. Therefore, the graph has horizontal asymptote $y = 0$.



24. $f(x) = \dfrac{1}{x^3 + 3x^2 + 4x + 1}$

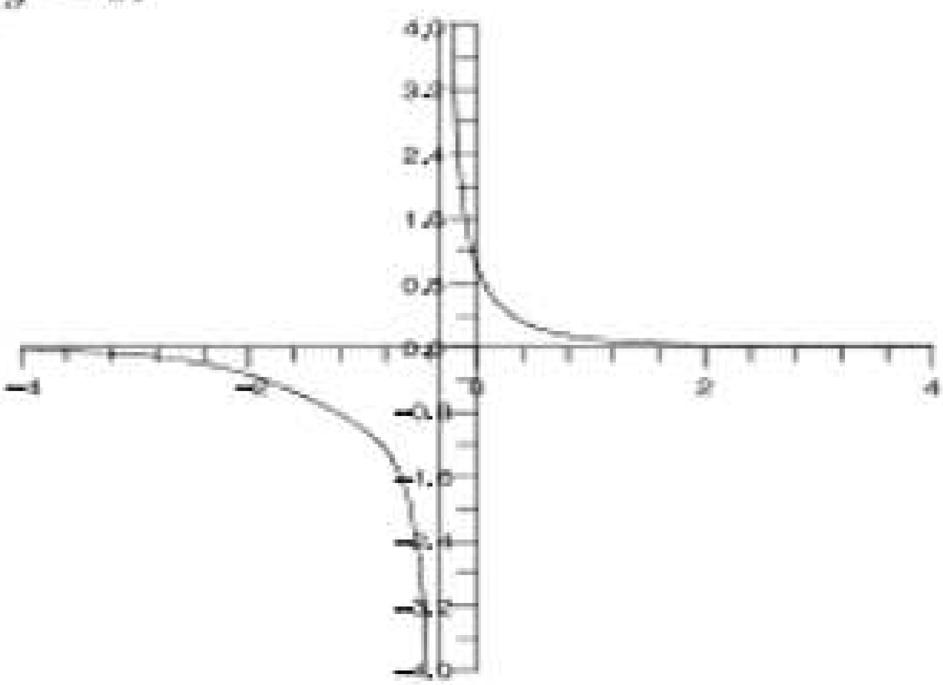
$f'(x) = -\dfrac{3x^2 + 6x + 4}{(x^3 + 3x^2 + 4x + 1)^2}$

Since $f'(x) = 0$ has no real roots, the graph has no extrema.

$f''(x) = \dfrac{12x^4 + 48x^3 + 78x^2 + 66x + 26}{(x^3 + 3x^2 + 4x + 1)^3}$

The Critical number is $x = -0.316722$. $f''(x) > 0$ on $(-0.3176722, \infty)$ so the graph is concave up on this interval. $f''(x) < 0$ on $(-\infty, -0.3176722)$ so the graph is concave down on this interval. the graph has a vertical asymptote at $x = -0.3176722$. $f(x) \to 0$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to \infty$.

Therefore, the graph has horizontal asymptote $y = 0$.



25. $f(x) = (x^3 - 3x^2 + 2x)^{2/3}$

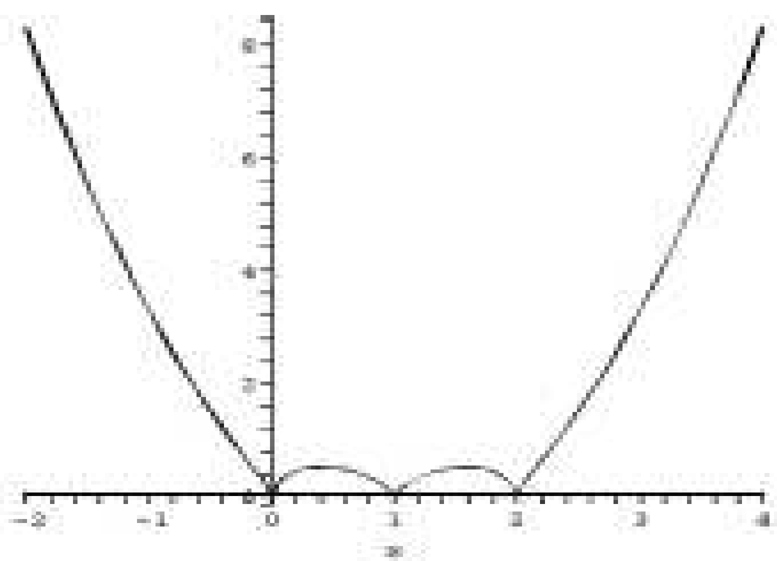
$$f'(x) = \frac{2(3x^2 - 6x + 2)}{3(x^3 - 3x^2 + 2x)^{1/3}}$$

There are critical numbers at $x = \dfrac{3 \pm \sqrt{3}}{3}$, 0, 1 and 2.

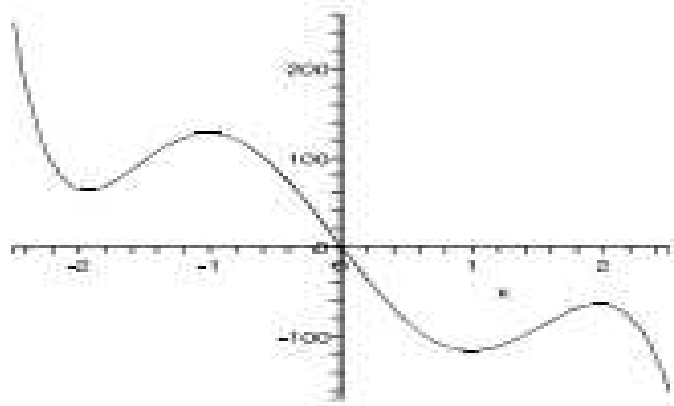
$$f''(x) = \frac{18x^4 - 72x^3 + 84x^2 - 24x - 8}{9(x^3 - 3x^2 + 2x)^{4/3}}$$

with critical numbers $x = 0$, 1 and 2 and $x \approx -0.1883$ and 2.1883. $f''(x)$ changes sign at these last two values, so these are inflection points. The Second Derivative test shows that $x = \dfrac{3 \pm \sqrt{3}}{3}$ are both maxima. Local minima occur at $x = 0$, 1 and 2.

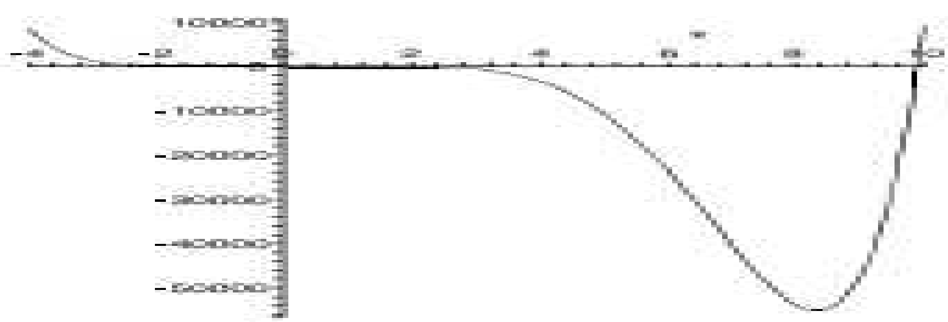
$f(x) \to \infty$ as $x \to \pm\infty$.



26. $f(x) = x^6 - 10x^5 - 7x^4 + 80x^3 + 12x^2 - 192x$
$f(x) \to \infty$ as $x \to \pm\infty$.
$f'(x) = 6x^5 - 50x^4 - 28x^3 + 240x^2 + 24x - 192$
Critical numbers at approximately $x = -1.9339, -1.0129, 1, 1.9644$, and 8.3158.
$f''(x) = 30x^4 - 200x^3 - 84x^2 + 480x + 24$
Critical numbers at approximately $x = -1.5534, -0.0496, 1.5430$, and 6.7267, and changes sign at each of these values, so these are inflection points. The Second Derivative Test shows that $x = -1.9339$, 1, and 8.3158 are local minima, and $x = -1.0129$ and 1.9644 are local maxima. The extreme near $x = 0$ look like this:



The inflection points, and the global behavior of the function can be seen on the following graph.



27. $f(x) = \dfrac{x^2+1}{3x^2-1}$

Note that $x = \pm\sqrt{1/3}$ are not in the domain of the function, but yield vertical asymptotes.

$$f'(x) = \frac{2x(3x^2-1)-(x^2+1)(6x)}{(3x^2-1)^2}$$
$$= \frac{(6x^3-2x)-(6x^3+6x)}{(3x^2-1)^2}$$
$$= \frac{-8x}{(3x^2-1)^2}$$

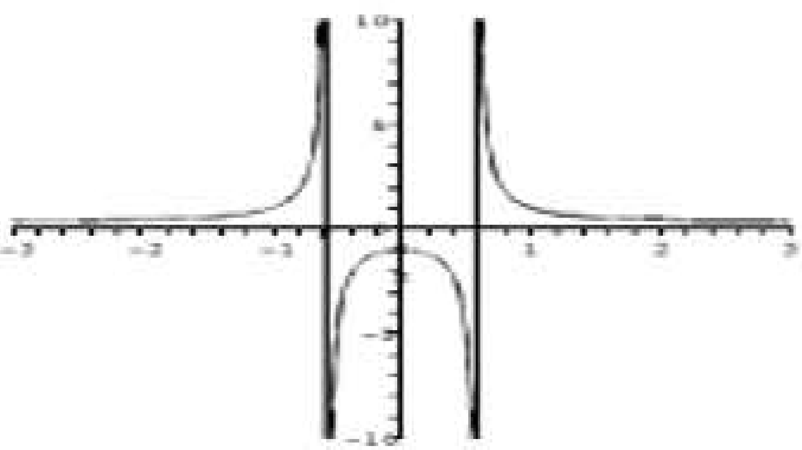
So the only critical point is $x = 0$.

$f'(x) > 0$ for $x < 0$
$f'(x) < 0$ for $x > 0$
so f is increasing on $(-\infty, -\sqrt{1/3})$ and on $(-\sqrt{1/3}, 0)$; decreasing on $(0, \sqrt{1/3})$ and on $(\sqrt{1/3}, \infty)$. Thus there is a local max at $x = 0$.

$$f''(x) = 8 \cdot \frac{9x^2+1}{(3x^2-1)^3}$$

$f''(x) > 0$ on $(-\infty, -\sqrt{1/3}) \cup (\sqrt{1/3}, \infty)$
$f''(x) < 0$ on $(-\sqrt{1/3}, \sqrt{1/3})$
Hence f is concave up on $(-\infty, -\sqrt{1/3})$ and on $(\sqrt{1/3}, \infty)$; concave down on $(-\sqrt{1/3}, \sqrt{1/3})$.

Finally, when $|x|$ is large, the function approached $1/3$, so $y = 1/3$ is a horizontal asymptote.



