

أوراق عمل كامل الوحدة السادسة تطبيقات على التكامل المحدود



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف الثاني عشر المتقدم ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

تاريخ إضافة الملف على موقع المناهج: 15-03-2026 16:22:08

ملفات اكتب للمعلم اكتب للطالب الاختبارات الكترونية الاختبارات ا حلول ا عروض بوربوينت ا أوراق عمل
منهج انجليزي ا ملخصات وتقارير ا مذكرات وبنوك ا الامتحان النهائي للمدرس

المزيد من مادة
رياضيات:

إعداد: عبد الله أبوالنجا

التواصل الاجتماعي بحسب الصف الثاني عشر المتقدم



صفحة المناهج
الإماراتية على
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

المواد على تلغرام

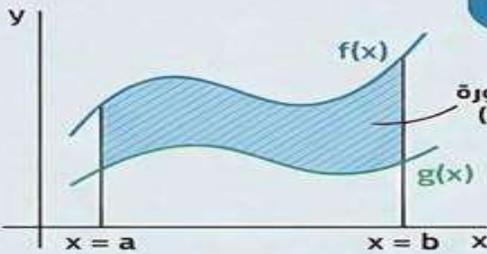
المزيد من الملفات بحسب الصف الثاني عشر المتقدم والمادة رياضيات في الفصل الثالث

أوراق عمل كامل الوحدة الخامسة التكامل	1
الخريطة الذهنية لمقرر وحدات ودروس الرياضيات الفصل الثالث	2
مقرر الوحدات والدروس المطلوبة للفصل الثالث	3
ملزمة أوراق عمل الوحدة الخامسة Differentiation of Applications منهج ريفيل Reveal متبوعة بمفاتيح الإجابات	4
حل ملزمة أوراق عمل الوحدة الخامسة التكامل	5

Unit 6: Applications of the definite Integration

وحدة 6: تطبيقات التكامل المحدود

6.1 المساحة بين منحنين (Area Between Curves)

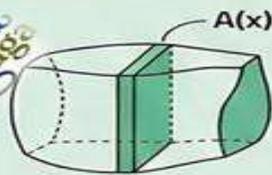


المنطقة المحصورة (Shaded Area)

حساب المنطقة المحصورة بين دالتين
يتم استخدام التكامل المحدود لإيجاد المساحة بين الدالة
الدالة $f(x)$ والدالة $g(x)$ على فترة محددة $[a, b]$.

$$A = \int_a^b [f(x) - g(x)] dx$$

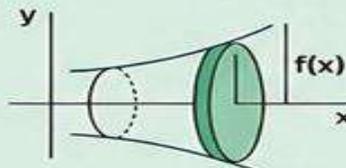
6.2 الحجم: الشرائح، الأقراص، والحلقات (Volume: Slicing, Disks & Washers)



طريقة الشرائح

إيجاد حجم الجسم بتقسيمه إلى
شرائح عرضية مساحة كل منها $A(x)$.
والصيغة هي:

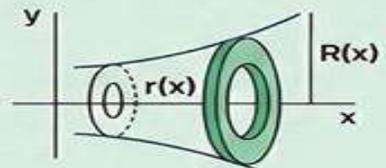
$$V = \int_a^b A(x) dx$$



طريقة الأقراص

تُستخدم للأجسام الصلبة الناتجة
عن الدوران، وتعتمد على الصيغة:

$$V = \int_a^b \pi [f(x)]^2 dx$$

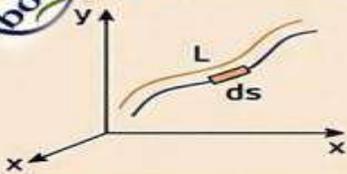


طريقة الحلقات

تُستخدم للأجسام الدوارية المفرغة
(ذات الفجوات)، وتعتمد على الفرق
بين نصف قطر خارجي وداخلي:

$$V = \int \pi ([R(x)]^2 - [r(x)]^2) dx$$

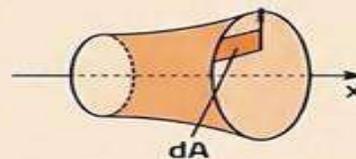
6.4 طول القوس والمساحة السطحية (Arc Length & Surface Area)



طول القوس

استخدام التكامل لتحديد
الطول الدقيق للمسارات
المنحنية (ds).

$$L = \int_a^b ds = \int \sqrt{1 + [f'(x)]^2} dx$$

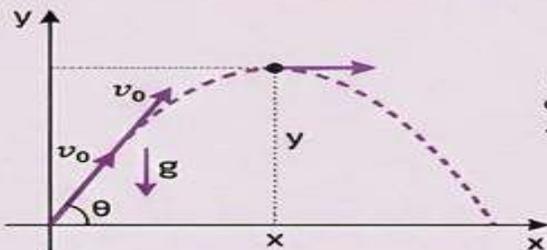


المساحة السطحية

حساب مساحة السطح
الخارجي للأجسام الناتجة
عن دوران منحنى حول
محور.

$$S = \int 2\pi f(x) ds$$

6.5 حركة المقذوفات (Projectile Motion)



تحليل المسار والسرعة

تطبيق التكامل المحدود لتحويل
التسارع (a) إلى سرعة (v) ثم إلى
إزاحة (s) في فضاء ثنائي الأبعاد.

$$v(t) = \int a(t) dt$$

معادلات الحركة التكاملية

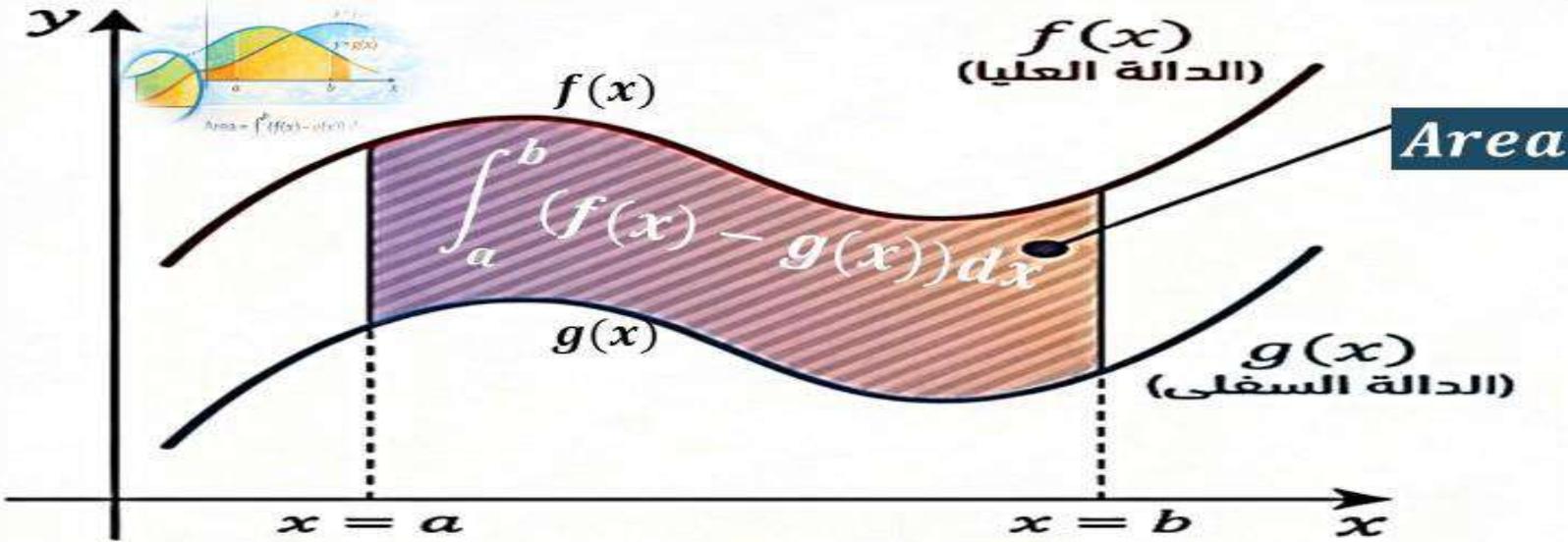
السرعة المتجهة، هي تكامل التسارع
 $v(t) = \int a(t) dt$ ، والإزاحة هي
تكامل السرعة $s(t) = \int v(t) dt$.

$$s(t) = \int v(t) dt$$



Unit6: Applications of the Definite Integrals

وحدة 6: تطبيقات التكامل المحدود



Lesson 1 : Area Between curves

الدرس الأول: المساحة المحصورة بين منحنين

Grade 12 Advanced

الثاني عشر متقدم

Mr. Abdalla
Abouelnaga



Mr. Abdalla Abouelnaga

0505114830



Student Name (اسم الطالب): _____

Class (الصف) _____

Unit 6

Application of the definite Integration

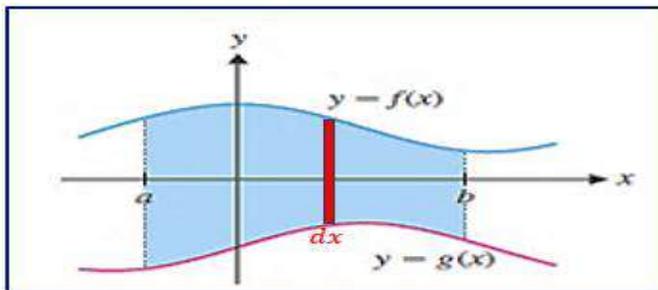
Lesson 1

Areas between Curves

Objectives

- Find the area between two curves using definite integration.
- Compute the area of a region using definite integration with y as a variable.

