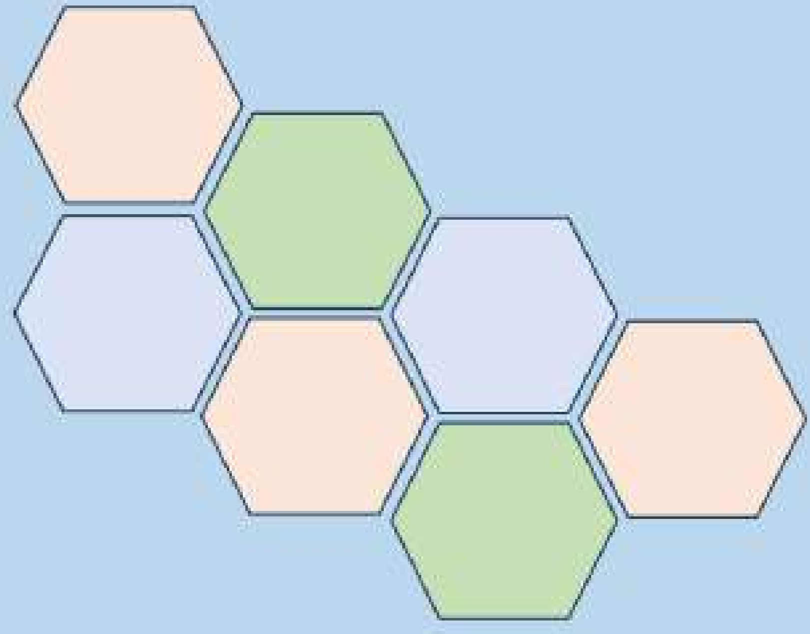
# ملزمة أوراق عمل الوحدة الخامسة Differentiation of Applications منهج ريفيل الإجابات بمفاتيح متبوعة Reveal

# Unit 5

# Applications of Differentiation



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**T3 - 2025/2026**

# L 5.1 Antiderivative

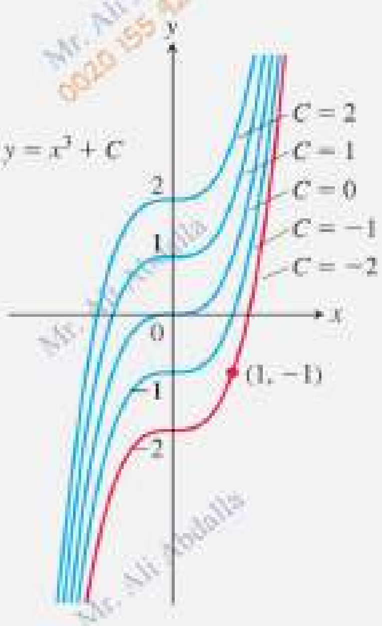
**1. Find the derivative of each function.**

**a)** $F(x) = x^3 + 3$

**b)** $F(x) = x^3 + 7$

**c)** $F(x) = x^3 - 25$

Find the antiderivative of $f(x) = 3x^2$



**Theorem 1.1**

Suppose that $F$ and $G$ are both antiderivatives of $f$ on an interval I . Then,

$$G(x) = F(x) + c,$$

for some constant $c$.

**Definition 1.1**

Let $F$ be any antiderivative of $f$ on an interval $I$ . The indefinite integral of $f(x)$ (with respect to $x$) on $I$ , is defined by

$$\int f(x)dx = F(x) + c$$

,where $c$ is an arbitrary constant (the constant of integration).

The process of computing an integral is called **integration**. Here, $f(x)$ is called the **integrand** and the term $dx$ identifies $x$ as the **variable of integration**.

Evaluate:

**2.** $\int 3x^2\, dx$

**3.** $\int t^5\, dx$

**Theorem 1.2 (Power Rule)**

For any rational power $n \neq -1$,

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Here, if $n < -1$, the interval $I$ on which this is defined can be any interval that does not include $x = 0$.

**By using Power rule Evaluate:**

**4.** $\int x^9 \, dx$

**5.** $\int \frac{1}{x^5} \, dx$

**6.** $\int \sqrt{x} \, dx$

**7.** $\int \sqrt[3]{x^2} \, dx$

## IMPORTANAT RULES

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad for \quad n \neq -1 \; (power \; rule)$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \csc^2 x \, dx = -\cot x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \csc x \cot x \, dx = -\csc x + c$$

$$\int e^x dx = e^x + c$$

$$\int e^{-x} dx = -e^{-x} + c$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$$

$$\int \frac{1}{|x|\sqrt{1-x^2}} dx = \sec^{-1} x + c$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

تعميم لبعض قواعد التكامل يمكن استخدامها مباشرة:

A generalization of some rules of integration can be used directly

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c \qquad \int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + c$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + c$$

بالمثل مع باقي النسب المثلثية

$$\int f'(x)\,e^{f(x)}\,dx = e^{f(x)} + c \qquad \int e^{ax+b}\,dx = \frac{1}{a}\,e^{ax+b} + c$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + c \qquad \int f'(x)\,a^{f(x)}\,dx = \frac{1}{\ln a}\,a^{f(x)} + c$$

**Theorem 1.3**

Suppose that $f(x)$ and $g(x)$ have antiderivatives. Then, for any constants, $a$ and $b$,

$$\int [a\,f(x) + b\,g(x)]\,dx = a\int f(x)\,dx + b\int g(x)\,dx$$

**8.** $\int (3\cos x + 6x^5)\,dx$

**9.** $\int (\sin x + \sqrt{x})\,dx$

**10.** $\int \left(3e^x - \frac{2}{1+x^2}\right)dx$

**11.** $\int \left(\frac{x^{1/3} - 3}{x^{2/3}}\right)dx$

**12.** $\int \left( \frac{x + x^{3/4}}{x^{5/4}} \right) dx$

**Theorem 1.4**

For $x \neq 0$ , $\frac{d}{dx}(\ln x) = \frac{1}{x}$

**13.** For any $x$ for which $\tan x \neq 0$, Evaluate : $\frac{d}{dx}(\ln|tan\, x|)$

**Corollary 1.1**

In any interval not containing 0,

$$\int \frac{1}{x}\, dx = \ln|x| + c$$

**Corollary 1.2**

In any interval not containing 0,

$$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$$

**Evaluate:**

**14.** $\int \frac{4x}{x^2 + 4}\, dx$

**15.** $\int \frac{\sec^2 x}{\tan x}\, dx$

**16.** $\int \frac{\cos x}{\sin x}\, dx$

**17.** $\int \frac{x}{(x-1)(x+1)}\, dx$

| | | |
|---|---|---|
| **18.** | $\int (x-3)(x+4)\,dx$ | **19.** $\int \frac{x^3+1}{x}\,dx$ |
| **20.** | $\int \frac{x^2+1}{\sqrt{x}}\,dx$ | |
| **21.** | $\int \frac{4x+6}{x^2+3x}\,dx$ | |

## Challenge

**22.** $\int \frac{x}{\sqrt{1-x^2}}\,dx$

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**23.** $\int \frac{2x-1}{x^2+1}\,dx$

## Finding the Position of a Falling Object Given Its Acceleration

**24.** If an object's downward acceleration is given by $y''(t) = -32ft/s^2$, find the position function $y(t)$. Assume that the initial velocity is $y'(0) = -100ft/s$ and the initial position is $y(0) = 100{,}000$ feet.

### Find the derivative.

**25.** $\dfrac{d}{dx}(\ln|\sec x + \tan x|)$

**26.** $\dfrac{d}{dx}(\ln|\sin x - 2|)$

**Find the general antiderivative.**

**25.** $\displaystyle\int \frac{\cos x}{\sin x}\,dx$

**26.** $\displaystyle\int \left(2\cos x - \sqrt{e^{2x}}\right) dx$

**27.** $\displaystyle\int \frac{e^{x}}{e^{x}+3}\,dx$

**28.** $\displaystyle\int \frac{e^{x}+3}{e^{x}}\,dx$

**29.** $\displaystyle\int x^{1/4}\left(x^{5/4}-4\right) dx$

**30.** $\displaystyle\int x^{2/3}\left(x^{-4/3}-3\right) dx$

**31.** $\displaystyle\int \csc x\,(\sec x \tan x - \cot x)\,dx$

**32.** $\int \frac{\sin^2 x}{1-\cos x}\,dx$

**33.** $\int \frac{1}{1-\sin x}\,dx$

**34.** $\int \sin x \cos x \, dx$

**35.** $\int (\cos^2 x - \sin^2 x)\,dx$

**36.** $\int \sin^2 x \, dx$

**37.** $\displaystyle\int \frac{1-\sin 2x}{\sin x-\cos x}\,dx$

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**38.** $\displaystyle\int \frac{4}{1+\cos 2x}\,dx$

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**39.** $\displaystyle\int \frac{e^{\tan x}}{1-\sin^2 x}\,dx$

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**40.** $\displaystyle\int e^{x^2+\ln x}\,dx$

## Challenge

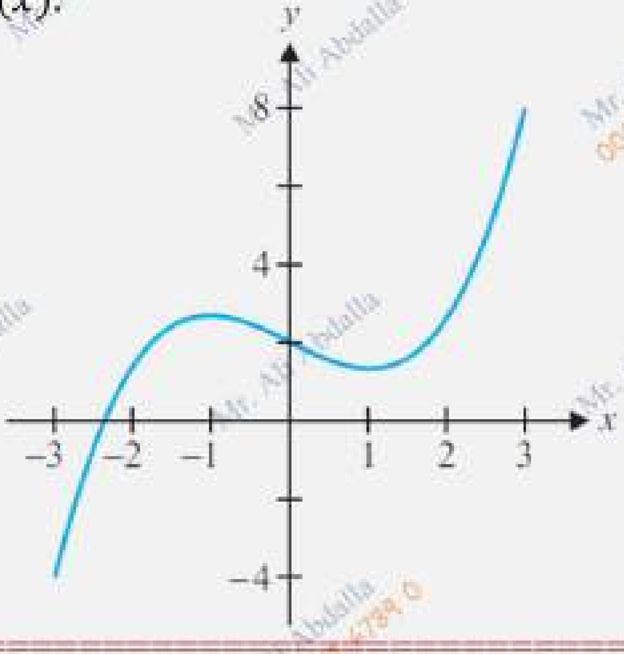
**41. Evaluate :**

$$\int \frac{x^3 + 4x}{x^4 + 1}\, dx$$

**42. Find all functions satisfying the given conditions.**

$$f'''(x) = \sin x - e^x$$

**43.** Sketch the graph of two functions $f(x)$ corresponding to the given graph of $y = f'(x)$.



**44.** Show that

$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1}x + c \quad \text{and} \quad \int \frac{-1}{\sqrt{1-x^2}}\,dx = -\sin^{-1}x + c$$

Explain why this does not imply that $\cos^{-1}x = -\sin^{-1}x$. Find an equation relating $\cos^{-1}x$ and $\sin^{-1}x$

## Challenge

**45.** **Evaluate :**

$$\int \sec x \; dx$$

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**46.** Determine the position function if the acceleration function is $a(t) = 3 \sin t + 1$, the initial velocity is $v(0) = 0$ and the initial position is $s(0) = 4$.

**47.** Find the function $f(x)$ satisfying the given conditions.

a) $f'(x) = 4\cos x \quad , f(0) = 3$

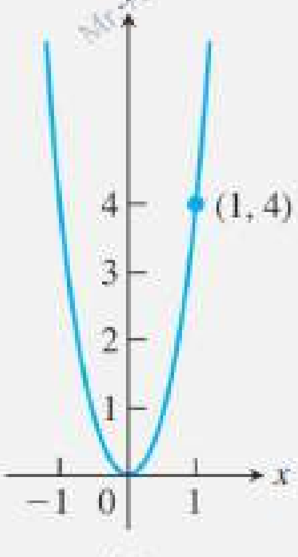
b) $f''(x) = 12x^2 + 2e^x \quad , f'(0) = 2 \, , \; f(0) = 3$

c) $f''(t) = 5 + 6t \quad , f(1) = 3 \; , \; f(-1) = -2$

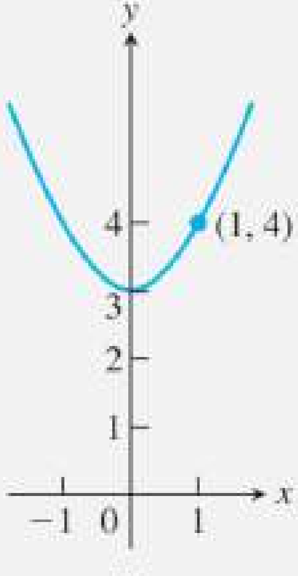
**48.** Find a function $f(x)$ such that the point $(1, 2)$ is on the graph of $y = f(x)$, the slope of the tangent line at $(1, 2)$ is $3$ and $f''(x) = x - 1$.

**49.** Which of the following graphs shows the solution of the initial value problem

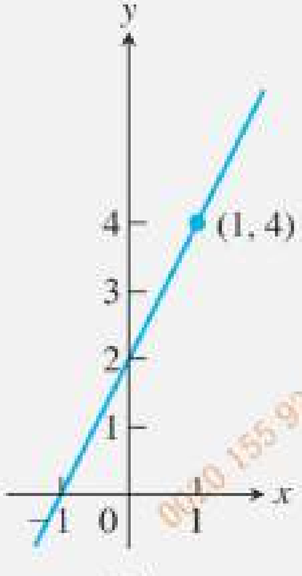
$$\frac{dy}{dx} = 2x\,, \quad y = 4 \text{ when } x = 1?$$



(a)



(b)



(c)

**50.** Find the general antiderivative.

**A)** $\int 4\,\frac{\cos x}{\sin^2 x}\,dx$

**B)** $\int \frac{4}{\sqrt{1-x^2}}\,dx$

**C)** $\int (2x^{-1} + \sin x)\,dx$

**D)** $\int (3\cos x - \sin x)\,dx$

**E)** $\int 5\sec^2 x\,dx$

**F)** $\int \left(3\cos x - \frac{1}{x}\right)dx$

**G)** $\int 2\sec x \tan x\,dx$

**I)** $\int \left(2x^{-2} + \frac{1}{\sqrt{x}}\right)dx$

## L 5.2 Sums and Sigma Notation

In general, for any real numbers $a_1, a_2, \ldots, a_n$, we have

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$$

Write each of the following without sigma notation sign:

**1)** $\sum_{i=1}^{5} 3i - 1 =$

**2)** $\sum_{i=1}^{10} i^2 =$

**3)** $\sum_{i=1}^{5} i^2 - i =$

**For questions (4-13): Write each of the following in summation notation**

**4)** $3^3 + 4^3 + 5^3 + \cdots + 45^3 = \sum$

**5)** $\sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{15} = \sum$

**6)** $\sqrt{2-1} + \sqrt{3-1} + \sqrt{4-1} + \cdots + \sqrt{50-1} = \sum$

**7)** $1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + 99 \times 100 = \sum$

**8)** $4 + 8 + 12 + \cdots + 400 = \sum$

**9)** $1 + 3 + 5 + \cdots + 101 = \sum$

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**10)** $5 + 10 + 15 + \cdots + 205 = \sum$

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**11)** $1.2 + 1.5 + 1.8 + \cdots + 31.2 = \sum$

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**12)** $2 + 4 + 8 + 16 + \cdots + 4048 = \sum$

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**13)** $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2048} = \sum$

## Sigma notation Rules and properties

for $n > 0$ if $a, b$ and $c$ are real numbers:

1) $\displaystyle\sum_{i=1}^{n} c = nc$ (Sum of constants)

2) $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ (Sum of the first $n$ positive integers)

3) $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ (Sum of the squares of the first $n$ positive integers).

4) $\displaystyle\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ (Sum of the cubes of the first $n$ positive integers).

5) $\displaystyle\sum_{i=1}^{n} i^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$

6) $\displaystyle\sum_{i=1}^{n} i^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$

7) $\displaystyle\sum_{i=1}^{n} ar^{i-1} = \frac{a(1-r^n)}{1-r}$ , $r \neq 1$ Geometric series

8) $\displaystyle\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$ , $|r| < 1$ Geometric series

9) $\displaystyle\sum_{i=1}^{n} (ca_i \pm db_i) = c\sum_{i=1}^{n} a_i \pm d\sum_{i=1}^{n} b_i$ For any constants $c$ and $d$

**Use summation rules to compute the sum.**

**14)** $\displaystyle\sum_{i=1}^{70} (3i-1) = \qquad \sum_{i=1}^{70} \qquad\qquad \sum_{i=1}^{70}$

**15)** $\sum_{i=1}^{40}(4-i^2) = \sum_{i=1}^{40} \qquad \sum_{i=1}^{40}$

**16)** $\sum_{n=1}^{100}(n^2-3n+2) = \sum_{n=1}^{100} \qquad \sum_{n=1}^{100} \qquad \sum_{n=1}^{100}$

**17)** $\sum_{k=0}^{20}(k^2+5) = \qquad + \sum$

$= \qquad + \sum \qquad + \sum$

**18)** $\sum_{k=5}^{20}(k^2-3) = \sum \qquad - \sum$

**REMEMBER**

**REMEMBER**

$\sum_{i=k}^{n} i$ The Number of terms $= n-k+1$

$\sum_{i=k}^{n} i = \sum_{i=1}^{n} i - \sum_{i=1}^{k-1} i$

$\sum_{i=0}^{n} f(i) = f(0) + \sum_{i=1}^{n} f(i)$

## Computing a Sum of Function Values

**19)** Sum the values of $f(x) = x^2 + 3$
evaluated $at\ x = 0.1, x = 0.2, \ldots, x = 1.0$

**20)** Sum the values of $f(x) = 3x^2 - 4x + 2$
evaluated at $x = 1.05,\ x = 1.15,\ x = 1.25, \ldots,\ x = 2.95$

**21)** Compute sums of the form $\sum_{i=1}^{n} f(x_i)\,\Delta x$ for the given values of $x_i$ .

$f(x) = x^2 + 4x$ , $x = 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0$ ; $\Delta x = 0.2$ ; $n = 5$

**22)** Compute sums of the form $\sum_{i=1}^{n} f(x_i)\,\Delta x$ for the given values of $x_i$ .

$f(x) = x^2 + 4x$ , $x = 2,\ 4,\ 6, \ldots ,\ 100$

**23)** Compute sums of the form $\sum_{i=1}^{n} f(x_i)\,\Delta x$ for the given values of $x_i$ .

$f(x) = x^3 + 4$ ; $x = 2.05,\ 2.15,\ 2.25,\ 2.35, \ldots ,\ 2.95$ ; $\Delta x = 0.1$ ; $n = 10$

**24)** Compute the sum and the limit of the sum as $n \to \infty$

**A)** $\sum_{i=1}^{n} \frac{3}{n}\left(\frac{i}{n} + 2\right)$

**B)** $\sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 + 2\left(\frac{i}{n}\right)\right]$

# Principle of Mathematical Induction

**25)** Use mathematical induction to prove that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

For $n = 1$, we have

as desired. So, the proposition is true for $n = 1$. Next, **assume** that

$$\sum_{i=1} i^2 = \underline{\qquad\qquad\qquad\qquad\qquad}$$ Induction assumption

for some integer $k \geq 1$.

In this case, we have by the induction assumption that for $n = k + 1$,

$$\sum_{i=1}^{n} i^2 = \sum_{i=1} i^2 = \sum i^2 + \sum i^2$$

## Translate each into summation notation and then compute the sum

**26)** The sum of the squares of the first 50 positive integers.

**27)** The square of the sum of the first 50 positive integers.

**28)** The sum of the square roots of the first 10 positive integers.

**29)** The square root of the sum of the first 10 positive integers.

## Use summation rules to compute the sum

**30)** $\sum_{i=3}^{30}[(i-3)^2+i-3]$

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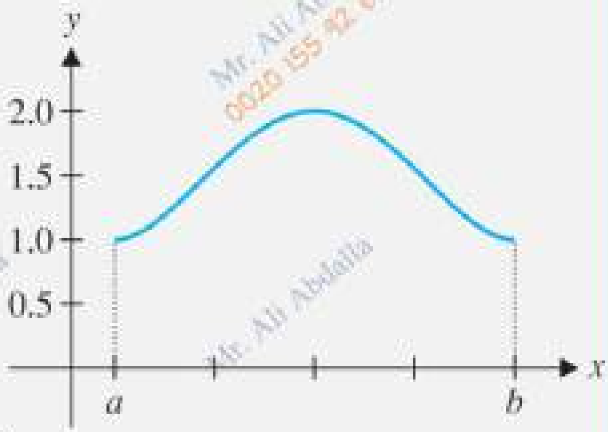
**31)** $\sum_{i=4}^{20}(i-3)(i+3)$

# L 5.3 Area



Assume that $f(x) \geq 0$ and $f$ is continuous on the interval $[a, b]$, as in Figure in the right.