28. $f(x) = \dfrac{5x}{x^3 - x + 1}$

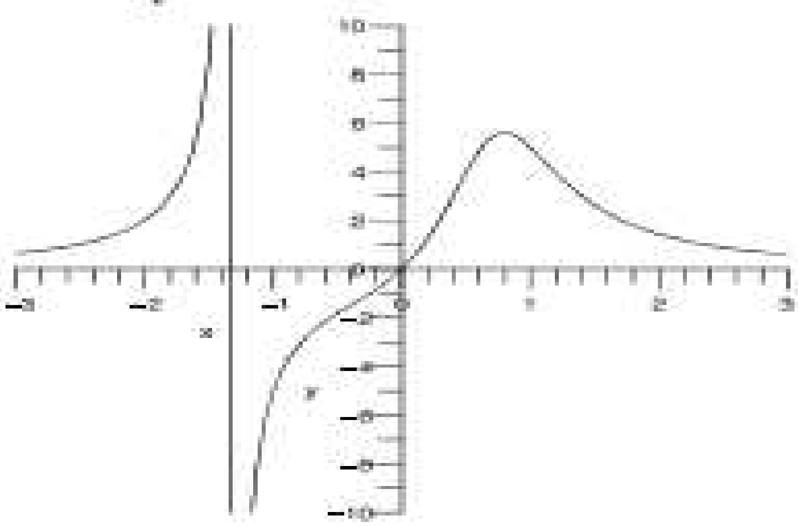
Looking at the graph of $x^3 - x + 1$, we see that there is one real root, at approximately -1.325; so the domain of the function is all x except for this one point, and $x = -1.325$ will be a vertical asymptote. There is a horizontal asymptote of $y = 0$.

$f'(x) = 5\dfrac{1 - 2x^3}{(x^3 - x - 1)^2}$

The only critical point is $x = \sqrt[3]{1/2}$. By the first derivative test, this is a local max.

$f''(x) = 10\dfrac{3x^5 + x^3 - 6x^2 + 1}{(x^3 - x + 1)^3}$

The numerator of f'' has three real roots, which are approximately $x = -.39018$, $x = .43347$, and $x = 1.1077$. $f''(x) > 0$ on $(-\infty, -1.325) \cup (-.390, .433) \cup (1.108, \infty)$
$f''(x) < 0$ on $(-1.325, -.390) \cup (.433, 1.108)$
So f is concave up on $(-\infty, -1.325) \cup (-.390, .433) \cup (1.108, \infty)$ and concave down on $(-1.325, -.390) \cup (.433, 1.108)$. Hence $x = -.39018$, $x = .43347$, and $x = 1.1077$ are inflection points.



في التمارين 49–52، جـــد دالة يوجد بتمثيلها البياني خطوط التقارب المعطاة.

49. $x = 1$, $x = 2$ and $y = 3$

49. One possibility:

$$f(x) = \frac{3x^2}{(x-1)(x-2)}$$

50. $x = -1$, $x = 1$ and $y = 0$

50. One possibility:

$$f(x) = \frac{x}{x^2 - 1}$$

51. $x = -1$, $x = 1$, $y = -2$ and $y = 2$

51. One possibility:

$$f(x) = \frac{2x}{\sqrt{(x-1)(x+1)}}$$

52. $x = 1$, $y = 2$ and $x = 3$

52. One possibility:

$$f(x) = \frac{2x^2}{(x-1)(x-3)}$$

حل مسائل اقتصادية وعلمية على القيم القصوى

ص 312 مثال 7-9

مثال 9.7 نمذجة التيار الكهربائي في السلك

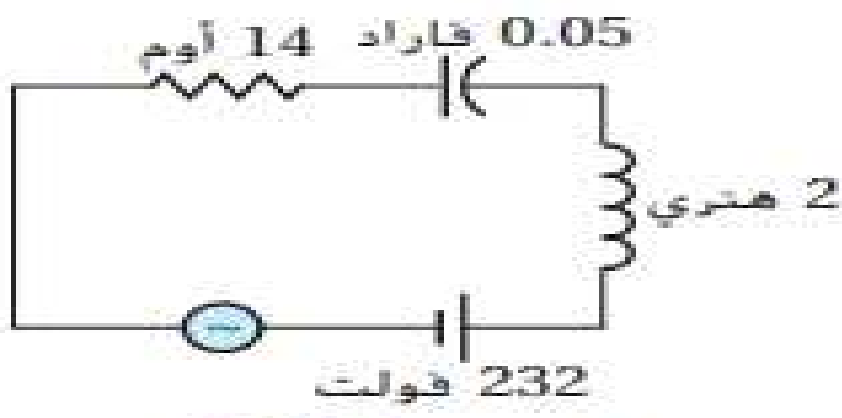
تتضمن الدارة الكهربائية المبينة في الشكل 4.103 مقاوم 14 أوم وأداة ومعابق 2 هنري، ومكثّف 0.05 − فاراد وبطارية إمداد 232 فولت من التيار المتردد المنمذج بالدالة المتذبذبة $232 \sin 2t$، حيث إن t تقاس بالثواني. فجــد التيار في الدارة عند أي t.

الحل يمكن إثبات أن الشحنة في هذه الدارة تعطى بالدالة (باستخدام القوانين الكهربائية الأساسية)

$$Q(t) = 10e^{-5t} + 2te^{-2t} + 3\sin 2t - 7\cos 2t$$

فالتيار إذن

$$Q'(t) = -50e^{-5t} + 2e^{-2t} - 4te^{-2t} + 6\cos 2t + 14\sin 2t$$



الشكل 4.103

دارة كهربائية بسيطة

ص 314 من 33 – 36

33. على فرض أن الشحنة في الدارة الكهربائية
$Q(t) = e^{-2t}(\cos 3t - 2\sin 3t)$ كولوم. جـــد التيار.

33. $Q'(t) = e^{-2t}\cdot(-2)(\cos 3t - 2\sin 3t)$
$+ e^{-2t}((-\sin 3t \cdot 3) - 2\cos 3t \cdot 3)$
$= e^{-2t}(-8\cos 3t + \sin 3t)$ amps

34. على فرض أن الشحنة في الدارة الكهربائية
$Q(t) = e^{t}(3\cos 2t + \sin 2t)$ كولوم. جـــد التيار

34. $Q'(t) = e^{t}(3\cos 2t + \sin 2t)$
$+ e^{t}(-6\sin 2t + 2\cos 2t)$
$= 5e^{t}(\cos 2t - \sin 2t)$ amps

35. على فرض أن الشحنة في مكان محدد في الدارة الكهربائية $Q(t) = e^{-3t}\cos 2t + 4\sin 3t$ كولوم. ماذا يحدث لهذه الدالة إذا كان $t \to \infty$؟ اشرح لماذا يسمى $t \to \infty$ الحد $e^{-3t}\cos 2t$ **حالة عابر** و $4\sin 3t$ يعرف بأنه **حالة ثابتة** أو قيمة **خط التقارب** لدالة الشحنة. جـــد قيم الحالة الثابتة والعابرة لدالة التيار.

35. As $t \to \infty$, $Q(t) \to 4\sin 3t$, so $e^{-3t}\cos 2t$ is called the transient term and $4\sin 3t$ is called the steady-state value.

$$Q'(t) = e^{-3t}\cdot(-3)\cos 2t + e^{-3t}(-\sin 2t \cdot 2) + 4\cos 3t \cdot 3$$
$$= e^{-3t}(-3\cos 2t - 2\sin 2t) + 12\cos 3t$$

The transient term is $e^{-3t}(-3\cos 2t - 2\sin 2t)$ and the steady-state value is $12\cos 3t$.

36. كما في التمرين 35. جـــد قيم الحالة الثابتة والعابرة إذا حُددت دالة الشحنة من خلال

$$Q(t) = e^{-2t}(\cos t - 2\sin t) + te^{-3t} + 2\cos 4t$$

36. $Q'(t) = -2e^{-2t}(\cos t - 2\sin t) + e^{-2t}(-\sin t - 2\cos t) + e^{-3t} - 3te^{-3t} - 8\sin 4t$

$Q'(t) = e^{-2t}(-4\cos t + 3\sin t) + e^{-3t}(1 - 3t) - 8\sin 4t$

The transient term is $e^{-2t}(-4\cos t + 3\sin t) + e^{-3t}(1 - 3t)$ and the steady-state value is $-8\sin 4t$.

إيجاد عكس المشتقة لدالة معطاة

ص 329 من 5 – 28

5. $\int (3x^4 - 3x)dx = \frac{3}{5}x^5 - \frac{3}{2}x^2 + c$

6. $\int (x^3 - 2)dx = \frac{1}{4}x^4 - 2x + c$

7. $\int \left(3\sqrt{x} - \frac{1}{x^4}\right) dx = 2x^{3/2} + \frac{x^{-3}}{3} + c$

8. $\int \left(2x^{-2} + \frac{1}{\sqrt{x}}\right) dx$
$= -2x^{-1} + 2x^{1/2} + c$

9. $\int \frac{x^{1/3} - 3}{x^{2/3}} dx = \int (x^{-1/3} - 3x^{-2/3})dx$
$= \frac{3}{2}x^{2/3} - 9x^{1/3} + c$

10. $\int \frac{x + 2x^{3/4}}{x^{5/4}} dx = \int (x^{-1/4} + 2x^{-1/2})dx$
$= \frac{4}{3}x^{3/4} + 4x^{1/2} + c$

11. $\int (2\sin x + \cos x)dx = -2\cos x + \sin x +$

12. $\int (3\cos x - \sin x)dx = 3\sin x + \cos x + c$

13. $\int 2\sec x \tan x dx = 2\sec x + c$

14. $\int \frac{4}{\sqrt{1-x^2}}dx = 4\arcsin x + c$

15. $\int 5\sec^2 x dx = 5\tan x + c$

16. $\int \frac{4\cos x}{\sin^2 x}dx = -4\csc x + c$

17. $\int (3e^x - 2)dx = 3e^x - 2x + c$

18. $\int (4x - 2e^x)dx = 2x^2 - 2e^x + c$

19. $\int (3\cos x - 1/x)dx = 3\sin x - \ln|x| + c$

20. $\int (2x^{-1} + \sin x)dx = 2\ln|x| - \cos x + c$

21. $\int \frac{4x}{x^2+4}dx = 2\ln|x^2+4| + c$

22. $\int \frac{3}{4x^2+4}dx = \frac{3}{4}\tan^{-1}x + c$

23. $\int \frac{\cos x}{\sin x}dx = \ln|\sin x| + c$

24. $\int (2\cos x - e^x)dx = 2\sin x - e^x + c$

25. $\int \frac{e^x}{e^x+3}dx = \ln|e^x + 3| + c$

26. $\int \frac{e^x+3}{e^x}dx = \int (1+3e^{-x})dx$
$= x - 3e^{-x} + c$

27. $\int x^{1/4}(x^{5/4} - 4)dx = \int (x^{3/2} - 4x^{1/4})dx$
$= \frac{2}{5}x^{5/2} - \frac{16}{5}x^{5/4} + c$

28. $\int x^{2/3}(x^{-4/3} - 3)dx = \int (x^{-2/3} - 3x^{2/3})d$
$= 3x^{1/3} - \frac{9}{5}x^{5/3} + c$

التعرف على مفهوم التكامل غير المحدود بصفته عكس المشتقة

ص 330 من 45 -48

45. حدد الدالة المكانية إذا كانت دالة السرعة المتجهة هي $v(t) = 3 - 12t$ والموقع الابتدائي هو $s(0) = 3$.