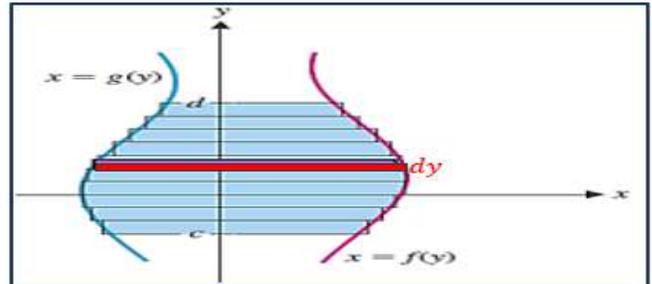
Vertical Slide



$$A = \int_a^b (\text{Upper function} - \text{Lower function}) dx$$

$$A = \int_a^b (f(x) - g(x)) dx$$

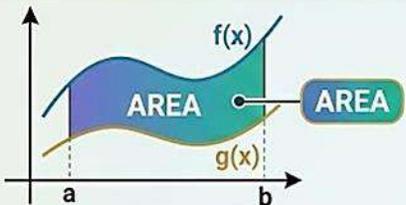
Horizontal Slide



$$A = \int_c^d (\text{Right function} - \text{Left function}) dy$$

$$A = \int_c^d (f(y) - g(y)) dy$$

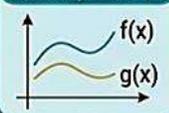
THE ESSENTIAL FORMULA



$$\int_a^b [f(x) - g(x)] dx$$

STEP-BY-STEP PROCESS

Identify Curves



Find Intersections

Set $f(x) = g(x)$
find limits $[a, b]$

Determine Upper/Lower

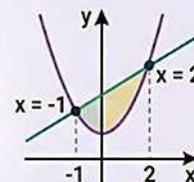
Use a test point

Set Up Integral

$$\int_a^b (\text{top} - \text{bottom}) dx$$

AREA BETWEEN TWO CURVES

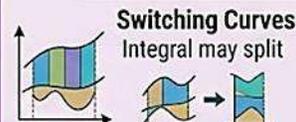
EXAMPLE: Finding the Area



e.g. $f(x) = x^2$
 $g(x) = x + 2$

$$\int_a^b (\text{top} - \text{bottom}) dx$$

IMPORTANT TIPS & ALTERNATIVES

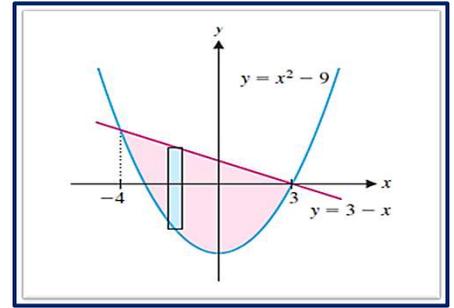


With respect to Y
 $x = f(y)$ and $x = g(y)$
 $\int_a^b [\text{right} - \text{left}] dy$

Example: 1

Find the area bounded by the graphs of $y = 3 - x$ and $y = x^2 - 9$.

Solution



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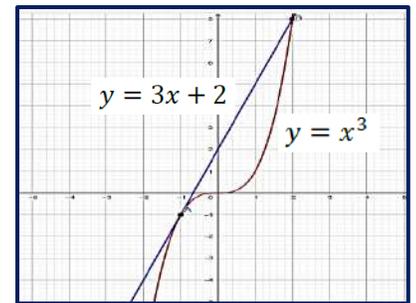
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Exercise: 1.1

Find the area of the region determined by the intersections of the curves.

$$y = x^3, y = 3x + 2$$

Solution



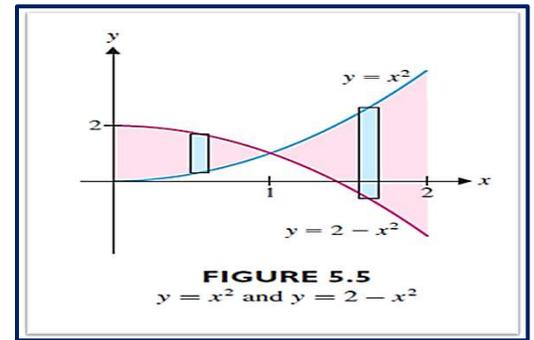
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Example: 2

Find the area bounded by the graphs of $y = x^2$ and $y = 2 - x^2$. $0 \leq x \leq 2$.

Solution



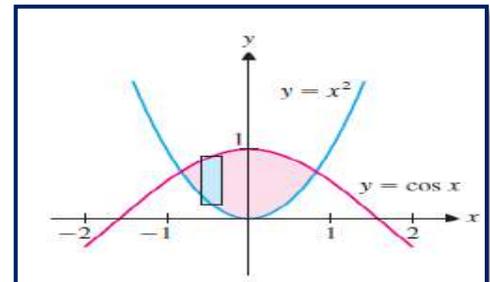
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Example: 3

Find the area of the region determined by the intersections of the curves.
 $y = \cos x$, $y = x^2$

Solution



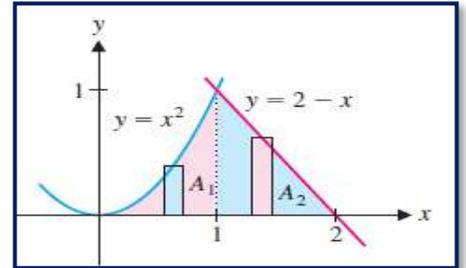
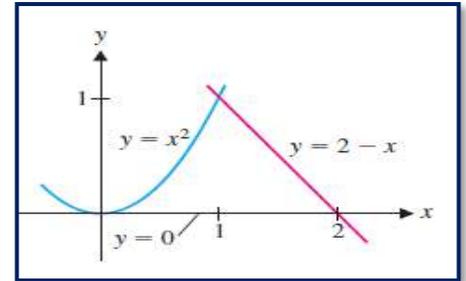
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Example: 4

Find the area of the region determined by the intersections of the curves.
 $y = x^2$ and $y = 2 - x$, $y = 0$

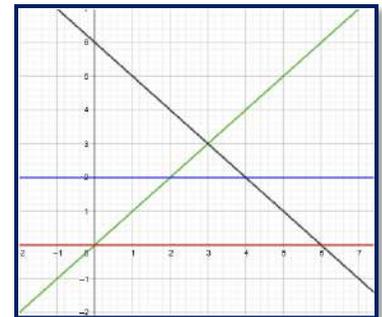
Solution



Exercise: 4.1

Find the area of the region determined by the intersections of the curves.
 $y = x$, $y = 2$, $y = 6 - x$, $y = 0$

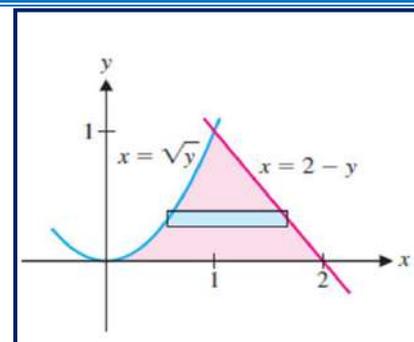
Solution



Example: 5

Find the area of the region determined by the intersections of the curves. respect to y

Solution



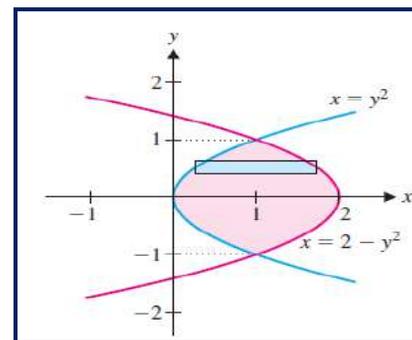
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Exercise 5.1

Find the area bounded by the graphs of $x = y^2$ and $x = 2 - y^2$.

Solution



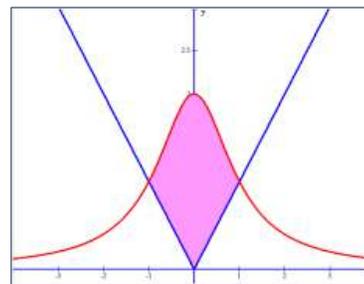
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Example: 6

Find the area bounded by the graphs of.

$$y = \frac{2}{x^2+1} \quad , \quad y = |x|$$

Solution

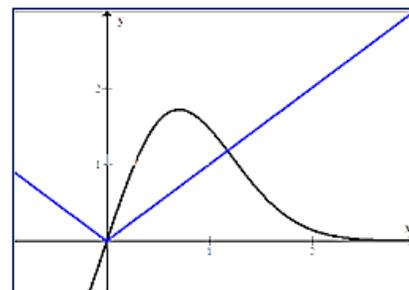
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Exercise 6.1

Find the area bounded by the graphs of.

$$y = 4xe^{-x^2} \quad , \quad y = |x|$$

Solution

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We can solve without graphing

Example: 7 Find the area bounded by the graphs of $y = x^3$ and $y = x^2 - 1$, $1 \leq x \leq 3$.

Solution

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Exercise 7.1 Find the area bounded by the graphs of $y = 3 - x^2$ and $y = 2x$, $0 \leq x \leq 2$.

Solution

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امتحان العام 2023/2022

1

Find the area bounded by the curves
 $x = 3y$ and $x = 2 + y^2$.

أوجد مساحة المنطقة المحصورة بين المنحنيات
 $x = 2 + y^2$ و $x = 3y$

A

$$\frac{8}{3}$$

B

$$\frac{1}{3}$$

C

$$\frac{4}{3}$$

D

$$\frac{1}{6}$$

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عزيزي الطالب : اجتهد في فهم ومذاكرة هذه الأمثلة والتمارين بالكتاب المدرسي تمنياتي للجميع بالنجاح والتفوق

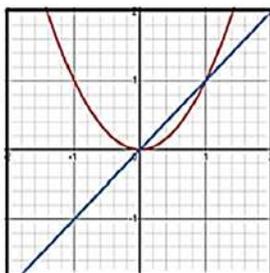
Solution Steps Before Choosi

امتحان العام 2023/2022

2

Find the area bounded by the curves
 $y = x$ and $y = x^2$.

أوجد مساحة المنطقة المحصورة بين المنحنيات
 $y = x^2$ و $y = x$



A

$$A = \int_{-1}^1 (x - x^2) dx$$

B

$$A = \int_0^1 (x^2 - x) dx$$

C

$$A = \int_0^1 (x - x^2) dx$$

D

$$A = \int_{-1}^1 (x^2 - x) dx$$

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عزيزي الطالب : اجتهد في فهم ومذاكرة هذه الأمثلة والتمارين بالكتاب المدرسي تمنياتي للجميع بالنجاح والتفوق

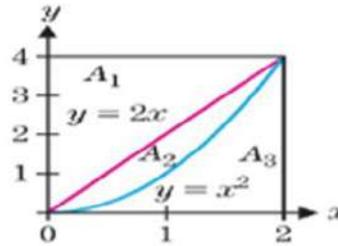
Solution Steps Before Choosi

3

In terms of A_1 , A_2 and A_3 , identify the area given by the integral.

$$\int_0^4 (2 - \sqrt{y}) dy$$

بدلالة A_1 و A_2 و A_3 ، حدد المساحة المعطاة بالتكامل.



$$\int_0^4 (2 - \sqrt{y}) dy$$

A

A_1

B

$A_1 + A_2$

C

A_3

D

A_2

A3

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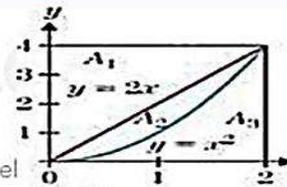
عزيزي الطالب : اجتهد في فهم ومذاكرة هذه الامثلة والتمارين بالكتاب المدرسي تمهيناتي للجميع بالنجاح والتفوق

Solution Steps Before Choosi

4

In terms of A_1 , A_2 , and A_3 , Identify

the area given by the integral $\int_0^4 (\sqrt{y} - \frac{y}{2}) dy$.