- We start by dividing the interval $[a, b]$ into $n$ equal pieces. This is called a regular partition of $[a, b]$.
- The width of each subinterval in the partition is then $\frac{b-a}{n}$, which we denote by $\Delta x$ (meaning a small change in $x$).
- The points in the partition are denoted by
  $x_0 = a, \quad x_1 = x_0 + \Delta x, \quad x_2 = x_1 + \Delta x \quad , \dots \dots \dots \quad , \; x_n = b$
- In general, $\quad x_i = x_0 + i\Delta x, \quad$ for $\; i = 1, 2, \dots, n.$

$$\Delta x = x_i - x_{i-1} = \frac{b-a}{n} \quad , \quad x_0 = a$$

Then

$$\boxed{x_i = a + i\,\Delta x = a + \left(\frac{b-a}{n}\right) i}$$

## Definition 3.2 ( to find approximation Area )

Let $\{x_0, x_1, \dots, x_n\}$ be a regular partition of the interval $[a, b]$, with $x_i - x_{i-1} = \Delta x = \frac{b-a}{n}$, for all $i$. Pick points $c_1, c_2, \dots, c_n$, where $c_i$ is any point in the subinterval $[x_{i-1}, x_i]$, for $i = 1, 2, \dots, n$. (These are called evaluation points.) **The Riemann sum** for this partition and set of evaluation points is

$$\sum_{i=1}^{n} f(c_i)\,\Delta x$$

**The Riemann sum**

Them the approximation area given by

$$A \approx \sum_{i=1}^{n} f(c_i)\,\Delta x = \Delta x \sum_{i=1}^{n} f(c_i) = \frac{b-a}{n} \sum_{i=1}^{n} f(c_i)$$

## Definition 3.1 ( to find exactly Area by limits )

For a function $f$ defined on the interval $[a, b]$, if $f$ is continuous on $[a, b]$ and $f(x) \geq 0$ on $[a, b]$, the area $A$ under the curve $y = f(x)$ on $[a, b]$ is given by:

$$A = \lim_{n \to \infty} A_n = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

## Computing approximation Area

To approximation the area, we have three methods in this lesson, and we have other methods in lesson 5-7

To approximate the area under a curve by using $n$ rectangles on the interval $[a, b]$ use the following summation.

$$\sum_{i=1}^{n} f(c_i)\,\Delta x \qquad , \qquad \Delta x = x_i - x_{i-1} = \frac{b-a}{n}$$

Where:

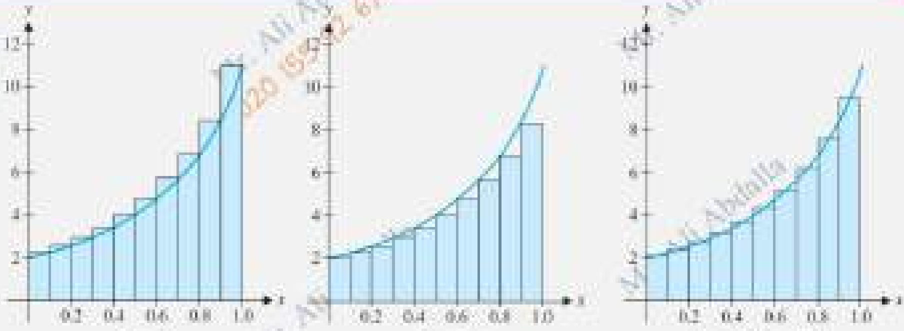
**1.** $c_i = x_i = a + \Delta x \cdot i$ The right-endpoint

**2.** $c_i = x_{i-1} = a + \Delta x \cdot (i-1)$ The left-endpoint

**3.** $c_i = \frac{1}{2}(x_{i-1} + x_i) = a + \Delta x \cdot \left(i - \frac{1}{2}\right)$ The midpoint

Exact area given by:

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$



$c_i = x_i$ $c_i = x_{i-1}$ $c_i = \frac{1}{2}(x_{i-1} + x_i)$

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We can find the area under a curve by using several methods, some method gives us approximate area and other give us exact area. We will summarize as following.

**Area**

- **Approximate area**
  - **rectangles**
    - **The right-endpoint**
    - **The left-endpoint**
    - **The midpoint**
  - **Trapezoid**
  - **Simpson**
- **Exact arae**
  - **limit of The Riemann sum**
  - **integration**

**1)** Approximate the area under the curve $y = f(x) = 2x - 2x^2$ on the interval [0, 1] by using 10 rectangles.

**A)** The right endpoint **B)** The left endpoint **C)** The midpoint

**Then find the exact area.**

Solution:

$\Delta x = \frac{1-0}{10} = 0.1$

| $x_i$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x_i)$ | | | | | | | | | | | |

The right-endpoint

The left-endpoint

The midpoint

| $c_i$ | 0.05 | 0.15 | 0.25 | 0.35 | 0.45 | 0.55 | 0.65 | 0.75 | 0.85 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(c_i)$ | | | | | | | | | | |

**Exact Area:**

$$A = \lim_{n\to\infty} A_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

**2)** Use the given function values to estimate the area under the curve using left-endpoint and right-endpoint evaluation.

| $x$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1.8 | 1.4 | 1.1 | 0.7 | 1.2 | 1.4 | 1.8 | 2.4 | 2.6 |

**Left-endpoint**

**Right-endpoint**

**3)** Use the given function values to estimate the area under the curve using left-endpoint and right-endpoint evaluation.

| $x$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 2.0 | 2.2 | 1.6 | 1.4 | 1.6 | 2.0 | 2.2 | 2.4 | 2.0 |

**Left-endpoint**

**Right-endpoint**

**4)** Approximate the area under the curve of the function $f(x) = 2x + 3$ and $x$-axis on the interval $[1, 5]$ using $n$ rectangles and the evaluation rules

(a) Left endpoint with $n = 20$ (b) Midpoint with $n = 10$

(c) Right endpoint with $n = 14$ (d) Exact Area

**The Riemann sum:** $\sum_{i=1}^{n} f(c_i)\,\Delta x$ , $\Delta x = x_i - x_{i-1} = \frac{b-a}{n}$

(a) left endpoint with $n = 20$

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

(b) midpoint with $n = 10$

(c) right endpoint with $n = 14$

(d) Exact Area

**5)** Find exact area under the curve of the function $f(x) = 4x - x^2$ and $x$-axis on the interval $[0, 4]$ by using limits of The Riemann sum.

$$A = \lim_{n\to\infty} A_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x \quad , \quad c_i = x_i = a + \frac{b-a}{n}\, i$$

**6)** Approximate the area under the curve on the given interval using $n$ rectangles and the evaluation rules

(a) left endpoint (b) midpoint (c) right endpoint.

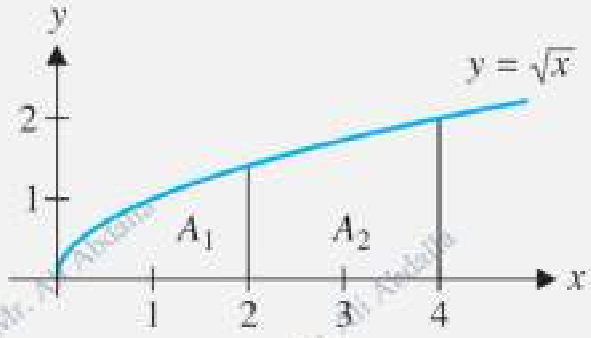
**A)** $y = \sqrt{x+2}$ on $[1,4]$ ; $n = 16$

**B)** $y = e^{-2x}$ on $[-1,1]$ ; $n = 16$

**C)** $y = \cos x$ on $\left[0, \frac{\pi}{2}\right]$ ; $n = 50$

**7)** In the figure, which area equals $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\sqrt{2}\left(\sqrt{1+\frac{i}{n}}\right)\left(\frac{2}{n}\right)$

Ans. $A_2$



Use right endpoint

Consider interval $[2,4]$, then $\Delta x=\frac{2}{n}$.
Use right endpoints as evaluation points, $x_i=\left(2+\frac{2i}{n}\right)$.

$$A=\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\sqrt{2+\frac{2i}{n}}\right)\frac{2}{n}\right]$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left[\sqrt{2}\left(\sqrt{1+\frac{i}{n}}\right)\frac{2}{n}\right]$$

Hence,

$$A_2=\lim_{n\to\infty}\sum_{i=1}^{n}\left[\sqrt{2}\left(\sqrt{1+\frac{i}{n}}\right)\frac{2}{n}\right].$$

**8)** In the figure, which area equals $\lim\limits_{n\to\infty}\sum\limits_{i=0}^{n-1}\frac{1}{\sqrt{n}}\left(\sqrt{1+2i}\right)\left(\frac{2}{n}\right)$

Ans. $A_1$

Use midpoint

$$\Delta x=\frac{2-0}{n}=\frac{2}{n}$$

$$c_i=a+\left(i-\frac{1}{2}\right)\Delta x$$

$$c_i=0+\frac{2}{n}\left(i-\frac{1}{2}\right)=\frac{1}{n}(2i-1)$$

$$A=\lim_{n\to\infty}\sum_{i=1}^{n}f(c_i)\Delta x$$

$$A=\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{\frac{1}{n}(2i-1)}\left(\frac{2}{n}\right)$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{\frac{1}{n}}\sqrt{2i-1}\left(\frac{2}{n}\right)$$

Let $i=k+1$ when $i=1\Rightarrow k=0$

when $i=n\Rightarrow k=n-1$

$$A=\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{1}{\sqrt{n}}\sqrt{2(k+1)-1}\left(\frac{2}{n}\right)$$

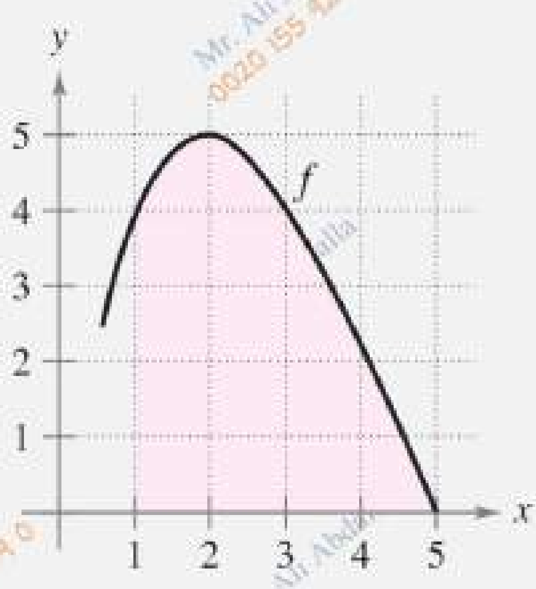
$$A=\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{1}{\sqrt{n}}\sqrt{2k+1}\left(\frac{2}{n}\right)$$

$$\Rightarrow A_1=\lim_{n\to\infty}\sum_{i=0}^{n-1}\frac{1}{\sqrt{n}}\sqrt{2i+1}\left(\frac{2}{n}\right)$$

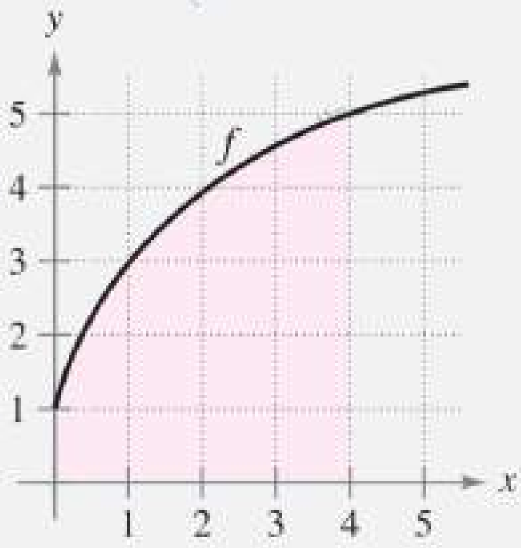
On $[0,2]$

In Exercises 9 and 10, use the given graph to estimate the left Riemann sum for the given interval with the stated number of subdivisions.

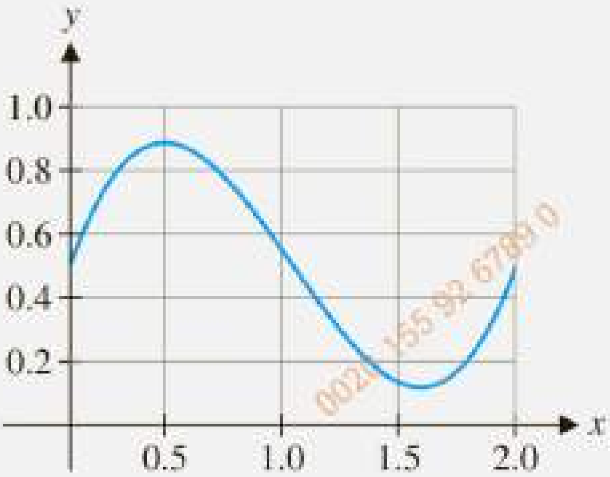
**9)** $[1,5], \quad n = 4$



**10)** $[0,4], \quad n = 4$



**11)** $[0,2], \quad n = 4$



# L 5.4 The Definite Integral

**Definition 4.1**

For any function $f$ defined on $[a, b]$, the **definite integral** of $f$ from $a$ to $b$ is:

Upper limit

Lower limit

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

whenever the limit exists and is the same for every choice of evaluation point, $c_1, c_2, \ldots, c_n$. When the limit exists, we say that $f$ is **integrable** on $[a, b]$.

## Properties of Definite Integral

**Definition**

$$\int_a^a f(x)\,dx = 0 \qquad \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

**Constant Multiple**

$$\int_a^b c\,dx = c(b-a) \qquad \int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$$

**Sum and Difference**

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

**Additivity**

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

**Integrals of Symmetric Functions**

If $f$ is even $f(-x) = f(x)$, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

If $f$ is odd $f(-x) = -f(x)$, then $\int_{-a}^{a} f(x)\,dx = 0$

**Comparison Property**

If $f(x) \ge 0$ for $a \le x \le b$, then $\int_a^b f(x)\,dx \ge 0$

If $f(x) \ge g(x)$ for $a \le x \le b$, then $\int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

If $m \le f(x) \le M$ for $a \le x \le b$, then $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$

**Rewrite each limit as definite integral form:**

**1)** $$\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\,\Delta x \quad , \quad [1,4]$$

**2)** $$\lim_{n\to\infty}\sum_{i=1}^{n}(2c_i^2-3c_i-2)\,\Delta x \quad , \quad [0,10]$$

**3)** $$\lim_{n\to\infty}\sum_{i=1}^{n}(2\cos c_i-\sin c_i)\,\Delta x \quad , \quad [0,\pi]$$

**Rewrite each definite integral as limit form:**

**4)** $$\int_0^1 (x^2-3x+2)dx$$

**5)** $$\int_0^3 \left(\frac{1}{1+x^2}\right)dx$$

Evaluate the integral by computing the limit of Riemann sums:

**6)** $\int_0^1 2x\ dx$

**7)** $\int_1^2 2x\ dx$

$$= \int \qquad dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

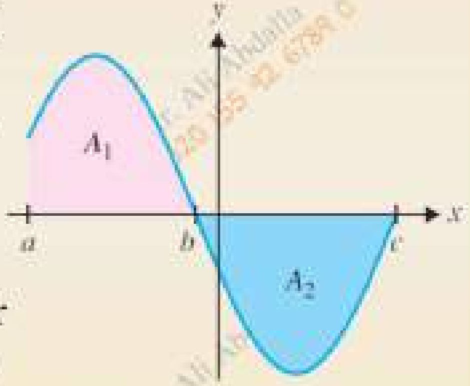
**8)** $\int_0^3 (x^2 - 3x)\ dx$

**9)** $\int_1^3 (x^2 - 3x)\ dx$

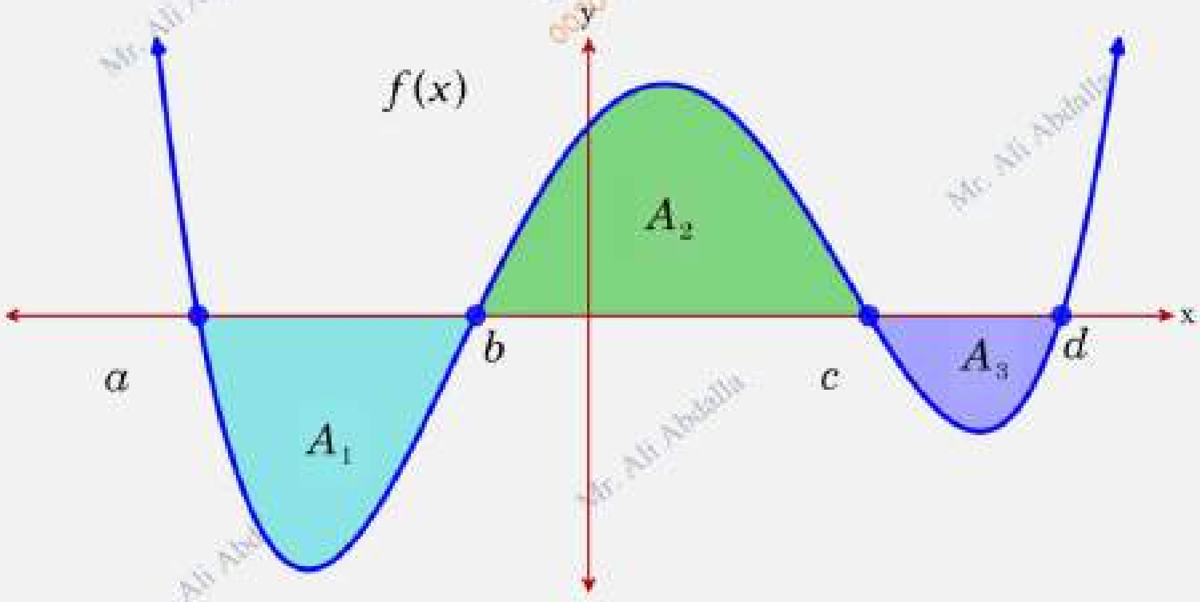
$$= \int \qquad dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

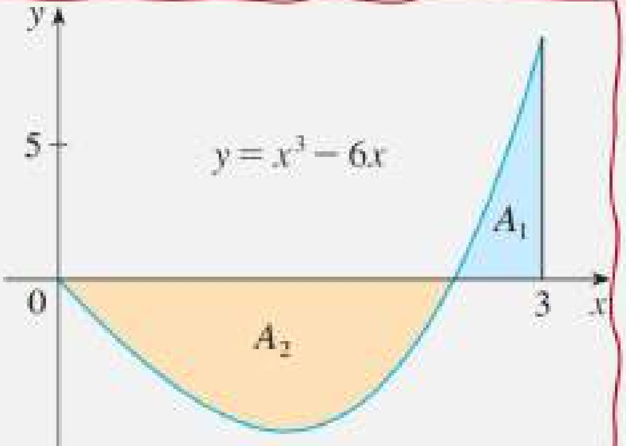
### Definition 4.2

Suppose that $f(x) \geq 0$ on the interval $[a, b]$ and $A_1$ is the area bounded between the curve $y = f(x)$ and the $x$-axis for $a \leq x \leq b$. Further, suppose that $f(x) \leq 0$ on the interval $[b, c]$ and $A_2$ is the area bounded between the curve $y = f(x)$ and the $x$-axis for $b \leq x \leq c$. The **signed area** between $y = f(x)$ and the $x$-axis for $a \leq x \leq c$ is $A_1 - A_2$, and the **total area** between $y = f(x)$ and the $x$-axis for $a \leq x \leq c$ is $A_1 + A_2$.



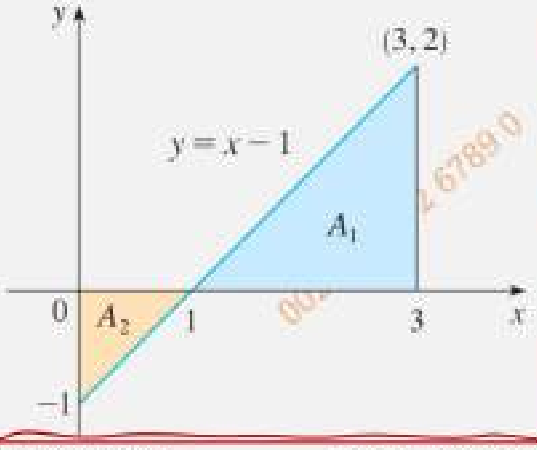
This means **signed area** is the difference between any areas lying above the $x$-axis and any areas lying below the $x$-axis, while the **total area** is the sum of the area bounded between the curve $y = f(x)$ and the $x$-axis.





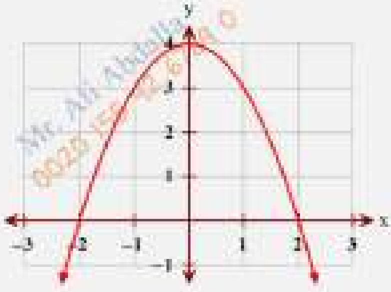
$$\int_0^3 (x^3 - 6x)\, dx = A_1 - A_2 = -6.75$$

$$\int_0^3 (x - 1)\, dx = A_1 - A_2$$
$$= \tfrac{1}{2}(2 \cdot 2) - \tfrac{1}{2}(1 \cdot 1) = 1.5$$

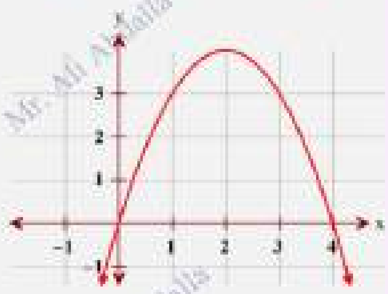


Write the given (**total**) area as an integral or sum of integrals.

