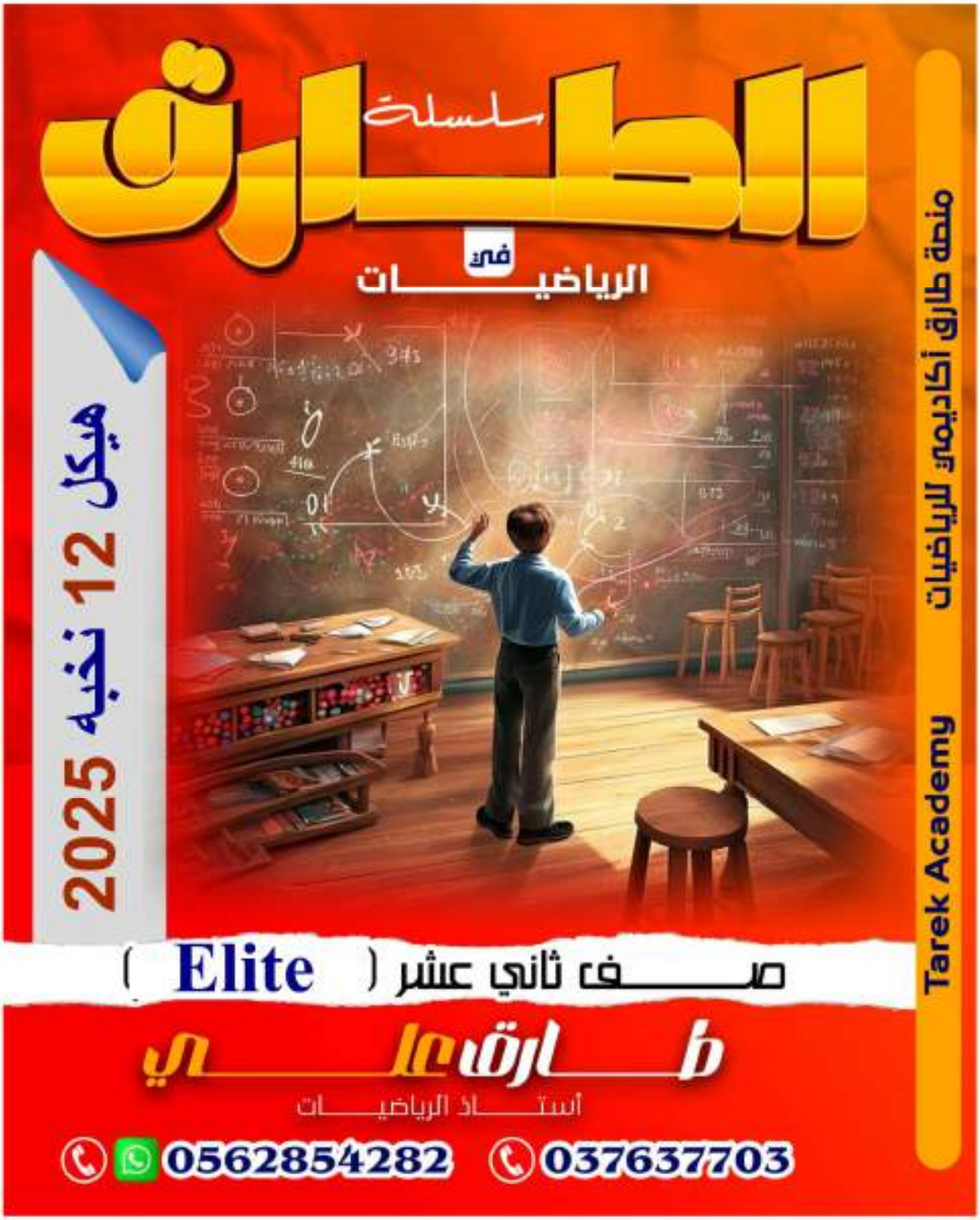
حل مراجعة وفق كامل الهيكل الوزاري الجديد منهج ريفيل المسار النخبة

تم تحميل هذا الملف من موقع المناهج الإماراتية	
موقع المناهج ⇦ المناهج الإماراتية ⇦ الصف الثاني عشر المتقدم ⇦ رياضيات ⇦ الفصل الأول ⇦ ملفات متنوعة ⇦ الملف	
تاريخ إضافة الملف على موقع المناهج: 2025-11-20 23:56:54	
ملفات ا كتب للمعلم ا كتب للطالب ا اختبارات الكترونية ا اختبارات ا حلول ا عروض بوربوينت ا أوراق عمل منهج انجليزي ا ملخصات وتقارير ا مذكرات وبنوك ا الامتحان النهائي ا للمدرس	المزيد من مادة رياضيات:
إعداد: طارق علي	

التواصل الاجتماعي بحسب الصف الثاني عشر المتقدم					
صفحة المناهج الإماراتية على فيسبوك					
المواد على تلغرام	التربية الاسلامية	اللغة العربية	اللغة الانجليزية	الرياضيات	

المزيد من الملفات بحسب الصف الثاني عشر المتقدم والمادة رياضيات في الفصل الأول	
1	حل كراسة تدريبية مراجعة وفق الهيكل الوزاري الجديد منهج بريدج
2	مراجعة شاملة امتحانات وزارية سابقة وفق الهيكل الوزاري الجديد باللغتين
3	كراسة تدريبية مراجعة وفق الهيكل الوزاري الجديد منهج بريدج
4	مراجعة الدرس السابع Functions Logarithmic and Exponential of Derivatives مشتقة الدوال الأسية والدوال اللوغاريتمية من الوحدة الثالثة (اختبر نفسك 11)
5	نموذج اختبار تجريبي وفق الهيكل الوزاري الجديد منهج بريدج


سلسلة
الطارق
في
الرياضيات
هيكل 2025 نخبة
منصة طارق أكاديمي للرياضيات
Tarek Academy
صف ثاني عشر (Elite)
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أستاذ الرياضيات
0562854282
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6.2

Q1 -MCQ

right

$A = \Delta x \cdot f(x)$

1.

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

$\Delta x_1 = 7-4 = 3$

$\Delta x_2 = 12-7 = 5$

$\Delta x_3 = 15-12 = 3$

A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$ where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9
(B) 68.2
(C) 114.9
(D) 116.6
(E) 118.2

$A = 3(6.2) + 5(5.9) + 3(5.6) = 64.9$

$50 + 64.9 = 114.9$

---

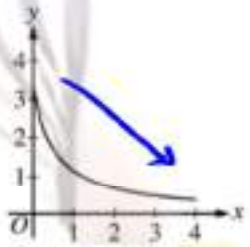
2. If a trapezoidal sum overapproximates $\int_0^4 f(x)dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

over

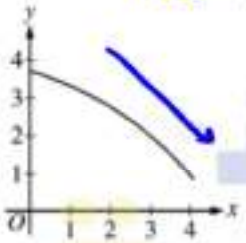
decreasing

concave up

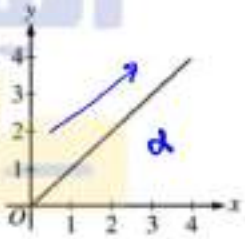
(A)



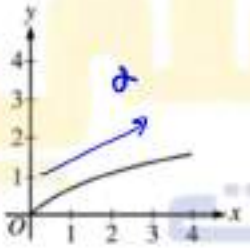
(C)



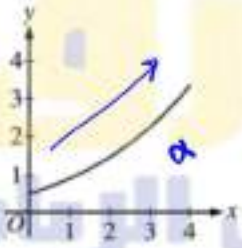
(E)



(B)



(D)



3. $h_1 = \Delta x$
$2 - 0 = 2$

Time (weeks)	0	2	6	10
Level	210	200	190	180

$h_2 = 6 - 2 = 4$
$h_3 = 10 - 6 = 4$

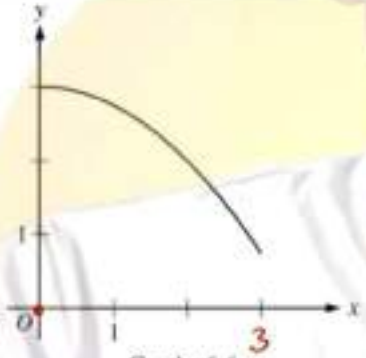
The table above gives the level of a person's cholesterol at different times during a 10-week treatment period. What is the average level over this 10-week period obtained by using a trapezoidal approximation with the subintervals [0, 2], [2, 6], and [6, 10] ?

(A) 188
(B) 193
(C) 195
(D) 198

$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}\left[2(210+200) + 4(200+190) + 4(190+180)\right]$
$= 1930 \div 10 = 193$

4.