45. Position is the antiderivative of velocity,
$s(t) = 3t - 6t^2 + c$.
Since $s(0) = 3$, we have $c = 3$. Thus,
$s(t) = 3t - 6t^2 + 3$.

46. حدد الدالة المكانية إذا كانت دالة السرعة المتجهة هي $v(t) = 3e^{-t} - 2$ والموقع الابتدائي هو $s(0) = 0$.

46. Position is the antiderivative of velocity,
$s(t) = -3e^{-t} - 2t + c$.
Since $s(0) = 0$, we have $-3 + c = 0$ and therefore $c = 3$. Thus,
$s(t) = -3e^{-t} - 2t + 3$.

47. حدد الدالة المكانية إذا كانت دالة التسارع هي $a(t) = 3\sin t + 1$.
والسرعة المنجهة الابتدائية هي $v(0) = 0$ والموقع الابتدائي هو
$s(0) = 4$.

47. First we find velocity, which is the antiderivative of acceleration,
$v(t) = -3\cos t + c_1$.
Since $v(0) = 0$ we have
$-3 + c_1 = 0$, $c_1 = 3$ and
$v(t) = -3\cos t + 3$.
Position is the antiderivative of velocity,
$s(t) = -3\sin t + 3t + c_2$.
Since $s(0) = 4$, we have $c_2 = 4$. Thus,
$s(t) = -3\sin t + 3t + 4$.

48. حدد الدالة المكانية إذا كانت دالة التسارع هي $a(t) = t^2 + 1$.
والسرعة المنجهة الابتدائية هي $v(0) = 4$ والموقع الابتدائي هو
$s(0) = 0$.

48. First we find velocity, which is the antiderivative of acceleration,
$v(t) = \frac{1}{3}t^3 + t + c_1$.
Since $v(0) = 4$ we have $c_1 = 4$ and
$v(t) = \frac{1}{3}t^3 + t + 4$.

Position is the antiderivative of velocity,
$s(t) = \frac{1}{12}t^4 + \frac{1}{2}t^2 + 4t + c_2$.
Since $s(0) = 0$, we have $c_2 = 0$. Thus,
$s(t) = \frac{1}{12}t^4 + \frac{1}{2}t^2 + 4t$.

استخدام رمز المجموع سيجما لإيجاد المجاميع البسيطة

ص 337 من 5 – 18

في التمارين 8–5، اكتب كل الحدود واحسب المجموع.

5. $\sum_{i=1}^{6} 3i^2$

6. $\sum_{i=3}^{7} (i^2 + i)$

5. $\sum_{i=1}^{6} 3i^2 = 3 + 12 + 27 + 48 + 75 + 108$
$= 273$

6. $\sum_{i=3}^{7} i^2 + i = 12 + 20 + 30 + 42 + 56$
$= 160$

---

7. $\sum_{i=6}^{10} (4i + 2)$
$= (4(6) + 2) + (4(7) + 2) + (4(8) + 2)$
$+ (4(9) + 2) + (4(10) + 2)$
$= 26 + 30 + 34 + 38 + 42$
$= 170$

8. $\sum_{i=6}^{8} (i^2 + 2)$
$= (6^2 + 2) + (7^2 + 2) + (8^2 + 2)$
$= 38 + 51 + 66 = 155$

في التمارين 18–9، استخدم قواعد المجموع لحساب المجموع.

$$9.\ \sum_{i=1}^{70}(3i-1) = 3\cdot\sum_{i=1}^{70} i - 70$$
$$= 3\cdot\frac{70(71)}{2} - 70 = 7,385$$

$$10.\ \sum_{i=1}^{45}(3i-4) = 3\sum_{i=1}^{45} i - 4\sum_{i=1}^{45} 1$$
$$= 3\left(\frac{45(46)}{2}\right) - 4(45) = 2925$$

$$11.\ \sum_{i=1}^{40}(4-i^2) = 160 - \sum_{i=1}^{40} i^2$$
$$= 160 - \frac{(40)(41)(81)}{6}$$
$$= 160 - 22,140 = -21,980$$

$$12.\ \sum_{i=1}^{50}(8-i) = 8\sum_{i=1}^{50} 1 - \sum_{i=1}^{50} i$$
$$= 8(50) - \frac{50(51)}{2} = -875$$

13. $\sum_{n=1}^{100}(n^2-3n+2)$

$$=\sum_{n=1}^{100}n^2-3\sum_{n=1}^{100}n+\sum_{n=1}^{100}2$$

$$=\frac{(100)(101)(201)}{6}-3\frac{100(101)}{2}+200$$

$$=338,350-15,150+200=323,400$$

14. $\sum_{n=1}^{140}(n^2+2n-4)$

$$=\sum_{n=1}^{140}n^2+2\sum_{n=1}^{140}n-\sum_{n=1}^{140}4$$

$$=\frac{(140)(141)(281)}{6}+2\left(\frac{140(141)}{2}\right)-4(140)$$

$$=943,670$$

15. $\sum_{i=3}^{30}\left[(i-3)^2+i-3\right]$

$$=\sum_{i=3}^{30}(i-3)^2+\sum_{i=3}^{30}(i-3)$$

$$=\sum_{n=0}^{27}n^2+\sum_{n=0}^{27}n \text{ (substitute } i-3=n)$$

$$=0+\sum_{n=1}^{27}n^2+0+\sum_{n=1}^{27}n$$

$$=\frac{27\,(28)\,(55)}{6}+\frac{27\,(28)}{2}=7308$$

16. $$\sum_{i=4}^{20}(i-3)(i+3)=\sum_{i=4}^{20}(i^2-9)$$

$$=\sum_{i=4}^{20}i^2-9\sum_{i=4}^{20}1$$

$$=\sum_{i=1}^{20}i^2-\sum_{i=1}^{3}i^2-9\sum_{i=4}^{20}1$$

$$=\frac{20(21)(41)}{6}-1-4-9-9(17)$$

$$=2703$$

17. $$\sum_{k=3}^{n}(k^2-3)$$

$$=\sum_{k=3}^{n}k^2+\sum_{k=3}^{n}(-3)$$

$$=\sum_{k=1}^{n}k^2-\sum_{k=1}^{2}k^2$$

$$+\sum_{k=1}^{n}(-3)-\sum_{k=1}^{2}(-3)$$

$$=\frac{n(n+1)(2n+1)}{6}-1-4$$

$$+(-3)n-(-3)(2)$$

$$=\frac{n(n+1)(2n+1)}{6}-5-3n+6$$

$$=\frac{n(n+1)(2n+1)}{6}-3n+1$$

18. $\sum_{k=0}^{n} (k^2 + 5)$

$$= \sum_{k=0}^{n} k^2 + \sum_{k=0}^{n} 5$$

$$= 0 + \sum_{k=1}^{n} k^2 + 5 + \sum_{k=1}^{n} 5$$

$$= \frac{n(n+1)(2n+1)}{6} + 5 + 5n$$

تقدير المساحة تحت المنحنى لدالة في فترة

محددة باستخدام المستطيلات

ص 345 من 35-38

في التمارين 35–38، استخدم قيم الدالة المعطاة لتقدير المساحة تحت المنحنى باستخدام قيم نقطة النهاية اليسرى ونقطة النهاية اليمنى.

35.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	2.0	2.4	2.6	2.7	2.6	2.4	2.0	1.4	0.6

35. Using left hand endpoints:

$L_8 = [f(0.0)+f(0.1)+f(0.2)+f(0.3)+f(0.4)+f(0.5)+f(0.6)+f(0.7)](0.1)$

$= (2.0+2.4+2.6+2.7+2.6+2.4+2.0+1.4)(0.1) = 1.81$

Right endpoints:

$R_8 = [f(0.1)+f(0.2)+f(0.3)+f(0.4)+f(0.5)+f(0.6)+f(0.7)+f(0.8)](0.2)$

$= (2.4+2.6+2.7+2.6+2.4+2.0+1.4+0.6)(0.1) = 1.67$

36.

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
$f(x)$	2.0	2.2	1.6	1.4	1.6	2.0	2.2	2.4	2.0

36. Using left hand endpoints:
$L_8 = [f().0) + f(0.2) + f(0.4) + f(0.6) + f(0.8) + f(1.0) + f(1.2) + f(1.4)](0.2)$
$= (2.0 + 2.2 + 1.6 + 1.4 + 1.6 + 2.0 + 2.2 + 2.4)(0.2) = 3.08$
Right endpoints:
$R_8 = [f(0.2) + f(0.4) + f(0.6) + f(0.8) + f(1.0) + f(1.2) + f(1.4) + f(1.6)](0.2)$
$= (2.2 + 1.6 + 1.4 + 1.6 + 2.0 + 2.2 + 2.4 + 2.0)(0.2) = 3.08$

37.

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$	1.8	1.4	1.1	0.7	1.2	1.4	1.8	2.4	2.6

37. Using left hand endpoints:
$L_8 = [f(1.0) + f(1.1) + f(1.2) + f(1.3) + f(1.4) + f(1.5) + f(1.6) + f(1.7)](0.1)$
$= (1.8 + 1.4 + 1.1 + 0.7 + 1.2 + 1.4 + 1.82 + 2.4)(0.1) = 1.182$

Right endpoints:
$R_8 = [f(1.1) + f(1.2) + f(1.3) + f(1.4) + f(1.5) + f(1.6) + f(1.7) + f(1.8)](0.1)$
$= (1.4 + 1.1 + 0.7 + 1.2 + 1.4 + 1.82 + 2.4 + 2.6)(0.1) = 1.262$

38.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6
$f(x)$	0.0	0.4	0.6	0.8	1.2	1.4	1.2	1.4	1.0

38. Using left hand endpoints:

$L_8 = [f(1.0)+f(1.2)+f(1.4)+f(1.6)+f(1.8)+f(2.0)+f(2.2)+f(2.4)](0.2)$

$= (0.0+0.4+0.6+0.8+1.2+1.4+1.2+1.4)(0.2) = 1.40$

Right endpoints:

$R_8 = [f(1.2)+f(1.4)+f(1.6)+f(1.8)+f(2.0)+f(2.2)+f(2.4)+f(2.6)](0.2)$

$= (0.4+0.6+0.8+1.2+1.4+1.2+1.4+1.0)(0.2) = 1.60$

التعرف على خصائص التكامل المحدود

ص 356 سؤال 23 و 24

في التمرينين 23 و24، احسب $\int_0^4 f(x)\,dx$.