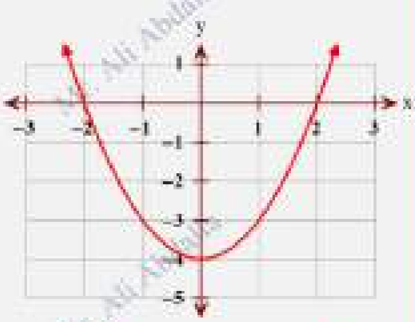
**10)** The area above the $x$-axis and below $y = 4 - x^2$



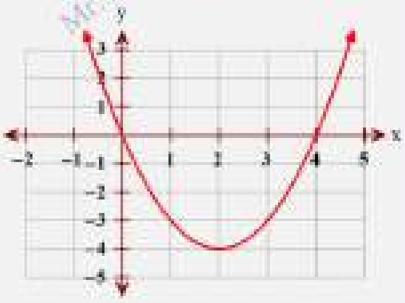
**11)** The area above the $x$-axis and below $y = 4x - x^2$



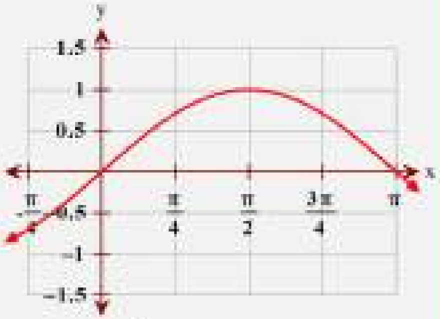
**12)** The area below the $x$-axis and above $y = x^2 - 4$



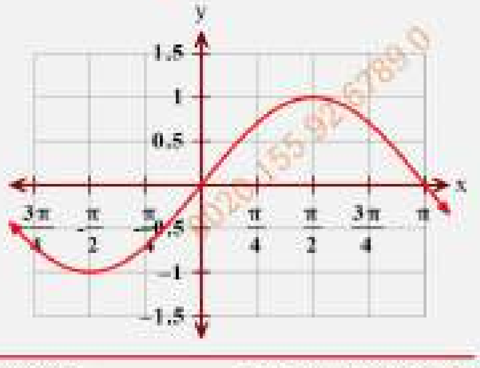
**13)** The area below the $x$-axis and above $y = x^2 - 4x$



**14)** The area between $y = \sin x$ and the $x$-axis for $0 \le x \le \pi$



**15)** The area between $y = \sin x$ and the $x$-axis for $-\frac{\pi}{2} \le x \le \frac{\pi}{4}$.



### Theorem 4.2

If $f$ and $g$ are integrable on $[a, b]$, then the following are true.

(i) For any constant $c$ and $d$ ,

$$\int_a^b [c\, f(x) + d\, g(x)]dx = c\int_a^b f(x)dx + d\int_a^b g(x)dx$$

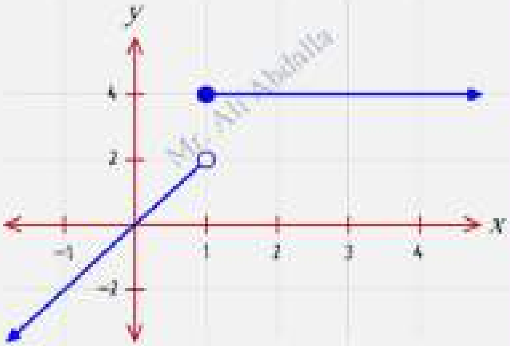
(ii) For any $c$ in $[a, b]$,

$$\int_a^b f(x)\, dx = \int_a^c f(x)dx + \int_c^b f(x)\, dx$$

(iii) $\int_b^a f(x)\, dx = -\int_a^b f(x)dx$ (iv) $\int_a^a f(x)\, dx = 0$

Integration at point

**16)** Evaluate $\int_0^4 f(x)\, dx$ , where $f(x)$ is defined by $f(x) = \begin{cases} 2x & , x < 1 \\ 4 & , x \geq 1 \end{cases}$



**17)** Write the expression as a single integral.

A) $\int_0^2 f(x)\, dx + \int_2^3 f(x)\, dx$

B) $\int_0^3 f(x)\, dx - \int_2^3 f(x)\, dx$

C) $\int_0^2 f(x)\, dx + \int_2^1 f(x)\, dx$

D) $\int_{-1}^2 f(x)\, dx + \int_2^3 f(x)\, dx$

**18)** Assume that $\int_1^3 f(x)\,dx = 3$ and $\int_1^3 g(x)\,dx = -2$ find

A) $\int_1^3 [f(x) + g(x)]\,dx$

B) $\int_1^3 [2f(x) - g(x)]\,dx$

C) $\int_1^3 [f(x) - g(x)]\,dx$

D) $\int_1^3 [4\,g(x) - 3f(x)]\,dx$

**Theorem 4.3**

Suppose that $g(x) \le f(x)$ for all $x \in [a, b]$ and that $f$ and $g$ are integrable on $[a, b]$. Then,

$$\int_a^b g(x)\,dx \le \int_a^b f(x)dx$$

## Average Value of a Function

**Average Value of a Function**

$$f_{ave} = \lim_{n\to\infty}\left[\frac{1}{b-a}\sum_{i=1}^{n} f(c_i)\,\Delta x\right] = \frac{1}{b-a}\int_a^b f(x)\,dx$$

**19)** Compute the average value of $f(x) = x^2 + 2x$ on the interval $[0, 1]$.

**20)** Compute the average value of $f(x) = \sin x$ on the interval $[0, \pi]$

## Squeeze property

Let $f$ any continuous function defined on $[a, b]$ and it has a minimum $m$, and a maximum $M$, on $[a, b]$, so that

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

This inequality used to approximate the value of the integration $\int_a^b f(x)\,dx$

Use the Integral Mean Value Theorem to estimate the value of the integral:

**21)** $\int_{\pi/3}^{\pi/2} 3\cos x^2\,dx$

**22)** $\int_0^{1/2} e^{-x^2}\,dx$

## Theorem 4.4 (Integral Mean Value Theorem)

If $f$ is continuous on $[a, b]$, then there is a number $c \in (a, b)$ for which

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Find a value of $c$ that satisfies the conclusion of the Integral Mean Value Theorem

**23)** $\int_0^2 3x^2\,dx \quad (= 8)$

**24)** $\int_{-1}^{1} (x^2 - 2x)\,dx \quad \left(= \frac{2}{3}\right)$

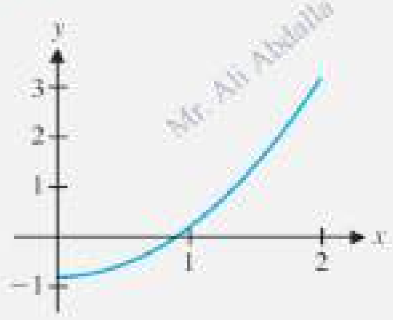
**25)** Find bounds for $\int_0^2 x^2\,e^{-\sqrt{x}}dx$

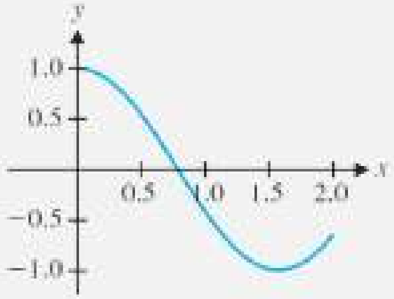
**26)** Express this limit as an integral:

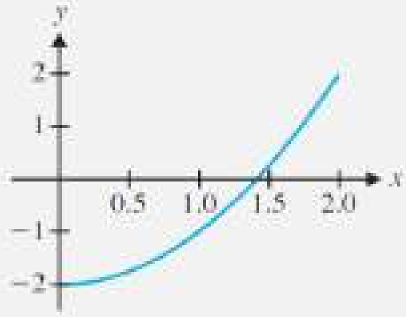
$$\lim_{n\to\infty}\frac{1}{n}\left[\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{n\pi}{n}\right]$$

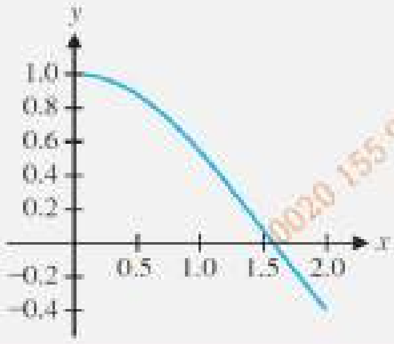
**27)** Show that the value of $\int_0^{\pi}\sqrt{1+\sin x}\,dx$ is between $\pi$ and $\sqrt{2}\,\pi$

**28)** Use the graph to determine whether $\int_0^{2} f(x)\,dx$ is positive or negative.



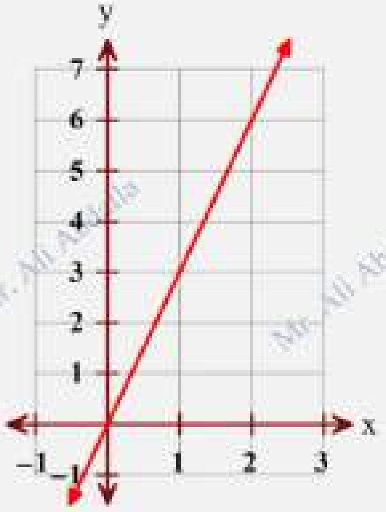




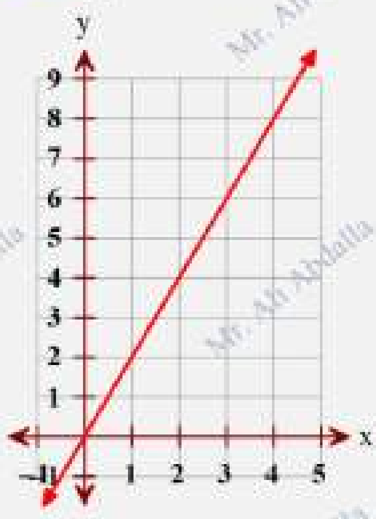


**29)** Use a geometric formula to compute the integral:

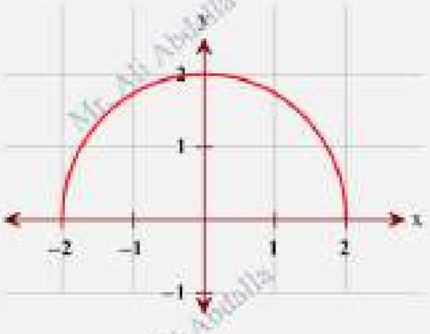
**A** $\int_0^2 3x\,dx$



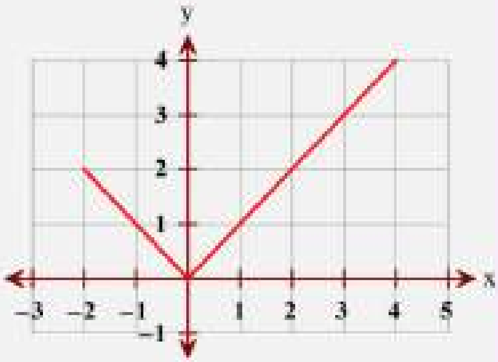
**B** $\int_1^4 2x\,dx$



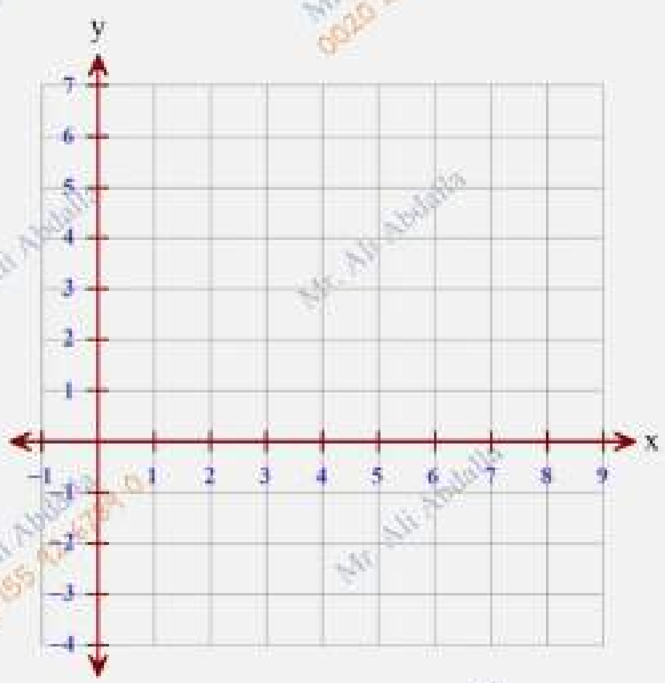
**C** $\int_0^2 \sqrt{4-x^2}\,dx$



**D** $\int_{-1}^{2} |x|\,dx$



**30)** Graph the function $f(x) = \begin{cases} 3x-3, & 0 \le x < 3 \\ 6 & , 3 \le x < 5 \\ 21-3x, & 5 \le x \le 8 \end{cases}$ then find $\int_0^8 f(x)\,dx$



**31)** If $\int_3^7 f(x)\,dx = 5$ and $\int_3^7 g(x)\,dx = 3$, then all of the following must be true except:

**(A)** $\int_3^7 f(x)g(x)dx = 15$ **(B)** $\int_3^7 [f(x) + g(x)]dx = 8$ **(C)** $\int_3^7 2\,f(x)\,dx = 10$

**(D)** $\int_3^7 [f(x) - g(x)]dx = 2$ **(E)** $\int_7^3 [g(x) - f(x)]dx = 2$

---

**32)** The expression $\frac{1}{20}\left(\sqrt{\frac{1}{20}} + \sqrt{\frac{2}{20}} + \sqrt{\frac{3}{20}} + \cdots + \sqrt{\frac{20}{20}}\right)$

is a Riemann sum approximation for:

A) $\int_0^1 \sqrt{\frac{x}{20}}\,dx$

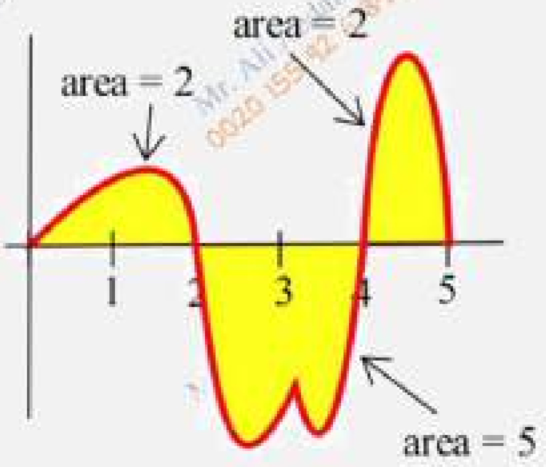
C) $\frac{1}{20}\int_0^1 \sqrt{\frac{x}{20}}\,dx$

B) $\int_0^1 \sqrt{x}\,dx$

D) $\frac{1}{20}\int_0^1 \sqrt{x}\,dx$

**33)** Use the graph to calculate: $\int_0^2 f(x)\,dx$, $\int_2^4 f(x)\,dx$ , $\int_4^5 f(x)\,dx$ and $\int_0^5 f(x)\,dx$

$\int_a^b f(x)dx$ = *area above − area below*



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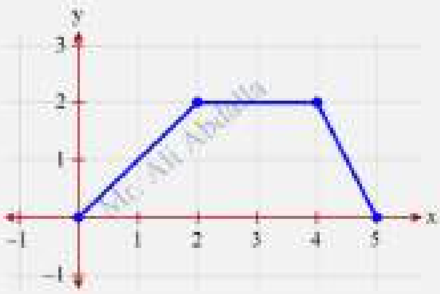
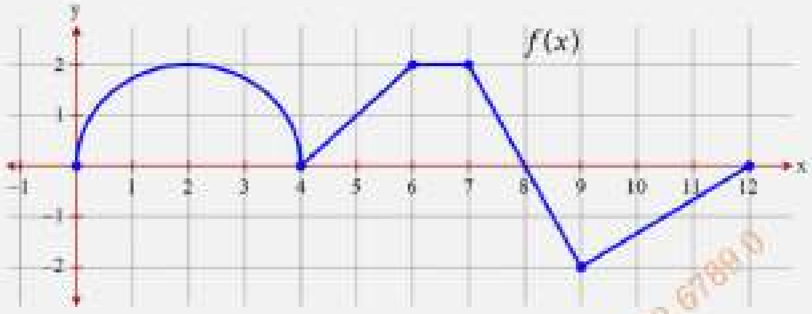
**34)** Let $A(x)$ represent the area bounded by the graph and the horizontal axis and vertical lines at $t = 0$ and $t = x$ for the graph shown. Evaluate $A(x)$ for $x = 1, 2, 3, 4,$ and $5$.



**35)** Use the graph to find



**A)** $\displaystyle\int_0^{12} f(x)\,dx$

**B)** $\displaystyle\int_0^{4} f(x)\,dx =$

**C)** $\displaystyle\int_{12}^{8} 3\,f(x)\,dx =$

**D)** $\displaystyle\int_0^{12} |f(x)|\,dx =$

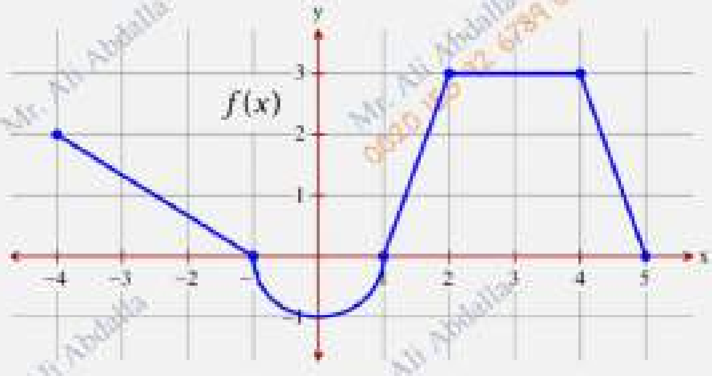
**36)** Use the graph to find

**A)** $\int_{-4}^{-1} f(x)\,dx$

**B)** $\int_{2}^{1} f(x)\,dx =$

**C)** $\int_{-4}^{1} |f(x)|\,dx =$

**D)** $\int_{-4}^{5} f(x)\,dx =$



**37)** Let $f$ and g be continuous functions that produce the following definite integral values. $\int_1^2 f(x)\,dx = -2$ , $\int_1^6 f(x)\,dx = 4$ , $\int_1^6 g(x)\,dx = 8$

Find the following:

| | | |
|---|---|---|
| A) $\int_2^2 g(x)\,dx$ | B) $\int_6^1 g(x)\,dx$ | C) $3\int_1^2 f(x)\,dx$ |
| D) $\int_2^6 f(x)\,dx$ | E) $\int_1^6 [f(x) - g(x)]\,dx$ | F) $\int_1^6 [3f(x) - g(x)]\,dx$ |
| G) $\int_1^6 \lvert f(x) - g(x)\rvert\,dx$<br>Cannot be determined | H) $\left\lvert \int_1^6 [f(x) - g(x)]\,dx \right\rvert$ | |

# L 5.5 The Fundamental Theorem of Calculus Part I

**Theorem 5.1**

If $f$ is continuous on $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

**Evaluate**

**1)** $\int_1^3 (3x^2 + 2x)\,dx$

**2)** $\int_0^{\frac{\pi}{2}} \sin x\,dx$

**3)** $\int_1^x (2t + 3)\,dt$

**4)** $\int_{-1}^1 e^{-2x}\,dx$

**5)** $\int_0^2 \frac{e^{2x} - 2e^{3x}}{e^{3x}}\,dx$

**6)** $\int_0^1 (\sin^2 x + \cos^2 x)\,dx$

**7)** $\int_0^{\frac{\pi}{4}} \sec x \tan x\,dx$

**8)** $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$

## Evaluate

**9)** $\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$

**10)** $\int_0^{\frac{\pi}{2}} \sin x \cos x\,dx$

**11)** $\int_0^{\frac{\pi}{4}} \cos^2 2x\,dx$

**12)** $\int_0^{1} \frac{2x+1}{1+x^2}\,dx$

### The Fundamental Theorem of Calculus, Part II

If $f$ is continuous on $[a,b]$ and $F(x) = \int_0^x f(t)\,dt$ ,
then $F'(x) = f(x)$ , on $[a,b]$.

