حلول كراسة تمارين التكامل بالتعويض الاستبدال في التفاضل والتكامل

General Steps:

1. Choose a new variable u (innermost expression).
2. Compute $du = \frac{du}{dx} dx$.
3. Replace all terms with u and du. Evaluate integral.
4. Replace u with corresponding x expression.

Choosing the correct substitution (u):

A) Brackets: $(x^2+1)^5 \to u = x^2+1$

B) Roots: $\sqrt{3x^2+2} \to u = 3x^2+2$

C) Exponents: $e^{x^2+2} \to u = x^2+2$

D) Angles: $\sin(3x^4) \to u = 3x^4$

I. Evaluate the Indefinite Integrals

1 $\int x^2(x^3+2)^{100} dx$

$u = x^3+2 \implies du = 3x^2 dx$

$= \frac{1}{3}\int u^{100} du$

$= \frac{1}{3}\frac{u^{101}}{101} + c$

$= \frac{(x^3+2)^{101}}{303} + c$

2 $\int (3x+4)^7 dx$

Linear inner function $(ax+b)$:

$= \frac{(3x+4)^8}{8 \cdot 3} + c$

$= \frac{(3x+4)^8}{24} + c$

3 $\int x\sin x^2 dx$

$u = x^2 \implies du = 2x dx$

$= \frac{1}{2}\int \sin(u) du$

$= -\frac{1}{2}\cos(u) + c$

$= -\frac{1}{2}\cos(x^2) + c$

4 $\int (3\tan x + 4)^5 \sec^2 x dx$

$u = 3\tan x + 4 \implies du = 3\sec^2 x dx$

$= \frac{1}{3}\int u^5 du$

$= \frac{1}{3}\frac{u^6}{6} + c$

$= \frac{(3\tan x+4)^6}{18} + c$

5 $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

$u = \sqrt{x} \implies du = \frac{1}{2\sqrt{x}} dx$

$= 2\int \sin(u) du$

$= -2\cos(u) + c$

$= -2\cos\sqrt{x} + c$

6 $\int \frac{x^5}{1+x^6} dx$

$u = 1+x^6 \implies du = 6x^5 dx$

$= \frac{1}{6}\int \frac{1}{u} du$

$= \frac{1}{6}\ln|u| + c$

$= \frac{1}{6}\ln(1+x^6) + c$

7 $\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}}dx$

$$u = \sin^{-1} x \implies du = \frac{1}{\sqrt{1-x^2}}dx$$

$$= \int u^3 du$$

$$= \frac{u^4}{4} + c$$

$$= \frac{(\sin^{-1} x)^4}{4} + c$$

8 $\int x\sqrt{2-x}dx$

$$u = 2 - x \implies x = 2 - u,\ dx = -du$$

$$= -\int (2-u)\sqrt{u}du$$

$$= \int (u^{3/2} - 2u^{1/2})du$$

$$= \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + c$$

$$= \frac{2}{5}(2-x)^{5/2} - \frac{4}{3}(2-x)^{3/2} + c$$

9 $\int \frac{x^3}{\sqrt{4-x^4}}dx$

$$u = 4 - x^4 \implies du = -4x^3dx$$

$$= -\frac{1}{4}\int u^{-1/2}du$$

$$= -\frac{1}{4}(2u^{1/2}) + c$$

$$= -\frac{1}{2}\sqrt{4-x^4} + c$$

10 $\int \frac{(\sqrt{x}+1)^4}{\sqrt{x}}dx$

$$u = \sqrt{x} + 1 \implies du = \frac{1}{2\sqrt{x}}dx$$

$$= 2\int u^4 du$$

$$= \frac{2}{5}u^5 + c$$

$$= \frac{2}{5}(\sqrt{x}+1)^5 + c$$

11 $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)}dx$

$$u = \sqrt{x} + 1 \implies du = \frac{1}{2\sqrt{x}}dx$$

$$= 2\int \frac{1}{u}du$$

$$= 2\ln|u| + c$$

$$= 2\ln(\sqrt{x}+1) + c$$

12 $\int \frac{1}{\sqrt{1+\sqrt{x}}}dx$

(Hint: Let $1 + \sqrt{x} = u$)