A

A_1

B

$A_1 + A_2$

C

A_3

D

A_2

A2

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عزيزي الطالب : اجتهد في فهم ومذاكرة هذه الامثلة والتمارين بالكتاب المدرسي تمهيناتي للجميع بالنجاح والتفوق

Solution Steps Before Choosi

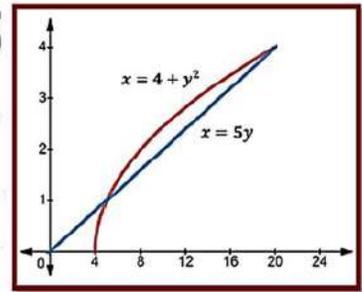
5

Find the area bounded by the curves

$x = 5y$ and $x = 4 + y^2$.

أوجد مساحة المنطقة المحصورة بين المنحنيات

$x = 4 + y^2$ و $x = 5y$



Exam2021.2022 T3

A $A = \int_1^4 (5y - (4 + y^2)) dy$

B $A = \int_5^{20} ((4 + y^2) - 5y) dy$

C $A = \int_5^{20} (5x - (4 + x^2)) dx$

D $A = \int_1^4 ((4 + x^2) - 5x) dx$

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A

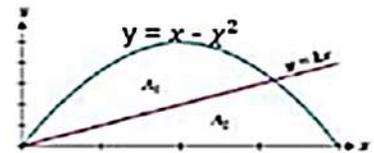
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عزيزي الطالب : اجتهد في فهم ومذاكرة هذه الأمثلة والتمارين بالكتاب المدرسي تمنياتي للجميع بالنجاح والتفوق

Solution Steps Before Choos

6

For $y = x - x^2$ and $y = Kx$ as shown, find A_1 such that $A_1 = A_2$.



- A** $\frac{1}{12}$
- B** $\frac{1}{6}$
- C** $\frac{1}{8}$
- D** $\frac{1}{10}$

Solution Steps

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A

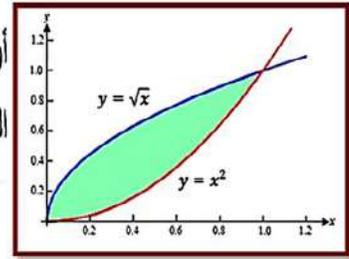
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عزيزي تلميذ : اولاً اجتهد في فهم ومذاكرة تمارين الكتاب (كتاب اول). تمنياتي للجميع بالنجاح والتفوق

2020/2021

Find the area bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$.

أوجد مساحة المنطقة المحصورة بين التمثيلين البيانيين $y = \sqrt{x}$ و $y = x^2$.



Exam2021.2022 T3

- A $A = \frac{1}{6}$
- B $A = \frac{1}{3}$
- C $A = \frac{8}{3}$
- D $A = \frac{16}{3}$

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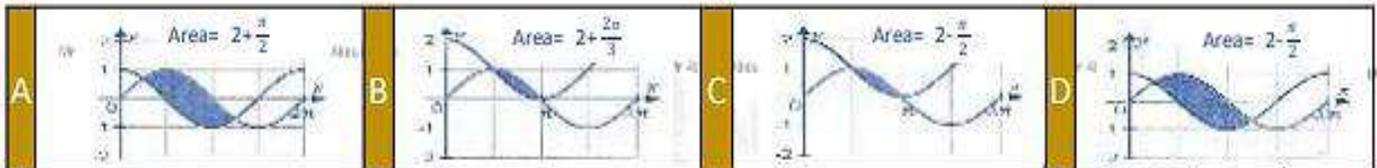
B

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عزيزي الطالب : اجتهد في فهم ومذاكرة هذه الأمثلة والتمارين بالكتاب المدرسي تمنياتي للجميع بالنجاح والتفوق

Solution Steps Before Choosi

Find The graph and Area between $y = \sin x$; $y = 1 + \cos x$; $\frac{\pi}{2} \leq x < \pi$



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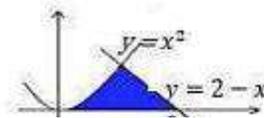
C

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عزيزي الطالب : اولاً اجتهد في فهم ومذاكرة تمارين الكتاب (كتاب اولاً) . تمنياتي للجميع بالنجاح والتفوق

2023/2024

Find the area bounded by the graph of $y = x^2$ and $y = 2 - x$ respect to y .



A $A = \int_0^1 (2 - x - x^2) dx$

B $A = \int_0^2 (2 - y - \sqrt{y}) dy$

C $A = \int_0^1 (2 - y - \sqrt{y}) dy$

D $A = \int_0^1 (2 - y + \sqrt{y}) dy$

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عزيزي الطالب : اجتهد في فهم ومذاكرة هذه الأمثلة والتمارين بالكتاب المدرسي تمنياتي للجميع بالنجاح والتفوق

Solution Steps Before Choosi

Sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral

$$x = 3 - y^2, \quad x = 2y^2$$

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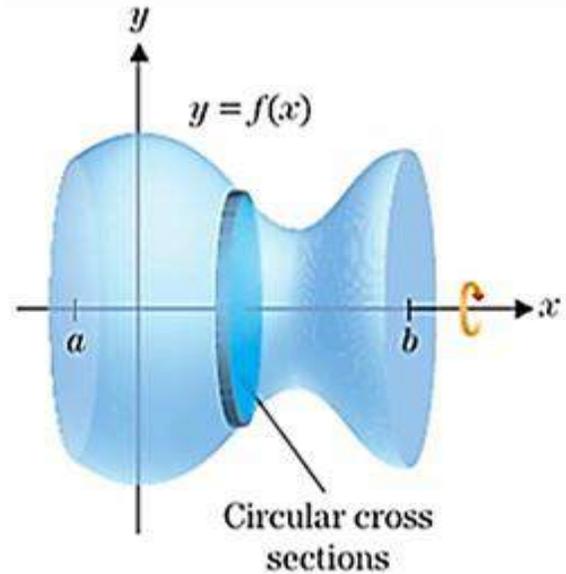
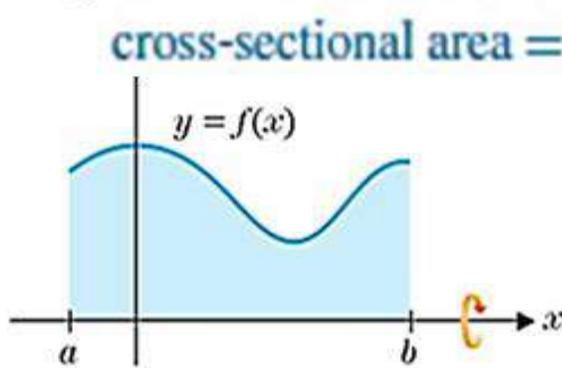
عزيزي الطالب : اجتهد في فهم ومذاكرة هذه الأمثلة والتمارين بالكتاب المدرسي تمنياتي للجميع بالنجاح والتفوق

Solution Steps Before Choosi

Unit 6: Applications of the Definite Integrals

وحدة 6: تطبيقات التكامل المحدود

$$V = \int_a^b \underbrace{\pi [f(x)]^2}_{\text{cross-sectional area} = \pi r^2} dx.$$



Lesson 2 : VOLUME: SLICING, DISKS AND WASHERS

الدرس الثاني: الحجم (شرائح - أقراص - حلقات)

Grade 12 Advanced
الثاني عشر متقدم

Mr. Abdalla
Abouelnaga



Mr. Abdalla Abouelnaga
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Student Name (اسم الطالب): _____

Class (الصف) _____

Unit 6

Application of the definite Integration

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Lesson 2

VOLUME: SLICING, DISKS AND WASHERS

Objectives

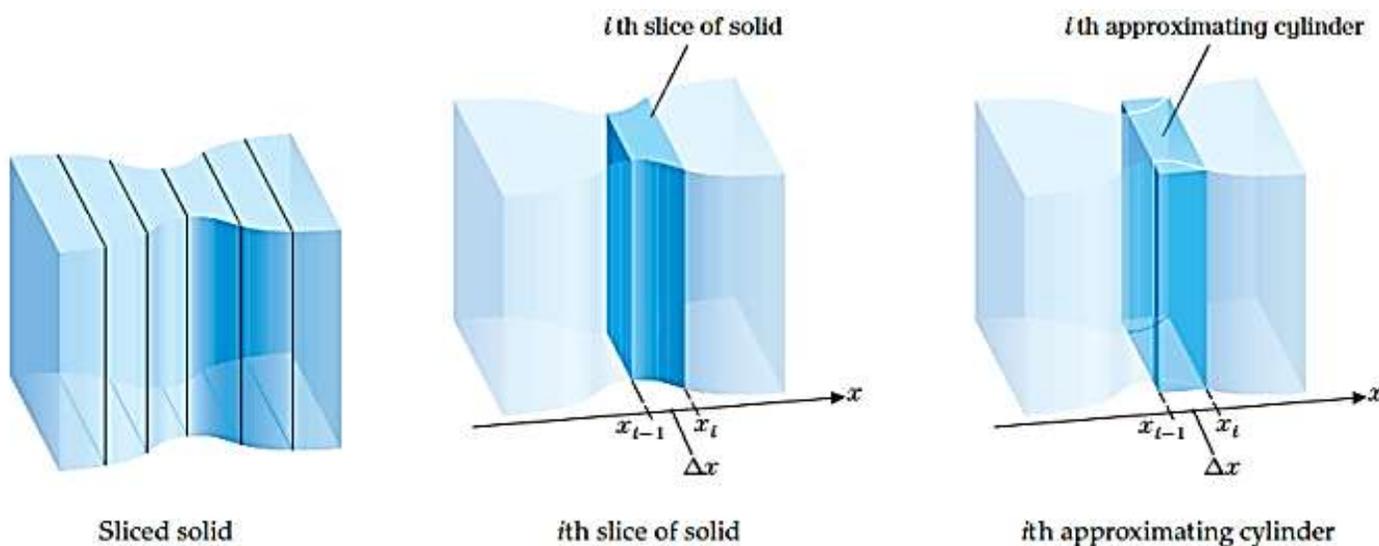
- Compute volume by means of definite integration using areas of cross sections.
- Find the volume of a solid of revolution using the method of disks.
- Find the volume of a solid of revolution by using the method of washers.