In general: if $g(x) = \int_0^{u(x)} f(t)\,dt$ , then $g'(x) = f\big(u(x)\big)\; u'(x)$ **or**

$$g'(x) = \frac{d}{dx}\int_0^{u(x)} f(t)\,dt = f\big(u(x)\big)\; u'(x) \quad \text{on } [a,b]$$

In general: if $g(x) = \int_{v(x)}^{u(x)} f(t)\,dt$ , on $[a,b]$ then

$$g'(x) = \frac{d}{dx}\int_{v(x)}^{u(x)} f(t)\,dt = f\big(u(x)\big)\; u'(x) - f\big(v(x)\big)\, v'(x)$$

## Examples:

**13)** For $f(x)=\int_1^x (t^2+t-1)\,dt$ compute $f'(x)$

**14)** For $f(x)=\int_3^{x^2} \sin t\,dt$ compute $f'(x)$

**15)** For $f(x)=\int_{3x}^{x^2} \sqrt{t^2+2t-4}\,dt$ compute $f'(x)$

**16)** For $f(x)=\int_{3x}^{\sin x} \sqrt{t^2+2t-4}\,dt$ compute $f'(x)$

**17)** For the function $F(x)=\int_4^{x^2} \ln(t^3+4)\,dt$ find an equation of the tangent line at $x=2$.

**18)** Identify all local extrema of $f(x) = \int_0^x (t^2 - 3t + 2)\, dt$

**19)** If $f(x) = \int_1^x \sqrt{5t^2 - 1}\, dt$ Find the value of $f'(1)$ and $f(1)$

**20)** If $\int_0^x f(t)\, dt = x(\ln x - 1)$ Find the value of $f(e^2)$.

**21)** For $f(x) = \int_{\sqrt{2x}}^1 e^{t^2}\, dt$ compute $f'(x)$

**22)** For $f(x) = \int_{x}^{\sin^{-1} x} \sin t \, dt$ where $x \in (-1,1)$. Compute $f'(x)$

**23)** For $f(x) = \int_{\cos x}^{\sin x} \sqrt{1 - t^2} \, dt$ Where $x \in \left[0, \frac{\pi}{2}\right]$. Show that $f'(x) = 1$

**24)** For $f(x) = x + \int_{0}^{\tan x} \frac{1}{1 + t^2} dt$ Show that $f'(x) = 2$

**25)** $\int_0^8 \left(\frac{1}{\sqrt{1+x}}\right) dx$

**A.** 1 **B.** 2 **C.** 4 **D.** $\frac{3}{2}$ **E.** 6

---

**26)** $\int_0^1 \left(\frac{x^2}{1+x^2}\right) dx$

**A.** $\frac{4-\pi}{4}$ **B.** $\ln 2$ **C.** 0 **D.** $\frac{1}{2}\ln 2$ **E.** $\frac{4+\pi}{4}$

---

**27)** If $F(x) = \int_0^x e^{-t^2}\, dt$ Then $F'(x)$

**A.** $2xe^{-x^2}$ **B.** $-2xe^{-x^2}$ **C.** $e^{-x^2}$

**D.** $e^{-x^2} - 1$ **E.** $\frac{e^{-x^2+1}}{-x^2+1} - e$

---

**28)** If $f(x) = \int_0^x \frac{1}{\sqrt{t^3+1}} dt.$ Which of the following is **FALSE**?

**A.** $f(0) = 0$ **B.** $f(1) > 0$ **C.** $f(-1) > 0$

**D.** $f'(1) = \frac{1}{\sqrt{2}}$ **E.** $f$ is continuous at for all $x \geq 0$

**29)** If $F$ and $f$ are continuous functions such that $F'(x) = f(x)$ for all $x$, then $\int_a^b f(x)\,dx$ is

**A.** $F'(a) - F'(b)$ **B.** $F'(b) - F'(a)$ **C.** $F(a) - F(b)$

**D.** $F(b) - F(a)$ **E.** none of the above

---

**30)** $\int_0^1 (x+1)e^{x^2+2x}\,dx$

**A.** $\frac{e^3}{2}$ **B.** $\frac{e^3-1}{2}$ **C.** $\frac{e^3-e}{2}$ **D.** $e^3 - 1$ **E.** $e^3 - e$

---

**31)** Given $f(x) = \begin{cases} x+1, & x < 0 \\ \cos \pi x\,, & x \geq 0 \end{cases}$ Then $\int_{-1}^1 f(x)\,dx$

**A.** $\frac{1}{2} + \frac{1}{\pi}$ **B.** $-\frac{1}{2}$ **C.** $\frac{1}{2} - \frac{1}{\pi}$ **D.** $\frac{1}{2}$ **E.** $-\frac{1}{2} + \pi$

---

**32)** $\int_{-1}^2 \frac{|x|}{x}\,dx$

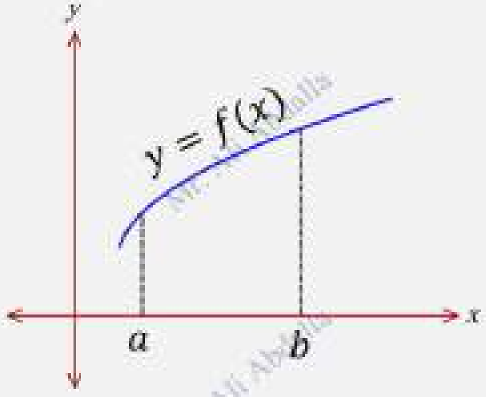
**A.** $-3$ **B.** 1 **C.** 2 **D.** 3 **E.** non-existent

**33)** If $n$ is a known positive integer, for what value of $k$ is: $\int_1^k x^{n-1}\,dx = \frac{1}{n}$

**A.** $0$ **B.** $\left(\frac{2}{n}\right)^{\frac{1}{n}}$ **C.** $\left(\frac{2n-1}{n}\right)^{\frac{1}{n}}$ **D.** $2^{\frac{1}{n}}$ **E.** $2^n$

---

**34)** If $f$ is the continuous, strictly increasing function on the interval $a \le x \le b$ as shown on the right, which of the following must be true?



I. $\int_a^b f(x)\,dx < f(b)(b-a)$

II. $\int_a^b f(x)\,dx > f(a)(b-a)$

III. $\int_a^b f(x)\,dx = f(c)(b-a)$ For some numbers $c$ such that $a < c < b$

**A.** I only **B.** II only **C.** III only
**D.** I and II only **E.** I, II and III

---

**35)** If $f(x) = 2x|x+1|$ find $\int_{-2}^{3} f(x)\,dx$

**36)** If $f(x) = 2[\![x+3]\!]$ , Where $[\![\ ]\!]$ the greatest integer function.

Find $\int_{-1}^{3} f(x)\ dx$

**37)** By using the table in the right
Find $h'(2)$ if

$$h(x) = \int_{2}^{g(x)} f(t)\,dt$$

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 0 | 2 | 1 | 10 | −2 |
| 1 | −5 | −8 | 5 | 1 |
| 2 | 15 | −1 | 1 | 3 |

Let $p(t)$ represent the function of population
Let $b(t)$ represent the birth rate **and** $a(t)$ represent the death rate
Rate of change in population is $p'(t)$
Then $p'(t) = b(t) - a(t)$
The rate of change over one year (12 months) is

$$\int_{0}^{12} p'(t)\,dt$$

**38)** Suppose that, for a particular population of organisms, the birth rate is given by $b(t) = 410 - 0.3t$ organisms per month and the death rate is given by $a(t) = 390 + 0.2t$ organisms per month.

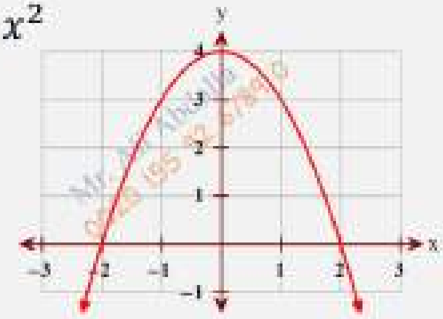
**A)** Explain why $\int_0^{12}[b(t) - a(t)]\,dt$ represents the net change in population in the first 12 months.

**B)** Determine for which values of $t$ it is true that $b(t) > a(t)$.

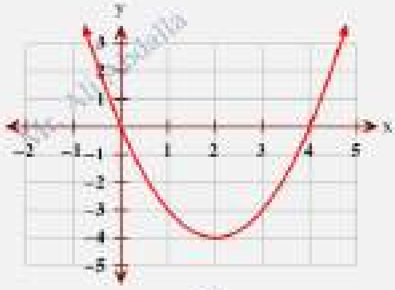
**C)** At which times is the population increasing? Decreasing?

**D)** Determine the time at which the population reaches a maximum.

**39)** The area above the $x$-axis and below $y = 4 - x^2$

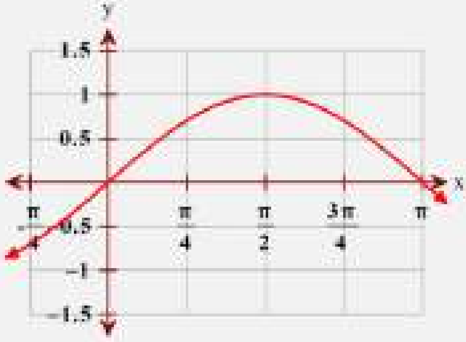


**40)** The area below the $x$-axis and above $y = x^2 - 4x$

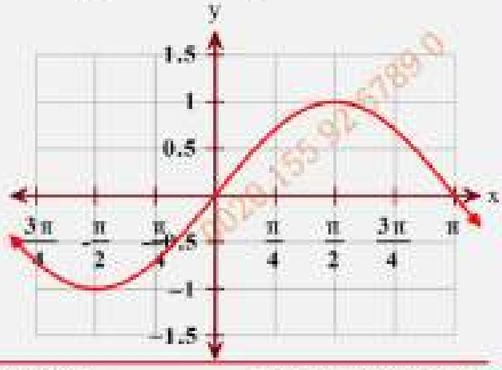


**41)** The area of the region bounded by $y = x^2$, $x = 2$ and the $x$-axis.

**42)** The area between $y = \sin x$ and the $x$-axis for $0 \le x \le \pi$



**43)** The area between $y = \sin x$ and the $x$-axis for $-\frac{\pi}{2} \le x \le \frac{\pi}{4}$

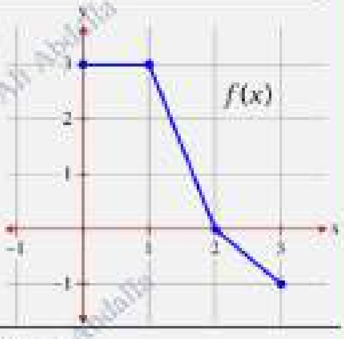


**44)** If $g(x) = \int_{\pi}^{\pi x} \cos t^2 \, dt$ then $g'(x) =$

A) $\sin(\pi^2 x^2)$ B) $\pi x \sin(\pi^2 x^2)$ C) $\pi \cos(\pi^2 x^2)$
D) $\pi x \cos(\pi^2 x^2)$ E) $\cos(\pi^2 x^2)$

---

**45)** The graph of $f$ is given, and g is an antiderivative of $f$.
If $g(3) = 6$ , find $g(0)$



(A) 1
(B) 2
(C) 4
(D) 5

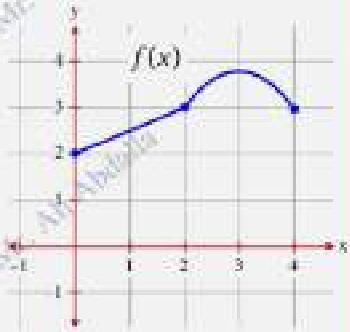
$$\int_0^3 f(x)\, dx = g(x)|_0^3 = g(3) - g(0)$$

$$\tfrac{1}{2}(1+2)(3) - \tfrac{1}{2}(1)(1) = 6 - g(0)$$

$$g(0) = 2$$

---

**46)** The graph of $f$ is given, and $F(x)$ is an antiderivative of $f$.
If $\int_2^4 f(x)\, dx = 7.5$ , find $F(4) - F(0)$ .



**A)** 1.5
**B)** 7.5
**C)** 12.5
**D)** 18.5

$$\int_0^4 f(x)\, dx = F(4) - F(0)$$

$$\int_0^2 f(x)\, dx + \int_2^4 f(x)\, dx = F(4) - F(0)$$

$$\tfrac{1}{2}(2+3)(2) + 7.5 = F(4) - F(0) \Rightarrow F(4) - F(0) = 12.5$$

---

**47)** Find $\int_{-2}^{2} f(x)\, dx$ if $f(x) = \begin{cases} 2x^2, & -2 \le x \le 0 \\ \sin 2x, & 0 < x \le 2 \end{cases}$

**A)** 4.507 **B)** 5.403 **C)** 6.161 **D)** 10.667

---

**48)** If $h(x) = \int_0^{2x} (e^{\cos t} - 1)\, dt$ on (3,6). On which interval(s) is $h$ decreasing?

**A)** (3.927, 5.498) **B)** (5.498, 6) **C)** (3, 4.712)
**D)** Always decreasing on (3, 6) **E)** Never decreasing on (3, 6)

**49)** $\lim\limits_{h\to0}\dfrac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h}$

(A) 0 (B) 1 (C) 3 (D) $2\sqrt{2}$ (E) does not exist

---

**50)** $\lim\limits_{x\to1}\dfrac{\int_1^{x}e^{t^2}\,dx}{x^2-1}$

(A) 0 (B) 1 (C) $\frac{e}{2}$ (D) $e$ (E) does not exist

---

**51)** The graph of $g'$ , the first derivative of the function g, consists of a semicircle of radius 2, and two line segments, as shown in the figure below. If $g(0) = 1$, what is $g(3)$ ?

$$\int_a^b g'(x)\,dx = g(b) - g(a)$$

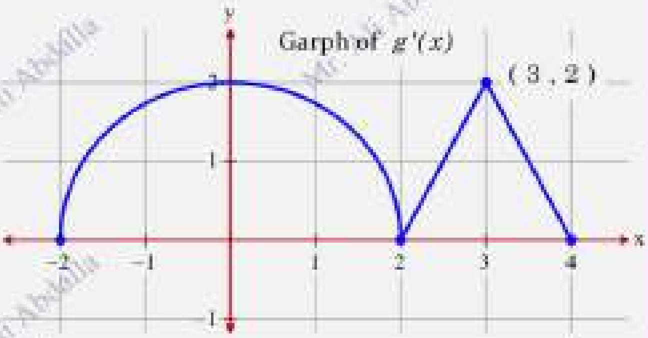
Fundomental theorem of calculus Part I

$$\int_0^3 g'(x)\,dx = g(3) - g(0)$$

$$\underbrace{\int_0^2 g'(x)\,dx}_{\text{Area of a quarter circle}} + \underbrace{\int_2^3 g'(x)\,dx}_{\text{Area of a triangle}} = g(3) - g(0)$$

$$\frac{\pi(2)^2}{4} + \frac{1}{2}(1)(2) = g(3) - 1$$

$$g(3) = \pi + 2$$

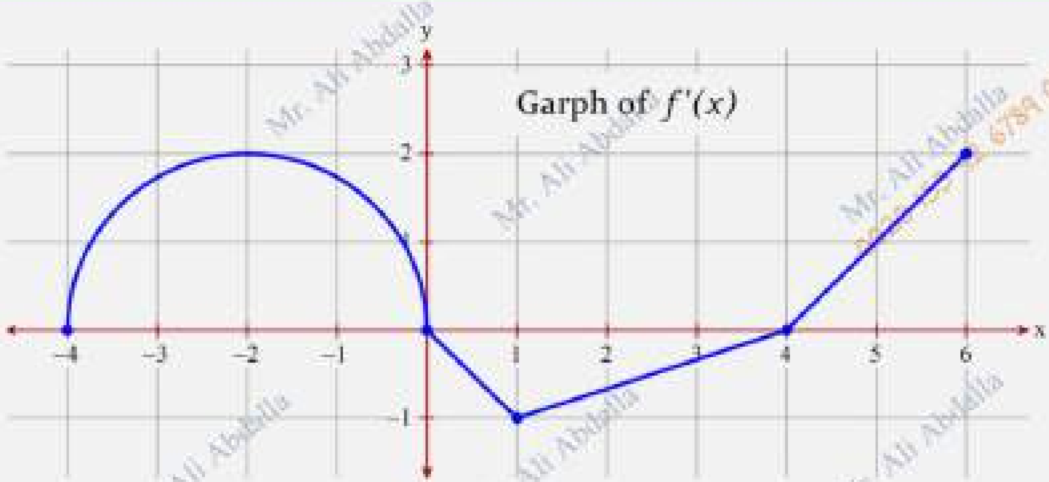


(A) $\pi + 1$ (B) $\pi + 2$
(C) $2\pi + 1$ (D) $2\pi + 2$

---

**52)** Let $f$ be the function given by $f(x) = \int_1^x (3t - 6t^2)\,dt$ .
What is the $x$-coordinate of the point of inflection of the graph of $f$?

A) $-\frac{1}{4}$ B) $\frac{1}{4}$ C) 0 D) $\frac{1}{2}$



**53)** The figure above represents the function $f'$ a continuous function, the derivative of $f$ over the interval $[-4,6]$ and satisfies $f(0)=4$. The graph of $f'$ consists of three line segments and a semi-circle.

A) Find the value of $f(-4)$.

$$\int_{-4}^{0} f'(x)\,dx = f(0) - f(-4) \Rightarrow f(-4) = \underbrace{f(0)}_{f(0)=4 \text{ given}} - \underbrace{\int_{-4}^{0} f'(x)\,dx}_{\text{Area of semi-circle}}$$

$$f(-4) = 4 - \frac{\pi(2)^2}{2} = 4 - 2\pi$$

B) On what interval(s) is $f$ decreasing and concave up? Justify your answer.

$f(x)$ is decreasing and concave up on the interval $(1,4)$ because $f'(x)$ is negative and increasing.

C) State all $x$-values where $f(x)$ has a horizontal tangent on the open interval $(-4,6)$. Explain whether $f$ has a relative minimum, relative maximum, or neither at each of those $x$-values.

At $x=0$ and $x=4$, $f(x)$ has a horizontal tangent.
At $x=0$, $f(x)$ has a relative maximum because $f'$ changes from positive to negative.
At $x=4$, $f(x)$ has a relative minimum because $f'$ changes from negative to positive.

D) Evaluate $\int_2^3 f''(2x)\,dx$

$$\int_2^3 f''(2x)\,dx = \frac{1}{2} f'(2x)\Big|_2^3 = \frac{1}{2}\left(f'(6) - f'(4)\right) = \frac{1}{2}(2-0) = 1$$

E) Critical numbers of the function $f(x)$

Critical numbers is the $x$-coordinates when the graph of $f'(x)$ intersect with $x$-axis and the end points if included in its domain.

then the critical numbers are: $x=-4$ , $x=0$ , $x=4, x=6$

| $x$ | 0 | 3 | 6 | 9 |
|---|---|---|---|---|
| $f(x)$ | 10 | 8 | 5 | 2 |

**54)** Let $g(x)$ be a twice-differentiable function defined by a differentiable function $f$, such that $g(x) = 2x + \int_1^{x^2} f(t)\,dt$ . Selected values of $f(x)$ are given in the table above.

A) Use a Left Riemann sum using the subintervals indicated by the table to approximate $g(3)$.

$$g(3) = 2(3) + \int_1^9 f(x)\,dx = 6 + \Delta x[f(x_0) + f(x_1) + f(x_2)]$$

$$g(3) \approx 6 + 3[\ 10 + 8 + 5\ ] = 75$$

B) Find $g'(3)$.

$$g(x) = 2x + \int_1^{x^2} f(t)\,dt \Rightarrow g'(x) = 2 + 2x\,f(x^2)$$

$$\Rightarrow g'(3) = 2 + 2(3)f(9) = 2 + 6(2)$$

$$\Rightarrow g'(3) = 14$$

C) Using the data in the table, estimate $f'(4)$.

$$f'(4) \approx \frac{f(6) - f(3)}{6 - 3} = \frac{5 - 8}{3} = -1$$

D) Explain why there must be a value of c, on $1 < x < 9$ such that $f(c) = 4$

Since $g(x)$ is twice differentiable, $g'(x)$ is continuous therefore IVT applies. There must be a value of $c$, on $1 < x < 9$, such that $g'(c) = f(c) = 4$ because $f(6) > 4 > f(9)$.

---

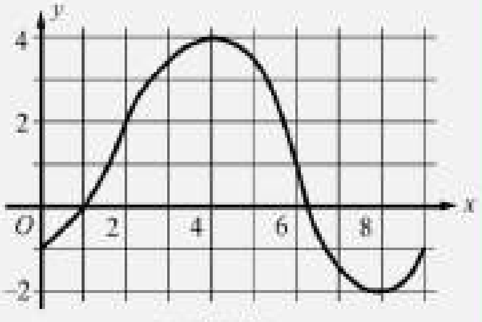
**55)** The graph of the function g , shown in the right figure, has horizontal tangents at $x = 4$ and $x = 8$ . If $f(x) = \int_0^{\sqrt{x}} g(t)\,dt$ , what is the value of $f'(4)$?

**A)** 0

**B)** $\frac{1}{2}$

**C)** $\frac{3}{4}$

**D)** $\frac{3}{2}$



graph of g

# L 5.6 Integration by Substitution

In this section, we significantly expand our ability to compute antiderivatives by developing a useful technique called **integration by substitution**.

**Integration by substitution consists of the following general steps:**

- ☑ Choose a new variable $u$: a common choice is the innermost expression or "inside" term of a composition of functions.
- ☑ Compute $du = \frac{du}{dx}dx$.
- ☑ Replace all terms in the original integrand with expressions involving $u$ and $du$.
- ☑ Evaluate the resulting $(u)$ integral. If you still cannot evaluate the integral, you may need to try a different choice of $u$.
- ☑ Replace each occurrence of $u$ in the antiderivative with the corresponding expression in $x$.