$$u = 1 + \sqrt{x} \implies dx = 2(u-1)du$$

$$= \int \frac{2(u-1)}{\sqrt{u}}du$$

$$= 2\int (u^{1/2} - u^{-1/2})du$$

$$= 2\left(\frac{2}{3}u^{3/2} - 2u^{1/2}\right) + c$$

$$= \frac{4}{3}(1+\sqrt{x})^{3/2} - 4(1+\sqrt{x})^{1/2} + c$$

13 $\int \frac{3}{\sqrt[4]{x}+x}dx$

$$u = x^{1/4} \implies dx = 4u^3du$$

$$= \int \frac{12u^3}{u+u^4}du = \int \frac{12u^2}{1+u^3}du$$

$$m = 1 + u^3 \implies dm = 3u^2du$$

$$= 4\int \frac{1}{m}dm = 4\ln(1+x^{3/4}) + c$$

14 $\int \frac{3\sqrt{x}}{1+x^3}dx$

$$u = x^{3/2} \implies u^2 = x^3,\ du = \frac{3}{2}\sqrt{x}dx$$

$$= 2\int \frac{1}{1+u^2}du$$

$$= 2\tan^{-1}(u) + c$$

$$= 2\tan^{-1}(x^{3/2}) + c$$

15 $\int \frac{1}{\sqrt{x}(1+x)}dx$

$u=\sqrt{x} \implies u^2=x,\ du=\frac{1}{2\sqrt{x}}dx$

$=2\int \frac{1}{1+u^2}du$

$=2\tan^{-1}(u)+c$

$=2\tan^{-1}(\sqrt{x})+c$

16 $\int xe^{x^2}dx$

$u=x^2 \implies du=2xdx$

$=\frac{1}{2}\int e^u du$

$=\frac{1}{2}e^u+c$

$=\frac{1}{2}e^{x^2}+c$

17 $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

$u=\sqrt{x} \implies du=\frac{1}{2\sqrt{x}}dx$

$=2\int e^u du$

$=2e^u+c$

$=2e^{\sqrt{x}}+c$

18 $\int \frac{\sqrt{\ln x}}{x}dx$

$u=\ln x \implies du=\frac{1}{x}dx$

$=\int u^{1/2}du$

$=\frac{2}{3}u^{3/2}+c$

$=\frac{2}{3}(\ln x)^{3/2}+c$

19 $\int \frac{\ln \sqrt{x}}{x}dx$

$\frac{\ln x^{1/2}}{x}=\frac{\frac{1}{2}\ln x}{x}$

$u=\ln x \implies du=\frac{1}{x}dx$

$=\frac{1}{2}\int udu=\frac{1}{4}u^2+c$

$=\frac{1}{4}(\ln x)^2+c$

20 $\int e^x\sqrt{e^x+4}dx$

$u=e^x+4 \implies du=e^x dx$

$=\int u^{1/2}du$

$=\frac{2}{3}u^{3/2}+c$

$=\frac{2}{3}(e^x+4)^{3/2}+c$

21 $\int \frac{\cos(1/x)}{x^2}dx$

$u=1/x \implies du=-\frac{1}{x^2}dx$

$=-\int \cos(u)du$

$=-\sin(u)+c$

$=-\sin(1/x)+c$

22 $\int \frac{x}{\sqrt{1-x^4}}dx$

(Hint: Let $u=x^2$)

$u=x^2 \implies u^2=x^4,\ du=2xdx$

$=\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}du$

$=\frac{1}{2}\sin^{-1}(u)+c$

$=\frac{1}{2}\sin^{-1}(x^2)+c$

23 $\int \frac{1+x}{1-x^2}dx$

(Hint: Factor denominator)