23. $f(x) = \begin{cases} 2x & , \quad x < 1 \\ 4 & , \quad x \geq 1 \end{cases}$

$$\int_0^4 f(x)dx$$
$$= \int_0^1 f(x)dx + \int_1^4 f(x)dx$$
$$= \int_0^1 2xdx + \int_1^4 4dx$$

$\int_0^1 2xdx$ is the area of a triangle with base 1 and height 2 and therefore has $area = \frac{1}{2}(1)(2) = 1.$

$\int_1^4 4dx$ is the area of a rectangle with base 3 and height 4 and therefore has $area = (3)(4) = 12.$

Therefore

$$\int_0^4 f(x)dx = 1 + 12 = 13$$

24. $f(x) = \begin{cases} 2 & x \le 2 \text{ إذا } \\ 3x & x > 2 \text{ إذا } \end{cases}$

$\int_0^4 f(x)dx$

$= \int_0^2 f(x)dx + \int_2^4 f(x)dx$

$= \int_0^2 2dx + \int_2^4 3xdx$

$\int_0^2 2dx$ is the area of a square with base 2 and height 2 (it is, after all, a square) and therefore has $area = 4$.

$\int_2^4 3xdx$ is a trapezoid with height 3 and bases 6 and 12 and therefore has area (using the formula in the front of the text)

$area = \frac{1}{2}(6 + 12)(2) = 18.$

Therefore

$\int_0^4 f(x)dx = 4 + 18 = 22$

في التمرينين 35 و36، استخدم النظرية 4.2 لكتابة تعبير في صورة تكامل منفرد.

35. (a) $\int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx$ (b) $\int_0^3 f(x)\,dx - \int_2^3 f(x)\,dx$

35. (a) $\int_0^2 f(x)dx + \int_2^3 f(x)dx = \int_0^3 f(x)dx$

(b) $\int_0^3 f(x)dx - \int_2^3 f(x)dx = \int_0^2 f(x)dx$

36. (a) $\int_0^2 f(x)\,dx + \int_2^1 f(x)\,dx$ (b) $\int_{-1}^2 f(x)\,dx + \int_2^3 f(x)\,dx$

36. (a) $\int_0^2 f(x)dx + \int_2^1 f(x)dx = \int_0^1 f(x)dx$

(b) $\int_{-1}^2 f(x)dx + \int_2^3 f(x)dx = \int_{-1}^3 f(x)dx$

في التمرينين 37 و38، فرضاً أنّ $\int_1^3 f(x)\,dx = 3$ و $\int_1^3 g(x)\,dx = -2$
اوجد

37. (a) $\int_1^3 [f(x) + g(x)]\,dx$ (b) $\int_1^3 [2f(x) - g(x)]\,dx$

37. (a) $\int_1^3 (f(x) + g(x))\,dx$

$$= \int_1^3 f(x)\,dx + \int_1^3 g(x)\,dx$$
$$= 3 + (-2) = 1$$

(b) $\int_1^3 (2f(x) - g(x))\,dx$

$$= 2\int_1^3 f(x)\,dx - \int_1^3 g(x)\,dx$$
$$= 2(3) - (-2) = 8$$

38. (a) $\int_1^3 [f(x) - g(x)]\,dx$ (b) $\int_1^3 [4g(x) - 3f(x)]\,dx$

38. (a) $\int_1^3 (f(x) - g(x))\,dx$

$$= \int_1^3 f(x)\,dx - \int_1^3 g(x)\,dx$$
$$= 3 - (-2) = 5$$

(b) $\int_1^3 (4g(x) - 3f(x))\,dx$

$$= 4\int_1^3 g(x)\,dx - 3\int_1^3 f(x)\,dx$$
$$= 4(-2) - 3(3) = -17$$

تطبيق نظرية القيمة المتوسطة في التكامل

ص 356 من 25 – 28

في التمارين 25–28، احسب القيمة المتوسطة للدالة في الفترة المعطاة.

25. $f(x) = 2x + 1, [0, 4]$

$$f_{ave} = \frac{1}{4}\int_0^4 (2x+1)dx$$

$$= 4 + 1 = 5$$

26. $f(x) = x^2 + 2x, [0, 1]$

$$26.\ f_{ave} = \frac{1}{1}\int_0^1 (x^2+2x)dx$$

$$= \frac{2}{6} + 2 = \frac{7}{3}$$

27. $f(x) = x^2 - 1, [1, 3]$

$$f_{ave} = \frac{1}{3-1}\int_1^3 (x^2 - 1)dx$$

$$= 2 + \frac{4}{3} = \frac{10}{3}$$

28. $f(x) = 2x - 2x^2, [0, 1]$

28. $f_{ave} = \frac{1}{1-0}\int_0^1 (2x - 2x^2)dx$

$$= 1 + \frac{2}{3} = \frac{5}{3}$$

في التمرينين 33 و34. جــد قيمة c التي تحقق نتيجة نظرية القيمة المتوسطة في التكامل.

33. $\int_0^2 3x^2\,dx\ (=8)$

We are looking for a value c, such that

$f(c) = \dfrac{1}{2-0}\displaystyle\int_0^2 3x^2dx$

Since $\displaystyle\int_0^2 3x^2dx = 8$, we want to find c so that

$f(c) = 4$ or, $3c^2 = 4$

Solving this equation using the quadratic formula gives $c = \pm\dfrac{2}{\sqrt{3}}$

We are interested in the value that is in the interval $[0, 2]$, so $c = \dfrac{2}{\sqrt{3}}$.

34. $\int_{-1}^1 (x^2 - 2x)\,dx\ (=\frac{2}{3})$

We are looking for a value c, such that

$f(c) = \dfrac{1}{1-(-1)}\displaystyle\int_{-1}^1 (x^2-2x)dx$

Since $\displaystyle\int_{-1}^1 (x^2-2x)dx = \dfrac{2}{3}$, we want to find c

so that $f(c) = \dfrac{1}{3}$ or, $c^2 - 2c = \dfrac{1}{3}$

We are interested in the value that is in the interval $[-1, 1]$, so $c = \dfrac{3-2\sqrt{3}}{3}$.

التعرف على النظرية الأساسية الاولي للتفاضل والتكامل

وتطبيقها على دوال متنوعة لإيجاد تكاملات محدودة

في التمارين 1–18، استخدم الجزء الأول من النظرية الأساسية لحساب كل تكامل بدقّة.

1. $\int_0^2 (2x - 3)dx = (x^2 - 3x)\Big|_0^2 = -2$

2. $\int_0^3 (x^2 - 2)dx = \left(\frac{x^3}{3} - 2x\right)\Big|_0^3 = 3$

3. $\int_{-1}^1 (x^3 + 2x)dx = \left(\frac{x^4}{4} + x^2\right)\Big|_{-1}^1 = 0$

4. $\int_0^2 (x^3 + 3x - 1)\,dx$

$= \left(\frac{x^4}{4} - \frac{3x^2}{2} - x\right)\Big|_0^2 = -4$

5. $\int_1^4 \left(x\sqrt{x} + \frac{3}{x}\right) dx$

$= \left(\frac{2}{5}x^{5/2} + 3\log x\right)\Big|_1^4$

$= \frac{2}{5} \cdot 32 + 3\log 4 - \frac{2}{5}.1 - 3\log 1$

$= \frac{62}{5} + 3\log 4$

6. $\int_1^2 \left(4x - \frac{2}{x^2}\right)dx = \left(2x^2 + \frac{2}{x}\right)\Big|_1^2 = 5$

7. $\displaystyle\int_0^1 \left(6e^{-3x}+4\right)dx = \left(\frac{6e^{-3x}}{-3}+4x\right)\Bigg|_0^1$

$= -\dfrac{2}{e^3}+4+2-0 = -\dfrac{2}{e^3}+6$

8. $\displaystyle\int_0^2 \left(\frac{e^{2x}-2e^{3x}}{e^{3x}}\right)dx$

$= \displaystyle\int_0^2 \left(e^{-x}-2\right)dx = \left(-e^{-x}-2x\right)\Bigg|_0^2$

$= -\dfrac{1}{e^2}-3$

9. $\displaystyle\int_{\pi/2}^{\pi} (2\sin x-\cos x)dx = -2\cos x-\sin x\Bigg|_{\pi/2}^{\pi}$

$= 3$

10. $\int_{\pi/4}^{\pi/2} 3 \csc x \cot x dx = (-3 \csc x)\Big|_{\pi/4}^{\pi/2}$
$= -3 + 3\sqrt{2}$

11. $\int_{0}^{\pi/4} (\sec t \tan t)\, dt = \sec t\Big|_{0}^{\pi/4}$
$= \sqrt{2} - 1$

12. $\int_{0}^{\pi/4} \sec^2 t dt = \tan t\Big|_{0}^{\pi/4} = 1$

13. $\int_{0}^{1/2} \frac{3}{\sqrt{1-x^2}} dx = 3\sin^{-1} x\Big|_{0}^{1/2}$
$= 3\left(\frac{\pi}{6} - 0\right) = \frac{\pi}{2}$

14. $\int_{-1}^{1} \frac{4}{1+x^2} dx = 4 \arctan x\Big|_{-1}^{1} = 2\pi$

15. $\int_{1}^{4} \frac{t-3}{t} dt$
$= \int_{1}^{4} \left(1 - 3t^{-1}\right) dt = (t - 3\ln|t|)\Big|_{1}^{4}$
$= 3 - 3\ln 4$

16. $\displaystyle\int_0^4 t\,(t-2)\,dt = \left(\frac{t^3}{3} - t^2\right)\Bigg|_0^4 = \frac{16}{3}$

17. $\displaystyle\int_0^t \left(e^{x/2}\right)^2 dx = (e^x)\Big|_0^t = e^t - 1$

18. $\displaystyle\int_0^t (\sin^2 x + \cos^2 x)\,dx$

$\displaystyle= \int_0^t 1dx = (x)\Big|_0^t = t$

الورقي (الكتابي)

حل مسائل رياضية وحياتية على القيم القصوى لإيجاد القيم المثلى

ص 296 1-7

1. يجب بناء سياج من ثلاثة جوانب بجوار الجزء المستقيم من النهر، الذي يشكل الجانب الرابع لمنطقة مستطيلة. المساحة المحاطة تساوي $1800\ \text{ft}^2$. جد أصغر قيمة ممكنة للمحيط المناظر لهذه المساحة.

3.7 Optimization

1. $A = xy = 1800$

$y = \dfrac{1800}{x}$

$P = 2x + y = 2x + \dfrac{1800}{x}$

$P' = 2 - \dfrac{1800}{x^2} = 0$

$2x^2 = 1800$

$x = 30$

$P'(x) > 0$ for $x > 30$

$P'(x) < 0$ for $0 < x < 30$

So $x = 30$ is min.

$y = \dfrac{1800}{x} = \dfrac{1800}{30} = 60$

So the dimensions are $30' \times 60'$ and the minimum perimeter is 120 ft.

2. يجب بناء سياج من ثلاثة جوانب بجوار الجزء المستقيم من النهر، الذي يشكل الجانب الرابع لمنطقة مستطيلة. يتوفر 96 ft من السياج. جـد القيمة العظمى للمساحة المحاطة بالسياج وأبعاد السياج المناظر لهذه المساحة.

2. If y is the length of fence opposite the river, and x is the length of the other two sides, then we have the constraint $2x + y = 96$. We wish to maximize

$A = xy = x(96 - 2x)$.

$A' = 96 - 4x = 0$ when $x = 24$.

$A'' = -4 < 0$ so this gives a maximum. Reasonable possible values of x range from 0 to 48, and the area is 0 at these extremes. The maximum area is $A = 1152$, and the dimensions are $x = 24$, $y = 48$.