How to choose the correct substitution to be $u$:

**A)** Inside brackets like $(x^2+1)^5$ use $u = x^2+1$

**B)** Inside roots like $\sqrt{3x^2+2}$ use $u = 3x^2+2$

**C)** The exponents like: $e^{x^2+2}$ use $u = x^2+2$

**D)** The angles like $\sin(3x^4)$ use $u = 3x^4$

And maybe other substitution

**Evaluate**

1) $\int x^2(x^3+2)^{100}\,dx$

2) $\int (3x+4)^7\,dx$

3) $\int x\ \sin x^2\,dx$

5) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

7) $\int \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}dx$

4) $\int (3\,tan\,x+4)^5\,sec^2x\ \ dx$

6) $\int \frac{x^5}{1+x^6}\,dx$

8) $\int x\sqrt{2-x}\,dx$

9) $\int \frac{x^3}{\sqrt{4-x^4}}dx$

11) $\int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}dx$

10) $\int \frac{\left(\sqrt{x}+1\right)^4}{\sqrt{x}}dx$

12) $\int \frac{1}{\sqrt{1+\sqrt{x}}}dx$

Let $1+\sqrt{x}=u$

$\Rightarrow$

13) $\int \frac{3}{\sqrt[4]{x}+x}dx$

Let $1+x^{3/4}=u$

$\Rightarrow \frac{3}{4}x^{-1/4}dx = du$

$\Rightarrow dx = \frac{4}{3}\sqrt[4]{x}\,du$

$= \int \frac{1}{\sqrt[4]{x}(1+x^{3/4})}\,dx = \int \frac{1}{\sqrt[4]{x}\;u}\left(\frac{4}{3}\sqrt[4]{x}\,du\right)$

$= \frac{4}{3}\int \frac{1}{u}\,du = \frac{4}{3}\ln u + c$

$= \frac{4}{3}\ln\left(1+x^{3/4}\right)+c$

14) $\int \frac{1}{\sqrt{x}(1+x)}dx$

Let $\sqrt{x}=u \Rightarrow x=u^2$

$\Rightarrow dx = 2u\,du$

$= \int \frac{1}{u(1+u^2)}\,2u\,du = 2\int \frac{1}{1+u^2}\,du$

$= 2\tan^{-1}u + c \; = 2\tan^{-1}\left(\sqrt{x}\right)+c$

## Challenge

15) $\int \frac{3\sqrt{x}}{1+x^3}\,dx$

Let $x^{3/2}=u \Rightarrow x^3=u^2$

$\Rightarrow \frac{3}{2}x^{1/2}dx = du$

$\Rightarrow dx = \frac{2}{3\sqrt{x}}\,du$

16) $\int x\,e^{x^2}\,dx$

17) $\int \left(\frac{e^{\sqrt{x}}}{\sqrt{x}}\right)dx$

18) $\int \left(\frac{\sqrt{\ln x}}{x}\right) dx$

19) $\int \left(\frac{\ln \sqrt{x}}{x}\right) dx$

20) $\int e^x \sqrt{e^x + 4}\, dx$

21) $\int \left(\frac{\cos(1/x)}{x^2}\right) dx$

22) $\int \frac{x}{\sqrt{1 - x^4}}\, dx$ Let $u = x^2$

$\Rightarrow$

23) $\int \frac{1 + x}{1 - x^2}\, dx$ Factor the denominator

24) $\int \frac{1+x}{1+x^2}\,dx$

25) $\int \sin^3 x \cos x \; dx$

26) $\int \frac{e^{\tan x}}{1-\sin^2 x}\,dx$

27) $\int x^2 \csc^2 x^3 \; dx$

28) $\int \tan 4x \; dx$

29) $\displaystyle\int \frac{\ln(\sin x)}{\tan x}\, dx$

30) $\displaystyle\int \frac{\cos(\ln x)}{x}\, dx$

31) $\displaystyle\int \sec^2 x\,\sqrt{1-2\tan x}\; dx$

32) $\displaystyle\int \frac{x-1}{1+2x-x^2}\, dx$

33) $\displaystyle\int \frac{x^2}{\sqrt{1-x^6}}\, dx$

Let $u = x^3$

$\Rightarrow$

34) $\int \frac{3x^2}{1+x^6}\ dx$

35) $\int \frac{x-2}{x+7}\ dx$

36) $\int \cos(\tan 3x)\sec^2 3x\ \ dx$

37) $\int \frac{x\sqrt{x}}{1+x^5}\ dx$

Let $x^{5/2} = u$

$\Rightarrow$

38) $\int \sec^2 x\sqrt{\tan x}\ \ dx$

39) $\int \frac{(1+\sin x)^5}{\sec x}\ dx$

40) $\displaystyle\int \frac{3}{(1+x^2)\tan^{-1}x}\,dx$

41) $\displaystyle\int \frac{\sqrt{1-\sin^2 x}}{1+\sin^2 x}\,dx$

***Challenge***

42) $\displaystyle\int \frac{2}{x^{\frac{2}{3}} - x^{\frac{5}{6}}}\,dx$

Let $u = x^{1/6}$

$\Rightarrow$

Evaluate each definite integral:

43) $\displaystyle\int_1^2 x^3\sqrt{x^4+1}\,dx$

44) $\displaystyle\int_1^e \frac{\ln x}{x}\,dx$

45) $\int_{1}^{e} \frac{1}{x\ln x + x}dx$

46) $\int_{-1}^{1} \frac{t}{(1+t^2)^2}\,dt$

47) $\int_{0}^{\ln 2} \frac{e^t}{1+e^{2t}}\,dt$

48) $\int_{0}^{\ln 2} \frac{e^t}{1+e^t}\,dt$

49) If $\int_0^1 f(x)\ dx = 3$ Find $\int_0^{\frac{\pi}{2}} \cos x\ f(\sin x)\ dx$

50) If $\int_1^2 f(x)\ dx = 4$ Find $\int_1^4 \frac{f(\sqrt{x})}{\sqrt{x}}\ dx$

51) If $\int_1^2 f(x)\ dx = 3$ Find **A)** $\int_2^4 f\left(\frac{x}{2}\right)\ dx$

**B)** $\int_0^{\ln 2} e^x\ f(e^x)\ dx$

52) **A)** For the integral $I = \int_0^{10} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{10-x}}\, dx$ , use a substitution to show that $I = \int_0^{10} \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}}\, dx$ Use these two representations of $I$ to evaluate $I$

Let $u = 10 - x \Rightarrow du = dx$

When $x = 0 \Rightarrow u = 10$ and $x = 10 \Rightarrow u = 0$

$$\Rightarrow \int_0^{10} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{10-x}}\, dx = \int_0^{10} \frac{\sqrt{10-u}}{\sqrt{10-u} + \sqrt{u}}\, du$$

The integration value does not change when replace variable with another variable

$$I = \int_0^{10} \frac{\sqrt{10-x}}{\sqrt{10-x} + \sqrt{x}}\, dx \qquad \to (1)$$

Frome the question

$$I = \int_0^{10} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{10-x}}\, dx \qquad \to (2)$$

Add (1) to (2)

$$I + I = \int_0^{10} \frac{\sqrt{10-x}}{\sqrt{10-x} + \sqrt{x}}\, dx + \int_0^{10} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{10-x}}\, dx$$

$$2I = \int_0^{10} \frac{\sqrt{10-x} + \sqrt{x}}{\sqrt{x} + \sqrt{10-x}}\, dx = \int_0^{10} 1\, dx$$

$$2I = x|_0^{10} = 10 - 0$$

$$2I = 10 \Rightarrow I = 5$$

$$\Rightarrow I = \int_0^{10} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{10-x}}\, dx = 5$$

**B)** Generalize to $I = \int_0^{a} \frac{f(x)}{f(x) + f(a-x)}\, dx$ for any positive, then find the value of $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$ and $\int_0^{5} \frac{f(x)}{f(x) + f(5-x)}\, dx$

53) When a patient is undergoing surgery, he is injected with anesthesia, and after $t$ hours the concentration of anesthetic in the patient's blood is

$$C(t) = \frac{2t}{\sqrt{(36+t^2)^3}} \quad mg/cm^2$$

Find the average concentration of anesthesia in the blood during the first eight hours after injection

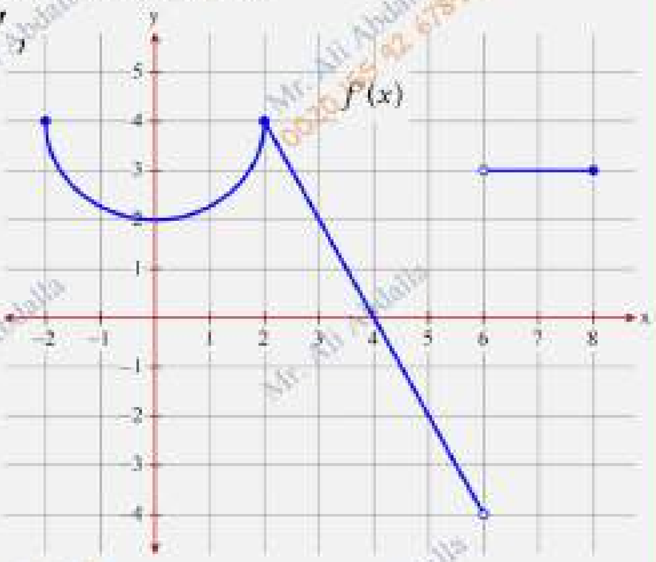
---

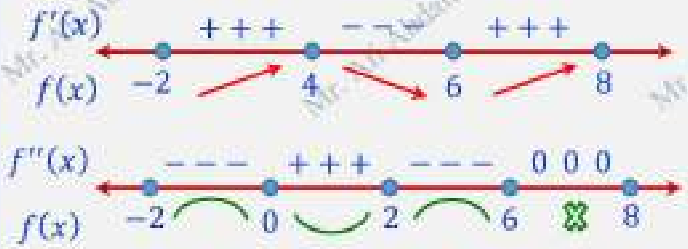
54) The Weather station observed the temperature $C$ in a city after midnight, so it was found that it can be modeled with as the following:

$$T(t) = 3 - \frac{1}{3}(t-5)^2 \quad °C$$

where $t$ is the time after midnight. Find the average temperature in the city from 10 AM to 3 PM

**55)** The function $f$ is continuous for all real values of $x$.
A portion of the graph of the function $f'$, the derivative of $f$, on $[-2, 8]$.
The graph of $f'$ is shown in the figure is shown on the right and consists of a semicircle and two linear pieces.





A) Find the $x$ coordinate of each critical point of $f$ on the interval $[-2, 8]$. Classify each critical point as a local maximum, a local minimum, or neither for $f$. Justify your answers.

Relative maximum at $x = 4$ since $f'$ changes from $(+)$ to $(-)$

Relative minimum at $x = 6$ since $f'$ changes from $(-)$ to $(+)$

B) Find the $x$ coordinate of each point of inflection for the graph off on the open interval $(-2, 6)$. Justify your answer.

Point of inflection at $x = 0$ since $f'$ changes from decreasing to increasing.

Point of inflection at $x = 2$ since $f'$ changes from increasing to decreasing.

C) Find $\lim\limits_{x\to4}\frac{\int_2^x f'(t)dt-4}{3(x-4)^2}$. Show the work that leads to your answer.

By direct substitution, Numerator is $\lim\limits_{x\to4}\int_2^x f'(t)dt - 4 = \underbrace{\int_2^4 f'(t)dt}_{\text{Area under curve of } f' \text{ from 2 to 4}} - 4 = 4 - 4 = 0$

Denominator: $\lim\limits_{x\to4} 3(x-4)^2 = 3(4-4)^2 = 0$

By using l'hopital twice:

$$\lim_{x\to4}\frac{\int_2^x f'(t)dt - 4}{3(x-4)^2} = \lim_{x\to4}\frac{f'(x)-0}{6(x-4)^1(1)}\overset{\text{Direct subs.}}{\left(\frac{0}{0}\right)} = \lim_{x\to4}\frac{f'(x)}{6x-24} = \lim_{x\to4}\frac{f''(x)}{6} = \overset{f''(4) \text{ from the graph of } f' \text{ slope at } x=4}{\frac{f''(4)}{6}} = -\frac{2}{6} = -\frac{1}{3}$$

D) Evaluate: $A)\ \int_{-2}^{1} f'(4-2x)dx$ $\qquad B)\ \int_{-2}^{3} f'(4-2x)dx$ B) Try it by

Let $u = 4-2x \Rightarrow du = -2dx \Rightarrow dx = -\frac{1}{2}du$ when $x = -2 \Rightarrow u = 8$ and when $x = 1 \Rightarrow u = 2$

$$\int_{-2}^{1} f'(4-2x)dx = -\frac{1}{2}\int_8^2 f'(u)\,du = \frac{1}{2}\int_2^8 f'(u)\,du = \frac{1}{2}\boxed{\underbrace{\int_2^8 f'(x)\,dx}_{\text{Area under curve of } f' \text{ from 2 to 8}}}$$

$$= \frac{1}{2}\left(\frac{1}{2}(2)(4) - \frac{1}{2}(2)(4) + (2)(3)\right) = 3$$

Fundamental theorem of calculus Part I

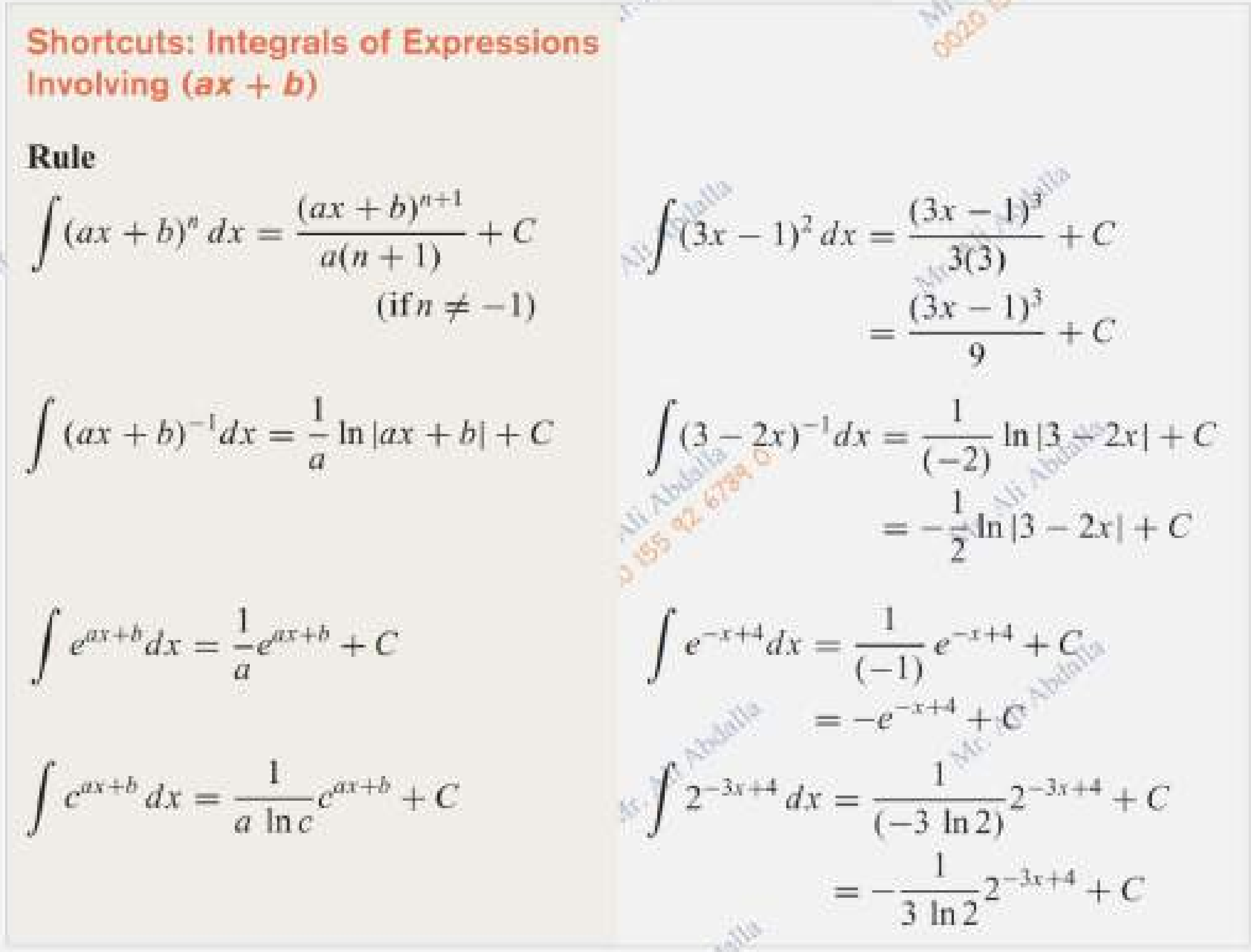
$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

E) Let $g(x) = f'(x)\cdot x^2$ find $g'(3)$

$$g'(x) = f''(x)\cdot x^2 + f'(x)\cdot 2x \Rightarrow g'(3) = f''(3)\cdot 3^2 + f'(3)\cdot 2(3)$$

$$\Rightarrow g'(3) = 9\underbrace{f''(3)}_{f''(3) \text{ from the graph of } f' \text{ slope at } x=3} + 6\underbrace{f'(3)}_{f'(3) \text{ from the graph of } f'\ f'(3)=2} = 9(-2) + 6(2) = -6 \Rightarrow \boxed{g'(3) = -6}$$

# For you



## Shortcuts: Integrals of Expressions Involving ($ax + b$)

| Rule | |
|---|---|
| $\int (ax+b)^n\,dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + C$ (if $n \neq -1$) | $\int (3x-1)^2\,dx = \dfrac{(3x-1)^3}{3(3)} + C = \dfrac{(3x-1)^3}{9} + C$ |
| $\int (ax+b)^{-1}dx = \dfrac{1}{a}\ln\lvert ax+b\rvert + C$ | $\int (3-2x)^{-1}dx = \dfrac{1}{(-2)}\ln\lvert 3-2x\rvert + C = -\dfrac{1}{2}\ln\lvert 3-2x\rvert + C$ |
| $\int e^{ax+b}dx = \dfrac{1}{a}e^{ax+b} + C$ | $\int e^{-x+4}dx = \dfrac{1}{(-1)}e^{-x+4} + C = -e^{-x+4} + C$ |
| $\int c^{ax+b}\,dx = \dfrac{1}{a\,\ln c}c^{ax+b} + C$ | $\int 2^{-3x+4}\,dx = \dfrac{1}{(-3\,\ln 2)}2^{-3x+4} + C = -\dfrac{1}{3\,\ln 2}2^{-3x+4} + C$ |

- $\int \sqrt{ax+b}\,dx = \dfrac{2}{3a}(ax+b)^{3/2} + C$

## Unit Review Exercises

**Find the antiderivative.**

**1)** $\int 4x \sec x^2 \tan x^2 \, dx$

**2)** $\int \tan x \, dx$

**3)** $\int \sqrt{3x+1} \, dx$

**4)** $\int e^x (1 - e^{-x}) \, dx$

**5)** $\int e^x (1 + e^x)^2 \, dx$

**6)** $\int 6x^2 \cos x^3 \, dx$

**7)** Find a function $f(x)$ satisfying $f(x) = e^{-2x}$ and $f(0) = 3$.

**8)** Determine the position function if the velocity is $v(t) = -32t + 10$ and the initial position is $s(0) = 2$.

**9)** Determine the position function if the acceleration is $a(t) = 6$ with initial velocity $v(0) = 10$ and initial position $s(0) = 0$.

**10)** Write out all terms and compute $\sum_{i=1}^{6}(i^2 + 3i)$ .

**11)** Use summation rules to compute the sum of $\sum_{i=1}^{100}(i^2 + 2i)$ .

**12)** Translate into summation notation and compute: the sum of the squares of the first 12 positive integers.

**13)** Compute the sum $\frac{1}{n^3}\sum_{i=1}^{n}(i^2 - i)$ and the limit of the sum as $n$ approaches $\infty$.

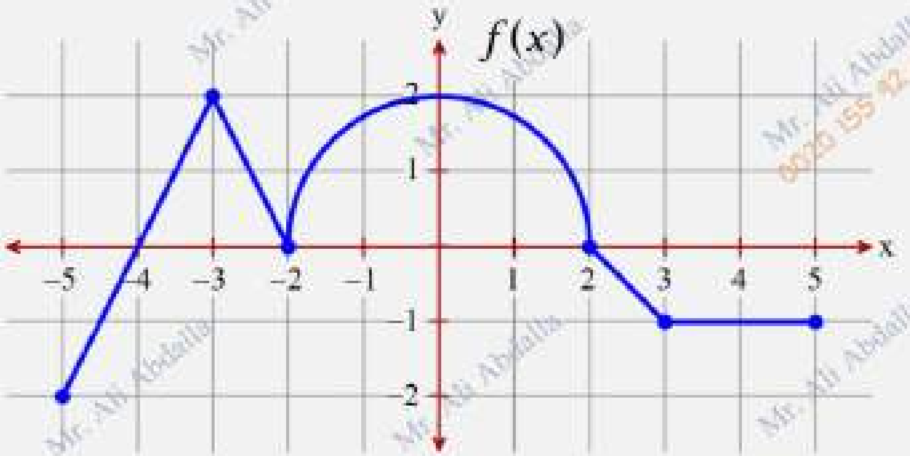
**14)** Use the velocity function to compute the distance traveled in the given time interval. $v(t) = 20e^{-t/2}$ , $[0,2]$

**15)** Find the derivative of:

**A)** $f(x) = \int_{2}^{x} (\sin t^2 - 2)\, dt$

**B)** $f(x) = \int_{0}^{x^2} \sqrt{t^2 + 1}\, dt$

**16)**



The graph of $y = f(x)$ consists of four-line segments and a semicircle as shown in the figure above. Evaluate each definite integral by using geometric formulas.

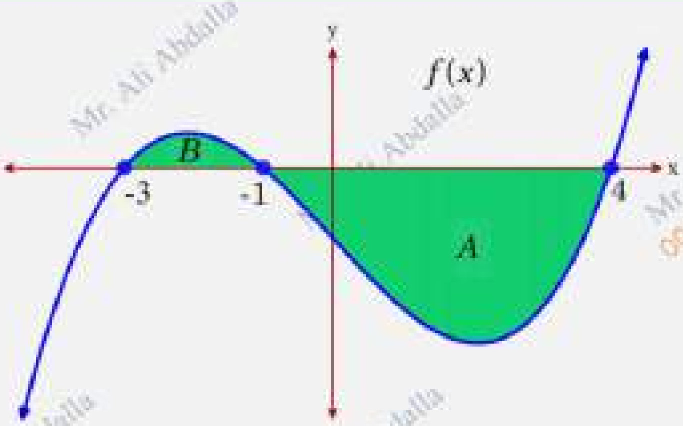
A) $\int_{-5}^{-2} f(x)\,dx$

B) $\int_{-2}^{2} f(x)\,dx$

C) $\int_{2}^{5} f(x)\,dx$

D) $\int_{-5}^{5} |f(x)|\,dx$

E) $\int_{-5}^{5} f(|x|)\,dx$



**17)** The graph of $y = f(x)$ is shown in the figure above. If $A$ and $B$ are positive numbers that represent the areas of the shaded regions, what is the value of $\int_{-3}^{4} f(x)\,dx - 2\int_{-1}^{4} f(x)\,dx$ in terms of $A$ and $B$?