Graph of f

over → left → concave up
under → Right → concave down

The graph of the function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_1^3 f(x)dx$ $\int_0^3$

(B) Left Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length — تدريب right

(C) Right Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length — تدريب left

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

6. Let f be the function given by $f(x) = 9^x$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation for $\int_0^2 f(x)dx$?

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

0 d
0.5
1
1.5
2

$A = \frac{1}{2}[f(0.5) + f(1) + f(1.5) + f(2)]$

$= \frac{1}{2}[9^{0.5} + 9^1 + 9^{1.5} + 9^2] = 60$

(A) 20
(B) 40
(C) 60
(D) 80
(E) 120

---

7. midPoint

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

$1 - 0 = 1$
$2 - 1 = 1$
$3 - 2 = 1$

The table above gives selected values for a continuous function f. If f is increasing over the closed interval [0,3], which of the following could be the value of $\int_0^3 f(x)dx$?

$1[4 + 18 + 40] = 62$

(A) 50
(B) 62
(C) 77
(D) 100
(E) 154

8.

x	2	3	5	8	13
$f(x)$	6	−2	−1	3	9 → d

$\Delta x_1 = 3 - 2 = 1$, $\Delta x_3 = 8 - 5 = 3$
$\Delta x_2 = 5 - 3 = 2$
$\Delta x_4 = 13 - 8 = 5$

The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$ what is the approximation of $\int_2^{13} f(x)dx$ obtained from a left Riemann sum?

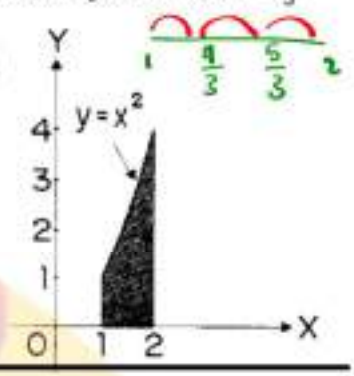
$1(6) + 2(-2) + 3(-1) + 5(3)$
$= 14$

(A) 6
(B) 14
(C) 28
(D) 32
(E) 50

Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$

$\frac{1}{2}\left[\left(\frac{4}{3}-1\right)\left(1^2+\left(\frac{4}{3}\right)^2\right)+\left(\frac{5}{3}-\frac{4}{3}\right)\left(\left(\frac{4}{3}\right)^2+\left(\frac{5}{3}\right)^2\right)+\left(2-\frac{5}{3}\right)\left(\left(\frac{5}{3}\right)^2+2^2\right)\right]$

(A) $\frac{50}{27}$
(B) $\frac{251}{108}$
(C) $\frac{7}{3}$
(D) $\frac{127}{54}$
(E) $\frac{77}{27}$



10.

x	2	5	7	8
$f(x)$	10	30	40	20

The function f is continuous on the closed interval [2,8] and has values that are given in the table above. Using the subintervals [2,5], [5, 7], and [7,8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

$\frac{1}{2}\left[(5-2)(10+30)+(7-5)(40+30)+(8-7)(20+40)\right]$

(A) 110
(B) 130
(C) 160
(D) 190
(E) 210

11.

x	2	5	10	14
$f(x)$	12	28	34	30

$\Delta x \cdot f(x)$

The function f is continuous on the closed interval [2,14] and has values as shown in the table above. Using the subintervals [2,5], [5,10], and [10,14], what is the approximation of $\int_2^{14} f(x)dx$ found by using a right Riemann sum?

$(5-2)(28)+(10-5)(34)+(14-10)(30)$

(A) 296
(B) 312
(C) 343
(D) 374
(E) 390

12.

$\frac{1}{2}h(b_1+b_2)$

$h = a_2 - a_1 = \Delta x$

x	0	2	4	6
$f(x)$	4	k	8	12

The function f is continuous on the closed interval $[0,6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)dx$ found with 3 subintervals of equal length is 52. What is the value of k?

(A) 2
(B) 6
(C) 7
(D) 10
(E) 14

$\frac{1}{2}\left[(2-0)(4+k)+(4-2)(k+8)+(6-4)(8+12)\right]=52$

$\frac{1}{2}\left[2(k+4)+2(k+8)+40\right]=52$ shift Solve =

---

15. $h_1 = 0.5 - 0 = 0.5$

$h_2 = 2 - 0.5 = 1.5$

$h_3 = 3 - 2 = 1$

t	0	0.5	2	3
$v(t)$	20	60	40	30

The table above gives the velocity $v(t)$, in miles per hour, of a truck at selected times t, in hours. Using a trapezoidal sum with the three subintervals indicated by the table, what is the approximate distance, in miles, the truck traveled from time $t = 0$ to $t = 3$?

(A) 140
(B) 130
(C) 125
(D) 120

$\frac{1}{2}\left[0.5(20+60)+1.5(60+40)+1(40+30)\right]$

16.

t (minutes)	0	4	7	9
$r(t)$ (gallons per minute)	9	6	4	3

$h_1 = 4 - 0 = 4$
$h_2 = 7 - 4 = 3$
$h_3 = 9 - 7 = 2$

Water is flowing into a tank at the rate r(t), where r(t) is measured in gallons per minute and t is measured in minutes. The tank contains 15 gallons of water at time t = 0. Values of r(t) for selected values of t are given in the table above. Using a trapezoidal sum with the three intervals indicated by the table, what is the approximation of the number of gallons of water in the tank at time t = 9 ?

(A) 52
(B) 57
(C) 67
(D) 77
(E) 79

$\frac{1}{2}\left[4(9+6) + 3(6+4) + 2(4+3)\right] = 52$

$\rightarrow 15 + 52 = 67$

19.

t (hours)	0	2	7	9
$R(t)$ (tons per hour)	15	9	5	4

$h_1 = 2 - 0 = 2$
$h_2 = 7 - 2 = 5$
$h_3 = 9 - 7 = 2$

On a certain day, the rate at which material is deposited at a recycling center is modeled by the function R, where $R(t)$ is measured in tons per hour and t is the number of hours since the center opened. Using a trapezoidal sum with the three subintervals indicated by the data in the table, what is the approximate number of tons of material deposited in the first 9 hours since the center opened?

(A) 68
(B) 70.5
(C) 85
(D) 136

$\frac{1}{2}\left[2(15+9) + 5(9+5) + 2(5+4)\right] = 68$

32. The definite integral $\int_0^4 \sqrt{x}dx$ is approximated by a left Riemann sum, a right Riemann sum, and a trapezoidal sum, each with 4 subintervals of equal width. If L is the value of the left Riemann sum, R is the value of the right Riemann sum, and T is the value of the trapezoidal sum, which of the following inequalities is true?

(A) $L < \int_0^4 \sqrt{x}dx < T < R$

(B) $L < T < \int_0^4 \sqrt{x}dx < R$

(C) $R < \int_0^4 \sqrt{x}dx < T < L$

(D) $R < T < \int_0^4 \sqrt{x}dx < L$

0 1 2 3 4

L ——— R

$T = \frac{1}{2}\left[(1-0)(\sqrt{0}+\sqrt{1}) + (2-1)(\sqrt{1}+\sqrt{1}) + (3-2)(\sqrt{3}+\sqrt{2}) + (4-3)(\sqrt{4}+\sqrt{3})\right]$

by Calc $\int_0^9 \sqrt{x}\,dx = 5.33$

32. The definite integral $\int_0^4 \sqrt{x}dx$ is approximated by a left Riemann sum, a right Riemann sum, and a trapezoidal sum, each with 4 subintervals of equal width. If L is the value of the left Riemann sum, R is the value of the right Riemann sum, and T is the value of the trapezoidal sum, which of the following inequalities is true?

(A) $L < \int_0^4 \sqrt{x}dx < T < R$

(B) $L < T < \int_0^4 \sqrt{x}dx < R$

(C) $R < \int_0^4 \sqrt{x}dx < T < L$

(D) $R < T < \int_0^4 \sqrt{x}dx < L$

0, 1, 1, 2, 3, 4

L ——— R

$T = \frac{1}{2}[(1-0)(\sqrt{0}+\sqrt{1}) + (2-1)(\sqrt{1}+\sqrt{2}) + (3-2)(\sqrt{3}+\sqrt{2}) + (4-3)(\sqrt{4}+\sqrt{3})]$

by Calc $\int_0^9 \sqrt{x}\,dx = 5.33$

---

26. Let f be a continuous function such that f changes from increasing to decreasing, and the graph of f changes from concave up to concave down. Which of the following is true about the midpoint Riemann sum approximation for $\int_1^3 f(x^2+x)dx$ using 4 subintervals of equal width?