$$\frac{1+x}{(1-x)(1+x)} = \frac{1}{1-x}$$

$$= \int \frac{1}{1-x}dx$$

$$= -\ln|1-x| + c$$

24 $\int \frac{1+x}{1+x^2}dx$

$$= \int \frac{1}{1+x^2}dx + \int \frac{x}{1+x^2}dx$$

$$= \int \frac{1}{1+x^2}dx + \frac{1}{2}\int \frac{2x}{1+x^2}dx$$

$$= \tan^{-1}(x) + \frac{1}{2}\ln(1+x^2) + c$$

25 $\int \sin^3 x \cos x dx$

$$u = \sin x \implies du = \cos x dx$$

$$= \int u^3 du$$

$$= \frac{u^4}{4} + c$$

$$= \frac{\sin^4 x}{4} + c$$

26 $\int \frac{e^{\tan x}}{1-\sin^2 x}dx$

$$1 - \sin^2 x = \cos^2 x$$

$$\frac{1}{\cos^2 x} = \sec^2 x$$

$$= \int e^{\tan x}\sec^2 x dx \implies u = \tan x$$

$$= \int e^u du$$

$$= e^{\tan x} + c$$

27 $\int x^2 \csc^2 x^3 dx$

$$u = x^3 \implies du = 3x^2 dx$$

$$= \frac{1}{3}\int \csc^2 u du$$

$$= -\frac{1}{3}\cot(u) + c$$

$$= -\frac{1}{3}\cot(x^3) + c$$

28 $\int \tan 4x dx$

$$= \int \frac{\sin 4x}{\cos 4x}dx$$

$$u = \cos 4x \implies du = -4\sin 4x dx$$

$$= -\frac{1}{4}\int \frac{1}{u}du$$

$$= -\frac{1}{4}\ln|\cos 4x| + c$$

29 $\int \frac{\ln(\sin x)}{\tan x}dx$

$$u = \ln(\sin x) \implies du = \frac{\cos x}{\sin x}dx = \frac{1}{\tan x}dx$$

$$= \int u du$$

$$= \frac{u^2}{2} + c$$

$$= \frac{1}{2}(\ln(\sin x))^2 + c$$

30 $\int \frac{\cos(\ln x)}{x}dx$

$$u = \ln x \implies du = \frac{1}{x}dx$$

$$= \int \cos(u)du$$

$$= \sin(u) + c$$

$$= \sin(\ln x) + c$$

31 $\int \sec^2 x\sqrt{1-2\tan x}\,dx$

$$u = 1 - 2\tan x \implies du = -2\sec^2 x dx$$

$$= -\frac{1}{2}\int u^{1/2} du$$

$$= -\frac{1}{3}u^{3/2} + c$$

$$= -\frac{1}{3}(1-2\tan x)^{3/2} + c$$

32 $\int \frac{x-1}{1+2x-x^2}dx$

$$u = 1 + 2x - x^2 \implies du = (2-2x)dx$$

$$du = -2(x-1)dx$$

$$= -\frac{1}{2}\int \frac{1}{u}du$$

$$= -\frac{1}{2}\ln|1+2x-x^2| + c$$

33 $\int \frac{x^2}{\sqrt{1-x^6}}dx$

(Let $u = x^3$)

$$u = x^3 \implies u^2 = x^6,\ du = 3x^2dx$$

$$= \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}du$$

$$= \frac{1}{3}\sin^{-1}(u) + c$$

$$= \frac{1}{3}\sin^{-1}(x^3) + c$$

34 $\int \frac{3x^2}{1+x^6}dx$

$$u = x^3 \implies u^2 = x^6,\ du = 3x^2dx$$

$$= \int \frac{1}{1+u^2}du$$

$$= \tan^{-1}(u) + c$$

$$= \tan^{-1}(x^3) + c$$

35 $\int \frac{x-2}{x+7}dx$

$$u = x + 7 \implies x = u - 7,\ dx = du$$

$$= \int \frac{u-9}{u}du$$

$$= \int \left(1 - \frac{9}{u}\right)du$$

$$= u - 9\ln|u| + c$$

$$= x + 7 - 9\ln|x+7| + c$$

36 $\int \cos(\tan 3x)\sec^2 3x dx$

$$u = \tan 3x \implies du = 3\sec^2 3x dx$$

$$= \frac{1}{3}\int \cos(u)du$$

$$= \frac{1}{3}\sin(u) + c$$

$$= \frac{1}{3}\sin(\tan 3x) + c$$

37 $\int \frac{x\sqrt{x}}{1+x^5}dx$

(Let $x^{5/2} = u$)