A) $-A-B$ B) $A+B$ C) $A-2B$ D) $A-B$

---

**18)** If $f$ is the antiderivative of $\frac{\sqrt{x}}{1+x^3}$ such that $f(1) = 2$, then $f(3) =$

A) 1.845 B) 2.397 C) 2.906 D) 3.234

---

**19)** If $f'(x) = \cos(2x-1)$ such that $f(1) = 2$, then $f(5) =$

A) 1.825 B) 1.338 C) 1.785 D) 5.482

---

**20)** If $\int_{-1}^{3} f(x+k)\,dx = 8$ where $k$ is constant, then $\int_{k-1}^{k+3} f(x)\,dx =$

A) $8-k$ B) $8+k$ C) 8 D) $k-8$

**21)** If $f$ is continuous and $\int_1^8 f(x)\,dx = 15$, find the value of $\int_1^2 x^2 f(x^3)\,dx$.

**22)** $\int_1^e \frac{\cos(\ln x)}{x}\,dx =$

(A) $\frac{1}{\sin 1}$ (B) $\frac{1}{\cos 1}$ (C) $\sin(e)$ (D) $\sin 1$

**23)** Which of the following limits is equal to $\int_1^3 x^3\,dx$ ?

(A) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}(1+\frac{i}{n})^3\frac{1}{n}$

(B) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}(1+\frac{i}{n})^3\frac{2}{n}$

(C) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}(1+\frac{2i}{n})^3\frac{1}{n}$

(D) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}(1+\frac{2i}{n})^3\frac{2}{n}$

تمت الوحدة الخامسة – بالتوفيق للجميع

# General Review on Unit 5 Term 3

1) $\int \frac{5}{1+x^2}\,dx$ is equal to

**A.** $5\ln(1+x^2)+c$ **B.** $5\tan^{-1}x+c$

**C.** $5\tan^{-1}x^2$ **D.** $\frac{5}{x}\ln(1+x^2)+c$

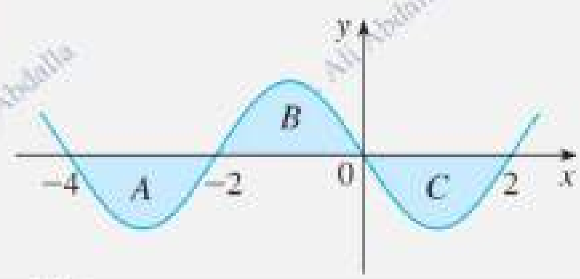
2) $\int x\sqrt{4-x^2}\,dx$ is equal to

**A.** $\frac{(4-x^2)^{3/2}}{3}+c$ **B.** $-\frac{(4-x^2)^{3/2}}{3}+c$

**C.** $-\frac{x^2(4-x^2)^{3/2}}{3}+c$ **D.** $-(4-x^2)^{3/2}+c$

3) Each of the region A, B and C are bounded by the graph of $f(x)$ and $x$-axis has area 3. Find the value of

$$\int_{-4}^{2}[f(x)+2x+5]\,dx$$



**A.** 15 **B.** 18

**C.** 12 **D.** 21

4) $\int_1^2 \frac{x-4}{x^2}\,dx$ Is equal to

**A.** 2 **B.** $\ln 2+2$

**C.** $\ln 2-2$ **D.** $\ln 2$

5) $\int_0^1 \frac{x+1}{x^2+2x+3}\,dx$ Is equal to

**A.** $\ln 2$ **B.** $-\ln 2$

**C.** $\frac{\ln 2}{2}$ **D.** $\frac{1-\ln 2}{2}$

6) The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

**A.** $\frac{\sqrt{2}}{3}$ **B.** $\frac{\sqrt{2}}{2}$

**C.** $\frac{2\sqrt{2}}{3}$ **D.** $\frac{4\sqrt{2}}{3}$

7) Suppose $g'(x) < 0$ for all $x > 0$ and $F(x) = \int_0^x t\, g'(t)dt$ Which of the following statement is **FALSE**?

**A.** $F$ takes on negative values **B.** $F(x) = x\, g(x) - \int_0^x g'(t)\, dt$

**C.** $F$ is continuous for all $x > 0$ **D.** $F'(x)$ exists for all $x > 0$

**E.** $F$ is an increasing function.

8) If $\frac{dy}{dx} = \cos 2x$ , then $y =$

**A.** $-\frac{1}{2}\cos 2x + c$ **B.** $-\frac{1}{2}\sin 2x + c$

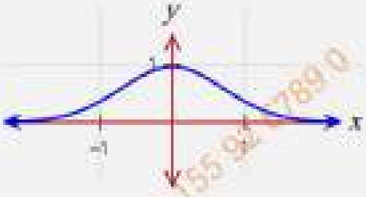
**C.** $\frac{1}{2}\sin 2x + c$ **D.** $\frac{1}{2}\sin^2 2x + c$

9) Which of the following is equal to ln 4

**A.** $\ln 3 + \ln 1$ **B.** $\ln 8 \div \ln 2$

**C.** $\int_1^4 e^t\, dt$ **D.** $\int_1^4 \frac{1}{t}\, dt$ **E.** $\int_1^4 \ln t\, dt$

10) If $\int_{-1}^{1} e^{-x^2}\, dx = k$ then $\int_{-1}^{0} e^{-x^2}\, dx =$



**A.** $\frac{1}{2}k$ **B.** $\frac{1}{2}k$

**C.** $-k$ **D.** $-2k$ **E.** $2k$

11) If $y = 10^{x^2-1}$ , then $\frac{dy}{dx} =$

**A.** $10^{x^2-1}\,(2x)\ln 10$ **B.** $10^{x^2-1}\,(2x)$

**C.** $10^{x^2-1}\,(x^2-1)$ **D.** $10^{x^2-1}\,\ln 10$ **E.** $10^{x^2-1}\,(x^2)\ln 10$

12) $\int_1^2 \frac{x^2-1}{x+1}\,dx =$

**A.** $\frac{1}{2}$ **B.** $\frac{5}{2}$

**C.** 1 **D.** 2 **E.** $\ln 3$

13) If $\int_{-2}^{2}(x^7+k)\;dx = 16$ then $k =$

**A.** 12 **B.** $-12$

**C.** 0 **D.** 4 **E.** $-4$

14) $\int_0^3 |x-1|\;dx =$

**A.** $\frac{3}{2}$ **B.** $\frac{5}{2}$

**C.** 0 **D.** 2 **E.** 6

15) $\int \tan 2x\;dx =$

**A.** $-2\ln|\cos 2x| + c$ **B.** $-\frac{1}{2}\ln|\cos 2x| + c$

**C.** $-2\ln|\cos 2x| + c$ **D.** $\frac{1}{2}\ln|\cos 2x| + c$

**E.** $-\frac{1}{2}\sec^2 2x + c$

16) $\int_0^{\frac{\pi}{3}} \sin 3x \ dx =$

**A.** $-\frac{2}{3}$ **B.** $\frac{2}{3}$

**C.** $-2$ **D.** $2$ **E.** $0$

17) Let $f$ be a continuous function on the closed interval $[0, 2]$

if $2 \leq f(x) \leq 4$ ,then the greatest possible value of $\int_0^2 f(x) \ dx =$

**A.** $0$ **B.** $2$

**C.** $4$ **D.** $8$

18) The average value of $f(x) = x^2\sqrt{x^3 + 1}$ on the closed interval $[0,2]$ is

**A.** $\frac{26}{9}$ **B.** $\frac{13}{3}$

**C.** $\frac{26}{3}$ **D.** $13$

19) $\frac{d}{dx}\left[\ln\left(\frac{1}{1 - x}\right)\right] =$

**A.** $\frac{1}{1 - x}$ **B.** $\frac{1}{x - 1}$

**C.** $x - 1$ **D.** $1 - x$

21) $\lim_{h\to 0} \frac{\int_1^{1+h} \sqrt{x^5 + 8}\, dx}{h}$ is

**A.** $0$ **B.** $1$

**C.** $3$ **D.** $2\sqrt{2}$

21) An antiderivative of $f(x) = e^{x+e^x}$ is

**A.** $\frac{e^{x+e^x}}{1+e^x}$ **B.** $(1+e^x)\,e^{x+e^x}$

**C.** $e^{e^x}$ **D.** $e^{x+e^x}$

22) If the substitution $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx =$

**A.** $\int_1^2 \frac{1-u^2}{u}\,du$ **B.** $\int_2^4 \frac{1-u^2}{u}\,du$

**C.** $\int_1^2 \frac{1-u^2}{2u}\,du$ **D.** $\int_2^4 \frac{1-u^2}{2u}\,du$

23) A particle with velocity at any time $t$ given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

**A.** $e^2 - 1$ **B.** $e - 1$

**C.** $2e$ **D.** $e^2$

24) The graph of $y = -\frac{5}{x-2}$ is concave downward for all values of $x$ such that

**A.** $x < 0$ **B.** $x < 2$

**C.** $x > 2$ **D.** $x > 0$

25) $\int \frac{x\,dx}{\sqrt{3x^2+5}} =$

**A.** $\frac{1}{3}\,(3x^2+5)^{\frac{1}{2}} + c$

**B.** $\frac{1}{6}\,(3x^2+5)^{\frac{1}{2}} + c$

**C.** $\frac{1}{3}\,(3x^2+5)^{\frac{3}{2}} + c$

**D.** $\frac{3}{2}\,(3x^2+5)^{\frac{1}{2}} + c$

26) $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

**A.** $-2(\sqrt{2}-1)$

**B.** $2\sqrt{2}$

**C.** $2(\sqrt{2}-1)$

**D.** $2(\sqrt{2}+1)$

27) If the function $f$ has a continuous derivative on $[0, c]$ ,

then $\int_0^{c} f'(x)\,dx =$

**A.** $f(c) - f(0)$

**B.** $|f(c) - f(0)|$

**C.** $f''(c) - f''(0)$

**D.** $f(x) + c$

28) For all $x > 1$ , if $f(x) = \int_1^x \frac{1}{t}\,dt$ Then $f(x) =$

**A.** $1$ **B.** $\ln x$

**C.** $\frac{1}{x}$ **D.** $e^x$

29) For $x > 0$ , $\int \left(\frac{1}{x}\int_1^x \frac{1}{t}\,dt\right) dx$

**A.** $\frac{1}{x^3} + c$

**B.** $\ln(\ln x) + c$

**C.** $\frac{(\ln x)^2}{2} + c$

**D.** $\frac{\ln(x^2)}{2} + c$

30) If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^{3} f(x)\,dx = 7$ then $\int_1^3 f(x)\,dx$

**A.** $-3$

**B.** $0$

**C.** $3$

**D.** $11$

31) If three equal subdivisions of $[-4, 2]$ are used, what is the trapezoidal approximation of $\int_{-4}^{2} \frac{e^{-x}}{2}\,dx$ ?

**A.** $\frac{1}{2}(e^4 + 2e^2 + 2e^0 + e^{-2})$

**B.** $e^4 + 2e^2 + 2e^0 + e^{-2}$

**C.** $\frac{1}{2}(e^4 + e^2 + e^0 + e^{-2})$

**D.** $e^4 + e^2 + e^0$

32) $\lim\limits_{n\to\infty} \frac{1}{n}\left[\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+\cdots+\sqrt{\frac{n}{n}}\right] =$

**A.** $\frac{1}{2}\int_0^1 \frac{1}{\sqrt{x}}\,dx$

**B.** $\int_0^1 \sqrt{x}\,dx$

**C.** $\int_1^2 x\sqrt{x}\,dx$

**D.** $\int_0^1 x\,dx$

33) If $\frac{dy}{dx} = \tan x$ then $y =$

**A.** $\frac{1}{2}\tan^2 x + C$

**B.** $\sec^2 x + C$

**C.** $\ln|\sec x| + C$

**D.** $\ln|\cos x| + C$

34) The acceleration of a particle moving along the $x$-axis at time $t$ is given by $a(t) = 6t^2 - 2$ . If the velocity is $25$ when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

**A.** $9t^2 + 1$

**B.** $t^3 - t^2 + 4t + 6$

**C.** $3t^2 - 2t + 4$

**D.** $t^3 - t^2 + 9t - 20$

35) If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers $x$ , which of the following **must** be true?

**I.** $\int_a^b f(x)\,g(x)\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$

**II.** $\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

**III.** $\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)\,dx}$

**A.** I only

**B.** II only

**C.** III only

**D.** II and III only

**E.** I, II and III

36) $\int_1^{500} (13^x - 11^x)\,dx + \int_2^{500} (11^x - 13^x)\,dx$

**A.** 14.946

**B.** 34.415

**C.** 46.000

**D.** 136.364

37) $\frac{d}{dx}\int_0^x \cos(2\pi u)\ du\ \ is$

**A.** $\frac{1}{2\pi}\sin x$

**B.** $\cos 2\pi x$

**C.** $\frac{1}{2\pi}\cos 2\pi x$

**D.** $2\pi \cos 2\pi x$

38) If $p$ is a polynomial of degree $n$, $n > 0$ , what is the degree of the polynomial $Q(x) = \int_0^x p(t)\ dt$ ?

**A.** $n-1$ **B.** $n$

**C.** $n+1$ **D.** $n-2$

39) A particle moves along the $x$-axis so that at any time $t \geq 0$ the acceleration of the particle is $a(t) = e^{-2t}$. If at $t = 0$ the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$ , then its position at any time $t > 0$ is $x(t) =$

**A.** $\frac{1}{4}e^{-2t} + 4$

**B.** $\frac{1}{2}e^{-2t} + 3t + \frac{15}{4}$

**C.** $\frac{1}{4}e^{-2t} + 3t + 4$

**D.** $4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$

40) Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\, dt$ at what value of $x$ is $f(x)$ a minimum?

**A.** $\frac{1}{2}$

**B.** $\frac{3}{2}$

**C.** 2

**D.** 3

41) If $f$ is continuous on the interval, $[a, b]$ then there exists $c$ such that $a < c < b$ and $\int_a^b f(x)\,dx =$

**A.** $\frac{f(a)-f(b)}{b-a}$ **B.** $(b-a)\,f'(c)$

**C.** $f(b) - f(b)$ **D.** $(b-a)f(c)$

42) If $\int_a^b f(x)\,dx = a + 2b$ then $\int_a^b (f(x) + 5)\,dx =$

**A.** $a + 2b + 5$ **B.** $5b - 5a$

**C.** $7b - 4a$ **D.** $7b - 5a$

43) $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{1 - \sin^2 x}\,dx =$

**A.** $e + 1$

**B.** $e - 1$

**C.** $e$

**D.** $1$

44) The average value of $\cos x$ on the interval $[-3, 5]$ is

**A.** $\frac{\sin 5 - \sin 3}{8}$

**B.** $\frac{\sin 5 + \sin 3}{8}$

**C.** $\frac{\sin 5 - \sin 3}{2}$

**D.** $\frac{\sin 5 + \sin 3}{2}$

45) The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}} + \sqrt{\frac{2}{50}} + \sqrt{\frac{30}{50}} + \cdots + \sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for:

**A.** $\int_0^1 \sqrt{\frac{x}{50}}\,dx$

**B.** $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$

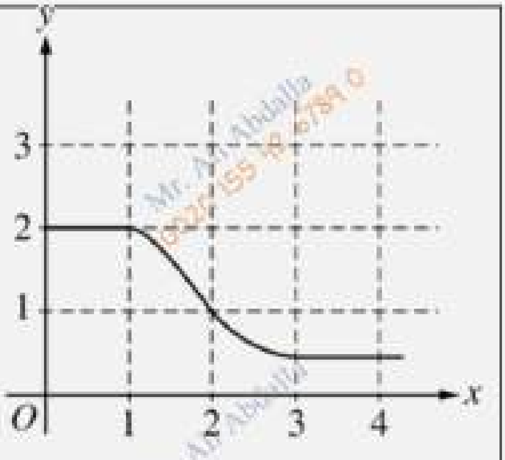
**C.** $\int_0^1 \sqrt{x}\,dx$

**D.** $\int_0^{50} \sqrt{x}\,dx$

Remember

$$\int_a^b f(x)\,dx = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

46) The graph of $f$ is shown in the right figure. If $F'(x) = f(x)$ and $\int_1^3 f(x)\,dx = 2.3$ then $F(3) - F(0) =$



**A.** 4.3

**B.** 3.3

**C.** 0.3

**D.** 1.3

47) At time $t \geq 0$, the acceleration of a particle moving on the $x$-axis is $a(t) = t + \sin t$ at $t = 0$, the velocity of the particle is $-2$. For what value $t$ will the velocity of the particle be zero?

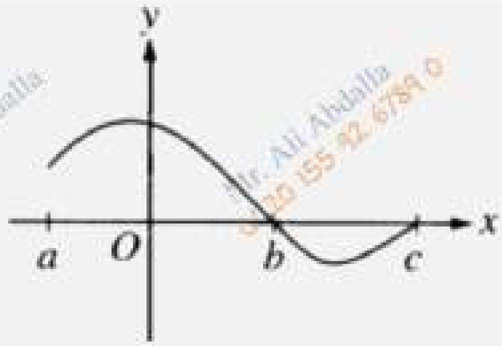
**A.** 1.02

**B.** 1.48

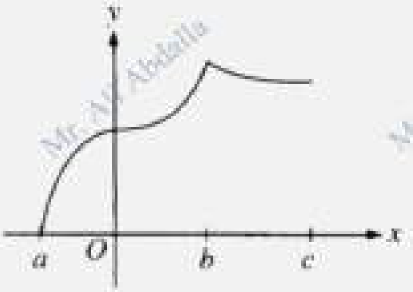
**C.** 1.85

**D.** 3.14

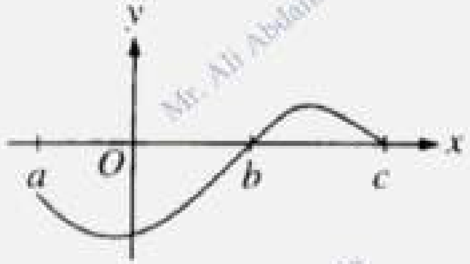
48) Let $f(x) = \int_a^x h(t)\,dt$ , where $h$ has the graph shown on the right. Which of the following could be the graph of $f$ ?



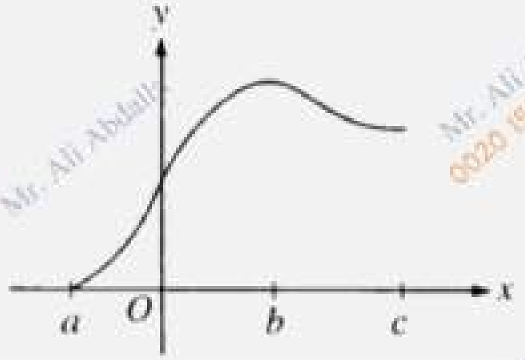
**A.**



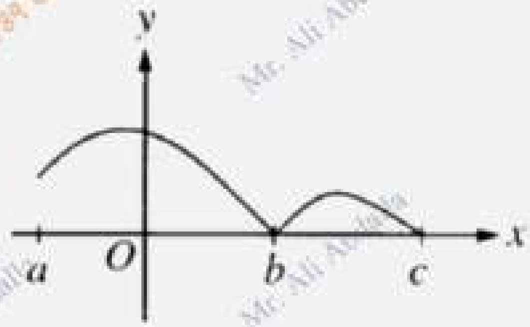
**B.**



**C.**



**D.**



49)

| $x$ | 0 | 0.5 | 1.0 | 1.5 | 2.0 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 3 | 5 | 8 | 13 |

A table of values for a continuous function $f$ is shown above. If four equal subintervals of $[0, 2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$

**A.** 8

**B.** 12

**C.** 16

**D.** 24

50) Which of the following are antiderivative of $f(x) = \sin x \cos x$ ?

**I.** $F(x) = \frac{\sin^2 x}{2}$ **II.** $F(x) = \frac{\cos^2 x}{2}$ **III.** $F(x) = -\frac{\cos 2x}{4}$

**A.** I only

**B.** II only

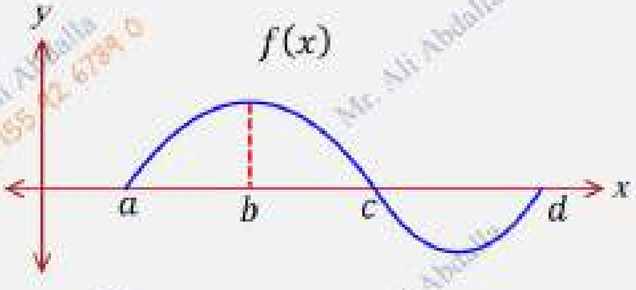
**C.** III only

**D.** I and III only

**E.** II and III only

51) The graph of $f(x)$ is shown in the right figure.

If $g(x) = \int_a^x f(t)\,dt$ for what value of $x$ does $g(x)$ have a maximum?



**A.** $a$ **B.** $b$

**C.** $c$ **D.** $d$

**E.** It cannot be determined from the information given.

52) Let $g$ be a continuously differentiable function with $g(1) = 6$ and $g'(1) = 3$. What is $\lim_{x \to 1} \frac{\int_1^x g(t)dt}{g(x) - 6}$ ?