$\Delta x = \frac{3-1}{4} = \frac{1}{2}$

1, 1.5, 2, 2.5, 3

(1, 1.5), (1.5, 2), (2, 2.5), (2.5, 3)

(A) $\frac{1}{2}\left(\frac{f(1^2+1)+f(1.5^2+1.5)}{2} + \frac{f(1.5^2+1.5)+f(2^2+2)}{2} + \frac{f(2^2+2)+f(2.5^2+2.5)}{2} + \frac{f(2.5^2+2.5)+f(3^2+3)}{2}\right)$ is the midpoint Riemann sum approximation and underestimates $\int_1^3 f(x^2+x)dx$.

mid $= \frac{1+1.5}{2} = 1.25$

(B) $\frac{1}{2}\left(\frac{f(1^2+1)+f(1.5^2+1.5)}{2} + \frac{f(1.5^2+1.5)+f(2^2+2)}{2} + \frac{f(2^2+2)+f(2.5^2+2.5)}{2} + \frac{f(2.5^2+2.5)+f(3^2+3)}{2}\right)$ is the midpoint Riemann sum approximation and overestimates $\int_1^3 f(x^2+x)dx$.

mid $= \frac{1.5+2}{2} = 1.75$

(C) $\frac{1}{2}\left(f(1.25^2+1.25) + f(1.75^2+1.75) + f(2.25^2+2.25) + f(2.75^2+2.75)\right)$ is the midpoint Riemann sum approximation and underestimates $\int_1^3 f(x^2+x)dx$.

(D) $\frac{1}{2}\left(f(1.25^2+1.25) + f(1.75^2+1.75) + f(2.25^2+2.25) + f(2.75^2+2.75)\right)$ is the midpoint Riemann sum approximation. There is not enough information to determine whether the approximation underestimates or overestimates $\int_1^3 f(x^2+x)dx$.

6.4

Q2-MCQ 6-4

1. Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x
(B) $\frac{1}{2}$
(C) $\frac{3}{2}$
(D) 2
(E) 3

$\dot{f}(x) = 0$

$\dot{f}(x) = e^{(x^2-3x)^2} \cdot (2x-3) = 0$

$e^{(x^2-3x)^2} \neq 0$

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

---

3. Which of the following is an antiderivative of $f(x) = \sqrt{1+x^3}$?

(A) $\frac{2}{3}(1+x^3)^{\frac{1}{2}}$
(B) $\frac{\frac{2}{3}(1+x^3)^{\frac{1}{2}}}{3x^2}$
(C) $\int_0^{1+x^3} \sqrt{t}\,dt$
(D) $\int_0^{x^3} \sqrt{1+t}\,dt$
(E) $\int_0^{x} \sqrt{1+t^3}\,dt$

$\frac{d}{dx}\int_a^{g(x)} f(x)\,dx = f(g(x)) \cdot \dot{g}(x)$

$\int_0^{x} \sqrt{1+t^3}\,dt$

---

4. The function f is given by $f(x) = \int_1^x \sqrt{t^3+2}\,dt$. What is the average rate of change of f over the interval $[0, 3]$?

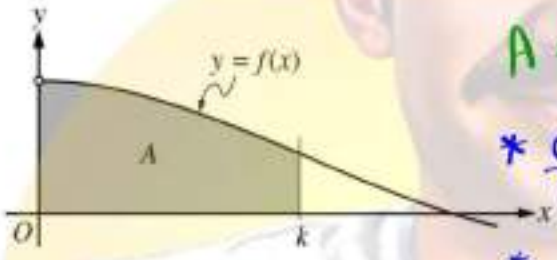
(A) 1.324
(B) 1.497
(C) 1.696
(D) 2.266
(E) 2.694

$avg = \frac{f(3) - f(0)}{3-0} = \frac{\int_1^3 \sqrt{x^3+2} - \int_1^0 \sqrt{x^3+2}}{3}$

$\frac{6.58 - -1.50}{3} = 2.69$

5. Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}\,dt$. Which of the following must be true on the interval $0 < x < 2$?

(A) g is increasing, and the graph of g is concave up.
(B) g is increasing, and the graph of g is concave down.
(C) g is decreasing, and the graph of g is concave up.
(D) g is decreasing, and the graph of g is concave down.
(E) g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.

6. 

The figure above shows the region A, which is bounded by the x- and y-axes, the graph of $f(x) = \frac{\sin x}{x}$ for $x > 0$, and the vertical line $x = k$. If k increases at a rate of $\pi/4$ units per second, how fast is the area of region A increasing when $k = \pi/6$?

(A) 0
(B) 3/4
(C) $3/\pi$
(D) $\frac{\sqrt{3}}{2}$
(E) $2\sqrt{3}$

7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 - 4$ for time $t \geq 0$. If the particle is at position $x = -2$ at time $t = 0$, what is the position of the particle at the time $t = 3$?

(A) 13
(B) 15
(C) 16
(D) 17
(E) 25

The graph of the differentiable function f is shown in the figure above. Let h be the function defined by $h(x) = \int_0^x f(t)dt$. Which of the following correctly orders $h(2)$, $h'(2)$, and $h''(2)$?

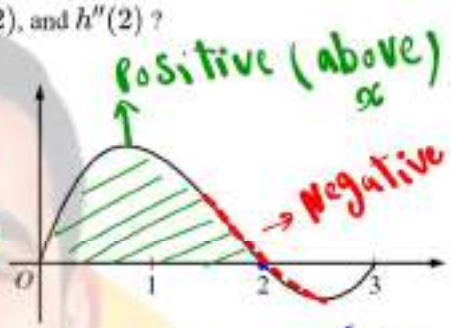
(A) $h(2) < h'(2) < h''(2)$
(B) $h'(2) < h(2) < h''(2)$
(C) $h'(2) < h''(2) < h(2)$
(D) $h''(2) < h(2) < h'(2)$
(E) $h''(2) < h'(2) < h(2)$

$* \acute{h}(x) = f(x)$

$\acute{h}(2) = F(2) = 0$

$* \acute{\acute{h}}(2) = \acute{F}(2) = m$ (+) $= (-)$

$* h(2) = \int_0^2 f(x)\,dt \rightarrow (+)$



Graph of f $= \acute{h}(x)$

$- < 0 < +$

$\acute{\acute{h}}(2) < \acute{h}(2) < h(2)$

---

9. Let F be the function given by $F(x) = \int_3^x (\tan(5t)\sec(5t) - 1)\,dt$. Which of the following is an expression for $F'(x)$?

(A) $\frac{1}{5}\sec(5x) - 1$
(B) $\frac{1}{5}\sec(5x) - x$
(C) $\tan(5x)\sec(5x)$
(D) $\tan(5x)\sec(5x) - 1$

$\acute{F}(x) = \tan 5x \cdot \sec 5x - 1$

---

10. For $x > 0$, $\frac{d}{dx}\left(\int_0^{2x} \ln(t^3+1)\,dt\right) =$

(A) $\ln(x^3+1)$
(B) $\ln(8x^3+1)$
(C) $2\ln(x^3+1)$
(D) $2\ln(8x^3+1)$
(E) $24x^2\ln(8x^3+1)$

$\ln((2x)^3+1) \cdot 2$

$= 2\ln(8x^3+1)$

12. For $x > 0$, $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{1}{1+t^2}dt =$

(A) $\frac{1}{2\sqrt{x}(1+x)}$

(B) $\frac{1}{2\sqrt{x}(1+\sqrt{x})}$

(C) $\frac{1}{1+x}$

(D) $\frac{\sqrt{x}}{1+x}$

(E) $\frac{1}{1+\sqrt{x}}$

$= \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$

---

14. If $y = 5 + \int_2^{2x} e^{-t^2}dt$, which of the following is true?

(A) $\frac{dy}{dx} = e^{-x^2}$ and y(0) = 5

(B) $\frac{dy}{dx} = e^{-x^2}$ and y(1) = 5

(C) $\frac{dy}{dx} = e^{-4x^2}$ and y(1) = 5

(D) $\frac{dy}{dx} = 2e^{-4x^2}$ and y(0) = 5

(E) $\frac{dy}{dx} = 2e^{-4x^2}$ and y(1) = 5

$y' = e^{-(2x)^2} \cdot 2$

$= 2e^{-4x^2}$

* $y(0) = 5 + \int_2^{2(0)} e^{-t^2}dt = 4.1$

* $y(1) = 5 + \int_2^{2(1)} e^{-t^2}dt = 4.6 \simeq 5$

---

16. For all values of x, the continuous function f is positive and decreasing. Let g be the function given by $g(x) = \int_2^x f(t)dt$. Which of the following could be a table of values for g ?