$$x\sqrt{x} = x^{3/2}$$

$$u = x^{5/2} \implies u^2 = x^5,\ du = \frac{5}{2}x^{3/2}dx$$

$$= \frac{2}{5}\int \frac{1}{1+u^2}du$$

$$= \frac{2}{5}\tan^{-1}(u) + c$$

$$= \frac{2}{5}\tan^{-1}(x^{5/2}) + c$$

38 $\int \sec^2 x\sqrt{\tan x}\,dx$

$$u = \tan x \implies du = \sec^2 x dx$$

$$= \int u^{1/2}du$$

$$= \frac{2}{3}u^{3/2} + c$$

$$= \frac{2}{3}(\tan x)^{3/2} + c$$

39 $\int \frac{(1+\sin x)^5}{\sec x}dx$

$\frac{1}{\sec x} = \cos x$

$u = 1+\sin x \implies du = \cos x dx$

$= \int u^5 du$

$= \frac{u^6}{6} + c$

$= \frac{1}{6}(1+\sin x)^6 + c$

40 $\int \frac{\sqrt{1-\sin^2 x}}{1+\sin^2 x}dx$

$\sqrt{\cos^2 x} = \cos x$

$u = \sin x \implies du = \cos x dx$

$= \int \frac{1}{1+u^2}du$

$= \tan^{-1}(u) + c$

$= \tan^{-1}(\sin x) + c$

41 $\int \frac{3}{(1+x^2)\tan^{-1} x}dx$

$u = \tan^{-1} x \implies du = \frac{1}{1+x^2}dx$

$= 3\int \frac{1}{u}du$

$= 3\ln|u| + c$

$= 3\ln|\tan^{-1} x| + c$

42 $\int \frac{2}{x^{2/3} - x^{5/6}}dx$

Challenge: Let $u = x^{1/6}$

$u = x^{1/6} \implies dx = 6u^5 du$

$= 12\int \frac{u}{1-u}du$

$= 12\int \left(-1 + \frac{1}{1-u}\right)du$

$= -12u - 12\ln|1-u| + c$

$= -12x^{1/6} - 12\ln|1-x^{1/6}| + c$

II. Evaluate Definite Integrals

43 $\int_1^2 x^3\sqrt{x^4+1}dx$

$u = x^4 + 1 \implies du = 4x^3 dx$

$x = 1 \to 2,\ x = 2 \to 17$

$= \frac{1}{4}\int_2^{17} u^{1/2}du$

$= \frac{1}{4}\left[\frac{2}{3}u^{3/2}\right]_2^{17}$

$= \frac{1}{6}(17\sqrt{17} - 2\sqrt{2})$

44 $\int_1^e \frac{\ln x}{x}dx$

$u = \ln x \implies du = \frac{1}{x}dx$

$x = 1 \to 0,\ x = e \to 1$

$= \int_0^1 u du$

$= \left[\frac{u^2}{2}\right]_0^1 = \frac{1}{2}$

45 $\int_1^e \frac{1}{x\ln x + x}dx$

$= \int_1^e \frac{1}{x(\ln x + 1)}dx$

$u = \ln x + 1 \implies du = \frac{1}{x}dx$

$x = 1 \to 1,\ x = e \to 2$

$= \int_1^2 \frac{1}{u}du$

$= [\ln u]_1^2 = \ln 2$

46 $\int_{-1}^1 \frac{t}{(1+t^2)^2}dt$

$f(-t) = \frac{-t}{(1+(-t)^2)^2}$

$= -f(t)$

Since $f(t)$ is an odd function

over a symmetric interval:

$= 0$

47 $\displaystyle\int_0^{\ln 2} \frac{e^t}{1+e^{2t}}dt$

$u = e^t \implies du = e^t dt$

$t = 0 \to 1,\ t = \ln 2 \to 2$

$= \displaystyle\int_1^2 \frac{1}{1+u^2}du$

$= [\tan^{-1} u]_1^2$

$= \tan^{-1}(2) - \frac{\pi}{4}$

48 $\displaystyle\int_0^{\ln 2} \frac{e^t}{1+e^t}dt$

$u = 1 + e^t \implies du = e^t dt$

$t = 0 \to 2,\ t = \ln 2 \to 3$

$= \displaystyle\int_2^3 \frac{1}{u}du$

$= [\ln u]_2^3$

$= \ln 3 - \ln 2 = \ln(1.5)$

III. Definite Integrals & Properties

49 If $\displaystyle\int_0^1 f(x)dx = 3$, Find $\displaystyle\int_0^{\pi/2} \cos x f(\sin x)dx$

$u = \sin x \implies du = \cos x dx$

$x = 0 \to 0,\ x = \pi/2 \to 1 \implies \displaystyle\int_0^1 f(u)du = 3$

50 If $\displaystyle\int_1^2 f(x)dx = 4$, Find $\displaystyle\int_1^4 \frac{f(\sqrt{x})}{\sqrt{x}}dx$

$u = \sqrt{x} \implies du = \frac{1}{2\sqrt{x}}dx \implies \frac{dx}{\sqrt{x}} = 2du$