3. يجب بناء إسطبل مكون من حظيرتين. يشكل مخطط الإسطبل مستطيلين متطابقين متجاورين. إذا كان هناك 120 ft من السياج متوفر. فما هي الأبعاد التي سيضيفها الإسطبل إلى المساحة المحاطة بالسياج؟

3. $P = 2x + 3y = 120$

$$3y = 120 - 2x$$

$$y = 40 - \frac{2}{3}x$$

$$A = xy$$

$$A(x) = x\left(40 - \frac{2}{3}x\right)$$

$$A'(x) = 1\left(40 - \frac{2}{3}x\right) + x\left(-\frac{2}{3}\right)$$

$$= 40 - \frac{4}{3}x = 0$$

$$40 = \frac{4}{3}x$$

$$x = 30$$

$A'(x) > 0$ for $0 < x < 30$

$A'(x) < 0$ for $x > 30$.

So $x = 30$ is max, $y = 40 - \frac{2}{3} \cdot 30 = 20$.

So the dimensions are $20' \times 30'$.

4. يجب أن تكون صالة عرض بمتجر متعدد الأقسام مستطيلة بثلاثة جدران في ثلاثة جوانب وفتحات باب 6 ft في الجانبين المتقابلين وفتحة باب 10 ft في الجدار المتبقي. يجب أن تكون مساحة أرضية صالة العرض $800\ ft^2$. ما هي الأبعاد التي ستكون أصغر طول للجدار المستخدم؟

4. Let x be the length of the sides facing each other and y be the length of the third side. We have the constraint that $xy = 800$, or $y = 800/x$. We also know that $x > 6$ and $y > 10$. The function we wish to minimize is the length of walls needed, or the side length minus the width of the doors.

$L = (y - 10) + 2(x - 6) = 800/x + 2x - 22.$

$L' = -800/x^2 + 2 = 0$ when $x = 20$.

$L'' = 1600/x^3 > 0$ when $x = 20$ so this is a minimum. Possible values of x range from 6 to 80. $L(6) = 123.3$, $L(80) = 148$, and $L(20) = 58$. To minimize the length of wall, the facing sides should be 20 feet, and the third side should be 40 feet.

5. بيّن أن المستطيل ذي المساحة العظمى الذي محيطه قيمة ثابتة P بشكل مربع دائما.

5.
$$A = xy$$
$$P = 2x + 2y$$
$$2y = P - 2x$$
$$y = \frac{P}{2} - x$$
$$A(x) = x\left(\frac{P}{2} - x\right)$$
$$A'(x) = 1 \cdot \left(\frac{P}{2} - x\right) + x(-1)$$
$$= \frac{P}{2} - 2x = 0$$
$$P = 4x$$
$$x = \frac{P}{4}$$
$A'(x) > 0$ for $0 < x < P/4$
$A'(x) < 0$ for $x > P/4$

So $x = P/4$ is max,
$$y = \frac{P}{2} - x = \frac{P}{2} - \frac{P}{4} = \frac{P}{4}$$
So the dimensions are $\frac{P}{4} \times \frac{P}{4}$. Thus we have a square.

6. بيّن أن المستطيل ذي المحيط الأصغر ومساحته قيمة ثابتة A بشكل مربع دائما.

6. We have a rectangle with sides x and y and area $A = xy$, and that we wish to minimize the perimeter,

$$P = 2x + 2y = 2x + 2 \cdot \frac{A}{x}.$$

$$P' = 2 - \frac{2A}{x^2} = 0 \text{ when } x = \sqrt{A}.$$

$P'' = 4A/x^3 > 0$ here, so this is a minimum. Possible values of x range from 0 to ∞. As x approaches these values the perimeter grows without bound. For fixed area, the rectangle with minimum perimeter has dimensions $x = y = \sqrt{A}$, a square.

7. يجب بناء صندوق مفتوح من الأعلى بواسطة لوح من الورق المقوى أبعاده 10 in - 6 in وذلك بقص مربعات قياس ضلعها x in من كل زاوية وطي الجوانب. جد قيمة x التي تحقق القيمة العظمى للصندوق.

7. $V = l \cdot w \cdot h$

$V(x) = (10 - 2x)(6 - 2x) \cdot x, \; 0 \leq x \leq 3$

$$\begin{aligned} V'(x) &= -2(6 - 2x) \cdot x + (10 - 2x)(-2) \cdot x \\ &\quad + (10 - 2x)(6 - 2x) \\ &= 60 - 64x + 12x^2 \\ &= 4(3x^2 - 16x + 15) \\ &= 0 \end{aligned}$$

$$x = \frac{16 \pm \sqrt{(-16)^2 - 4 \cdot 3 \cdot 15}}{6} = \frac{8}{3} \pm \frac{\sqrt{19}}{3}$$

$$x = \frac{8}{3} + \frac{\sqrt{19}}{3} > 3.$$

$V'(x) > 0$ for $x < 8/3 - \sqrt{19}/3$

$V'(x) < 0$ for $x > 8/3 - \sqrt{19}/3$

So $x = \dfrac{8}{3} - \dfrac{\sqrt{19}}{3}$ is a max.

8. يجب بناء صندوق مفتوح من الأعلى بأخذ لوح من الورق المقوى مساحته 12 in –في– 16 in وقص مربعات مساحة كل منها x in^2 من كل زاوية وطي الجوانب. جـــد قيمة x تحقق القيمةالعظمى لحجم الصندوق.

8. If we cut squares out of the corners of a 12" by 16" sheet and fold it into a box, the volume of the resulting box will be

$$V = x(12 - 2x)(16 - 2x)$$
$$= 4x^3 - 56x^2 + 192x,$$

where the value of x must be between 0 and 6.
$V' = 12x^2 - 112x + 192 = 0$
when $x = \frac{14 \pm 2\sqrt{13}}{3} \approx 7.07$ and 2.26. The critical value $x = \frac{14+2\sqrt{13}}{3}$ is outside of the reasonable range. The volume is 0 when x is 0 or 6. The First Derivative Test shows that $x = \frac{14-2\sqrt{13}}{3}$ gives the maximum volume.

9. (a) تم بناء صندوق مفتوح من الأعلى بأخذ قطعة من الورق المقوى مساحتها 6 in–في–6 in وقص مربعات بحجم x-in من كل زاوية وطي الجوانب. ثم تم لصق المربعات الأربعة بمساحة x in^2 معًا لتشكل صندوقًا ثانيًا (مفتوح من الأعلى أو سفلي).
جـــد قيمة x تحقق القيمة العظمى لأحجام الصناديق.
(b) كرر المسألة بدءًا بقطعة من الورق المقوى مساحتها 4 in–في–6 in

9. (a) $V = l \cdot w \cdot h$

The volume of the first box (without top) is

$V_1 = V_1(x) = (6-2x)^2(x) = 4x(3-x)^2$

where $0 < x < 3$. The volume of the second box (without top and bottom) is

$V_2 = V_2(x) = x^3$.

Thus, we find the absolute maximum of

the continuous function
$V = V(x) = V_1(x) + V_2(x) = 4x(3-x)^2 + x^3$
on the interval $0 < x < 3$.

$$V'(x) = 4(3-x)^2 + 4x\left(2\left(3-x\right)(-1)\right) + 3x^2$$
$$= 4\left(9 - 6x + x^2\right) - 8x\left(3-x\right) + 3x^2$$
$$= 15x^2 - 48x + 36$$
$$= (x-2)(15x-18)$$

Now compare the value of the function at the critical points.

$V(1.2) = 17.28$

$V(2) = 16$

Therefore, the value $x = 1.2$ maximizes the sum of volumes of the boxes.

(b) The volume of the first box (without top) is
$V_1 = V_1(x) = (6-2x)(4-2x)(x)$
$= 4x(3-x)(2-x)$, where $0 < x < 2$
The volume of the second box (without top and bottom) is
$V_2 = V_2(x) = x^3$.
Thus, we find the absolute maximum of the continuous function
$V = V(x) = V_1(x) + V_2(x)$
$= 4x(3-x)(2-x) + x^3$, on the interval $0 < x < 2$.
We have,

$$V'(x) = 4\left(3-x\right)\left(2-x\right) + 4x\left(2-x\right)(-1)$$
$$+ 4x\left(3-x\right)(-1) + 3x^2$$
$$= 4\left(6 - 5x + x^2\right) - 4x\left(2-x\right)$$
$$- 4x\left(3-x\right) + 3x^2$$
$$= 15x^2 - 40x + 24$$

Now compare the value of the function at the critical points.

$V(0.91169) = 9.0$

$V(1.75496) = 5.4$

Therefore, $x = 0.91169$ maximizes the sum of volumes of the boxes.

حل مسائل رياضية وحياتية على المعدلات المرتبطة

ص 303 من 1 –13

1. يتسرب النفط من ناقلة النفط بمعدل 120 gl/min. ينتشر النفط في دائرة بسمك $\frac{1}{4}''$. نظرًا لأن $1\ ft^3$ يساوي 7.5 براميل، حدد معدل تزايد نصف قطر التسرب عند وصول نصف القطر إلى 100 m (a) و 200 m (b) اشرح سبب تناقص المعدل بتزايد نصف القطر.

3.8 Related Rates

1. $V(t) = \text{(depth)(area)} = \frac{\pi}{48}[r(t)]^2$
(units in cubic feet per min)
$V'(t) = \frac{\pi}{48} 2r(t)r'(t) = \frac{\pi}{24} r(t)r'(t)$
We are given $V'(t) = \frac{120}{7.5} = 16$.
Hence $16 = \frac{\pi}{24} r(t)r'(t)$ so
$r'(t) = \frac{(16)(24)}{\pi r(t)}$.

(a) When $r = 100$,
$$r'(t) = \frac{(16)(24)}{100\pi} = \frac{96}{25\pi} \approx 1.2223 \text{ ft/min},$$

(b) When $r = 200$,
$$r'(t) = \frac{(16)(24)}{200\pi} = \frac{48}{25\pi} \approx 0.61115 \text{ ft/min}$$

2. يتسرب النفط من ناقلة النفط بمعدل 90 gl/min. ينتشر النفط في دائرة بسمك $\frac{1}{8}''$. حدد معدل تزايد نصف قطر التسرب عند وصول نصف القطر إلى 100 ft

2. $V = (\text{depth})(\text{area})$. $\frac{1}{8}'' = \frac{1}{96}'$, so
$V(t) = \frac{1}{96}\pi r(t)^2$.
Differentiating we find $\frac{dV}{dt} = \frac{2\pi}{96} r(t)\frac{dr}{dt}$.
Using 1 ft^3 = 7.5 gal, the rate of change of volume is $\frac{90}{7.5} = 12$. So when $r(t) = 100$,
$12 = \frac{2\pi}{96} 100 \frac{dr}{dt}$, and
$\frac{dr}{dt} = \frac{144}{25\pi}$ feet per minute.