$g(1) < g(2) < g(3)$

* $g(1) = \int_2^1 f(t)dt$

(A)

x	g(x)
1	−2
2	0
3	1

+2 increase +1

(B)

x	g(x)
1	−2
2	0
3	3

increase +3

(C)

x	g(x)
1	1
2	0
3	−2

decrease

(D)

x	g(x)
1	2
2	0
3	−1

decreasing → positive

g(x) → increasing

(E)

x	g(x)
1	3
2	0
3	2

decrease

$g(2) = \int_2^2 f(t)dt = 0$

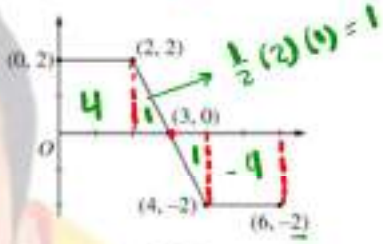
The graph of a function f, consisting of three line segments, is shown above. The function f is defined on the closed interval $[0, 6]$. Let $g(x) = \int_2^x f(t)dt$. What is the maximum value of $g(x)$ for $0 \le x \le 6$?

(A) 0
(B) 1
(C) 5
(D) 10

$g'(x) = f(x)$ * $g'(3) = 1$

* $g'(0) = f(0) = -4$
* $g'(6) = f(6) = -4$

$g'(x) = 0 \rightarrow f(x) = 0$ at $x = 3$



Graph of f

---

18. For $x > 0$, $\int \left(\frac{1}{x} \int_1^x \frac{du}{u} \right) dx =$

(A) $\frac{1}{x^3} + C$
(B) $\frac{8}{x^4} - \frac{2}{x^2} + C$
(C) $\ln(\ln x) + C$
(D) $\frac{\ln(x^2)}{2} + C$
(E) $\frac{(\ln x)^2}{2} + C$

$\int \frac{1}{x} \cdot \ln u \Big|_1^x$ $\ln 1 = 0$

$\int \frac{1}{x} \cdot (\ln x - \ln 1)\, dx$

$\int \frac{1}{x} \cdot \ln x \, dx$

$u = \ln x \rightarrow du = \frac{1}{x} dx \rightarrow$

$dx = x\, du$

$\int \frac{1}{x} \cdot u \cdot x\, du = \int u\, du$

$\frac{u^2}{2} + C = \frac{(\ln x)^2}{2} + C$

6.5

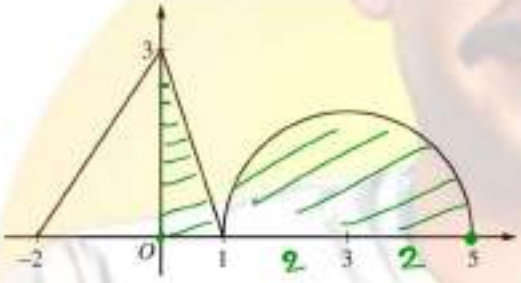
Q3-MCQ 6-5

1. If $0 \le b \le 2$, for what value of b is $\int_0^b \cos(e^x)dx$ a minimum?

(A) 0
(B) 0.452
(C) 1.145
(D) 1.550
(E) 2

Skip

2.



Graph of f

$g(5) = \int_0^5 f(t)\,dt$
$= \frac{1}{2}(1)(3) + \frac{1}{2}\pi(2)^2$
$= \frac{3}{2} + 2\pi$

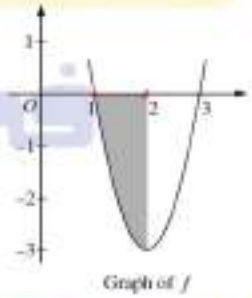
The graph of the function f shown above consists of two line segments and a semicircle. Let g be defined by $g(x) = \int_0^x f(t)\,dt$. What is the value of $g(5)$?

(A) 0
(B) $-1.5 + 2\pi$
(C) 2π
(D) $1.5 + 2\pi$
(E) $4.5 + 2\pi$

The figure above shows the graph of the function f. If $g(x) = \int_1^x f(t)\,dt$ and the shaded region has an area of 2, what is the value of g(2)?

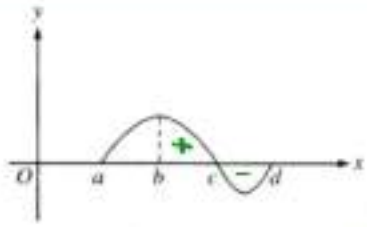
$= \int_1^2 f(t)\,dt$
down
-2

(A) -3
(B) -2
(C) 0
(D) 1
(E) 2



Graph of f

5.

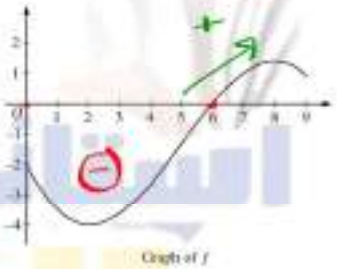


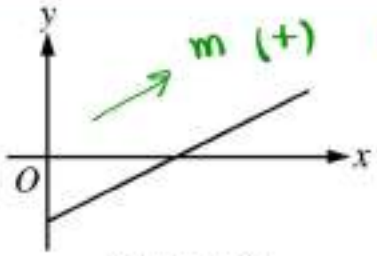
The graph of f is shown in the figure above. If $g(x) = \int_a^x f(t)dt$, for what value of x does $g(x)$ have a maximum?

(A) a
(B) b
(C) c
(D) d
(E) It cannot be determined from the information given.

The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$
(B) $h(6) < h''(6) < h'(6)$
(C) $h'(6) < h(6) < h''(6)$
(D) $h''(6) < h(6) < h'(6)$
(E) $h''(6) < h'(6) < h(6)$

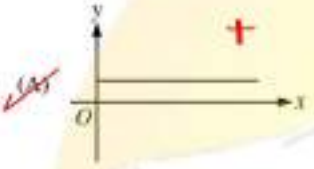




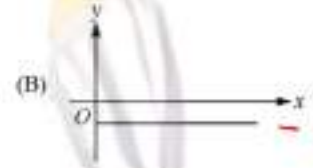
Graph of f

linear
Constant
Horizontal
$\Rightarrow f(x) = x$
$f(x) = 1$

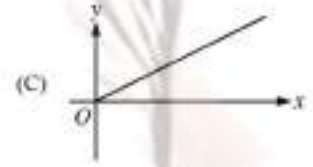
The figure above shows the graph of f. If $f(x) = \int_2^x g(t)dt$, which of the following could be the graph of $y = g(x)$?



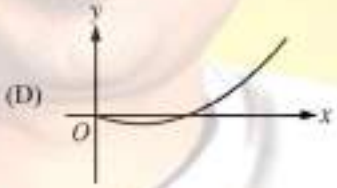
(A)



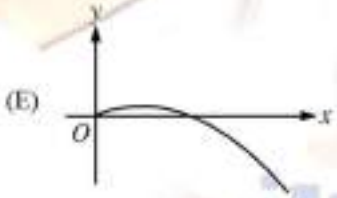
(B)



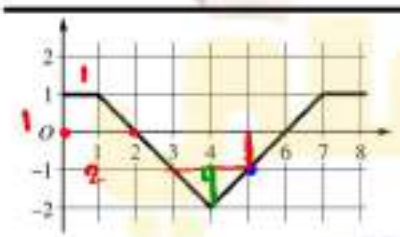
(C)



(D)



(E)



Graph of f

$y - y_1 = m(x - x_1)$
$x = 5$
$y = g(5) = \int_0^5 F(t)dt$

$m = g'(x) = f(x)$
$= g'(5) = F(5)$
$= -1$

The graph of the function f in the figure above consists of four line segments. Let g be the function defined by $g(x) = \int_0^x f(t)dt$. Which of the following is an equation of the line tangent to the graph of g at $x = 5$?

(A) $y+1=x-5$
(B) $y-2=x-5$
(C) $y-2=-1(x-5)$
(D) $y+2=x-5$
(E) $y+2=-1(x-5)$

$x_1=5$ $\quad m=-1$

$g(5)=y_1=\int_0^5 f(t)\,dt$

$=\frac{1}{2}(1)(1+2)-\frac{1}{2}(1)(3+2)-\frac{1}{2}(2)(1)$

$=-2$

$y-(-2)=-1(x-5) \rightarrow y+2=-1(x-5)$

---

10. If $f'(x)>0$ for all real numbers x and $\int_4^7 f(t)dt=0$ which of the following could be a table of values for the function f?