$x = 1 \to 1,\ x = 4 \to 2 \implies 2\displaystyle\int_1^2 f(u)du = 2(4) = 8$

IV. Word Problems & Generalizations

51A If $\displaystyle\int_1^2 f(x)dx = 3$, find: $\displaystyle\int_2^4 f(\frac{x}{2})dx$

$A)\ u = x/2 \implies dx = 2du \implies 2\displaystyle\int_1^2 f(u)du = 2(3) = 6$

51B If $\displaystyle\int_1^2 f(x)dx = 3$, find: $\displaystyle\int_0^{\ln 2} e^x f(e^x)dx$

$B)\ u = e^x \implies du = e^x dx \implies \displaystyle\int_1^2 f(u)du = 3$

52A For $I = \displaystyle\int_0^{10} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}dx$, show that $I = \displaystyle\int_0^{10} \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$.

$u = 10 - x \implies x = 10 - u,\ dx = -du \quad (0 \to 10,\ 10 \to 0)$

$I = \displaystyle\int_{10}^0 \frac{\sqrt{10-u}}{\sqrt{10-u}+\sqrt{u}}(-du) = \int_0^{10} \frac{\sqrt{10-x}}{\sqrt{10-x}+\sqrt{x}}dx \quad \text{(Proven)}$

52B Generalize to $I = \displaystyle\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx$ for any positive a, then find $\displaystyle\int_0^5 \frac{f(x)}{f(x)+f(5-x)}dx$ and $\displaystyle\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}dx$.

$2I = \displaystyle\int_0^a \frac{f(x)+f(a-x)}{f(x)+f(a-x)}dx = \int_0^a 1dx = a \implies I = \frac{a}{2}$

First: $a = 5 \implies I = \frac{5}{2}$ Second: $a = \pi/2 \implies I = \frac{\pi}{4}$

53 **When a patient is undergoing surgery, he is injected with anesthesia, and after t hours the concentration of anesthetic in the patient's blood is $C(t) = \frac{2t}{\sqrt{(36+t^2)^3}} mg/cm^3$. Find the average concentration of anesthesia in the blood during the first eight hours after injection.**

$$\text{Avg} = \frac{1}{8}\int_0^8 \frac{2t}{(36+t^2)^{3/2}}dt$$

$$u = 36 + t^2 \implies du = 2tdt\ (0 \to 36,\ 8 \to 100)$$

$$= \frac{1}{8}\int_{36}^{100} u^{-3/2}du = \frac{1}{8}\left[-2u^{-1/2}\right]_{36}^{100} = -\frac{1}{4}\left(\frac{1}{10} - \frac{1}{6}\right) = \frac{1}{60}\ \text{mg/cm}^3$$

54 **The Weather station observed the temperature C in a city after midnight, so it was found that it can be modeled with as the following: $T(t) = 3 - \frac{1}{3}(t-5)^2$ °C where t is the time after midnight. Find the average temperature in the city from 10 AM to 3 PM.**

$$t_1 = 10, \quad t_2 = 15 \implies \text{Avg} = \frac{1}{5}\int_{10}^{15}\left(3 - \frac{1}{3}(t-5)^2\right)dt$$

$$= \frac{1}{5}\left[3t - \frac{1}{9}(t-5)^3\right]_{10}^{15} = \frac{1}{5}\left(15 - \frac{875}{9}\right) = -\frac{148}{9}{}^{\circ}C$$

55 **If both $F(x) = \frac{-1}{x^2+1}$ and $G(x) = \frac{x^2}{x^2+1}$ are antiderivatives of the same function $f(x)$, then the value of the constant C by which the two functions F and G differ is:**