**A.** 0 **B.** 1 **E.** The limit does not exist.

**C.** 2 **D.** $\frac{1}{2}$

53) If $f$ is the antiderivative of $\frac{x^2}{1 + x^6}$ such that $f(1) = 0$, then $f(4) =$

**A.** $-0.012$ **B.** $0.376$

**C.** $0.256$ **D.** $0.629$

54) What are all values of $k$ for which $\int_{-2}^{k} x^2\,dx = 0$

**A.** $-2$

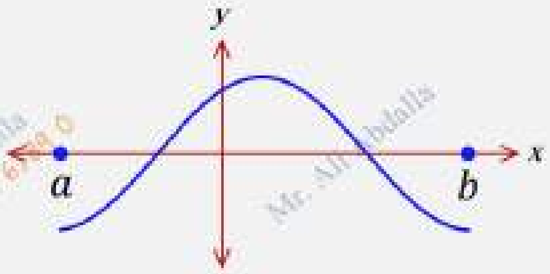
**B.** $-2$ and $2$

**C.** $2$

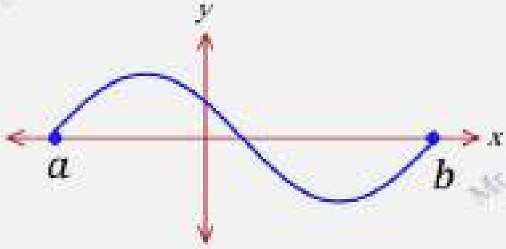
**D.** $0$

55) The graph of $f$ is shown in the right figure.

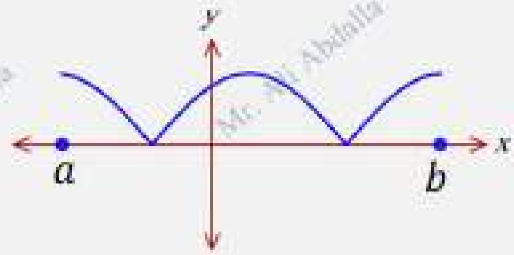
Which of the following could be the graph of the derivative of $f$ ?



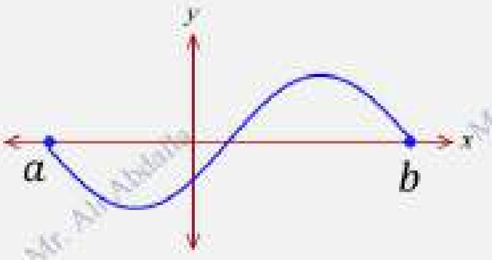
**A.**



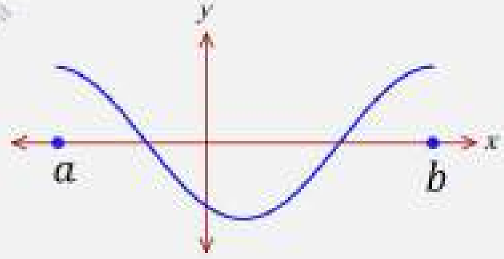
**B.**



**C.**



**D.**



56) $\lim\limits_{x\to 1} \dfrac{\int_1^x e^{t^2}\,dx}{x^2-1}$ is

**A.** $0$

**B.** $1$

**C.** $\frac{e}{2}$

**D.** $e$

57) The function $f(x)$ is continuous on the closed interval $[0,6]$ and has values that are given in the table below.

| $x$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| $f(x)$ | 4 | k | 8 | 12 |

If three equal subintervals of $[0,6]$ are used, if $\int_0^6 f(x)\,dx$ by using the trapezoidal method $= 52$. Find the value of $k$

**A.** 6

**B.** 10

**C.** 7

**D.** 14

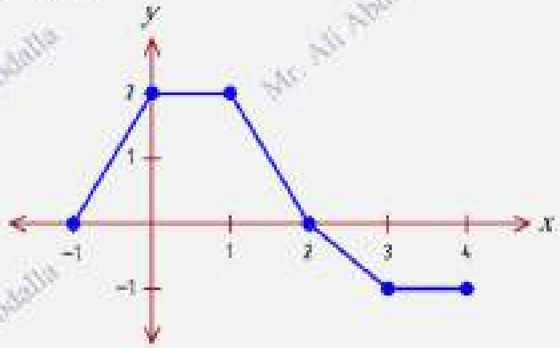
58) The graph of a piecewise-linear function $f$ for $-1 \le x \le 4$, is shown on the right. What is the value of $\int_{-1}^{4} f(x)\,dx$ ?

**A.** 2.5

**B.** 4

**C.** 5.5

**D.** 8



59) If $f$ is a continuous function and if $F'(x) = f(x)$ for all real numbers $x$, then $\int_1^3 f(2x)\,dx = 0$

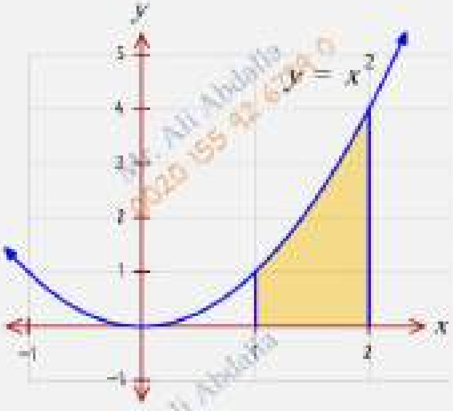
**A.** $2F(6) - 2F(2)$

**B.** $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

**C.** $F(6) - F(2)$

**D.** $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

60) Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$



**A.** $\frac{50}{27}$ **B.** $\frac{251}{108}$

**C.** $\frac{7}{3}$ **D.** $\frac{127}{54}$ **E.** $\frac{77}{27}$

61)

| $t\ (sec)$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| $a(t)\ \ (ft/sec^2)$ | 5 | 2 | 8 | 3 |

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

**A.** 26 ft/sec **B.** 41 ft/sec **C.** 30 ft/sec **D.** 37 ft/sec

62) If $f(x) = g(x) + 7$ for $3 \le x \le 5$, then $\int_3^5 [f(x) + g(x)]\,dx =$

**A.** $2\int_3^5 g(x)\,dx + 7$

**B.** $2\int_3^5 g(x)\,dx + 14$

**C.** $2\int_3^5 g(x)\,dx + 28$

**D.** $\int_3^5 g(x)\,dx + 7$

**E.** $\int_3^5 g(x)\,dx + 14$

63) If $f$ is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

**A.** $0$
**B.** $\frac{ab}{2}$
**C.** $b - a$
**D.** $1$
**E.** $\frac{b^2 - a^2}{2}$

64) If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$,
what the value of $y$ when $x = 0$

**A.** $\frac{1}{3}$
**B.** $0$
**C.** $-1$
**D.** $-\frac{1}{3}$
**E.** $1$

65) Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) = \cdots$

**A.** 0.048
**B.** 5.827
**C.** 1640.250
**D.** 0.144
**E.** 23.308

66) If $f(x) = \int_0^x \frac{1}{\sqrt{t^3 + 1}}\,dt$ Which of the following is **FALSE**?

**A.** $f(0) = 0$
**B.** $f(1) > 0$
**C.** $f(-1) > 0$
**D.** $f'(1) = \frac{1}{\sqrt{2}}$
**E.** $f$ is continuous at for all $x \geq 0$

67) If $F$ and $f$ are continuous functions such that $F'(x) = f(x)$ for all $x$, then $\int_a^b f(x)\,dx$ is

**A.** $F'(a) - F'(b)$

**B.** $F(b) - F(a)$

**C.** $F'(b) - F'(a)$

**D.** $F(a) - F(b)$

**E.** none of the above

68) $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

**A.** $\frac{e^3}{2}$

**B.** $e^3 - e$

**C.** $\frac{e^3 - e}{2}$

**D.** $\frac{e^3 - 1}{2}$

**E.** $e^3 - 1$

69) Given $f(x) = \begin{cases} x+1, & x < 0 \\ \cos \pi x, & x \geq 0 \end{cases}$ Then $\int_{-1}^1 f(x)\,dx$

**A.** $\frac{1}{2} + \frac{1}{\pi}$

**B.** $\frac{1}{2}$

**C.** $-\frac{1}{2} + \pi$

**D.** $\frac{1}{2} - \frac{1}{\pi}$

**E.** $-\frac{1}{2}$

70) $\int_{-1}^{2} \frac{|x|}{x}\,dx \;=$

**A.** $-3$

**B.** $2$

**C.** $3$

**D.** $1$

**E.** non-existent

71) If $n$ is a known positive integer, for what value of $k$ is: $\int_{1}^{k} x^{n-1}\,dx = \frac{1}{n}$

**A.** $0$

**B.** $\left(\frac{2n-1}{n}\right)^{\frac{1}{n}}$

**C.** $2^{n}$

**D.** $2^{\frac{1}{n}}$

**E.** $\left(\frac{2}{n}\right)^{\frac{1}{n}}$

72) If $\int_{1}^{2} f(x-c)\,dx = 5$ Where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx =$

**A.** $5+c$

**B.** $5$

**C.** $5-c$

**D.** $c-5$

**E.** $-5$

73) For $f(x) = x + \int_0^{\tan x} \frac{1}{1+t^2}\,dt$ then $f'(x) =$

**A.** 2

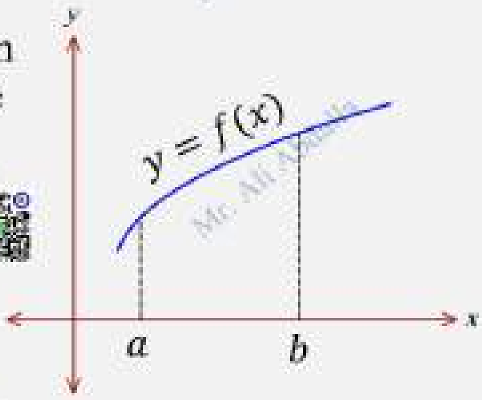
**B.** $1 + \sec^2 x$

**C.** $1 + \frac{\sec^2 x}{1 + \tan x}$

**D.** $1 + \frac{1}{1 + \tan^2 x}$

**E.** 0

74) If $f$ is the continuous, strictly increasing function on the interval $a \le x \le b$ as shown on the right, which of the following must be true?



**I.** $\int_a^b f(x)\,dx < f(b)(b-a)$

**II.** $\int_a^b f(x)\,dx > f(a)(b-a)$

**III.** $\int_a^b f(x)\,dx = f(c)(b-a)$ For some numbers $c$ such that $a < c < b$

**A.** I only

**B.** II only

**C.** III only

**D.** I and II only

**E.** I, II and III

75) If $f(x) = \int_{2x}^{\sin x} \cos t^3 \, dt$ , then $f'(x)$ is equal to

**A.** $\cos(\sin^3 x)\cos x - 2\cos 8x^3$

**B.** $\sin(\sin^3 x)\sin x - 2\sin 8x^3$

**C.** $\cos(\cos^3 x)\cos x - 2\cos x^3$

**D.** $\cos(\sin^3 x)\cos x - \cos 8x^3$

76) Let $f$ be a differentiable function for all real numbers $x$, and

$f(1) = 4$ then the value of $\lim\limits_{x\to 1}\frac{\int_4^{f(x)} 2t\,dt}{x-1}$ If $f'(1) = 2$ is

**A.** 16

**B.** 8

**C.** 4

**D.** 2

77) $\int 2\,\sin^2 x\,dx =$

**B.** $2x - \frac{1}{2}\cos 2x + \mathrm{c}$

**C.** $2x - \frac{1}{2}\cos 2x + \mathrm{c}$

**D.** $x - \frac{1}{2}\cos 2x + \mathrm{c}$

78) $\int \frac{e^{\sec^2 x}}{e^{\tan^2 x}}\,dx$

**A.** $x + c$

**B.** $e\,x + c$

**C.** $e^x + c$

**D.** $e + c$

79) A sensor measures the position $f(t)$ of a particle $t$ microseconds after a collision as given in the table. Estimate the position of the particle at $t = 8$

| $x$ | 5 | 10 | 15 |
|---|---|---|---|
| $f(x)$ | 8 | 14 | 18 |

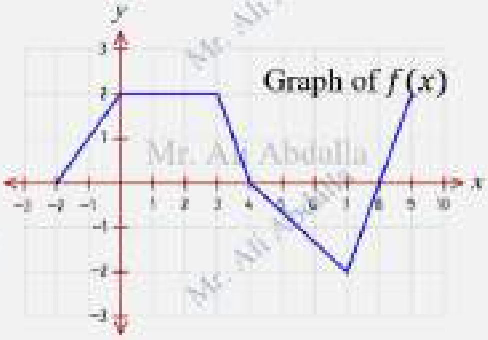
**A.** 11.2

**B.** 11.6

**C.** 16.2

**D.** 13.1

80) Let g be the function given by $g(x) = \int_{-2}^{2} f(t)\,dt$ The graph of the function $f(x)$, shown on the right, consists of five lines segment, which of the following intervals the function $g(x)$ is decreasing



**A.** $(-2, 4)$

**B.** $(3, 7)$

**C.** $(4, 8)$

**D.** $(3, 4)$

81) $\int \dfrac{e^{3\ln 2x} + 5e^{2\ln 2x}}{e^{4\ln x} + 5e^{3\ln x} - 7e^{2\ln x}}\,dx$

**A.** $8\ln|x^2 + 5x - 7| + c$

**B.** $4\ln|x^2 + 5x - 7| + c$

**C.** $8\ln|x^2 + 10x - 7| + c$

**D.** $4\ln|x^2 + 10x - 7| + c$

82) $\int \dfrac{x}{(x-1)(x+1)}\,dx$

**A.** $\ln|x^2 - 1| + c$

**B.** $2\ln|x^2 - 1| + c$

**C.** $\ln|x - 1| - x\ln|x + 1| + c$

**D.** $\ln|x - 1| + c$

83) $\int \cot x \, dx$

**A.** $\ln|\sin x| + c$

**B.** $-\ln|\cos x| + c$

**C.** $-\csc^2 x + c$

**D.** $\ln|\sec x| + c$

84) $\int \frac{x}{\sqrt{1-x^2}} \, dx$

**A.** $-\sqrt{1-x^2} + c$

**B.** $-\frac{1}{2}\sqrt{1-x^2} + c$

**C.** $\sqrt{1-x^2} + c$

**D.** $\frac{1}{2}(1-x)^{3/2} + c$

85) $\int \frac{e^x}{e^{2x}+1} dx$

**A.** $\ln|e^{2x}+1| + c$

**B.** $\tan^{-1}(e^x) + c$

**C.** $\tan^{-1}(e^{2x}) + c$

**D.** $\ln|e^x + 1| + c$

86) $\int \frac{\sin^2 x}{1-\cos x} dx$

**A.** $x + \sin x + c$

**B.** $x + \cos x + c$

**C.** $x - \sin x + c$

**D.** $1 + \sin x + c$

87) $\int \frac{x^3 + 4x}{x^4 + 1}\, dx$

**A.** $\frac{1}{4}\ln|x^4+1| + 2\tan^{-1}2x + c$

**B.** $\ln|x^4+1| + 2\tan^{-1}2x + c$

**C.** $\frac{1}{4}\ln|x^4+1| - 2\tan^{-1}2x + c$

**D.** $4\ln|x^4+1| + 2\tan^{-1}2x + c$

88) Find the function $f(x)$ satisfying the given conditions:
$f''(x) = 12x^2 + 2e^x$ , $f'(0) = 2$ , $f(0) = 3$

**A.** $x^4 + 2e^x + 1$

**B.** $x^4 - 2e^x + 1$

**C.** $x^4 + 2e^x + 5$

**D.** $8x^4 + 6e^x + 1$

89) Find a function $f(x)$ such that the point $(1,2)$ is on the graph of $y = f(x)$, the slope of the tangent line at $(1,2)$ is 3 and $f''(x) = x - 1$

**A.** $\frac{1}{6}x^3 - \frac{1}{2}x^2 + \frac{7}{2}x - \frac{7}{6}$

**B.** $\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{7}{2}x + \frac{7}{6}$

**C.** $x^3 - \frac{1}{2}x^2 + \frac{7}{2}x - 7$

**D.** $x^3 - x^2 + x - 7$

90) $\int e^{x^2 + \ln x}\, dx$

**A.** $e^{x^2} + c$

**B.** $2e^{x^2} + c$

**C.** $\frac{1}{2}e^{x^2} + c$

**D.** $\frac{1}{2}x^2 + c$

91) $\sum_{i=1}^{100}(i^2-3i+2)=$

**A.** 323000

**B.** 323400

**C.** 300400

**D.** 323200

92) $\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{2}{n}\left(\frac{i}{n}+2\right)\right)$

**A.** 5

**B.** 50

**C.** 500

**D.** 5000

93) Use the given function values to estimate the area under the curve of $f(x)$ using left-endpoint evaluation.

| $x$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1.8 | 1.4 | 1.1 | 0.7 | 1.2 | 1.4 | 1.8 | 2.4 | 2.6 |

**A.** $\frac{50}{59}$

**B.** $\frac{59}{50}$

**C.** $\frac{63}{50}$

**D.** $\frac{50}{63}$

94) If $f(x)=3x^2$ Find a value of $c$ that satisfies the conclusion of the Integral Mean Value Theorem on the interval $[0,2]$

**A.** $\frac{2}{3}$

**B.** $\frac{2}{\sqrt{3}}$

**C.** $-\frac{2}{\sqrt{3}}$

**D.** $\pm\frac{2}{\sqrt{3}}$

95) The value of $\int_0^1 3x^2\sqrt{1+x^2}\,dx$ Is between

**A.** $\frac{3}{2}$ and 2

**B.** 0 and $\sqrt{3}$

**C.** 1 and $\sqrt{2}$

**D.** $\frac{1}{2}$ and $\frac{4}{3}$

$$0 \le x \le 1$$
$$0 \le x^2 \le 1$$
$$1 \le 1 + x^2 \le 2$$
$$1 \le \sqrt{1+x^2} \le \sqrt{2}$$
$$3x^2 \le 3x^2\sqrt{1+x^2} \le 3\sqrt{2}\,x^2$$
$$\int_0^1 3x^2\,dx \le \int_0^1 3x^2\sqrt{1+x^2}\,dx \le \int_0^1 3\sqrt{2}\,x^2\,dx$$
$$x^3\Big|_0^1 \le \int_0^1 3x^2\sqrt{1+x^2}\,dx \le \sqrt{2}\,x^3\Big|_0^1$$
$$1 \le \int_0^1 3x^2\sqrt{1+x^2}\,dx \le \sqrt{2}$$

Or: $m(b-a) \le \int_0^1 3x^2\sqrt{1+x^2}\,dx \le M(b-a)$

$$0(1-0) \le \int_0^1 3x^2\sqrt{1+x^2}\,dx \le 3\sqrt{2}(1-0)$$
$$0 \le \int_0^1 3x^2\sqrt{1+x^2}\,dx \le 3\sqrt{2}$$

96) If $f(x) = \int_0^x (t^2 - 3t + 2)\,dt$ Which of the following is true?

**A.** $f$ has local maximum at $x = 2$ and local minimum at $x = 1$

**B.** $f$ has local maximum at $x = 1$ and local minimum at $x = 2$

**C.** $f$ has no local maximum and local minimum at $x = 2$

**D.** $f$ has no local minimum and maximum local at $x = 1$

97) If $\int_0^x f(t)\,dt = x(\ln x - 1)$ What is the value of $f(e^2)$?