3. يتسرب النفط من ناقلة النفط بمعدل g برميل في الدقيقة. ينتشر النفط في دائرة بسمك $\frac{1}{4}''$. (a). على فرض أن نصف قطر التسرب يتزايد بمعدل 0.6 m/min عندما يساوي نصف القطر 100 ft فحدد قيمة g. (b). إذا تضاعف سمك النفط. فكيف يتغير معدل تزايد نصف القطر؟

3. (a) From #1,
$V'(t) = \frac{\pi}{48} 2r(t)r'(t) = \frac{\pi}{24} r(t)r'(t)$,
so $\frac{g}{7.5} = \frac{\pi}{24}(100)(.6) = 2.5\pi$,
so $g = (7.5)(2.5)\pi$
$= 18.75\pi \approx 58.905$ gal/min.

(b) If the thickness is doubled, then the rate of change of the radius is halved.

4. على فرض أن المنطقة المصابة بإصابة ما دائرية. (a). فإذا كان نصف قطر المنطقة المصابة 3 mm وتزداد بمعدل 1 mm/hr. فما هو معدل تزايد المنطقة المصابة؟ (b) جـــد معدل تزايد المنطقة المصابة عند وصول نصف القطر إلى 6 mm. اشرح بمنطق سليم سبب كون هذا المعدل أكبر من معدل الجزء (a).

4. (a) t = hours elapsed since injury
r = radius of the infected area
A = area of the infection
$A = \pi r^2$
$A'(t) = 2\pi r(t) \cdot r'(t)$
When $r = 3$ mm, $r' = 1$ mm/hr,
$A' = 2\pi(3)(1) = 6\pi$ mm^2/hr

(b) We have $A'(t) = 2\pi r r'(t)$, and $r'(t) = 1$ mm/hr, so when the radius is 6 mm we have
$A'(t) = 2\pi \cdot 6 \cdot 1 = 12\pi$ mm^2/hr.
This rate is larger when the radius is larger because the area is changing by the same amount along the entire circumference of the circle. When the radius is larger, there is more circumerence, so the same change in radius causes a larger change in area.

5. على فرض أن قطرة مطر تتبخر بطريقة تحافظ معها على شكلها الكروي. علماً ان حجم شكل كروي بنصف قطر r هو $V = \frac{4}{3}\pi r^3$ وأن مساحة سطحه هي $A = 4\pi r^2$. فإذا تغير نصف القطر مع الزمن، واصبح الحجم $V' = Ar'$. إذا كان معدل التبخر (V') يتناسب مع مساحة السطح. بيّن أن نصف القطر يتغير بمعدل ثابت.

5. $V(t) = \frac{4}{3}\pi[r(t)]^3$

$V'(t) = 4\pi[r(t)]^2 r'(t) = Ar'(t)$

If $V'(t) = kA(t)$, then

$$r'(t) = \frac{V'(t)}{A(t)} = \frac{kA(t)}{A(t)} = k.$$

6. على فرض أن حريق غابات ينتشر في دائرة بنصف قطر يتغير بمعدل 5 ft/min عندما يصل نصف القطر إلى 200 ft. فما هو معدل تزايد مساحة المنطقة المحترقة؟

6. We have $A'(t) = 2\pi r r'(t)$, and $r'(t) = 5$ ft/min, so when the radius is 200 ft we have $A'(t) = 2\pi \cdot 200 \cdot 5 = 2,000\pi$ ft^2/min.

7. يرتكز سلم بطول 10 ft على جانب المبنى كما في المثال 8.2. فإذا تم سحب الجزء السفلي من السلم بعيدًا عن الجدار بمعدل 3 ft/s وبقي السلم ملامسًا للجدار.(a) جـد المعدل الذي يسقط به الجزء العلوي من السلم عندما يكون الجزء السفلي بعيدًا بمقدار 6 ft عن الجدار. (b). جـد معدل تغير الزاوية بين السلم وسطح الأرض عندما يبعد أسفل السلم 6 ft من الجدار.

7. (a) $10^2 = x^2 + y^2$

$$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$= -\frac{6}{8}(3)$$

$$= -2.25 \text{ ft/s}$$

(b) We have

$$\cos\theta(t) = \frac{x(t)}{10}.$$

Differentiating with respect to t gives

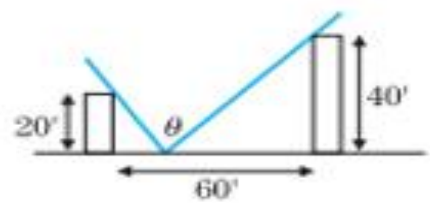
$$-\sin\theta(t)\cdot\theta'(t) = \frac{x'(t)}{10}.$$

When the bottom is 6 feet from the wall, the top of the ladder is 8 feet from the floor and this distance is the opposite side of the triangle from *theta*. Thus, at this point, $\sin\theta = 8/10$. So

$$-\frac{8}{10}\theta'(t) = \frac{3}{10}$$

$$\theta'(t) = -\frac{3}{8} \text{ rad/s.}$$

8. مبنيان ارتفاعهما 20 ft و 40 ft على التوالي. والمسافة بينهما 60 ft على فرض أن شدة الضوء في نقطة معينة بين المبنيين تتناسب طرديا مع الزاوية θ في الشكل (a). إذا تحرك شخص ما من اليمين إلى اليسار بمعدل 4 ft/s. فما معدل تغير θ عندما يكون الشخص في منتصف المسافة بين المبنيين بالضبط ؟ (b) جـــد الموقع الذي يكون قياس الزاوية θ أكبر ما يمكن.



8. (a) $\theta = \pi - \tan^{-1}\left(\frac{40}{60-x}\right) - \tan^{-1}\left(\frac{20}{x}\right)$

$$\frac{d\theta}{dx} = -\frac{40\left(\frac{1}{60-x}\right)^2}{1+\left(\frac{40}{60-x}\right)^2} + \frac{\frac{20}{x^2}}{1+\left(\frac{20}{x}\right)^2}$$

When $x = 30$, this becomes

$$\frac{d\theta}{dx} = -\frac{40\left(\frac{1}{30}\right)^2}{1+\left(\frac{40}{30}\right)^2} + \frac{\frac{20}{900}}{1+\left(\frac{20}{30}\right)^2}$$

$$= -\frac{1}{1625}\ \text{rad/ft}$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dx}\frac{dx}{dt}$$

$$= \left(-\frac{1}{1625}\right)(4)$$

$$\approx -0.00246\ \text{rad/s}$$

(b) As in the solution to #8(a), let x be the distance from the $20'$ building to the person. To find the maximum θ, we set $\frac{d\theta}{dx} = 0$ and solve for x:

$$0 = -\frac{40\left(\frac{1}{60-x}\right)^2}{1+\left(\frac{40}{60-x}\right)^2} + \frac{\frac{20}{x^2}}{1+\left(\frac{20}{x}\right)^2}$$

$$\frac{20}{x^2+40} = \frac{40}{(60-x)^2+1}$$

$$0 = 20x^2 + 2400x - 56000$$

$$0 = x^2 + 120x - 2800$$

Using the quadratic formula, we find two roots:

$x = -60 \pm 80$

We discard the x value obtained from the minus sign as it is negative and does not make sense for our problem. The other value is $x = 20$. We find $\theta'(10) > 0$ and $\theta'(30) < 0$, so $x = 20$ must be a maximum as desired.

.9 تقع طائرة على بعد $x = 40$ mi (أفقيًا). عن المطار وارتفاع h كيلومتر مكعب. يوجد رادار في المطار $s(t)$ يكشف المسافة بين الطائرة والمطار ويتغير بمعدل $s'(t) = -240$. (a) -240 mph إذا حلقت الطائرة نحو المطار بارتفاع ثابت $h = 4$. فما هي السرعة $|x'(t)|$ للطائرة؟ (b) كرر العملية بارتفاع 6 mi. استنادًا إلى إجاباتك. ما أهمية معرفة الارتفاع الفعلي للطائرة؟

9. (a) We know $[x(t)]^2 + 4^2 = [s(t)]^2$. Hence $2x(t)x'(t) = 2s(t)s'(t)$, so
$x'(t) = \dfrac{s(t)s'(t)}{x(t)} = \dfrac{-240s(t)}{x(t)}$. When $x = 40$, $s = \sqrt{40^2 + 4^2} = 4\sqrt{101}$, so at that moment
$x'(t) = \dfrac{(-240)(4\sqrt{101})}{40} = -24\sqrt{101}$.
So the speed is $24\sqrt{101} \approx 241.2$mph.

(b) From #9(a), we have
$x'(t) = \dfrac{s(t)s'(t)}{x(t)} = \dfrac{-240s(t)}{x(t)}$.
This time the height is 6 miles, so $s = \sqrt{40^2 + 6^2} = 2\sqrt{409}$, so at that moment
$x'(t) = \dfrac{(-240)(2\sqrt{409})}{40} = -12\sqrt{409}$.

So the speed is $12\sqrt{409} \approx 242.7$mph. The difference in height does not make a large difference in the speed of the plane.

.10 (a) أعد صياغة المثال 8.3 إذا كانت سيارة الشرطة لا تتحرك. هل هذا يجعل قياس الرادار أكثر دقة؟ (b) بيّن أن الرادار المذكور في المثال 8.3 يحدد السرعة الصحيحة إذا كانت سيارة الشرطة تقع في نقطة الأصل.

10. (a) If the police car is not moving, then $x'(t) = 0$, but all the other data are unchanged. So

$$d'(t) = \frac{x(t)x'(t) + y(t)y'(t)}{\sqrt{[x(t)]^2 + [y(t)]^2}}$$
$$= \frac{-(1/2)(50)}{\sqrt{1/4 + 1/16}}$$
$$= \frac{-100}{\sqrt{5}} \approx -44.721.$$

This is more accurate.

(b) If the police car is at the intersection, then the rate of change the police car measures is

$$\frac{0 \cdot (-40) + \frac{1}{2} \cdot (-50)}{\sqrt{\frac{1}{4} + 0}} = -50,$$

the true speed of the car.

11. بين أن الرادار المذكور في المثال 8.3 يحدد السرعة الصحيحة $x = \frac{1}{2}$ إذا كانت سيارة الشرطة تتحرك بسرعة $50(\sqrt{2}-1)$ mph.

11. $$d'(t) = \frac{x(t)x'(t) + y(t)y'(t)}{\sqrt{[x(t)]^2 + [y(t)]^2}}$$
$$= \frac{-(1/2)(\sqrt{2}-1)(50) - (1/2)(50)}{\sqrt{1/4 + 1/4}}$$
$$= -50.$$

12. جـد موقع وسرعة الرادار المذكور في المثال 8.3 عندما تكون قراءته أبطأ من السرعة الفعلية.

12. The radar gun will read less than the actual speed if the police car is not at the intersection, and is travelling away from the intersection.

13. تنفق شركة صغيرة الآلاف سنويًا على الإعلانات، على فرض أن مبيعاتها السنوية AEDX بآلاف من الدراهم تساوي $s = 60 - 40e^{-0.05x}$. تتضح أعداد إعلاناتها السنوية في الثلاث سنوات الأخيرة في الجدول التالي.