(A)

x	$f(x)$
4	−4
5	−3
7	0

$\frac{-3-(-4)}{5-4}=1$

(B)

x	$f(x)$
4	−4
5	−2
7	5

$\frac{-2-(-4)}{5-4}=2$ $\quad - \rightarrow +$

(C)

x	$f(x)$
4	−4
5	6
7	3

$\frac{6-(-4)}{5-4}=10$

(D)

x	$f(x)$
4	0
5	0
7	0

$m=0$

(E)

x	$f(x)$
4	0
5	4
7	6

increasing

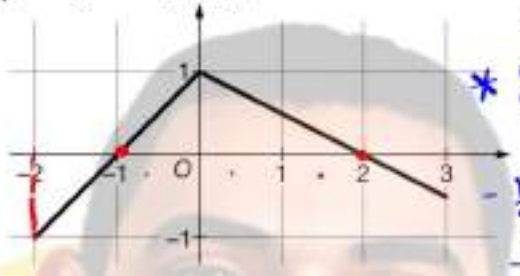
12. $g' = 0$

$g(x) = F(x) = 0$

at $x = -1$

$x = 2$

+ → − max



Graph of f

$g(2)$, $g(-2)$, $g(3)$

$* g(2) = \int_{-2}^{2} f(t)\,dt$

$= \frac{1}{2}(1)(1) + \frac{1}{2}(3)(1)$

$= \frac{1}{2} + 1.5 = \boxed{1}$

The graph of the function f consists of two line segments, as shown in the figure above. Let g be the function defined by $g(x) = \int_{-2}^{x} f(t)dt$. At what value of x does g attain its global maximum on the closed interval $[-2, 3]$?

(A) -2

(B) -1

(C) 0

(D) 2

$* g(-2) = \int_{-2}^{-2} f(t) = 0$ Same

$* g(3) = \int_{-2}^{3} f(t)\,dt = 1 - \frac{1}{2}(1)(\frac{1}{2})$

$= 0.75$

$* g(-1) = \int_{-2}^{-1} f(t)\,dt = -\frac{1}{2}(1)(1) = -0.5$

The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-5}^{x} f(t)dt$, how many zeros does g have on the closed interval $[-5, 6]$?

(A) One

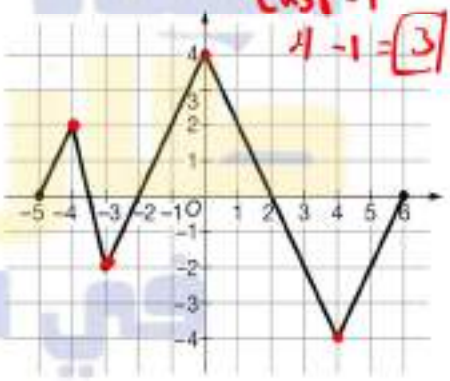
(B) Three

(C) Four

(D) Five

$g(x) = 0$ when

$g(-5) = \int_{-5}^{-5} f(t)\,dt = 0$



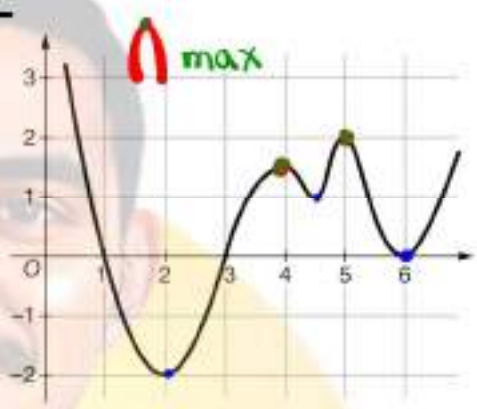
Graph of f

Cusp -1

$4 - 1 = \boxed{3}$

The graph of the function f is shown above. Let g be the function defined by $g(x) = \int_1^x f(t)dt$. At what values of x in the interval $0.5 < x < 6.5$ does g have a relative maximum?

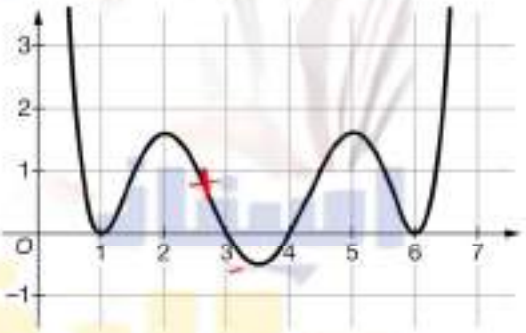
(A) 1 only
(B) 3 only
(C) 4 and 5
(D) 1, 3, and 6



Graph of f

The graph of the function f is shown above. Let g be the function defined by $g(x) = \int_1^x f(t)dt$. At what values of x in the interval $0.5 < x < 6.5$ does g have a relative maximum?

(A) 3 only
(B) 4 only
(C) 2 and 5
(D) 1, 3, 4, and 6



Graph of f

6.6

Q4 MCQ 6-6

1. If $\int_4^{-10} g(x)dx = -3$ and $\int_4^6 g(x)dx = 5$, then $\int_{-10}^6 g(x)dx =$

(A) -8
(B) -2
(C) 2
(D) 8

$-\int_{-10}^{4} g(x)dx + \int_{4}^{6} g(x)dx$

$-(-3) + 5$

8

---

2. Let f and g be continuous functions for $a \le x \le b$. If $a < c < b$, $\int_a^b f(x)dx = P$, $\int_c^b f(x)dx = Q$, $\int_a^b g(x)dx = R$, and $\int_c^b g(x)dx = S$, then $\int_a^c (f(x) - g(x))\,dx =$

(A) $P - Q + R - S$
(B) $P - Q - R + S$
(C) $P - Q - R - S$
(D) $P + Q - R - S$
(E) $P + Q - R + S$

$\left[\int_a^b f(x)dx - \int_b^c f(x)\right] - \left[\int_a^b g(x)dx - \int_b^c g(x)dx\right]$

$(P - Q) - (R - S) = P - Q - R + S$

---

4. Let f and g be continuous functions such that $\int_0^6 f(x)dx = 9$, $\int_3^6 f(x)dx = 5$, and $\int_3^0 g(x)dx = -7$. What is the value of $\int_0^3 \left(\frac{1}{2}f(x) - 3g(x)\right)dx$?

(A) -23
(B) -19
(C) $-\frac{17}{2}$
(D) 19
(E) 23

$\frac{1}{2}\int_0^3 f(x)dx - 3\int_0^3 g(x)dx$

$\frac{1}{2}[9 - 5] - 3\left(-\int_3^0 g(x)dx\right)$

$2 + 3(-7)$

$2 - 21 = -19$

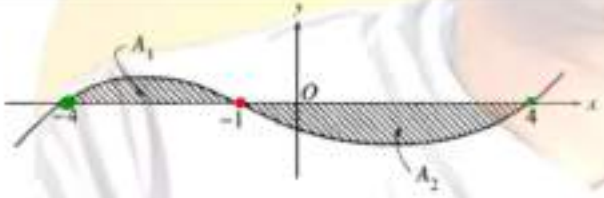
If $\int_0^{-1} f(x)dx = 2$ and $\int_0^{-3} f(x)dx = -3$, then $\int_1^4 (3f(x)+2)dx =$

(A) -13
(B) -9
(C) -7
(D) 3
(E) 21

$3\int_1^4 f(x)dx + \int_1^4 2\,dx$

$3\left[-\int_1^0 f(x)\,dx + \int_0^4 f(x)\,dx\right] + \int_1^4 2\,dx$ بالأقل

$3(-2 + -3) + 6 = -9$



The graph of $y=f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$\int_{-4}^4 f(x)dx - 2\int_{-1}^4 f(x)dx =$ $(A_1 - A_2) - 2(-A_2)$

(A) A_1
(B) A_1-A_2
(C) $2A_1-A_2$
(D) A_1+A_2
(E) A_1+2A_2

$= A_1 - A_2 + 2A_2$

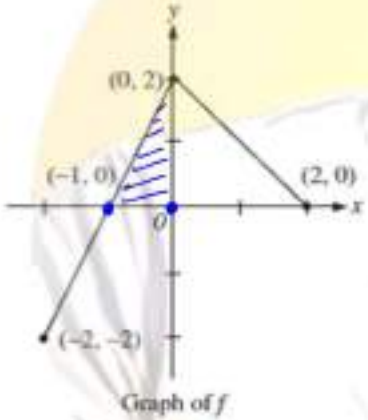
$= A_1 + A_2$

10. If $\int_0^3 f(x)dx = 6$ and $\int_3^5 f(x)dx = 4$, then $\int_0^5 (3 + 2f(x))dx =$

(A) 10
(B) 20
(C) 23
(D) 35
(E) 50

$\int_0^5 3dx + 2\int_0^5 f(x)dx$

$15 + 2[6 + 4] = 15 + 20$



Graph of f

The graph of the function f shown above consists of two line segments. If g is the function defined by $g(x) = \int_0^x f(t)dt$, then $g(-1) =$

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

$g(-1) = \int_0^{-1} f(t)dt = -\int_{-1}^{0} f(t)dt$

$-\frac{1}{2}(2)(1)$

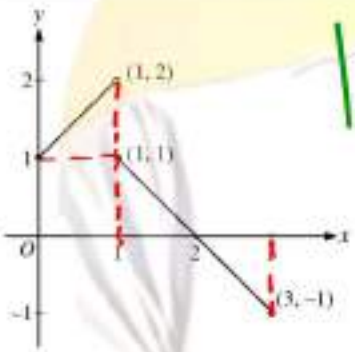
$= -1$

13. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)dx$?