**A.** $2 - \ln 2$

**B.** $\ln 2$

**C.** $e^2$

***D.*** 2

98) If $f(x) = \int_{\cos x}^{\sin x} \sqrt{1-t^2}\,dt$ Where $0 \le x \le \frac{\pi}{2}$ then $f'(x) =$

**A.** $2\sin x$

**B.** $\sin^2 x$

**C.** $\sin^2 x - \cos^2 x$

**D.** 1

99) By using the table in the right **Find** $h'(2)$ if

$$h(x) = \int_{2}^{g(x)} f(t)\,dt$$

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 0 | 2 | 1 | 10 | $-2$ |
| 1 | $-5$ | $-8$ | 5 | 1 |
| 2 | 15 | $-1$ | 1 | 3 |

**A.** $-5$

**B.** $3$

**C.** $-15$

**D.** $15$

100) The area between $y = \sin x$ and the $x$-axis for $\frac{\pi}{4} \le x \le \frac{\pi}{2}$ is

**A.** $\frac{\sqrt{3}}{3}$

**B.** $\sqrt{2}$

**C.** $\frac{\sqrt{2}}{2}$

**D.** $\frac{2\sqrt{3}}{3}$

101) The area above the $x$-axis and below $y = 4x - x^2$

**A.** $\frac{32}{3}$

**B.** $\frac{16}{3}$

**C.** $\frac{32}{9}$

**D.** $\frac{16}{5}$

102) $\int \frac{\sqrt{1 - 2\tan x}}{\cos^2 x}\, dx =$

**A.** $-\frac{1}{3}(1 - 2\tan x)^{\frac{3}{2}} + c$

**B.** $\frac{1}{3}(1 - 2\tan x)^{\frac{3}{2}} + c$

**C.** $-3(1 - 2\tan x)^{\frac{3}{2}} + c$

**D.** $-\frac{1}{3}(1 - 2\tan x)^{\frac{1}{2}} + c$

103) $\displaystyle\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx =$

**A.** $\frac{4}{3}(1+\sqrt{x})^{\frac{3}{2}} - 4\sqrt{1+\sqrt{x}} + c$

**B.** $\frac{3}{2}(1+\sqrt{x})^{\frac{3}{2}} - 2\sqrt{1+\sqrt{x}} + c$

**C.** $(1+\sqrt{x})^{\frac{3}{2}} - 4\sqrt{1+\sqrt{x}} + c$

**D.** $\frac{4}{3}(1+\sqrt{x})^{\frac{3}{2}} + 4\sqrt{1+\sqrt{x}} + c$

104) $\displaystyle\int \frac{(1+\sin x)^5}{\sec x}\,dx =$

**A.** $6(1+\sin x)^6 + c$

**B.** $\frac{1}{6}(1+\sin x)^6 + c$

**C.** $\frac{1}{6}\cos^6 x + c$

**D.** $\frac{1}{6} + \frac{1}{6}\sin^6 x + c$

105) If $\displaystyle\int_1^2 f(x)\,dx = 4$ Then $\displaystyle\int_1^4 \frac{f(\sqrt{x})}{\sqrt{x}}\,dx =$

**A.** 4

**B.** 6

**C.** 8

**D.** 10

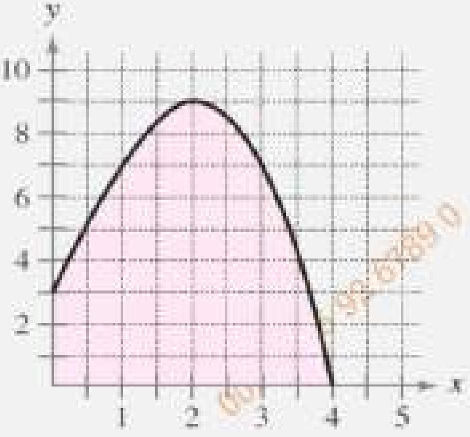
106) Use the graph to estimate shaded area $\left(\int_0^2 f(x)\,dx\right)$ by using Simpson's Rule with $n = 4$

**A.** $\frac{77}{3}$

**B.** $\frac{77}{2}$

**C.** $77$

**D.** $\frac{77}{6}$



107) Determine the number of steps that will guarantee an accuracy of at least $10^{-7}$ for using each of Simpson's Rule to approximate $\int_1^4 \frac{1}{x}\,dx$

**A.** 135

**B.** 136

**C.** 137

**D.** 134

108) Determine the number of steps that will guarantee an accuracy of at least $10^{-7}$ for using each of Trapezoidal Rule to approximate $\int_1^4 \frac{1}{x}\,dx$

**A.** 6708

**B.** 6709

**C.** 135

**D.** 136

109) Use Trapezoidal Rule to estimate $\int_0^2 f(x)\,dx$ from the given data:

| $x$ | 0.0 | 0.50 | 1.00 | 1.50 | 2.00 |
|---|---|---|---|---|---|
| $f(x)$ | 4.0 | 5.2 | 5.0 | 4.4 | 4.0 |

**A.** $\frac{31}{5}$

**B.** $\frac{93}{10}$

**C.** $\frac{93}{5}$

**D.** $\frac{186}{5}$

110) Use Simpson's Rule with $n = 4$ to estimate ln 5

**A.** $\frac{73}{45}$

**B.** $\frac{73}{30}$

**C.** $\frac{3}{2}$

**D.** $\frac{73}{15}$

111) $\int \frac{xe^x}{(1+x)^2}\,dx =$

**A.** $\frac{e^x}{1+x} + c$

**B.** $\frac{e^x}{(1+x)^2} + c$

**C.** $e^x \ln|1+x| + c$

**D.** $\frac{e^x}{2(1+x)} + c$

112) If $\int \frac{1+\cos 4x}{\cot x - \tan x}\,dx = k\cos 4x + c$ then $k =$

Challenge

**A.** $-\frac{1}{4}$

**B.** $-\frac{1}{2}$

**C.** $\frac{1}{4}$

**D.** $-\frac{1}{8}$

113) If $\int \frac{4x^3 + a\,4^x}{4^x + x^4}\,dx = \ln|x^4 + 4^x| + c$ then $a =$

**A.** $\ln 4$

**B.** $\log_4 e$

**C.** $1$

**D.** $4$

114) If $\int \frac{\sqrt{\cot x}}{\sin x \cos x}\,dx = a\sqrt{\cot x} + c$ then $a =$

**A.** $-1$

**B.** $1$

**C.** $-2$

**D.** $2$

115) $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}}\, dx =$

**A.** $10^x + x^{10}$

**B.** $10^x - x^{10}$

**C.** $\log_e(10^x + x^{10})$

**D.** $\frac{1}{10^x + x^{10}}$

116) If $\int_0^x f(t)\, dt = x + \int_x^1 t\, f(t)\, dt$ , then $f(1) =$

**A.** $\frac{1}{2}$

**B.** $0$

**C.** $1$

**D.** $-\frac{1}{2}$

117) $\int_0^{\frac{\pi}{3}} \frac{\cos x + \sin x}{\sqrt{1 + \sin 2x}}\, dx =$

**A.** $\frac{\pi}{3}$

**B.** $\frac{2\pi}{3}$

**C.** $\frac{4\pi}{3}$

**D.** $\pi$

118) If $f(x) = \begin{cases} x^2, & x < 0 \\ -1, & x = 0 \\ x, & x > 0 \end{cases}$ , then $\int_{-1}^{1} f(x)\, dx =$

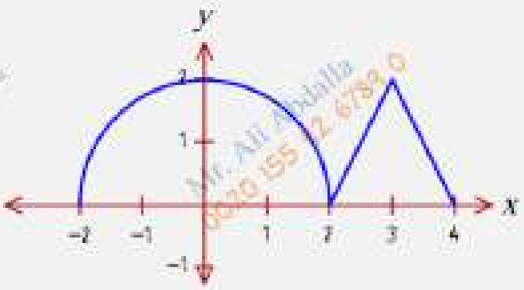
**A.** $\frac{5}{6}$

**B.** $\frac{2}{3}$

**C.** $-\frac{1}{6}$

**D.** nonexistent

119) The graph of g′ , the first derivative of the function g, consists of a semicircle of radius 2 and two line segments, as shown in the right figure. If $g(0) = 1$, what is the value of $g(3)$?



**A.** $\pi + 1$ **C.** $2\pi + 1$

**B.** $\pi + 2$ **D.** $2\pi + 2$

120) Let g be a twice differentiable, increasing function of $t$ .If $g(0) = 20$ and $g(10) = 220$ ,which of the following must be true on the interval $0 < t < 10$?

**A.** $g'(t) = 0$ for some $t$ in the interval.

**B.** $g'(t) = 20$ for some $t$ in the interval.

**C.** $g''(t) = 0$ for some $t$ in the interval.

**D.** $g''(t) = 0$ for some $t$ in the interval.

121) If $\int_1^{x^3} \frac{1}{1+\ln t}\,dt$ For $x > 1$ then $f'(2) =$

**A.** $\frac{1}{1+\ln 2}$ **C.** $\frac{1}{1+\ln 8}$

**B.** $\frac{12}{1+\ln 2}$ **D.** $\frac{12}{1+\ln 8}$

122) Which of the following limits is equal to $\int_3^5 x^4\,dx$ ?

**A.** $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(3+\frac{i}{n}\right)^4\left(\frac{1}{n}\right)$

**B.** $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(3+\frac{i}{n}\right)^4\left(\frac{2}{n}\right)$

**C.** $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(3+\frac{2i}{n}\right)^4\left(\frac{1}{n}\right)$

**D.** $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(3+\frac{2i}{n}\right)^4\left(\frac{2}{n}\right)$

123) If $\frac{d}{dx}\left(\int_{e}^{x^3} \ln(t^2+1)\,dt\right) =$

**A.** $\ln(x^6+1)$

**B.** $2x^2\ln(x^6+1)$

**C.** $2x^2\ln(x^2+1)$

**D.** $\ln(x^6+1) - \ln(e^6+1)$

124) Using the substitution $u = 1 + x$ then $\int \frac{x}{\sqrt{1+x}}\,dx$ is equivalent to

**A.** $\int \frac{1}{u+1}\,du$

**B.** $\int u^{-\frac{1}{2}}\,du$

**C.** $\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du$

**D.** $(u-1)\int u^{-\frac{1}{2}}\,du$

125) $\lim_{x\to 2}\frac{x^2-4}{\int_2^x \cos\pi t\,dt}$ *is*

**A.** 0

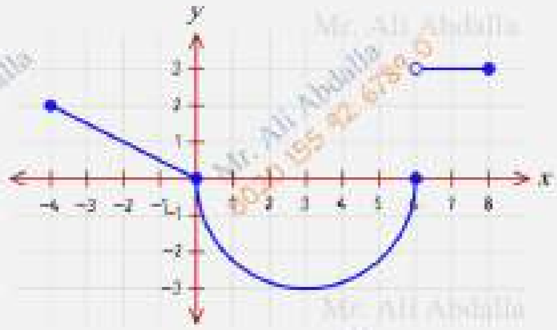
**B.** 2

**C.** 4

**D.** non-existent

126) The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the right figure. Let $f$ be the function defined by:

$$f(x) = 3x + \int_0^x g(t)\ dt$$



Graph of g(x)

Find the value of $f(7)$ and $f'(7)$

**A.** $f(7) = 24 - \frac{9\pi}{2}$ , $f'(7) = 6$

**B.** $f(7) = 24 + \frac{9\pi}{2}$ , $f'(7) = 6$

**C.** $f(7) = 24 - \frac{9\pi}{2}$ , $f'(7) = 9$

**D.** $f(7) = 21 - \frac{9\pi}{2}$ , $f'(7) = 9$

127) $\lim_{x\to-2} \dfrac{f(x) + 7}{e^{3x+6} - 1}$

Use information in question 126

**A.** $\frac{4}{3}$

**B.** $\infty$

**C.** $-\frac{5}{3}$

**D.** $-\frac{4}{3}$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 0.25 | 0.48 | 0.68 | 0.84 | 0.95 | 1 |

128) For the function whose value are given in the table above, $\int_0^6 f(x)\,dx$ is approximated by a Riemann sum using the value at the midpoint of each of three intervals of width 2. The approximation is

**A.** 2.64

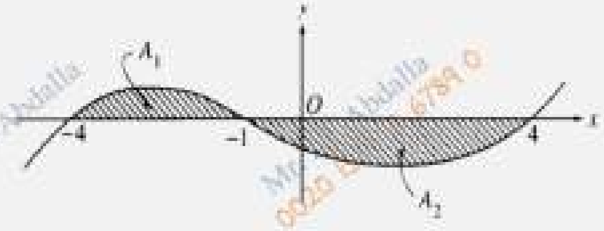
**B.** 3.64

**C.** 3.76

**D.** 4.64

129) The graph of $y = f(x)$ is shown in the right figure. If $A_1$ and $A_2$ are positive numbers that represent the areas of the shaded regions, then in terms of $A_1$ and $A_2$,

$$\int_{-4}^{4} f(x)\,dx - 2\int_{-1}^{4} f(x)\,dx =$$



**A.** $A_1 - A_2$

**B.** $A_1 + A_2$

**C.** $A_1 - 3A_2$

**D.** $A_1 + 2A_2$

130) If $\int_a^b f(x)\,dx = 5$ and $\int_a^b g(x)\,dx = -1$ Which of the following must be true?

**I.** $f(x) > g(x)$ for $a \le x \le b$

**II.** $\int_a^b (f(x) + g(x))\,dx = 4$

**III.** $\int_a^b (f(x)\,g(x))\,dx = -5$

**A.** I only

**B.** II only

**C.** III only

**D.** II and III only

**E.** I, II and III

131) Find the area under the curve $f(x) = 3x^2$ on $[0, 1]$, $n = 4$ using Simpson's rule

**A.** 0.8

**B.** 1

**C.** 6

**D.** 12

132) If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$ then $G(4) =$

**A.** $f'(4) - 7$

**B.** $\int_2^4 (f(x) - 7)\, dt$

**C.** $-7 + \int_2^4 f(x)\, dt$

**D.** $f'(4)$

133) The function $f$ is continuous and $\int_0^8 f(u)\ du = 6$ What is the value of $\int_1^3 x\, f(x^2 - 1)\ dx$ ?

**A.** $\frac{3}{2}$

**B.** 3

**C.** 6

**D.** 12

134) Approximate the area under the curve $y = x^2 + 2$ from $x = 1$ to $x = 2$ using four midpoint rectangles.

**A.** 4.333

**B.** 4.328

**C.** 4.719

**D.** 4.344

135) If the substitution $u = \sqrt{x}$ is made, the integral $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$

**A.** $2\int_1^{16} e^u\, du$

**B.** $2\int_1^{4} e^u\, du$

**C.** $\frac{1}{2}\int_1^{2} e^u\, du$

**D.** $2\int_1^{2} e^u\, du$

136) The function $f$ is defined by $f(x) = \begin{cases} 2 & ,x < 3 \\ x - 1 & ,x \geq 3 \end{cases}$

What is the value of $\int_{1}^{5} f(x)\,dx =$

**A.** 2 **B.** 8

**C.** 6 **D.** 10

137) Let $f$ be a function such that $g'(x) = \int_{6}^{12} f(2x)\,dx = 10$

Which of the following must be true?

**A.** $\int_{12}^{24} f(u)\,du = 5$ **C.** $\int_{6}^{12} f(u)\,du = 5$

**B.** $\int_{12}^{24} f(u)\,du = 20$ **D.** $\int_{6}^{12} f(u)\,du = 20$

**E.** $\int_{3}^{6} f(u)\,du = 5$

138) If $\int_{0}^{x} f\left(\frac{x}{c}\right)dx =$

**A.** $\frac{1}{c}\int_{a}^{b} f(x)\,dx$ **C.** $c\int_{a}^{b} f(x)\,dx$

**B.** $\int_{a}^{b} f(x)\,dx$ **D.** $\int_{ac^2}^{bc^2} f(x)\,dx$

139) Let $f(x) = \int_{1}^{x} \sqrt{2 - t^2}\,dt$ , then the real roots of the equation $x^2 - f'(x) = 0$ are

**A.** $\pm 1$ **C.** $\pm\frac{1}{\sqrt{2}}$

**B.** $\pm\frac{1}{2}$ **D.** $0, -1$ and $1$

140) $\int \left(2\sqrt{x}\cos x + \frac{1}{\sqrt{x}}\sin x\right)\, dx =$ … … … …

**A.** $2\sqrt{x}\cos x + C$

**B.** $2\sqrt{x}\sin x + C$

**C.** $\sqrt{x}\cos x + C$

**D.** $\sqrt{x}\sin x + C$

141) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n} 3\left(c_i^2 - 3\right)\Delta x$ on the interval $[1, 2]$ equivalent to

**A.** $\int_1^2 (3x^2 - 9)dx$

**B.** $\int_1^2 (3x^2 - 6)dx$

**C.** $\int_1^2 (3x^2)dx$

**D.** $\int_1^2 (3x^2 + 9)dx$

142) Let $\int_a^2 f(x)\, dx = 12$ , and the average value of $f(x)$ on the interval $[a, 2]$ is 4. Find the value of $a$?

**A.** $-1$

**B.** 0

**C.** 1

**D.** 2

143) If $f'(x) = e^{-x}$ and $f(0) = 3$ then $f(x) =$

**A.** $2 - e^{-x}$

**B.** $4 - e^{-x}$

**C.** $4 + e^{-x}$

**D.** $2 + e^{-x}$

144) $\frac{d}{dx}\left(\ln\sqrt{x^2+1}\right) =$ ……………

**A.** $\frac{2x}{x^2+1}$

**B.** $2x$

**C.** $\frac{x}{x^2+1}$

**D.** $x^2 + 1$

145) $\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt$

**A.** $\frac{x}{\sqrt{1+x^2}}$

**B.** $\sqrt{1+x^2}$

**C.** $\sqrt{1+x^2} - 5$

**D.** $\frac{x}{\sqrt{1+x^2}} - \frac{1}{\sqrt{5}}$

146) Which of the following gives the bounded of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\cos x^2 \;dx$

**A.** $-1 \le \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\cos x^2 \;dx \le 0.5$

**B.** $-1.23 \le \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\cos x^2 \;dx \le 0.72$

**C.** $-1.23 \le \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\cos x^2 \;dx \le 1$

**D.** $-1.23 \le \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\cos x^2 \;dx \le 0.5$

147) $\int_0^1 \sqrt{x^2 - 2x + 1}\ dx = \dots\dots\dots$

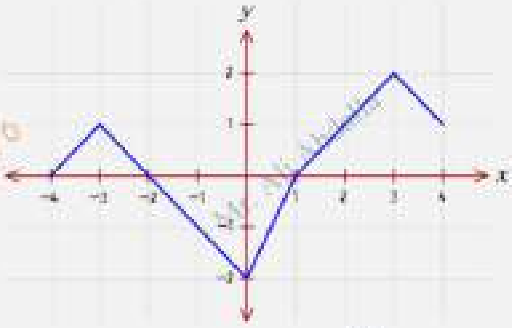
**A.** $-\frac{1}{2}$

**B.** $-1$

**C.** $\frac{1}{2}$

**D.** $1$

148) If $f(x)$ is continuous on $-4 \leq x \leq 4$ and the graph of $f(x)$ consist of five lines segment as shown in the right figure find the average value of $f(x)$ on the average on $[-4, 4]$



**A.** $\frac{1}{8}$

**B.** $\frac{3}{16}$

**C.** $\frac{5}{16}$

**D.** $\frac{3}{2}$

149) By using Reimann Sum which of the following represent the following integral: $\int_3^5 x^4\ dx$

**A.** $\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + \frac{i}{n}\right)^4 \frac{1}{n}$

**B.** $\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + \frac{i}{n}\right)^4 \frac{2}{n}$

**C.** $\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + \frac{2i}{n}\right)^4 \frac{1}{n}$

**D.** $\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + \frac{2i}{n}\right)^4 \frac{2}{n}$

150) $\int 3e^{2\ln x}\ dx =$

**A.** $x^3 + c$

**B.** $3e^{2x} + c$

**C.** $\frac{3}{x} + c$

**D.** $\ln x^3 + c$

151)

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|---|
| $g'(x)$ | $2$ | $3$ | $0$ | $-3$ | $-2$ | $-1$ | $0$ | $3$ | $2$ |

The derivative g′ of a function g is continuous and lies exactly two zeros. Selected value of g′ are given in the table above.
If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

**A.** $-2 \le x \le 2$ only

**B.** $-1 \le x \le 1$ only

**C.** $x \ge -2$

**D.** $x \ge 2$ only

**E.** $x \le -2$ only $x \ge 2$

152) Let $f$ and g be continuous functions such that $\int_0^{10} f(x)\,dx = 21$

$, \int_0^{10} \frac{1}{2}\,g(x)\,dx = 8 \ \ and \ \int_3^{10} (f(x) - g(x))\,dx = 2$

What is the value of $\int_0^{3} (f(x) - g(x))\,dx$

**A.** 3

**B.** 7

**C.** 11

**D.** 15

تم بحمد الله

سُبْحَانَكَ اللَّهُمَّ وَبِحَمْدِكَ، أَشْهَدُ أَنْ لا إِلَهَ

إِلا أَنْتَ، أَسْتَغْفِرُكَ وَأَتُوبُ إِلَيْكَ

# Answers keys

| Q# | Answer | Q# | Answer | Q# | Answer | Q# | Answer |
|---|---|---|---|---|---|---|---|
| 1 | B | 2 | B | 3 | A | 4 | C |
| 5 | C | 6 | C | 7 | E | 8 | C |
| 9 | D | 10 | B | 11 | A | 12 | A |
| 13 | D | 14 | B | 15 | B | 16 | B |
| 17 | D | 18 | A | 19 | A | 20 | C |
| 21 | C | 22 | A | 23 | A | 24 | C |
| 25 | A | 26 | C | 27 | A | 28 | B |
| 29 | C | 30 | D | 31 | A | 32 | B |
| 33 | D | 34 | B | 35 | B | 36 | A |
| 37 | D | 38 | C | 39 | C | 40 | B |
| 41 | D | 42 | C | 43 | B | 44 | B |
| 45 | C | 46 | A | 47 | B | 48 | C |
| 49 | B | 50 | D | 51 | C | 52 | C |
| 53 | C | 54 | A | 55 | A | 56 | A |
| 57 | B | 58 | A | 59 | B | 60 | D |
| 61 | B | 62 | B | 63 | A | 64 | D |
| 65 | B | 66 | C | 67 | B | 68 | D |
| 69 | B | 70 | D | 71 | D | 72 | B |
| 73 | A | 74 | E | 75 | A | 76 | A |
| 77 | A | 78 | C | 79 | B | 80 | A |
| 81 | A | 82 | A | 83 | B | 84 | A |
| 85 | A | 86 | A | 87 | A | 88 | C |
| 89 | B | 90 | A | 91 | B | 92 | A |
| 93 | C | 94 | B | 95 | D | 96 | D |
| 97 | C | 98 | C | 99 | A | 100 | A |
| 101 | A | 102 | B | 103 | C | 104 | A |
| 105 | B | 106 | B | 107 | B | 108 | A |
| 109 | A | 110 | D | 111 | B | 112 | C |
| 113 | C | 114 | A | 115 | A | 116 | A |
| 117 | B | 118 | B | 119 | D | 120 | D |
| 121 | B | 122 | C | 123 | C | 124 | A |
| 125 | A | 126 | C | 127 | B | 128 | B |
| 129 | B | 130 | C | 131 | B | 132 | B |
| 133 | D | 134 | D | 135 | B | 136 | C |
| 137 | A | 138 | B | 139 | A | 140 | C |
| 141 | A | 142 | C | 143 | B | 144 | B |
| 145 | C | 146 | B | 147 | D | 148 | A |
| 149 | A | 150 | A | 151 | A | 152 | A |
| 153 |  | 154 |  | 155 |  | 156 |  |
| 157 |  | 158 |  | 159 |  | 160 |  |