13. From the table, we see that the recent trend is for advertising to increase by $2000 per year. A good estimate is then $x'(2) \approx 2$ (in units of thousands). Starting with the sales equation
$s(t) = 60 - 40e^{-0.05x(t)}$,
we use the chain rule to obtain
$$s'(t) = -40e^{-0.05x(t)}[-0.05x'(t)]$$
$$= 2x'(t)e^{-0.05x(t)}.$$
Using our estimate that $x'(2) \approx 2$ and since $x(2) = 20$, we get $s'(2) \approx 2(2)e^{-1} \approx 1.471$. Thus, sales are increasing at the rate of approximately $1471 per year.

مثال 9.7 نمذجة التيار الكهربائي في السلك

تتضمن الدارة الكهربائية المبينة في الشكل 4.103 مقاوم 14 أوم وأداة ومعايق 2 هنري، ومكثّف 0.05 – فاراد وبطارية إمداد 232 فولت من التيار المتردد المنمذج بالدالة المتذبذبة $232 \sin 2t$، حيث إن t تقاس بالثواني. فجـــد التيار في الدارة عند أي t.

الحل يمكن إثبات أن الشحنة في هذه الدارة تعطى بالدالة (باستخدام القوانين الكهربائية الأساسية)

$$Q(t) = 10e^{-5t} + 2te^{-2t} + 3\sin 2t - 7\cos 2t$$

فالتيار إذن

$$Q'(t) = -50e^{-5t} + 2e^{-2t} - 4te^{-2t} + 6\cos 2t + 14\sin 2t$$

الشكل 4.103

دارة كهربائية بسيطة

ص 314 سؤال 37

37. على فرض أن النمو السكاني وفقًا للمعادلة اللوجستية هو $p'(t) = 4p(t)[5 - p(t)]$. جـــد التعداد الســكاني الذي يصل فيه معدل النمو إلى القيمة العظمى.

37. The rate of population growth is given by
$f(p) = 4p(5 - p) = 4(5p - p^2)$
$f'(p) = 4(5 - 2p)$,
so the only critical number is $p = 2.5$. Since the graph of f is a parabola opening down, this must be a max.

ص 315 سؤال 38

38. على فرض أن النمو السكاني وفقًا للمعادلة اللوجستية هو $p'(t) = 2p(t)[7 - 2p(t)]$ جـــد التعداد السكاني الذي يصل فيه معدل النمو إلى القيمة العظمى.

38. The rate of growth $R = 2p(7 - 2p)$, so $R' = 14 - 8p = 0$ when $p = 7/4$. This is a maximum because $R'' = -8 < 0$.

إيجاد المساحة تحت المنحنى للدالة باستخدام المجاميع والنهايات

ص 341 مثال 2 – 3

المثال 3.2 إيجاد قيمة المساحة بدقة

أوجد المساحة تحت المنحنى $y = f(x) = 2x - 2x^2$ على الفترة $[0, 1]$.

الحل باستخدام n فترة جزئية متساوية الطول، يكون لدينا

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}$$

وبهذا، $x_0 = 0, x_1 = \frac{1}{n}$ ، $x_2 = x_1 + \Delta x = \frac{2}{n}$ وهكذا. إذًا، $x_i = \frac{i}{n}$ ، حيث $i = 0, 1, 2, \ldots, n$. من (3.1)، تساوي المساحة تقريبًا

$$\begin{aligned}
A \approx A_n &= \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\left(\frac{1}{n}\right) = \sum_{i=1}^{n}\left[2\frac{i}{n} - 2\left(\frac{i}{n}\right)^2\right]\left(\frac{1}{n}\right) \\
&= \sum_{i=1}^{n}\left[2\left(\frac{i}{n}\right)\left(\frac{1}{n}\right)\right] - \sum_{i=1}^{n}\left[2\left(\frac{i^2}{n^2}\right)\left(\frac{1}{n}\right)\right] \\
&= \frac{2}{n^2}\sum_{i=1}^{n} i - \frac{2}{n^3}\sum_{i=1}^{n} i^2 \\
&= \frac{2}{n^2}\frac{n(n+1)}{2} - \frac{2}{n^3}\frac{n(n+1)(2n+1)}{6} \qquad \text{من نظرية 2.1 (ii) و (iii)} \\
&= \frac{n+1}{n} - \frac{(n+1)(2n+1)}{3n^2} \\
&= \frac{(n+1)(n-1)}{3n^2}
\end{aligned}$$

بما أنه لدينا صيغة لـ A_n ، لأي n، فإنه يمكننا إيجاد قيم متعددة بسهولة.

$$A_{200} = \frac{(201)(199)}{3(40,000)} = 0.333325$$

$$A_{500} = \frac{(501)(499)}{3(250,000)} = 0.333332$$

أخيرًا، يمكننا إيجاد النهاية لـ A_n. لدينا

$$\lim_{n\to\infty} A_n = \lim_{n\to\infty}\frac{n^2-1}{3n^2} = \lim_{n\to\infty}\frac{1-1/n^2}{3} = \frac{1}{3}$$

المساحة الدقيقة في الشكل 5.8 هي 1/3 ، كما كنا نقدر. ■

في التمارين 11–14، استخدم مجموع ريمان ونهاية لإيجاد قيمة المساحة الدقيقة تحت المنحنى.

11. $y = x^2 + 1$ on (a) $[0, 1]$; (b) $[0, 2]$; (c) $[1, 3]$

a) $\Delta x = \frac{1}{n}$. We will use right endpoints as evaluation points, $x_i = \frac{i}{n}$.

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{i}{n}\right)^2 + 1\right] = \frac{1}{n^3}\sum_{i=1}^{n} i^2 + 1$$

$$= \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + 1$$

$$= \frac{8n^2 + 3n + 1}{6n^2}$$

Now to compute the exact area, we take the limit as $n \to \infty$:

$$A = \lim_{n\to\infty} A_n \quad = \lim_{n\to\infty} \frac{8n^2 + 3n + 1}{6n^2}$$

$$= \lim_{n\to\infty} \frac{8}{6} + \frac{3}{6n} + \frac{1}{6n^2} = \frac{4}{3}$$

(b) $\Delta x = \dfrac{2}{n}$. We will use right endpoints as evaluation points, $x_i = \dfrac{2i}{n}$.

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \frac{2}{n}\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^2 + 1\right]$$
$$= \frac{2}{n}\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^2 + 1\right]$$
$$= \frac{2}{n}\sum_{i=1}^{n}\left(\frac{2i}{n}\right)^2 + \frac{2}{n}\sum_{i=1}^{n} 1$$
$$= \frac{8}{n^3}\sum_{i=1}^{n} i^2 + 2$$
$$= \frac{8}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + 2$$
$$= \frac{8}{n^2}\left[\frac{(n+1)(2n+1)}{6}\right] + 2$$
$$= \frac{4}{3n^2}(2n^2 + 3n + 1) + 2$$
$$= \frac{14n^2 + 12n + 4}{3n^2}$$

Now, to compute the exact area, we take the limit as $n \to \infty$:

$$A = \lim_{n\to\infty} A_n$$
$$= \lim_{n\to\infty} \frac{14n^2 + 12n + 4}{3n^2}$$
$$= \frac{14}{3}$$

(c) $\Delta x = \frac{2}{n}$ We will use right endpoints as evaluation points, $x_i = 1 + \frac{2i}{n}$.

$$
\begin{aligned}
A_n &= \sum_{i=1}^{n} f(x_i)\,\Delta x \\
&= \sum_{i=1}^{n} (x_i{}^2 + 1)\left(\frac{2}{n}\right) \\
&= \frac{2}{n}\sum_{i=1}^{n}\left(\left(1+\frac{2i}{n}\right)^2 + 1\right) \\
&= \frac{2}{n}\sum_{i=1}^{n}\left(2+\frac{4i}{n}+\frac{4i^2}{n^2}\right) \\
&= 4 + \frac{8}{n^2}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^2 \\
&= 4 + \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right) \\
&\quad + \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) \\
&= 4 + \left(\frac{4n+4}{n}\right) + \left[\frac{8n^2+12n+4}{3n^2}\right]
\end{aligned}
$$

Now, to compute the exact area, we take the limit as $n \to \infty$:

$$
\begin{aligned}
A &= \lim_{n\to\infty} A_n \\
&= \lim_{n\to\infty}\left(4 + \frac{4n+4}{n} + \frac{8n^2+12n+4}{3n^2}\right)
\end{aligned}
$$

$$= 4 + 4 + \frac{8}{3} = \frac{32}{3}$$

12. $y = x^2 + 3x$ on (a) $[0, 1]$; (b) $[0, 2]$; (c) $[1, 3]$

12. (a) $\Delta x = \frac{1}{n}$. We will use right endpoints as evaluation points, $x_i = \frac{i}{n}$.

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{i}{n}\right)^2 + 3\left(\frac{i}{n}\right)\right]$$

$$= \frac{1}{n^3}\sum_{i=1}^{n} i^2 + \frac{3}{n^2}\sum_{i=1}^{n} i$$

$$= \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$+ \frac{3}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{11n^2 + 12n + 1}{6n^2}$$

Now to compute the exact area, we take the limit as $n \to \infty$:

$$A = \lim_{n\to\infty} A_n$$

$$= \lim_{n\to\infty} \frac{11n^2 + 12n + 1}{6n^2}$$

$$= \lim_{n\to\infty} \frac{11}{6} + \frac{12}{6n} + \frac{1}{6n^2} = \frac{11}{6}$$

(b) $\Delta x = \dfrac{2}{n}$. We will use right endpoints as evalution points, $x_i = \dfrac{2i}{n}$.

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \frac{2}{n}\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^2 + 3\left(\frac{2i}{n}\right)\right]$$
$$= \frac{8}{n^3}\sum_{i=1}^{n} i^2 + \frac{12}{n^2}\sum_{i=1}^{n} i$$
$$= \frac{8}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$
$$+ \frac{12}{n^2}\left[\frac{n(n+1)}{2}\right]$$
$$= \left[\frac{(8n^2+12n+4)}{3n^2}\right] + \left[\frac{6n+6}{n}\right]$$

Now, to compute the exact area, we take the limit as $n \to \infty$: $A = \lim\limits_{n\to\infty} A_n$

$$= \lim_{n\to\infty}\left(\frac{(8n^2+12n+4)}{3n^2} + \frac{6n+6}{n}\right)$$
$$= \frac{8}{3} + 6 = \frac{26}{3}$$

(c) $\Delta x = \frac{2}{n}$. We will use right endpoints as evalution points, $x_i = 1 + \frac{2i}{n}$.

$$
\begin{aligned}
A_n &= \sum_{i=1}^{n} f(x_i)\Delta x \\
&= \sum_{i=1}^{n} \left[x_i{}^2 + 3x_i\right] \frac{2}{n} \\
&= \frac{2}{n} \sum_{i=1}^{n} \left[\left(1 + \frac{2i}{n}\right)^2 + 3\left(1 + \frac{2i}{n}\right)\right] \\
&= \frac{2}{n} \sum_{i=1}^{n} \left(4 + \frac{10i}{n} + \frac{4i^2}{n^2}\right) \\
&= 8 + \frac{20}{n^2} \sum_{i=1}^{n} i + \frac{8}{n^3} \sum_{i=1}^{n} i^2 \\
&= 8 + \frac{20}{n^2}\left(\frac{n(n+1)}{2}\right) \\
&\quad + \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) \\
&= 8 + \frac{10}{n}(n+1) + \frac{4}{3n^2}\left(2n^2 + 3n + 1\right)
\end{aligned}
$$

Now, to compute the exact area, we take the limit as $n \to \infty$:

$$
\begin{aligned}
A &= \lim_{n\to\infty} A_n \\
&= \lim_{n\to\infty} \\
&\left[8 + \frac{10}{n}(n+1) + \frac{4}{3n^2}\left(2n^2 + 3n + 1\right)\right] \\
&= 8 + 10 + \frac{8}{3} = \frac{62}{3}
\end{aligned}
$$

13. $y = 2x^2 + 1$ on (a) $[0, 1]$; (b) $[-1, 1]$; (c) $[1, 3]$

13. (a) $\Delta x = \frac{1}{n}$. We will use right endpoints as evalution points, $x_i = \frac{i}{n}$.