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Volume by Slicing



Volume of a solid with cross – sectional area $A(x)$

$$V = \int_a^b A(x) dx$$

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Example: 1

Find the volume of the solid with cross-section area $A(x)$.

$$A(x) = x + 2, \quad -1 \leq x \leq 3$$

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Exercise: 1.1

$$A(x) = 10e^{0.01x}, \quad 0 \leq x \leq 10$$

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Exercise: 1.2

Find the volume of the solid with cross-section area $A(x)$.

$$A(x) = \pi(4 - x)^2, \quad 0 \leq x \leq 2$$

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Exercise: 1.3

$$A(x) = 2(x + 1)^2, \quad 1 \leq x \leq 4$$

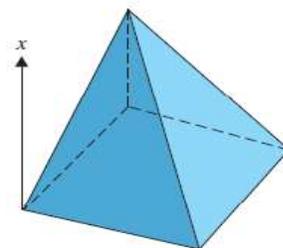
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Example: 2

The Pyramid Arena in Memphis has a square base of side approximately 180 m and a height of approximately 100 m. Find the volume of the pyramid with these measurements.

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A Pyramid

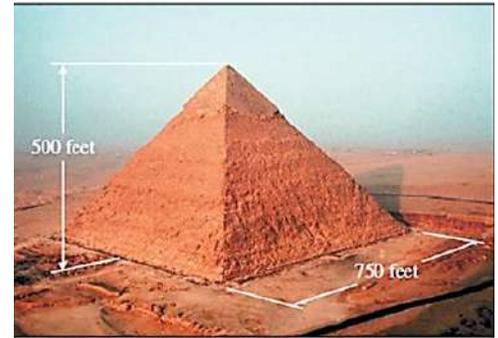
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Exercise: 2.1

The great pyramid at Gizeh is 500 feet high, rising from a square base of side 750 feet. Compute its volume using integration.



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Exercise: 2.2

A church steeple is 30 feet tall with square cross sections. The square at the base has side 3 feet, the square at the top has side 6 inches and the side varies linearly in between compute the volume.

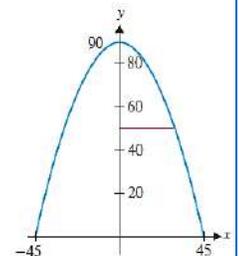


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Example: 3

Suppose that a dome has circular cross sections, with outline $y = \frac{-2}{45}x^2 + 90$, for $-45 \leq x \leq 45$. (In units of centimetres, this gives dimensions similar to the Capitol Dome in the Figure .A graph is shown in the Figure Find the volume of the dome.



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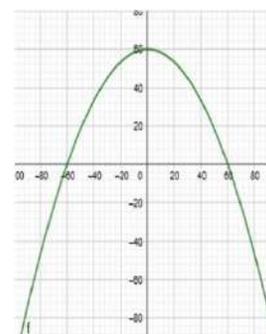
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Exercise: 3.1

The outline of a dome is given by $y = 60 - \frac{x^2}{60}$ for $-60 \leq x \leq 60$ (units of feet), with circular cross-sections perpendicular to the y -axis. Find its volume.

- A. 339292.0066 ft^3 B. $108000\pi \text{ ft}^3$
 C. $10800\pi \text{ ft}^3$ D. $18000\pi \text{ ft}^3$



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B

Example: 4

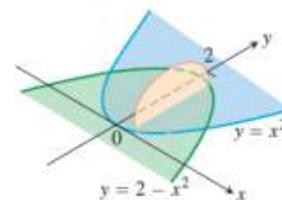
The base of a solid V is the region bounded by the functions

$$y = x^2 \text{ and } y = 2 - x^2 \text{ . Find the volume if } V \text{ has}$$

a) Square cross sections

b) Semicircular cross sections

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c) Equilateral triangle cross sections perpendicular to the x - axis.

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d) Circular cross sections

e) Square cross sections and diameter is perpendicular to the x - axis.

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Exercise 4.1

A pottery jar has circular cross sections of radius $4 + \sin \frac{x}{2}$ inches for $0 \leq x \leq 2\pi$. compute the volume.

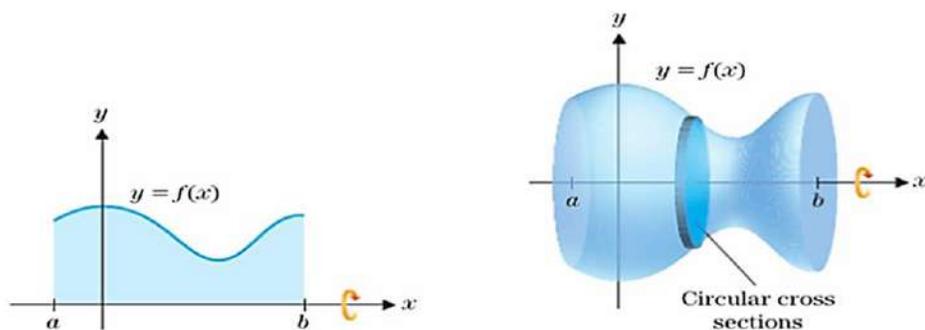
The Method of Disks

A) Horizontal

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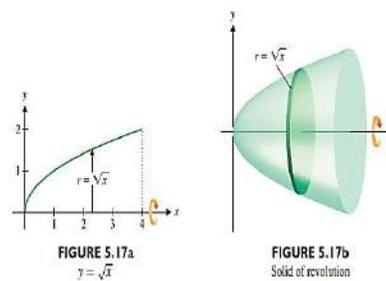
$$V = \int_a^b \underbrace{\pi [f(x)]^2 dx}_{\text{cross-sectional area} = \pi r^2}$$

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Example: 5

Revolve the region under the curve $y = \sqrt{x}$ on the interval $[0, 4]$ about the x-axis and find the volume of the resulting solid of revolution.



Exercise 5.1

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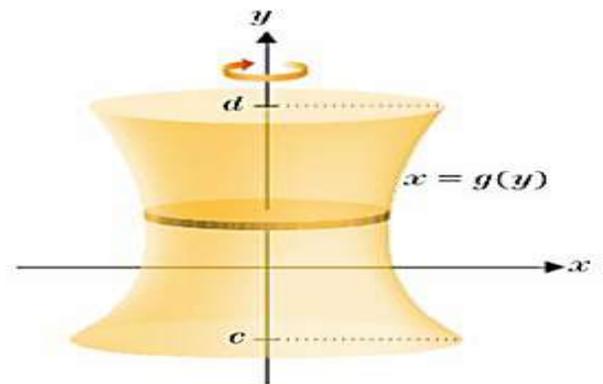
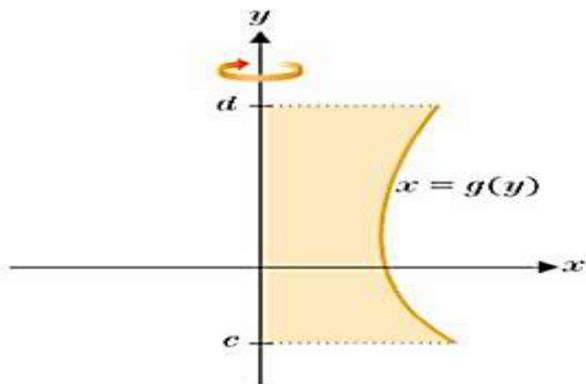
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Compute the volume of the solid formed by revolving the given region about the given line.

Region bounded by $y=2-x$, $y=0$ and $x=0$ about the x -axis

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B) Vertical



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$$V = \int_c^d \underbrace{\pi [g(y)]^2}_{\text{cross-sectional area} = \pi r^2} dy.$$

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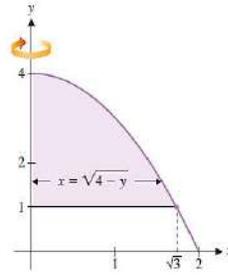
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Example: 6

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Find the volume of the solid resulting from revolving the region bounded by the curves $y = 4 - x^2$ and $y = 1$ from $x = 0$ to $x = \sqrt{3}$ about the y -axis.

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FIGURE 5.19a
 $y = 4 - x^2$

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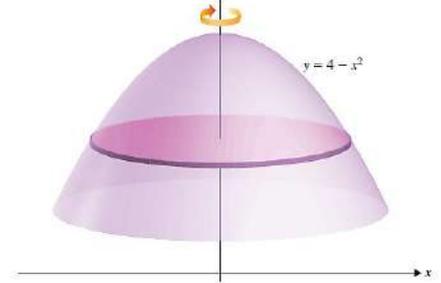
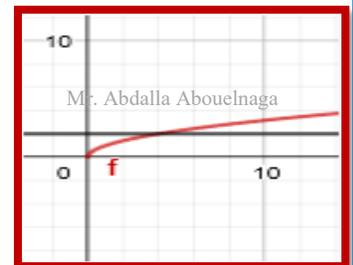


FIGURE 5.19b

Solid of revolution
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Exercise: 6.1

Compute the volume of the solid formed by revolving the given region about the given line.
Region bounded by $y = \sqrt{x}$, $y=2$ and $x=0$ about the y -axis.



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The Method of Washers

One complication that occurs in computing volumes is that the solid may have a cavity or “hole” in it. Another occurs when a region is revolved about a line other than the x-axis. Neither case will present you with any significant difficulties, if you look carefully at the figures. We illustrate these ideas in the following examples .



Computing Volumes of Solids with and without Cavities

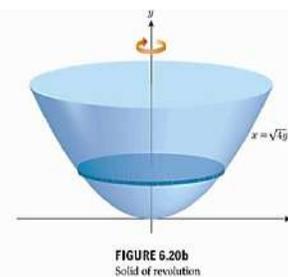
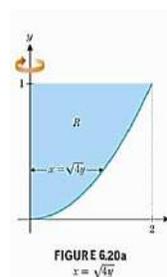
Example: 7

Let **R** be the region bounded by the graphs of $y = 1/4x^2$, $x = 0$ and $y = 1$. Compute the volume of the solid formed by revolving **R** about

- (a) the y-axis (b) the x-axis (c) the line $y = 2$.

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In parts (b) and (c) the volume will compute by **subtracting** an **inner volume** from an **outer volume** to compensate for a cavity inside the solid. This technique is a slight generalization of the method of disks and is referred to as the **method of washers**, since the cross sections of the solids look like washers.

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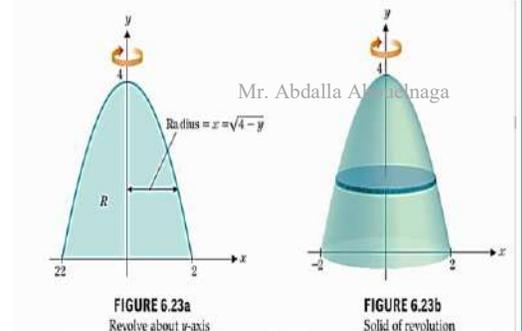
Example: 8

Let R be the region bounded by $y = 4 - x^2$ and $y = 0$. Find the volume of the solids obtained by revolving R about each of the following:

- (a) the y -axis
- (b) the line $y = -3$
- (c) the line $y = 7$
- (d) the line $x = 3$.