A. $C = 1$

B. $C = -1$

C. $C = \pm 1$ ✓

D. Cannot be determined

$$c = G(x) - F(x)$$

$$= \frac{x^2}{x^2+1} - \frac{-1}{x^2+1} = \frac{x^2+1}{x^2+1} = 1$$

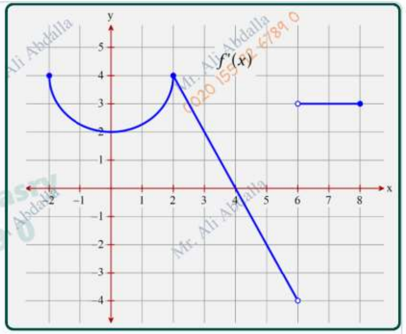
Or $c = F(x) - G(x)$

$$= \frac{-1}{x^2+1} - \frac{x^2}{x^2+1} = \frac{-1-x^2}{x^2+1} = \frac{-(1+x^2)}{x^2+1} = -1$$

$$\therefore c = \pm 1$$

V. Graphical Analysis

56 **The function f is continuous for all real values of x. A portion of the graph of the function f', the derivative of f, on $[-2, 8]$ is shown on the right and consists of a semicircle and two linear pieces.**



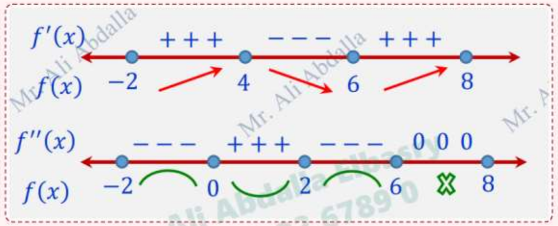
A) Find the x coordinate of each critical point of f on $[-2, 8]$. Classify each as a local max/min or neither.

B) Find the x coordinate of each point of inflection for f on $(-2, 6)$.

C) Find $\lim_{x \to 4} \dfrac{\int_2^x f'(t)dt - 4}{3(x-4)^2}$.

D) Evaluate: $\int_{-2}^{1} f'(4-2x)dx$ and $\int_{-2}^{3} f'(4-2x)dx$

E) Let $g(x) = f'(x) \cdot x^2$ find $g'(3)$



A) $f'(x) = 0 \implies x = 4 \text{ (Max)},\ x = 6 \text{ (Min)}$

B) Inflection at extrema of f' : $x = 0,\ x = 2$

C) L'Hôpital: $\lim_{x\to 4} \frac{f'(x)}{6(x-4)} = \lim_{x\to 4} \frac{f''(x)}{6} = \frac{f''(4)}{6} = \frac{-2}{6} = -\frac{1}{3}$

D) $u = 4 - 2x \implies dx = -\frac{1}{2}du \implies -\frac{1}{2}\int_8^2 f'(u)du = \frac{1}{2}\int_2^8 f'(u)du = \frac{1}{2}(4) = 2$

E) $g'(x) = f''(x)x^2 + 2xf'(x) \implies g'(3) = f''(3)(9) + 6f'(3) = (-2)(9) + 6(2) = -6$

VI. Shortcuts: Integrals Involving $(ax + b)$

Rule	Example
$\int (ax+b)^n \ dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + C$	$\int (3x-1)^2 \ dx = \dfrac{(3x-1)^3}{9} + C$
$\int (ax+b)^{-1} \ dx = \dfrac{\ln\lvert ax+b\rvert}{a} + C$	$\int (3-2x)^{-1} \ dx = -\dfrac{1}{2}\ln\lvert 3-2x\rvert + C$
$\int e^{ax+b} \ dx = \dfrac{e^{ax+b}}{a} + C$	$\int e^{-x+4} \ dx = -e^{-x+4} + C$
$\int k^{ax+b} \ dx = \dfrac{k^{ax+b}}{a \ln k} + C$ where k is constant	$\int 2^{-3x+4} \ dx = -\dfrac{2^{-3x+4}}{3\ln 2} + C$
$\int \sqrt{ax+b} \ dx = \dfrac{2(ax+b)^{3/2}}{3a}(ax+b)^{3/2} + C$	$\int \sqrt{2x+5} \ dx = \dfrac{(2x+5)^{3/2}}{3} + C$

VII. Challenge Training

T1 $\int x^8\left(2+\frac{3}{x}\right)^8 dx$

$$x^8\left(2+\frac{3}{x}\right)^8 = \left(x\left(2+\frac{3}{x}\right)\right)^8 = (2x+3)^8$$

$$= \int (2x+3)^8 dx = \frac{(2x+3)^9}{9\times 2} + c = \frac{(2x+3)^9}{18} + c$$