(A) 2
(B) 6
(C) 8
(D) 10
(E) 12

$\int_1^3 f(x)dx + \int_3^5 f(x)\,dx$

$\int_1^3 2\,dx + \int_3^5 (x-1)\,dx$



Graph of f

$\left|\frac{1}{2}(1)(1)\right| + \left|\frac{1}{2}(1)(1)\right| + \left|-\frac{1}{2}(1)(1)\right| + |1(1)|$

$1.5 + 1 = 2.5$

$= \frac{5}{2}$

The graph of the function f consists of two line segments, as shown in the figure above. The value of $\int_0^3 |f(x)|dx$ is

(A) $-\frac{3}{2}$
(B) $\frac{1}{2}$
(C) $\frac{3}{2}$
(D) $\frac{5}{2}$
(E) nonexistent

Q5-MCQ 6-7

1. If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

(A) $f'(4)$

(B) $-7 + f'(4)$

(C) $\int_2^4 f(t)dt$

(D) $\int_2^4 (-7 + f(t))dt$

(E) $-7 + \int_2^4 f(t)dt$

Big +

$g(4) = -7 + \int_2^4 f(x)dx$

---

4. Let f be the function defined by $f(x) = \int_0^x (2t^3 - 15t^2 + 36t)\,dt$. On which of the following intervals is the graph of $y = f(x)$ concave down?

(A) $(-\infty, 0)$ only

(B) $(-\infty, 2)$

(C) $(0, \infty)$

(D) $(2, 3)$ only

(E) $(3, \infty)$ only

$f = 0$

$* f(x) = 2x^3 - 15x^2 + 36x$

$* f'(x) = 6x^2 - 30x + 36 = 0$

$x = 2 \quad , \quad x = 3$

---

5. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$

(B) -1

(C) 0

(D) 1

(E) $\sqrt{2}$

$v(t) = \cos t - -\sin t$

$= \cos t + \sin t = 0$

$* \cos t = -\sin t$

Radian

$t = \frac{3\pi}{4}$

$a(t) = -\sin t + \cos t$

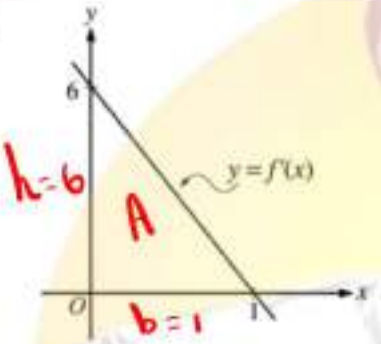
$= -\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4} = -\sqrt{2}$

6. Let f be a differentiable function such that $f(0) = -5$ and $f'(x) \leq 3$ for all x. Of the following, which is not a possible value for $f(2)$?

(A) -10
(B) -5
(C) 0
(D) 1
(E) 2

b = -5 m = 3 y = mx + b
y = 3x - 5
y(2) = 3(2) - 5 = 1
f(2) not > 1

7.



h = 6
A
b = 1
$f(1) = 5 + \int_0^1 f(x)\,dx$
b = 5
Big → +

The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0
(B) 3
(C) 6
(D) 8
(E) 11

$5 + \frac{1}{2}(1)(6) = 8$

8. $\int_0^1 \sqrt{x}(x+1)dx =$

(A) 0
(B) 1
(C) $\frac{16}{15}$ = 1.60
(D) $\frac{7}{5}$
(E) 2

9. What are all values of k for which $\int_{-3}^{k} x^2 dx = 0$?

(A) -3
(B) 0
(C) 3
(D) -3 and 3
(E) -3, 0, 3

$\frac{x^3}{3}\Big|_{-3}^{k} = 0$

$\frac{k^3}{3} - \frac{(-3)^3}{3} = 0$

$k = -3$

shift solve =

---

10. $\int_{1}^{2} \frac{x-4}{x^2} dx$ $= -1.30685$

(A) $-\frac{1}{2}$
(B) ln2-2 $= -1.30685$
(C) ln2
(D) 2
(E) ln2+2

---

11. $\int_{2}^{x} (3t^2 - 1)\, dt =$

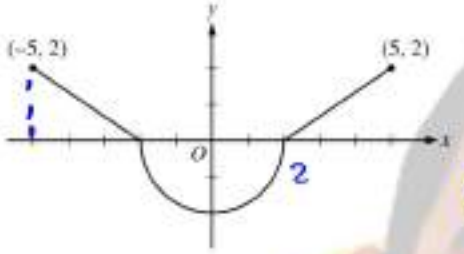
(A) $x^3 - x - 6$
(B) $x^3 - x$
(C) $3x^2 - 12$
(D) $3x^2 - 1$
(E) $6x - 12$

$\frac{3t^3}{3} - t\Big|_{2}^{x} = F(x) - F(2)$

$\left(\frac{3x^3}{3} - x\right) - \left(\frac{3(2)^3}{3} - 2\right)$

$x^3 - x - 6$

12.



Graph of f'

$F(-3) = 1 - \int_{-5}^{-3} f(x)dx$

$1 - \left[\frac{1}{2}(3)(2) - \frac{1}{2}\pi(2)^2\right]$

$1 - (3 - 2\pi)$

The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

(A) $2\pi - 2$

(B) $2\pi - 3$

(C) $2\pi - 5$

(D) $6 - 2\pi$

(E) $4 - 2\pi$

↳ Small → −

$1 - 3 + 2\pi = -2 + 2\pi$

13.

x	−4	−3	−2	−1
$f(x)$	0.75	−1.5	−2.25	−1.5
$f'(x)$	−3	−1.5	0	1.5

The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$ what is the value of $\int_{-4}^{-1} f'(x)dx$?

(A) −4.5

(B) −2.25

(C) 0

(D) 2.25

(E) 4.5

$= F(-1) - F(-4)$

$= -1.5 - 0.75 = -2.25$

16. If n is a known positive integer, for what value of k is $\int_1^k x^{n-1}dx = \frac{1}{n}$?

(A) 0
(B) $(\frac{2}{n})^{1/n}$
(C) $(\frac{2n-1}{n})^{1/n}$
(D) $2^{1/n}$
(E) 2^n

$n+n$

$\frac{x^n}{n}\Big|_1^k = \frac{1}{n}$

$\frac{x^n}{n}\Big|_1^k = \frac{1}{n} \rightarrow \frac{k^n}{n} - \frac{1^n}{n} = \frac{1}{n}$

$k^n - 1 = 1 \rightarrow k^n = 2$

$k = 2^{\frac{1}{n}}$ Flip Power

---

20. If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_1^3 f(x)dx =$

(A) $-\frac{14}{3}$
(B) -2
(C) 2
(D) 4
(E) $\frac{14}{3}$

$\int_1^3 g'(x)\,dx = g(x)\Big|_1^3 = F(3) - F(1)$

$(3^2 - 3(3) + 4) - (1^2 - 3(1) + 4)$

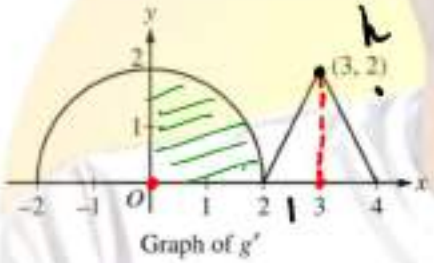
21. $f(x) = \begin{cases} x^2 & \text{for } x < 0 \\ -1 & \text{for } x = 0 \\ x & \text{for } x > 0 \end{cases}$

Let f be the function defined above. What is $\int_{-1}^{1} f(x)dx$? =

(A) $\frac{5}{6}$
(B) $\frac{2}{3}$
(C) $-\frac{1}{6}$
(D) nonexistent

$\int_{-1}^{0} x^2\,dx + \int_{0}^{1} x\,dx = \frac{5}{6}$

22.