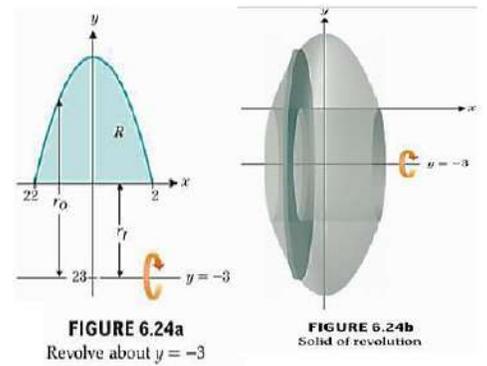
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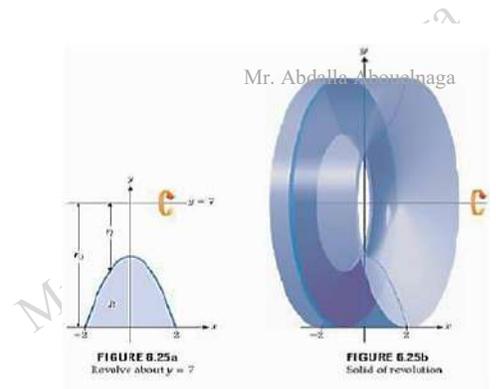
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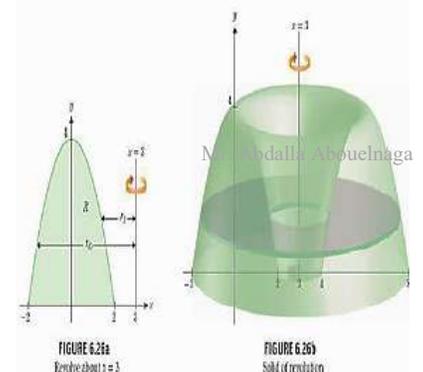
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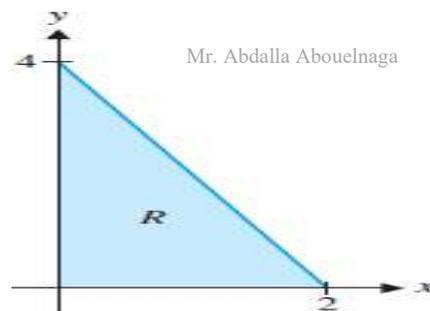
Exercise: 8.1

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Let R be the region bounded by $y = 4 - 2x$, the x -axis and the y -axis. Compute the volume of the solid formed by revolving R about the given line.

- (a) the y -axis
- (b) the x -axis
- (c) $y = 4$
- (d) $y = -4$
- (e) $x = 2$
- (f) $x = -2$



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- (a) $\frac{16\pi}{3}$ (b) $\frac{32\pi}{3}$ (c) $\frac{64\pi}{3}$ (d) $\frac{128\pi}{3}$ (e) $\frac{32\pi}{3}$ (f) $\frac{64\pi}{3}$

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Find the volume of the solid with cross sectional area

$A(x) = x + 2$ for $-1 \leq x \leq 3$.

أوجد حجم الجسم مع مساحة المقطع العرضي $A(x) = x + 2$ لكل $-1 \leq x \leq 3$.

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A $V = 12\pi$

B $V = 6$

C $V = 12$

D $V = 4$

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Q1

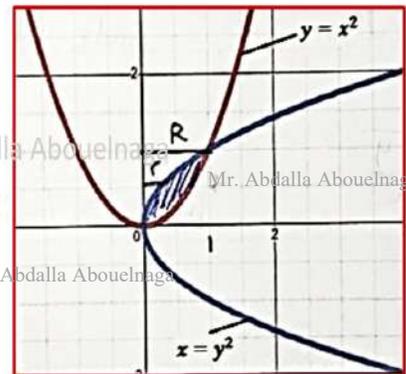
Part 3 Writing (كتابة) الجزء الثالث

Compute the volume of the solid formed by revolving the region bounded by $y = x^2$ and $x = y^2$ about

- a) the y -axis,
- b) about $x = 1$.

احسب حجم الجسم الذي تكوّن من دوران المنطقة المحددة بواسطة $y = x^2$ و $x = y^2$ حول المحور y ،
حول $x = 1$ (b)

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Let R be the region bounded by
 $y = 2 - x, y = 0$ and $x = 0$.

Compute the volume of the solid
formed by revolving R about the
 $y = 3$.

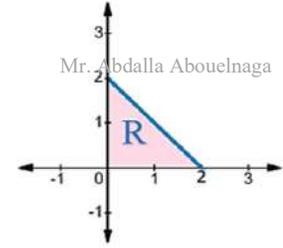
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لتكن R هي المنطقة المحدودة بواسطة

$$.x = 0 \text{ و } y = 0, y = 2 - x$$

احسب حجم الجسم الذي تكون من دوران R

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حول $y = 3$ 

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Exam2021.2022 T3

A

$$V = \int_0^2 \pi (3)^2 dx - \int_0^2 \pi (2 - x)^2 dx$$

B

$$V = \int_0^2 \pi (3 - (2 - x))^2 dx$$

C

$$V = \int_0^2 \pi (3)^2 dx - \int_0^2 \pi (3 - (2 - x))^2 dx$$

D

$$V = \int_0^2 \pi (3) dx - \int_0^2 \pi (3 - (2 - x))^2 dx$$

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Let R be the region bounded by
 $y = x^2, y = 0$ and $x = 1$.
Compute the volume of the solid
formed by revolving R about the
 x -axis.

لتكن R هي المنطقة المحدودة بواسطة

$$.x = 1 \text{ و } y = 0, y = x^2$$

احسب حجم الجسم الذي تكون من دوران R

حول المحور x .

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A

$$V = \frac{1}{6} \pi$$

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B

$$V = \frac{1}{5} \pi$$

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C

$$V = \frac{1}{2} \pi$$

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D

$$V = \frac{32}{5} \pi$$

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Find the volume of the solid with
cross sectional area

$$A(x) = 2(x + 1)^2 \text{ for } 1 \leq x \leq 4.$$

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 $A(x) = 2(x + 1)^2$ لكل $1 \leq x \leq 4$.

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A

$$V = \frac{21}{2}$$

B

$$V = 21$$

C

$$V = 39$$

D

$$V = 78$$

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Exam2021.2022 T3

الأستاذ عبد الله أبو النجا معلم أول الرياضيات 0505114830

D

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Find the volume of a pottery jar that has circular cross sections of radius
 $(4 - \sin \frac{x}{2})$ cm . for $0 \leq x \leq 2\pi$.

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أوجد حجم إناء فخاري له مقاطع عرضية دائرية
بنصف قطر $(4 - \sin \frac{x}{2})$ cm لكل
 $0 \leq x \leq 2\pi$

A

$$V = 65 \pi \text{ cm}^3$$

B

$$V = 33 \pi^2 \text{ cm}^3$$

C

$$V = 32 \pi \text{ cm}^3$$

D

$$V = (33\pi^2 - 32\pi) \text{ cm}^3$$

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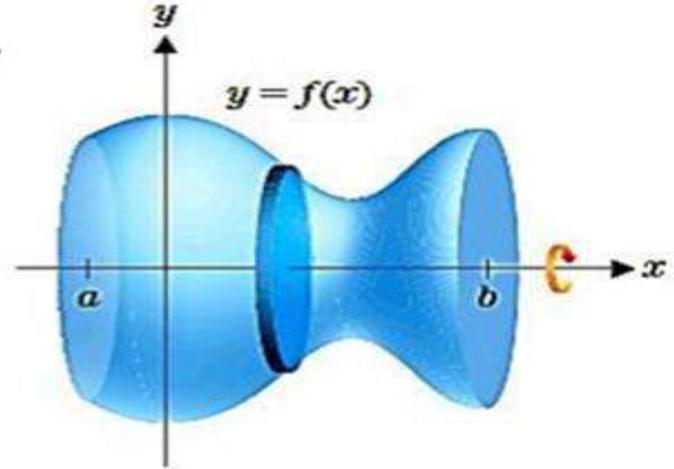
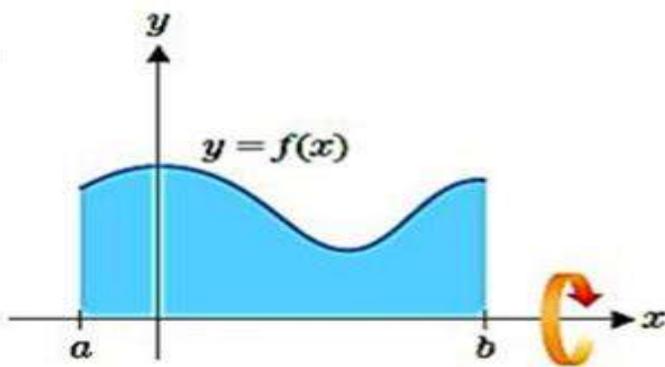
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Unit6: Applications of the Definite Integrals

وحدة 6: تطبيقات التكامل المحدود

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx$$



Lesson 4 : Arc Length and Surface الدرس الرابع: طول القوس ومساحة السطح

Grade 12 Advanced
الثاني عشر متقدم

Mr. Abdalla
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Student Name (اسم الطالب): _____

Class (الصف) _____

Unit 6

Application of the definite Integration

Lesson 4

Arc Length and Surface Area Mr. Abdalla Abouelnaga

Objectives

- Find arc length in a given interval using definite integration.
- Find surface area of a solid of revolution using definite integration.
- Solve mathematical problems involving applications on arc length or surface area.

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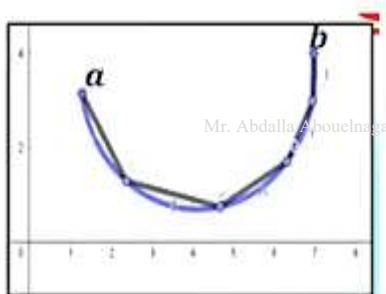
Arc length of $y = f(x)$ on the interval $[a, b]$

$$s = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

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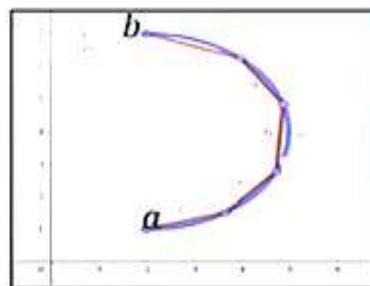
X-axis

Y-axis



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If f' is continuous on $[a, b]$ then the length of



$$s = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

The curve $y=f(x)$, $a \leq x \leq b$ is	The curve $x=g(y)$, $a \leq y \leq b$ is
--	--

$$s = \int_a^b \sqrt{1 + (f'(y))^2} dy$$

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Example: 1

Find the arc length of the portion of the curve $y = \sin x$ with $0 \leq x \leq \pi$

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Exercise: 1.1

Find the arc length of the portion of the curve $y = x^2$ with $0 \leq x \leq 1$.