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \frac{1}{n}\sum_{i=1}^{n}\left[2\left(\frac{i}{n}\right)^2 + 1\right]$$
$$= \frac{2}{n^3}\sum_{i=1}^{n} i^2 + 1$$
$$= \frac{2}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + 1$$
$$= \frac{(5n^2 + n + 1)}{3n^2}$$

Now, to compute the exact area, we take the limit as $n \to \infty$:

$$A = \lim_{n\to\infty} A_n$$
$$= \lim_{n\to\infty}\left[\frac{(5n^2 + n + 1)}{3n^2}\right] = \frac{5}{3}.$$

(b) $\Delta x = \frac{2}{n}$. We will use right endpoints as evalution points, $x_i = -1 + \frac{2i}{n}$.

$$A_n = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \sum_{i=1}^{n} \left(2{x_i}^2 + 1\right)\left(\frac{2}{n}\right)$$

$$= \frac{2}{n}\sum_{i=1}^{n}\left(2\left(-1 + \frac{2i}{n}\right)^2 + 1\right)$$

$$= \frac{2}{n}\sum_{i=1}^{n}\left(3 - \frac{8i}{n} + \frac{8i^2}{n^2}\right)$$

$$= 6 - \frac{16}{n^2}\sum_{i=1}^{n} i + \frac{16}{n^3}\sum_{i=1}^{n} i^2$$

$$= 6 - \frac{16}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{16}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= 6 - \left(\frac{8n+8}{n}\right) + \left(\frac{16n^2 + 24n + 8}{3n^2}\right)$$

Now, to compute the exact area, we take the limit as $n \to \infty$:

$$A = \lim_{n\to\infty} A_n$$

$$= \lim_{n\to\infty}\left[6 - \left(\frac{8n+8}{n}\right) + \left(\frac{16n^2 + 24n + 8}{3n^2}\right)\right]$$

$$= 6 - 8 + \frac{16}{3} = \frac{10}{3}$$

(c) $\Delta x = \frac{2}{n}$. We will use right endpoints as evaluation points, $x_i = 1 + \frac{2i}{n}$.

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \frac{2}{n}\sum_{i=1}^{n} 2\left(1 + \frac{2i}{n}\right)^2 + 1$$

$$= \frac{2}{n}\sum_{i=1}^{n}\left(\frac{8i^2}{n^2} + \frac{8i}{n} + 3\right)$$

$$= \frac{16}{n^3}\sum_{i=1}^{n} i^2 + \frac{16}{n^2}\sum_{i=1}^{n} i + 6$$

$$= \frac{16}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{16}{n^2}\left(\frac{n(n+1)}{2}\right) + 6$$

$$= \frac{16n(n+1)(2n+1)}{6n^3} + \frac{16n(n+1)}{2n^2} + 6$$

Now to compute the exact area, we take the limit as $n \to \infty$:

$$A = \lim_{n\to\infty} A_n$$

$$= \lim_{n\to\infty}\left(\frac{16n(n+1)(2n+1)}{6n^3} + \frac{16n(n+1)}{2n^2} + 6\right)$$

$$= \lim_{n\to\infty} \frac{32}{6} + \frac{16}{2} + 6 \quad = \frac{58}{3}$$

14. $y = 4x^2 - x$ on (a) $[0, 1]$; (b) $[-1, 1]$; (c) $[1, 3]$

14. (a) $\Delta x = \frac{1}{n}$. We will use right endpoints as evalution points, $x_i = \frac{i}{n}$.

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} (4x_i^2 - x_i)\frac{1}{n}$$
$$= \frac{1}{n}\sum_{i=1}^{n}\left[4\left(\frac{i}{n}\right)^2 - \left(\frac{i}{n}\right)\right]$$
$$= \frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{4i^2}{n^2} - \frac{i}{n}\right)\right]$$
$$= \frac{4}{n}\sum_{i=1}^{n}\frac{i^2}{n^2} - \frac{1}{n}\sum_{i=1}^{n}\frac{i}{n}$$
$$= \frac{4}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{1}{n^2}\left(\frac{n(n+1)}{2}\right)$$
$$= \frac{2}{3n^2}(2n^2+3n+1) - \frac{1}{2n}(n+1)$$
$$= \frac{5}{6} + \frac{3}{2n} + \frac{2}{3n^2}$$

Now, to compute the exact area, we take the limit as $n \to \infty$:

$$A = \lim_{n\to\infty} A_n$$
$$= \lim_{n\to\infty}\left(\frac{5}{6} + \frac{3}{2n} + \frac{2}{3n^2}\right)$$
$$= \frac{5}{6}$$

(b) $\Delta x = \frac{2}{n}$. We will use right endpoints as evalution points, $x_i = -1 + \frac{2i}{n}$.

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left[4x_i^2 - x_i\right]\frac{2}{n}$$
$$= \frac{2}{n}\sum_{i=1}^{n}\left[4\left(-1+\frac{2i}{n}\right)^2 - \left(-1+\frac{2i}{n}\right)\right]$$

$$= \frac{2}{n}\sum_{i=1}^{n}\left(5 - \frac{18i}{n} + \frac{16i^2}{n^2}\right)$$

$$= \frac{10}{n}\sum_{i=1}^{n}1 - \frac{36}{n^2}\sum_{i=1}^{n}i + \frac{32}{n^3}\sum_{i=1}^{n}i^2$$

$$= 10 - \frac{36}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{32}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= 10 - \frac{18}{n}(n+1) + \frac{16}{3n^2}\left(2n^2+3n+1\right)$$

$$= \frac{8}{3} - \frac{2}{n} + \frac{16}{3n^2}$$

Now, to compute the exact area, we take the limit as $n \to \infty$:

$$A = \lim_{n\to\infty} A_n$$

$$= \lim_{n\to\infty}\left(\frac{8}{3} - \frac{2}{n} + \frac{16}{3n^2}\right)$$

$$= \frac{8}{3}$$

(c) $\Delta x = \frac{2}{n}$. We will use right endpoints as evaluation points $x_i = 1 + \frac{2i}{n}$.

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} \left[4{x_i}^2 - x_i\right] \frac{2}{n}$$

$$= \frac{2}{n}\sum_{i=1}^{n} \left[4\left(1 + \frac{2i}{n}\right)^2 - \left(1 + \frac{2i}{n}\right)\right]$$

$$= \frac{2}{n}\sum_{i=1}^{n} \left(3 + \frac{14i}{n} + \frac{16i^2}{n^2}\right)$$

$$= \frac{6}{n}\sum_{i=1}^{n} 1 + \frac{28}{n^2}\sum_{i=1}^{n} i + \frac{32}{n^3}\sum_{i=1}^{n} i^2$$

$$= 6 + \frac{28}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{32}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= 6 + \frac{14}{n}(n+1) + \frac{16}{3n^2}(2n^2 + 3n + 1)$$

$$= \frac{92}{3} + \frac{30}{n} + \frac{16}{3n^2}$$

Now, to compute the exact area, we take the limit as $n \to \infty$:

$$A = \lim_{n\to\infty} A_n$$

$$= \lim_{n\to\infty}\left(\frac{92}{3} + \frac{30}{n} + \frac{16}{3n^2}\right)$$

$$= \frac{92}{3}$$

التعرف على النظرية الأساسية الثانية للتفاضل والتكامل وتطبيقها

على دوال معرفة لإيجاد كتكاملات محدودة لإيجاد مشتقاتها

ص 366 من 25 – 32

في التمارين 25–32، جد المشتقة $f'(x)$.

25. $f(x) = \int_0^x (t^2 - 3t + 2)\, dt$

$$f'(x) = x^2 - 3x + 2$$

26. $f(x) = \int_2^x (t^2 - 3t - 4)\, dt$

$$f'(x) = x^2 - 3x - 4$$

27. $f(x) = \int_0^{x^2} (e^{-t^2} + 1)\, dt$

$$f'(x) = \left(e^{-(x^2)^2} + 1\right) \frac{d}{dx}(x^2)$$
$$= 2x\left(e^{-x^4} + 1\right)$$

28. $f(x) = \int_x^2 \sec t \, dt$

$f'(x) = -\sec x$

29. $f(x) = \int_{e^x}^{2-x} \sin t^2 \, dt$

$$f(x) = \int_{e^x}^{0} \sin t^2 \, dt + \int_0^{2-x} \sin t^2 \, dt$$

$$\begin{aligned} f'(x) &= -\sin e^{2x} \frac{d}{dx}(e^x) \\ &\quad + \sin (2-x)^2 \frac{d}{dx}(2-x) \\ &= -e^x \sin e^{2x} - \sin (2-x)^2 \end{aligned}$$

30. $f(x) = \int_{2-x}^{xe^x} e^{2t}\,dt$

$$f(x) = \int_{2-x}^{0} e^{2t}\,dt + \int_{0}^{xe^x} e^{2t}\,dt$$

$$f'(x) = -e^{2(2-x)}\frac{d}{dx}(2-x) + e^{2(xe^x)}\frac{d}{dx}(xe^x) = e^{4-2x} + e^{2xe^x}(xe^x + e^x)$$

31. $f(x) = \int_{x^2}^{x^3} \sin(3t)\,dt$

$$f(x) = \int_{x^2}^{0} \sin(2t)\,dt + \int_{0}^{x^3} \sin(2t)\,dt$$

$$f'(x) = -\sin(2x^2)\frac{d}{dx}(x^2) + \sin(2x^3)\frac{d}{dx}(x^3)$$

$$= -2x\sin(2x^2) + 3x^2\sin(2x^3)$$

32. $f(x) = \int_{3x}^{\sin x} (t^2 + 4)\, dt$

$$f(x) == \int_{3x}^{0} (t^2 + 4)\, dt + \int_{0}^{\sin x} (t^2 + 4)\, dt$$

$$= -\int_{0}^{3x} (t^2 + 4)\, dt + \int_{0}^{\sin x} (t^2 + 4)\, dt$$

$$f'(x) = -(9x^2 + 4) \frac{d}{dx}(3x)$$

$$+ (\sin^2 x + 4) \frac{d}{dx}(\sin x)$$

$$= -27x^2 - 12 + \sin^2 x \cos x + 4\cos x$$