Graph of g'

$g(3) = 1 + \int_{0}^{3} g'(x)\,dx$

$1 + \frac{1}{2}(\pi)(2)^2 + \frac{1}{2}(1)(2)$

The graph of g', the first derivative of the function g, consists of a semicircle of radius 2 and two line segments, as shown in the figure above. If $g(0) = 1$, what is $g(3)$?

(A) $\pi + 1$
(B) $\pi + 2$
(C) $2\pi + 1$
(D) $2\pi + 2$

$2 + \pi$

---

29. If the function f has a continuous derivative on [0,c] , then $\int_0^c f'(x)\,dx$=

(A) $f(c) - f(0)$
(B) $|f(c) - f(0)|$
(C) $f(c)$
(D) $f(x) + c$
(E) $f'(c) - f'(0)$

$F(c) - F(0)$

30. $\int_0^{\frac{\pi}{3}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

Radian , by Calc

$= 0.828$

(A) $-2\left(\sqrt{2}-1\right)$

(B) $-2\sqrt{2}$

(C) $2\sqrt{2}$

(D) $2\left(\sqrt{2}-1\right) = 0.828$

(E) $2\left(\sqrt{2}+1\right)$

---

31. $\int_{-2}^{1} (8x^3 - 3x^2)\,dx =$

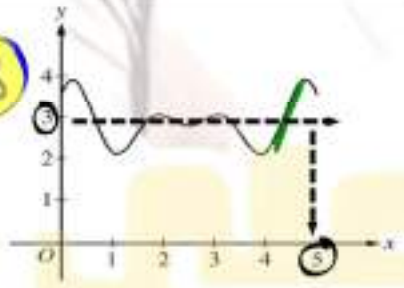
(A) -561

(B) -90

(C) -39

(D) 81

23.



Graph of f'

$f(5) = 20 + \int_0^5 f'(x)\,dx$

$20 + f'(5)$ → slope = avg

$20 + 3(5)$

The graph of f', the derivative of f, is shown in the figure above. If $f(0) = 20$ which of the following could be the value of $f(5)$?

(A) 15

(B) 20

(C) 25

(D) 35

(E) 40

$20 + 15$

35

Q6 MCQ 6-9

1. If $\int_0^k \frac{x}{x^2+4}dx = \frac{1}{2}\ln 4$, where $k > 0$, then $k =$

(A) 0
(B) $\sqrt{2}$
(C) 2
(D) $\sqrt{12}$
(E) $\frac{1}{2}\tan(\ln\sqrt{2})$

$u = x^2+9$
$du = 2x\,dx$
$dx = \frac{du}{2x}$

$\int_0^k \frac{x}{u}\cdot\frac{du}{2x}$

$\frac{1}{2}\int_0^k \frac{1}{u}du \Rightarrow \frac{1}{2}\ln u\Big|_0^k = \frac{1}{2}\ln 4$

$\ln(x^2+4)\Big|_0^k = \ln 4$

$\ln(k^2+4) - \ln(0^2+4) = \ln 4$

$\ln(k^2+4) - \ln 4 = \ln 4 \rightarrow$

shift → solve

$k \approx 3.46$

---

3. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u du$
(B) $2\int_1^4 e^u du$
(C) $2\int_1^2 e^u du$
(D) $\frac{1}{2}\int_1^2 e^u du$
(E) $\int_1^4 e^u du$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$dx = 2\sqrt{x}\,du$

$x = 1 \rightarrow u = \sqrt{1} = 1$

$x = 4 \rightarrow u = \sqrt{4} = 2$

$\int_1^2 \frac{e^u}{\sqrt{x}}\cdot 2\sqrt{x}\,du = 2\int_1^2 e^u du$

5. If $dy/dx = \tan x$, then $y =$

(A) $12\tan^2 x + C$

(B) $\sec^2 x + C$

(C) $\ln|\sec x| + C$

(D) $\ln|\cos x| + C$

(E) $\sec x \tan x + C$

$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx \quad u = \cos x$

$du = -\sin x\,dx$

$dx = \frac{du}{-\sin x}$

$\int \frac{\sin x}{u} \cdot \frac{du}{-\sin x} = -\int \frac{1}{u}\,du$

$= -\ln|u| = -\ln|\cos x| = \ln\frac{1}{\cos x} = \ln \sec x$

---

6. Which of the following definite integrals has the same value as $\int_0^4 xe^{x^2}\,dx$?

(A) $\frac{1}{2}\int_0^4 e^u\,du$

(B) $\frac{1}{2}\int_0^{16} e^u\,du$

(C) $2\int_0^2 e^u\,du$

(D) $2\int_0^4 e^u\,du$

(E) $2\int_0^{16} e^u\,du$

$u = x^2 \to du = 2x\,dx \to dx = \frac{du}{2x}$

$x = 0 \to u = 0^2 = 0$

$x = 4 \to u = 4^2 = 16$

$\int_0^{16} x\,e^u \cdot \frac{du}{2x} = \frac{1}{2}\int_0^{16} e^u\,du$

---

8. $\int_0^1 (3x-2)^2\,dx =$

(A) $-\frac{7}{3}$

(B) $-\frac{7}{9}$

(C) $\frac{1}{9}$

(D) 1

(E) 3

---

9. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

Radian by Calc

(A) $-2\left(\sqrt{2}-1\right)$

(B) $-2\sqrt{2}$

(C) $2\sqrt{2}$

(D) $2\left(\sqrt{2}-1\right)$

(E) $2\left(\sqrt{2}+1\right)$

10. $\int \cos(3x)dx =$

$\frac{\sin(3x)}{3} + C$

$\frac{1}{3}\sin(3x) + C$

(A) $-3\sin(3x) + c$
(B) $-\frac{1}{3}\sin(3x) + c$
(C) $\frac{1}{3}\sin(3x) + c$
(D) $\sin(3x) + c$
(E) $3\sin(3x) + c$

---

12. $\int (\sin(2x) + \cos(2x))dx =$

$-\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$

(A) $\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$
(B) $-\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$
(C) $2\cos(2x) + 2\sin(2x) + C$
(D) $2\cos(2x) - 2\sin(2x) + C$
(E) $-2\cos(2x) + 2\sin(2x) + C$

---

13. $\int \frac{x}{x^2-4}dx =$

$u = x^2 - 4 \rightarrow du = 2x\,dx \rightarrow dx = \frac{du}{2x}$

$\int \frac{x}{u} \cdot \frac{du}{2x}$

$\frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2 - 4| + C$

(A) $\frac{-1}{4(x^2-4)^2} + C$
(B) $\frac{1}{2(x^2-4)} + C$
(C) $\frac{1}{2}\ln|x^2-4| + C$
(D) $2\ln|x^2-4| + C$
(E) $\frac{1}{2}\arctan(\frac{x}{2}) + C$

14. Using the substitution $u = \sin(2x)$, $\int_{\pi/6}^{\pi/2} \sin^5(2x)\cos(2x)\,dx$ is equivalent to

(A) $-2\int_{1/2}^{1} u^5\,du$

(B) $\frac{1}{2}\int_{1/2}^{1} u^5\,du$

(C) $\frac{1}{2}\int_{0}^{\sqrt{3}/2} u^5\,du$

(D) $\frac{1}{2}\int_{\sqrt{3}/2}^{0} u^5\,du$

(E) $2\int_{\sqrt{3}/2}^{0} u^5\,du$

$u = \sin 2x \quad , du = 2\cos 2x\,dx$

$dx = \dfrac{du}{2\cos 2x}$

$x = \frac{\pi}{6} \rightarrow u = \sin\frac{2\pi}{6} = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{2} \rightarrow u = \sin\frac{2\pi}{2} = 0$

$$\int_{\frac{\sqrt{3}}{2}}^{0} u^5 \cdot \cancel{\cos 2x} \cdot \frac{du}{2\cancel{\cos 2x}} = \frac{1}{2}\int_{\frac{\sqrt{3}}{2}}^{0} u^5\,du$$