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Exercise: 1.2

Find the arc length of the portion of the curve $y = x^4$ with $0 \leq x \leq 1$ and compare to the arc length of the portion of the curve $y = x^2$ on the same interval.

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Example: 2

Find the arc length of the portion of the curve $x = y^2$ with $0 \leq y \leq 4$.

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Exercise: 2.1

Find the function of the curve that passes through (1,1) and its arc length is given by

$$L = \int_1^4 \sqrt{1 + \frac{1}{4x}} dx$$

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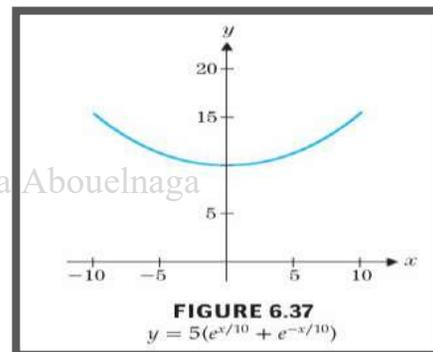
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Example: 3

A cable is to be hung between two poles of equal height that are 20 feet apart. It can be shown that such a hanging cable assumes the shape of a *catenary*, the general form of which is $y = a \cosh x/a = \frac{a}{2}(e^{x/a} + e^{-x/a})$. In this case, suppose that the cable takes the shape of $y = 5(e^{x/10} + e^{-x/10})$, for $-10 \leq x \leq 10$. How long is the cable?

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Exercise: 3.1

set up the integral for arc length and then approximate the integral with a numerical method. $v = 2x - x^2$, $0 \leq x \leq 2$

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$$\mathbf{A} = \int_0^2 \sqrt{1 + (2 - x)^2} dx = 2.9578$$

$$\mathbf{B} = \int_0^2 \sqrt{1 + 4(1 - x)^2} dx = 2.9578$$

$$\mathbf{C} = \int_0^2 \sqrt{2 + 2(1 - x)^2} dx = 2.9578$$

$$\mathbf{D} = \int_0^2 \sqrt{2 + (2 - 2x)^2} dx = 2.9578$$

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Exercise: 3.2

set up the integral for arc length and then approximate the integral with a numerical method. $y = x^3$, $-1 \leq x \leq 1$

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$$\mathbf{A} = \int \sqrt{1 + x^6} dx = 3.0957$$

$$\mathbf{B} = \int \sqrt{1 + 9x^2} dx = 3.0957$$

$$\mathbf{C} = \int \sqrt{1 + 3x^4} dx = 3.0957$$

$$\mathbf{D} = \int \sqrt{1 + 9x^4} dx = 3.0957$$

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Exercise: 3.3

Find the arc length of the portion of the curve

$$y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}} \quad \text{from } x=0 \text{ to } x=3$$

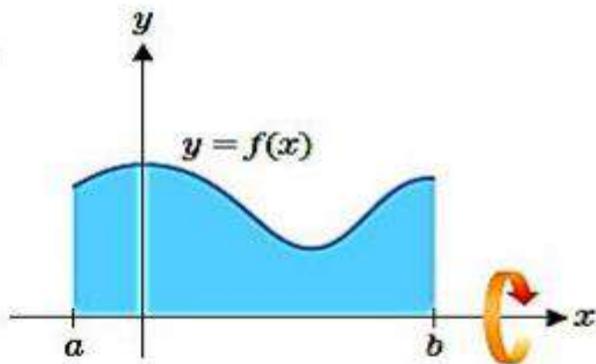
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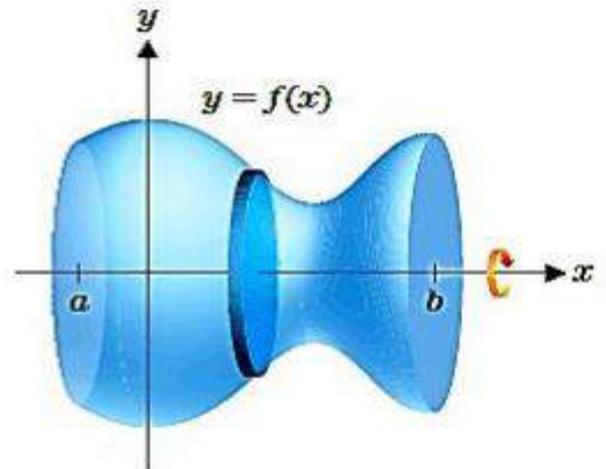
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Surface Area



Revolve about x-axis



Surface of revolution

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx,$$

Revolution about x axis

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx$$

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Revolution about y axis

$$S = \int_c^d 2\pi g(y) \sqrt{1 + [g'(y)]^2} dy$$

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Example: 4

Find the surface area of the surface generated by revolving $y = x^4$, for $0 \leq x \leq 1$, about the x-axis.

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Exercise:4.1

Find the surface area of the surface generated by revolving $y = x^2$, for $0 \leq x \leq 3$, about the x-axis.

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Exercise:4.2

Find the surface area of the surface generated by revolving $y = \sqrt{x}$, for $0 \leq x \leq 2$, about the x-axis.

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Term 3

امتحان العام 2023

الأسئلة المتضمنة في امتحان الوزارة للفصل الدراسي الثالث 2024
Questions which included in MOE Exam Term3 (2024)

Mathematics G12 Advanced

Final Revision Grade12 Adv

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Find the surface area of the surface generated by revolving $y = \sin x$, for $0 \leq x \leq \pi$, about the x -axis.

أوجد مساحة السطح المتولد من تدوير المنحنى $y = \sin x$ ، لكل $0 \leq x \leq \pi$ ، حول المحور x .

A $S = \int_0^{\pi} 4\pi \sin x \sqrt{1 + \cos x} dx$

B $S = \int_0^{\pi} \sin x \sqrt{1 + \cos^2 x} dx$

C $S = \int_0^{\pi} 2\pi \sin x \sqrt{1 + \cos^2 x} dx$

D $S = \int_0^{\pi} 2\pi \cos x \sqrt{1 + \cos^2 x} dx$

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Term 3

امتحان العام 2023

الأسئلة المتضمنة في امتحان الوزارة للفصل الدراسي الثالث 2024
Questions which included in MOE Exam Term3 (2024)

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Find the arc length of the portion of the curve $y = \ln x$ with $1 \leq x \leq 3$.

أوجد طول القوس لجزء من المنحنى $y = \ln x$ مع $1 \leq x \leq 3$.

A $s = \int_1^3 \sqrt{1 + \ln x^2} dx$

B $s = \int_1^3 \sqrt{1 + \frac{1}{x^2}} dx$

C $s = \int_1^3 \sqrt{1 + \frac{1}{x}} dx$

D $s = \int_1^3 \left(1 + \frac{1}{x^2}\right) dx$

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Q4

Part 3 Writing (كتابة) الجزء الثالث

Find the arc length of the portion of the curve $f(x) = \int_0^x \sqrt{t^2 + 4t + 3} dt$ with $0 \leq x \leq 2$.

أوجد طول القوس لجزء من المنحنى

مع $f(x) = \int_0^x \sqrt{t^2 + 4t + 3} dt$. $0 \leq x \leq 2$

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A rope is to be hung between two poles 40 m apart. If the rope assumes the shape of the catenary $y = 10(e^{x/20} + e^{-x/20})$, $-20 \leq x \leq 20$, compute the length of the rope.

تم تعليق حبل بين عمودين البعد بينهما 40 m . إذا كان الحبل يبدو أنه يتخذ شكل سلسلة معادلتها $y = 10(e^{x/20} + e^{-x/20})$ ، $-20 \leq x \leq 20$ ، احسب طول الحبل.

A $s = 20(e - e^{-1}) m$

B $s = 20(e^{-1} - e) m$

C $s = 10(e - e^{-1}) m$

D $s = 40(e - e^{-1}) m$

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Find the arc length of the portion of
the curve $y = 3x + 2$ with
 $0 \leq x \leq 2$.

أوجد طول القوس لجزءه من المنحنى
 $y = 3x + 2$ مع $0 \leq x \leq 2$.

A $s = \sqrt{10}$

B $s = 2\sqrt{10}$

C $s = \sqrt{5}$

D $s = 2\sqrt{5}$

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Find the surface area of the surface
generated by revolving $y = x^2$, for
 $0 \leq x \leq 1$, about the x -axis.

أوجد مساحة السطح المتولد من تدوير المنحنى
 $y = x^2$ لكل $0 \leq x \leq 1$ حول المحور x .

A $S = \int_0^1 4\pi x \sqrt{1 + 2x} dx$

B $S = \int_0^1 4\pi x \sqrt{1 + (2x)^2} dx$

C $S = \int_0^1 2\pi x^2 \sqrt{1 + (2x)^2} dx$

D $S = \int_0^1 2\pi x^2 \sqrt{1 + 2x} dx$

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Exam2020.2021 T3

Identify the integral for the surface area of the surface of revolution for the shape described by $y = \sqrt{x}$, $1 \leq x \leq 2$, revolved about the x -axis. حدد التكامل لمساحة السطح الدوراني الذي يتكون بدوران $y = \sqrt{x}$ ، حيث $1 \leq x \leq 2$ حول x -axis.

- A $s = 2\pi \int_1^2 \sqrt{1 + \frac{1}{4x}} dx$ B $s = 2\pi \int_1^2 \sqrt{x} \sqrt{1 + \frac{1}{4x}} dx$ C $s = \int_1^2 \sqrt{x} \sqrt{1 + \frac{1}{4x}} dx$ D $s = 2\pi \int_1^2 \sqrt{x} \sqrt{1+x} dx$

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Exam2020.2021 T3

Find the arc length of the curve $y = \tan x$ for $0 \leq x \leq \frac{\pi}{4}$. أوجد طول قوس لجزء من المنحنى $0 \leq x \leq \frac{\pi}{4}$, $y = \tan x$.