T2 $\int x^9\left(2-\frac{3}{x}\right)^8 dx$

$$= \int x\cdot x^8\left(2-\frac{3}{x}\right)^8 dx = \int x(2x-3)^8 dx$$

$$u = 2x-3 \implies x = \frac{u+3}{2},\ dx = \frac{du}{2}$$

$$= \int\left(\frac{u+3}{2}\right)u^8\frac{du}{2} = \frac{1}{4}\int(u^9+3u^8)du = \frac{(2x-3)^{10}}{40} + \frac{(2x-3)^9}{12} + c$$

VIII. Past Exams Questions

1 $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \ \dots + c$

- A. $\frac{1}{2e^{\sqrt{x}}}$
- B. $\frac{2}{e^{\sqrt{x}}}$
- C. $\frac{1}{2}e^{\sqrt{x}}$
- D. $2e^{\sqrt{x}}$ ✓

$$u = \sqrt{x} \implies du = \frac{1}{2\sqrt{x}}dx$$

$$2\int e^u du = 2e^{\sqrt{x}} + c$$

2 $\int x^3\sqrt{x^4+3}dx = \ \dots + c$

- A. $\frac{1}{2}(x^4+3)^{3/2}$
- B. $\frac{1}{6}x^4(x^5+3)^{3/2}$
- C. $\frac{1}{6}(x^4+3)^{1/2}$
- D. $\frac{1}{6}(x^4+3)^{3/2}$ ✓

$$u = x^4+3 \implies \frac{1}{4}\int u^{1/2}du$$

$$= \frac{1}{6}(x^4+3)^{3/2} + c$$

3 $\int \frac{3\sqrt{x}}{1+x^3}dx$

$$u = x^{3/2} \implies du = \frac{3}{2}\sqrt{x}dx$$

$$2\int\frac{1}{1+u^2}du = 2\tan^{-1}(x^{3/2}) + c$$

4 $\int x(x^2+1)^2 dx = \ \dots + c$

- A. $\frac{x^4}{4} + \frac{x^2}{2}$
- B. $\frac{x^6}{6} + \frac{x^4}{4} + \frac{x^2}{2}$ ✓
- C. $\frac{x^5}{5} + \frac{2x^3}{3} + x$
- D. $\frac{x^6}{6} + \frac{x^4}{2} + \frac{x^2}{2}$