- A $s = \int_0^{\frac{\pi}{4}} \sqrt{1 + \sec^4 x} dx$ B $s = \int_0^{\frac{\pi}{4}} (1 + \sec^4 x) dx$ C $s = \int_0^{\frac{\pi}{4}} \sqrt{1 + \sec^2 x} dx$ D $s = \pi \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x} dx$

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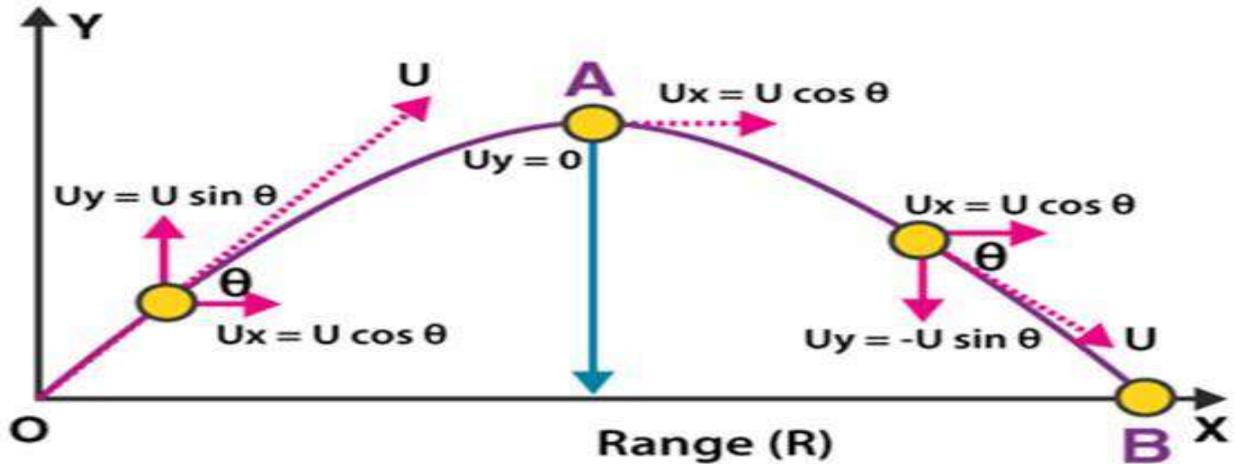
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Unit 6: Applications of the Definite Integrals

وحدة 6: تطبيقات التكامل المحدود

PROJECTILE MOTION



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Lesson 5 : Projectile Motion

الدرس الخامس: حركة المقذوفات

Grade 12 Advanced

الثاني عشر متقدم

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Unit 6

Application of the definite Integration

Lesson 5

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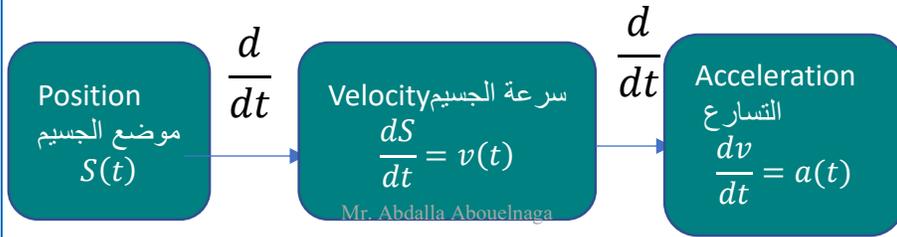
Projectile Motion

Objectives

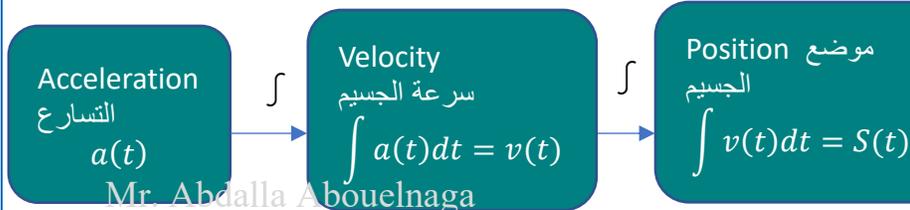
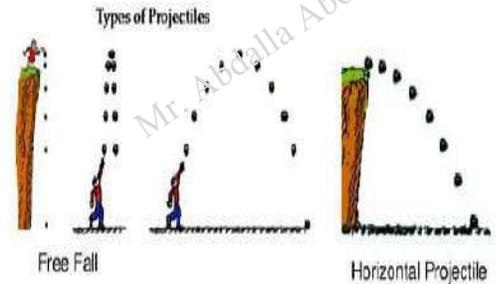
- Solve physical problems involving velocity.
- Solve problems on projectiles.

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Introduction



- There are several different types of projectiles. We will only study horizontal projectiles and objects in free fall.



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Newton's Second Law of Motion

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Newton's second law of motion
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States that: $F = ma$

ignore any other forces, such as air resistance.

$$F = -mg,$$

where the minus sign indicates that the force is acting downward

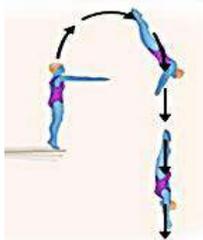
Where: F is the sum of the forces acting on an object,
 m is the mass of the object
 a is the acceleration of the object.

$$a(t) = h''(t)$$

$$h''(t) = -g.$$

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The force due to gravity is weight,

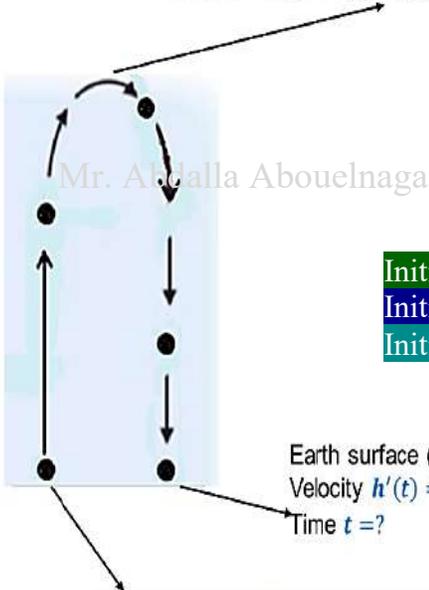
$$W = mg,$$

g is the gravitational constant.

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A) (In one Dimension) Vertical Motion

The highest height $h(t) = y(t) = ?$ And the velocity $h'(t) = y'(t) = v(t) = 0$ and time $t = ?$



$$a = h''(t) = -9.8 \text{ m/s}^2$$

$$a = h''(t) = -32 \text{ ft/s}^2$$

Initial velocity $h'(0) = y'(0) = v(0) = ?$

Initial height $h(0) = y(0) = ?$

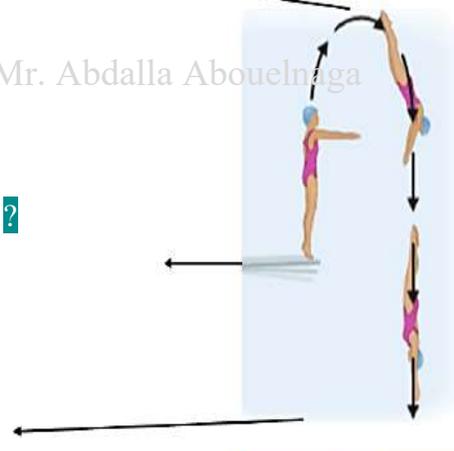
Initial time $t_0 = 0$

Earth surface (at impact): height $h(t) = y(t) = 0$

Velocity $h'(t) = y'(t) = v(t) = ?$

Time $t = ?$

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Initial velocity $h'(0) = y'(0) = v(0) = ?$

Initial height $h(0) = y(0) = ?$

Initial time $t_0 = 0$

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Initial Condition: Initial height $h(0)$
Initial velocity $h'(0)$

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Free Fallen

Acceleration $h''(t) = a(t) = -g$

Velocity $h'(t) = y'(t) = v(t) = -gt + v_0$

Position $h(t) = y(t) = s(t) = -\frac{1}{2}gt^2 + v_0t + h_0$

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m/s^2

ft/s^2

Acceleration $h''(t) = a(t) = -9.8 \text{ m/s}^2$

Velocity $h'(t) = y'(t) = v(t) = -9.8t + v_0 \text{ m/s}$

Position $h(t) = y(t) = s(t) = -4.9t^2 + v_0t + h_0 \text{ m}$

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Acceleration $h''(t) = a(t) = -32 \text{ ft/s}^2$

Velocity $h'(t) = y'(t) = v(t) = -32t + v_0 \text{ ft/s}$

Position $h(t) = y(t) = s(t) = -16t^2 + v_0t + h_0 \text{ ft}$

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Example: 1 Mr. Abdalla Abouelnaga

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Identify the Initial Conditions:

For the following examples we are going to identify the initial height $h(0)$ and initial velocity $h'(0)$

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1. An object is dropped from a height of 80ft.

Answer:

2. An object released from a height of 60ft with an upward velocity 10ft/s

Answer:

3. An object released from height of 20ft with a downward velocity of 4ft/s

Answer:

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Note: the height and velocity are given in ft so you need to change it into meters to use: $h''(t) = -9.8m/s^2$ or you can use $h''(t) = -32ft/s^2$

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Example: 2 If a diving board is 4.5 m above the surface of the water and a diver start with initial velocity 2.4m/s upward, what is the diver velocity at impact?

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Exercise: 2.1

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Question 34 MOE Exam 2019/2020

A diver drops from 36 meters above the water. What is the diver's velocity at impact?

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A

26.6 m/s

B

-26.6 m/s

C

36 m/s

D

-36 m/s

Solution Steps

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B

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Exercise: 2.2

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A ball is propelled straight upward from the ground with initial velocity 19.6m/s. Ignoring the air resistance find the equation for the height of the ball, determine the maximum height and the amount of time the ball spend in the air.

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Example: 3

One of the Basketball players has a vertical jump of 20 in, what is the initial velocity required to jump this height?

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Exercise: 3.1

It has been reported that former basketball star Michael Jordan had a vertical leap of 135cm. Ignoring air resistance, what is the initial velocity required to jump this high?

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Exercise: 3.2

A ball is propelled straight upward from the roof of a 30 m height building with initial velocity 10m/s Find:

1) An equation for the velocity on the ball at any time t ?

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2) An equation for the height of the ball at any time t ?

3) The time it takes for the ball to reach the Earth.

4) The velocity of the ball when it hits the ground?

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5) When does the ball reach its maximum height?

6) The maximum height of the ball?

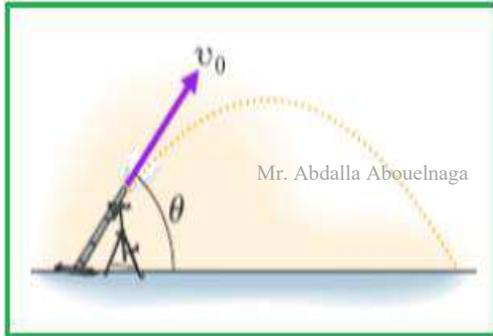
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Here we apply Newton's Second law to vertical and horizontal directions separately.