$$u = x^2+1$$

$$\frac{1}{2}\int u^2 du = \frac{1}{6}(x^2+1)^3$$

5 $\int \cos^4 x \sin x dx = \ \dots \ + c$

A. $\frac{\cos^5 x}{5}$

B. $-\frac{\cos^5 x}{5}$

C. $\frac{\sin^5 x}{5}$

D. $-\frac{\sin^5 x}{5}$

$u = \cos x \implies du = -\sin x dx$

$-\int u^4 du = -\frac{1}{5}\cos^5 x + c$

6 $\int \frac{x^3}{1+x^8} dx = \ \dots \ + c$

A. $\frac{1}{4}\tan^{-1}(x^4)$

B. $\frac{1}{4}\tan^{-1}(x^2)$

C. $4\tan^{-1}(x^2)$

D. $4\tan^{-1}(x^4)$

$u = x^4$

$\frac{1}{4}\int \frac{1}{1+u^2} du = \frac{1}{4}\tan^{-1}(x^4) + c$

7 $\int 4xe^{-x^2} dx = \ \dots \ + c$

A. $2x^2 e^{-x^3/3}$

B. $-2e^{-x^3/3}$

C. $2x^2 e^{-x^2}$

D. $-2e^{-x^2}$

$u = -x^2 \implies du = -2x dx$

$-2\int e^u du = -2e^{-x^2} + c$

8 $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx = \ \dots \ + c$

A. $-2\sin\sqrt{x}$

B. $-2\cos\sqrt{x}$

C. $2\cos\sqrt{x}$

D. $2\sin\sqrt{x}$

$u = \sqrt{x}$

$2\int \sin u du = -2\cos\sqrt{x} + c$

9 $\int \sin^3 x \cos x dx = \ \dots \ + c$

A. $-\frac{\sin^4 x}{4}$

B. $\frac{\sin^4 x}{4}$

C. $\frac{\cos^4 x}{4}$

D. $3\sin^2 x \cos x$

$u = \sin x$

$\int u^3 du = \frac{1}{4}\sin^4 x + c$

10 $\int \frac{\ln x}{x} dx = \ \dots \ + c$

A. $\ln x$

B. $\frac{1}{x^2}$

C. $\frac{1}{2}(\ln x)^2$

D. $\frac{1}{x}$

$u = \ln x$

$\int u du = \frac{1}{2}(\ln x)^2 + c$

11 $\int_0^1 \frac{x}{\sqrt{4-x^2}} dx = \ \dots$

A. $2 - \cos(\frac{\pi}{6})$

B. $2 + \cos(\frac{\pi}{6})$

C. $-2 + 2\sin(\frac{\pi}{3})$

D. $2 - 2\sin(\frac{\pi}{3})$

$u = 4 - x^2 \implies -\frac{1}{2}\int_4^3 u^{-1/2} du$

$= 2 - \sqrt{3}$

$2 - 2\sin(\frac{\pi}{3}) = 2 - \sqrt{3}$

12 $\int m\sin(mx) dx = \ \dots \ + c$

A. $-\cos(mx)$

B. $\cos(mx)$

C. $-\sin(mx)$

D. $\frac{1}{m}\cos(mx)$

$u = mx \implies du = m dx$

$\int \sin u du = -\cos(mx) + c$

13 $\int \frac{1}{x^2}\cos(\frac{1}{x})dx = \ \dots + c$

A. $-\sin(\frac{1}{x^2})$
B. $-\sin(\frac{1}{x})$
C. $\sin(\frac{1}{x})$
D. $\sin(\frac{1}{x^2})$

$u = \frac{1}{x} \implies du = -\frac{1}{x^2}dx$

$-\int \cos u du = -\sin(\frac{1}{x}) + c$

14 $\int e^{5x}dx = \ \dots + c$

A. $5e^{5x}$
B. $\frac{1}{5}e^{x}$
C. e^{5x}
D. $\frac{1}{5}e^{5x}$

$u = 5x \implies du = 5dx$

$= \frac{1}{5}\int e^u du = \frac{1}{5}e^{5x} + c$

15 $\int \frac{2x}{x^2+1}dx = \ \dots + c$

A. $\frac{1}{2}\ln(x^2+1)$
B. $\ln(x^2+1)$
C. $\ln(2x)$
D. $2\ln(x^2+1)$

$u = x^2+1 \implies du = 2xdx$

$= \int \frac{1}{u}du = \ln|x^2+1| + c$

16 $\int \sin(3x)dx = \ \dots + c$

A. $\frac{1}{3}\cos(3x)$
B. $3\cos(3x)$
C. $-\frac{1}{3}\cos(3x)$
D. $-3\cos(3x)$

$u = 3x \implies du = 3dx$

$= \frac{1}{3}\int \sin(u)du = -\frac{1}{3}\cos(3x) + c$

17 $\int \sec^2(2x)dx = \ \dots + c$

A. $\frac{1}{2}\tan(2x)$
B. $2\tan(2x)$
C. $\tan(2x)$
D. $\frac{1}{2}\sec(2x)$

$u = 2x \implies du = 2dx$

$= \frac{1}{2}\int \sec^2(u)du = \frac{1}{2}\tan(2x) + c$

18 $\int \frac{e^x}{e^x+1}dx = \ \dots + c$

A. $\ln(e^x+1)$
B. $\frac{1}{e^x+1}$
C. $e^x\ln(e^x+1)$
D. e^x+1

$u = e^x+1 \implies du = e^x dx$

$= \int \frac{1}{u}du = \ln|e^x+1| + c$

19 $\int x\cos(x^2)dx = \ \dots + c$

A. $-\frac{1}{2}\sin(x^2)$
B. $2\sin(x^2)$
C. $\frac{1}{2}\sin(x^2)$
D. $\sin(x^2)$

$u = x^2 \implies du = 2xdx$

$= \frac{1}{2}\int \cos(u)du = \frac{1}{2}\sin(x^2) + c$

20 $\int (2x+1)^3dx = \ \dots + c$

A. $\frac{(2x+1)^3}{3}$
B. $\frac{(2x+1)^4}{8}$
C. $\frac{(2x+1)^4}{4}$
D. $\frac{(2x+1)^4}{2}$

$u = 2x+1 \implies du = 2dx$

$= \frac{1}{2}\int u^3du = \frac{1}{2}\cdot\frac{u^4}{4} = \frac{(2x+1)^4}{8} + c$