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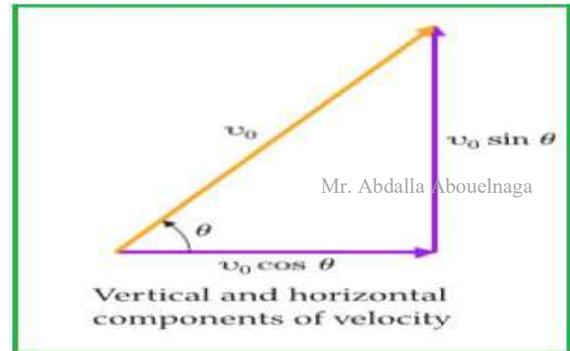


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Vertical Force (gravity)

$$y''(t) = -9.8m/s^2$$

Initial vertical velocity

$$y'(0) = v_y = v_0 \sin \theta$$

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No horizontal force acts on the object

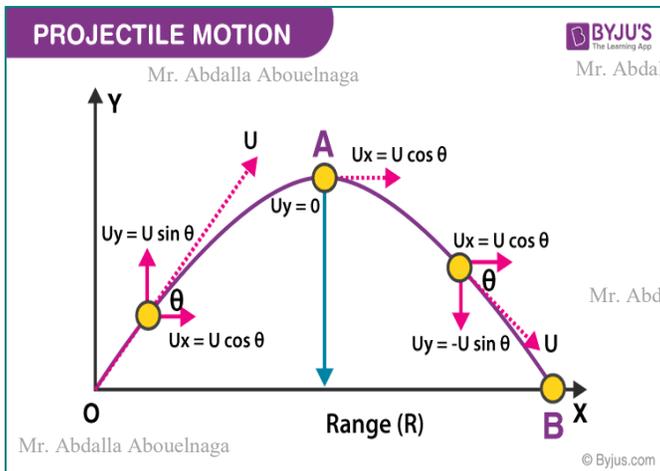
$$x''(t) = 0$$

Initial horizontal velocity

$$x'(0) = v_x = v_0 \cos \theta$$

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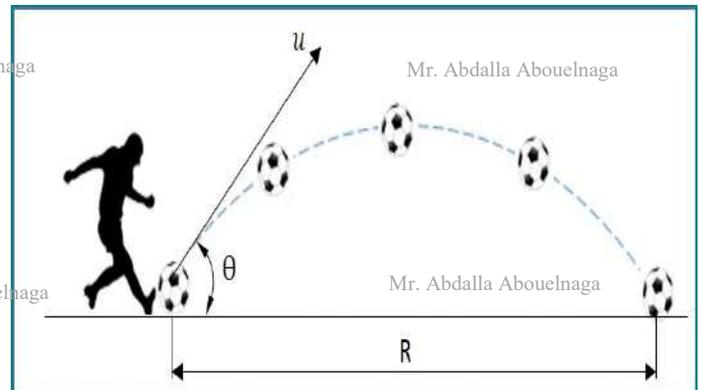
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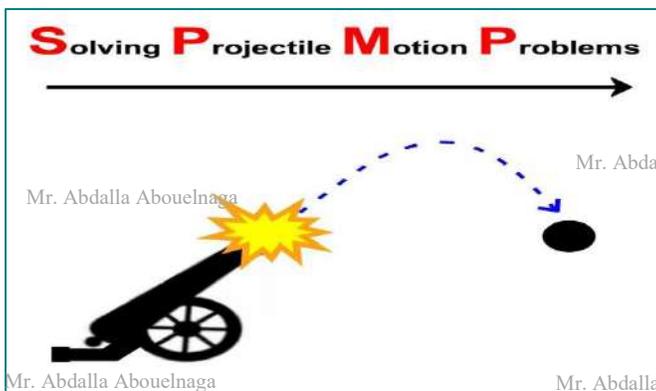
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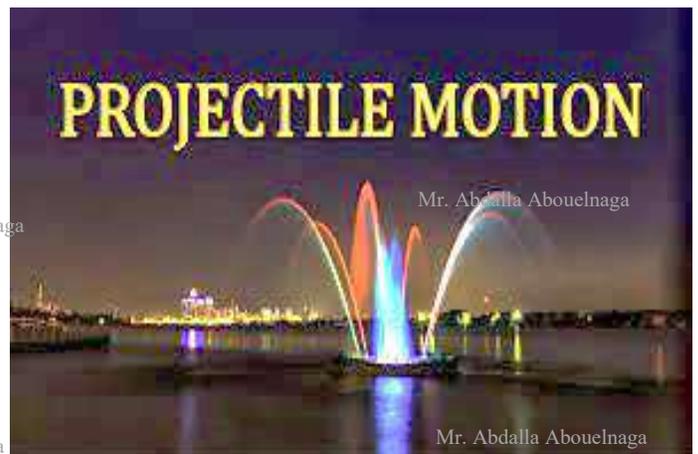


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Initial vertical velocity $y'(t) = v_0 \sin \theta$

Horizontal velocity $x'(t) = v_0 \cos \theta$

Horizontal Distance

Horizontal →

$$x''(t) = 0$$

$$x'(t) = v_0 \cos \theta$$

$$x(t) = v_0 \cos \theta t$$

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Height/vertical →

$$y''(t) = a(t) = -9.8$$

$$y'(t) = -9.8 t + v_0 \sin \theta$$

Or

$$y'(t) = -32 t + v_0 \sin \theta$$

$$y(t) = -4.9 t^2 + v_0 \sin \theta t + h_0$$

or

$$y(t) = -16 t^2 + v_0 \sin \theta t + h_0$$

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Example 4

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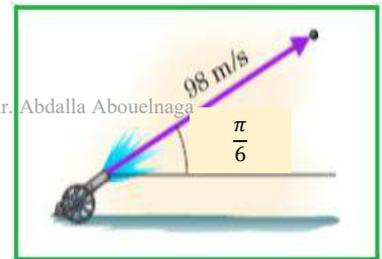
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An object is launched at angle $\theta = \frac{\pi}{6}$ from the horizontal with an initial speed of 98m/s. determine the time of the flight and the horizontal range.

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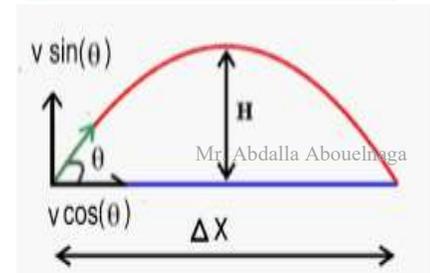
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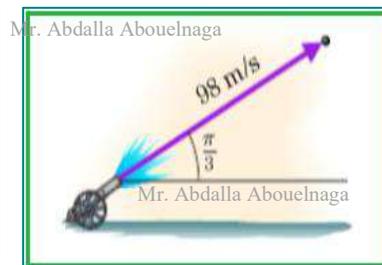
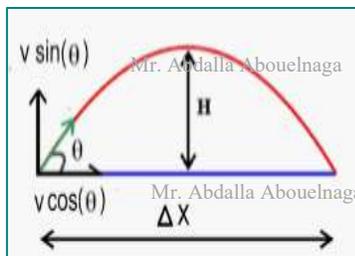
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Exercise 4.1

An object is launched at angle $\theta = \frac{\pi}{3}$ from the horizontal with an initial speed of 98m/s . determine the time of the flight and the horizontal range.

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Example:5

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Venus Williams has one of the fastest serves in women's tennis. Suppose that she hits a serve from a height of 10 feet at an initial speed of 120 mph and at an angle of 7° below the horizontal. The serve is "in" if the ball clears a 3'-high net that is 39' away and hits the ground in front of the service line 60' away. (We illustrate this situation in Figure 6.46.) Determine whether the serve is in or out.

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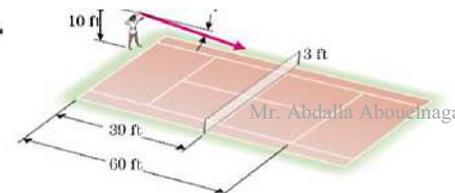


FIGURE 6.46
Height of tennis serve

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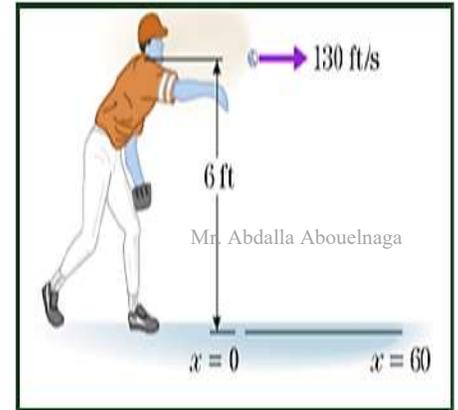
Exercise 5.1

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A baseball pitcher releases the ball horizontally from height of $6ft$ with initial velocity of $130ft/s$, find the height of the ball when it reaches home plate $60ft$ away.



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Example:6

Consider a knuckleball (see example 5.7) with lateral motion satisfying the initial value problem $x''(t) = -25 \sin(4\omega t + \theta_0)$, $x'(0) = x(0) = 0$. With $\omega = 1$, $\theta_0 = \pi/2$, find an equation for $x(t)$

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Term 3

امتحان العام 2023

الأسئلة المتضمنة في امتحان الوزارة للفصل الدراسي الثالث 2024
Questions which included in MOE Exam Term 3 (2024)

Mathematics G2 Advanced

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An object is launched from the ground at an angle of 20° with an initial speed of 48 ft/s. Find the time of flight. (ignore air resistance)

يتم إطلاق جسم ما من الأرض بزاوية 20° بسرعة ابتدائية 48 ft/s. أوجد زمن التحليق. (تجاهل مقاومة الهواء)

A $t = 1.026 s$

B $t = 16.4 s$

C $t = 2.03 s$

D $t = 45.1 s$

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عزيزي الطالب : اجتهد في فهم ومذاكرة هذه الأمثلة والتمارين بالكتاب المدرسي تمنياتي للجميع بالنجاح والتفوق

Term 3

امتحان العام 2021.2022

الأسئلة المتضمنة في امتحان الوزارة للفصل الدراسي الثالث 2024
Questions which included in MOE Exam Term3 (2024)

Mathematics G12 Advanced

22

Find the time of flight of an object launched at angle 30° with initial speed 40 m/s.

أوجد زمن التحليق لجسم أطلق بزاوية 30° مع سرعة ابتدائية 40 m/s.

A $t = 4.08 s$

B $t = 7.07 s$

C $t = 10 s$

D $t = 17.3 s$

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An object is dropped from a height
of 50 ft.

أسقط جسم من ارتفاع 50 ft .
حدد الشروط الابتدائية $y(0)$ و $y'(0)$.

Identify the initial conditions $y(0)$
and $y'(0)$.

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Exam2021.2022 T3

A $y(0) = 50 , y'(0) = 0$

B $y(0) = -50 , y'(0) = 0$

C $y(0) = 0 , y'(0) = 50$

D $y(0) = 0 , y'(0) = -50$

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Exam2020.2021 T3

A diver drops from 120 ft above the water (about the height of an Olympic platform
dive). What is the diver's velocity at impact ? (gravitational constant 32 ft/sec^2).

A -32 ft/sec

B $\frac{15}{2} \text{ ft/sec}$

C $-32 \frac{15}{2} \text{ ft/sec}$

D 120 ft/sec

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Exam2020.2021 T3

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