---

15. $\int xe^{x^2}dx =$

(A) $\frac{1}{2}e^{x^2} + C$

(B) $e^{x^2} + C$

(C) $xe^{x^2} + C$

(D) $\frac{1}{2}e^{2x} + C$

(E) $e^{2x} + C$

$u = x^2 \quad du = 2x\,dx \;, dx = \dfrac{du}{2x}$

$$\int x \cdot e^{u} \cdot \frac{du}{2x}$$

$$\frac{1}{2}\int e^{u}\,du = \frac{1}{2}e^{u} + C$$

$$\frac{1}{2}e^{x^2} + C$$

16. $\int (3x+1)^5 dx =$

(A) $\frac{(3x+1)^6}{18} + C$

(B) $\frac{(3x+1)^6}{6} + C$

(C) $\frac{(3x+1)^6}{2} + C$

(D) $\frac{\left(\frac{3x^2}{2}+x\right)^6}{2} + C$

(E) $\left(\frac{3x^2}{2}+x\right)^5 + C$

$\frac{1}{3}\frac{(3x+1)^6}{6} + C$

$\frac{(3x+1)^6}{18} + C$

---

17. $\int x^2 \cos(x^3) dx =$

(A) $-\frac{1}{3}\sin(x^3) + C$

(B) $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3) + C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

$u = x^3 \rightarrow du = 3x^2 dx$

$dx = \frac{du}{3x^2}$

$\int x^2 \cos u \cdot \frac{du}{3x^2}$

$= \frac{1}{3}\int \cos u \, du = \frac{1}{3}\sin u + C$

$\frac{1}{3}\sin(x^3) + C$

---

18. $\int_0^1 x\sqrt{1+8x^2} dx =$

(A) $\frac{1}{24}$

(B) $\frac{13}{12}$

(C) $\frac{9}{8}$

(D) $\frac{52}{3}$

(E) 18

19. Let f be a function such that $\int_6^{12} f(2x)dx = 10$. Which of the following must be true?

(A) $\int_{12}^{24} f(t)dt = 5$

(B) $\int_{12}^{24} f(t)dt = 20$

(C) $\int_6^{12} f(t)dt = 5$

(D) $\int_6^{12} f(t)dt = 20$

(E) $\int_3^6 f(t)dt = 5$

---

20. $\int_2^4 \frac{dx}{5-3x} =$

(A) $-\ln 7$

(B) $-\frac{\ln 7}{3}$

(C) $\frac{\ln 7}{3}$

(D) $\ln 7$

(E) $3\ln 7$

---

21. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x(x^2-3)^5 dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5\,du$

(B) $\int_{-2}^{13} u^5\,du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5 du$

(D) $\int_{-1}^{4} u^5 du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5 du$

40. If $\int_{1}^{2} f(x-c)dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)dx =$

(A) 5+c
(B) 5
(C) 5-c
(D) c-5
(E) -5

$u = x - c$

$du = 1\,dx$

$x = 1 \rightarrow u = 1 - c$

$x = 2 \rightarrow u = 2 - c$

$\int_{1-c}^{2-c} f(u)\,du = 5$

Q7-MCQ 6-10

1. $\int_1^e \frac{x^2+1}{x}\,dx =$ 4.19 by Calc

(A) $\frac{e^2-1}{2}$

(B) $\frac{e^2+1}{2}$ = 4.19

(C) $\frac{e^2+2}{2}$

(D) $\frac{e^2-1}{e^2}$

(E) $\frac{2e^2-8e+6}{3e}$

---

2. An antiderivative for $\frac{1}{x^2-2x+2}$ is

$\int \frac{1}{(x-1)^2+1}\,dx$ Completing the Square

$\tan^{-1}(x-1)+C$

$*\frac{b}{2} = \frac{-2}{2} = -1$

$*(x-1)^2-(-1)^2+2$

$(x-1)^2+1$

(A) $-(x^2-2x+2)^{-2}$

(B) $\ln(x^2-2x+2)$

(C) $\ln\left|\frac{x-2}{x+1}\right|$

(D) arcsec(x-1)

(E) arctan(x-1)

---

3. $\int_1^e (\frac{x^2-1}{x})dx =$ 2.18

(A) $e-\frac{1}{e}$

(B) e^2-e

(C) $\frac{e^2}{2}-e+\frac{1}{2}$

(D) e^2-2

(E) $\frac{e^2}{2}-\frac{3}{2}$ = 2.18

---

4. $\int_1^e (\frac{x^2-1}{x})dx =$ مكرر

(A) $e-\frac{1}{e}$

(B) e^2-e

(C) $\frac{e^2}{2}-e+\frac{1}{2}$

(D) e^2-2

(E) $\frac{e^2}{2}-\frac{3}{2}$

6. $\int \frac{1}{x^2+4x+5}dx =$

(A) $\arctan(x+2)+C$

(B) $\arcsin(x+2)+C$

(C) $\ln|x^2+4x+5|+C$

(D) $\frac{1}{\frac{1}{3}x^3+2x^2+5x}+C$

$b=\frac{4}{2}=2 \rightarrow (x+2)^2-2^2+5$

$(x+2)^2+1$

$\int \frac{1}{(x+2)^2+1}dx$

$\tan^{-1}(x+2)+C$

---

7. $\int \frac{6x^2-4x-25}{x-2}dx =$

(A) $3x^2+8x-9\ln|x-2|+C$

(B) $3x^2+8x+\frac{9}{(x-2)^2}+C$

(C) $(2x^3-2x^2-25x)\ln|x-2|+C$

(D) $\frac{2x^3-2x^2-25x}{\frac{x^2}{2}-2x}+C$

$\int 6x+8-\frac{9}{x-2}dx$

$x-2 \overline{)6x^2-4x-25}$ $\quad 6x+8$

$-(6x^2-12x)$

$8x-25$

$-(8x-16)$

-9

$\frac{6x^2}{2}+8x-9\ln|x-2|+C$

$=3x^2+8x-9\ln|x-2|+C$

---

8. $\int (x^3+2)^2dx =$

(A) $\frac{(x^3+2)^3}{9x^2}+C$

(B) $\frac{1}{3}(x^3+2)^3+C$

(C) $6x^2(x^3+2)+C$

(D) $\frac{1}{7}x^7+x^4+4x+C$

$\int x^6+4x^3+4dx$

$\frac{x^7}{7}+\frac{4x^4}{4}+4x+C$

---

9. $\int \frac{12x^2}{2x+1}dx =$

(A) $2x^3\ln|2x+1|+C$

(B) $4x^3\ln|2x+1|+C$

(C) $3x^2-3x+\frac{3}{2}\ln|2x+1|+C$

(D) $3x^2-3x+3\ln|2x+1|+C$

$\int 6x-3+\frac{3}{2x+1}dx$

$2x+1 \overline{)12x^2}$ $\quad 6x-3$

$-(12x^2+6x)$

$-6x$

$-(-6x-3)$

3

$\frac{6x^2}{2}-3x+\frac{3}{2}\ln|2x+1|+C$

$3x^2-3x+\frac{3}{2}\ln|2x+1|+C$

10. $\int \frac{8}{\sqrt{12-x^2-4x}}dx =$

$-(x^2+4x-12)$ $\frac{b}{2} = \frac{4}{2} = 2 \rightarrow -((x+2)^2 - 2^2 - 12)$

$-((x+2)^2 - 16)$

$\int \frac{8}{\sqrt{-(x+2)^2+16}}dx = \frac{1}{4}\int \frac{8\ dx}{\sqrt{-\left(\frac{x+2}{4}\right)^2+1}}$

(A) $16\sqrt{12-x^2-4x}+C$

(B) $2\sin^{-1}\left(\frac{x+2}{4}\right)+C$

(C) $8\sin^{-1}\left(\frac{x-2}{4}\right)+C$

(D) $8\sin^{-1}\left(\frac{x+2}{4}\right)+C$

$\frac{1}{4} \cdot 8 \sin^{-1}\left(\frac{x+2}{4}\right) \cdot 4 + C$

---

11. $\int \frac{4}{x^2+4x+8}dx =$

$\frac{b}{2} = \frac{4}{2} = 2 \rightarrow (x+2)^2 - 2^2 + 8$

$(x+2)^2 + 4$

$\frac{1}{4}\int \frac{4\ dx}{\left(\frac{x+2}{2}\right)^2+1}$

(A) $4\ln|x^2+4x+8|+C$

(B) $\tan^{-1}\left(\frac{x+2}{2}\right)+C$

(C) $4\tan^{-1}(x+2)+C$

(D) $2\tan^{-1}\left(\frac{x+2}{2}\right)+C$

$4 \cdot \frac{1}{4} \tan^{-1}\left(\frac{x+2}{2}\right) \cdot 2 = 2\tan^{-1}\left(\frac{x+2}{2}\right)+C$

---

12. $\int_{-1}^{1} \frac{3x^2+2x+1}{x+2}dx =$ 1.8875

(A) $\frac{8}{15}$

(B) 1

(C) $-8+9\ln 3$ = 1.8875

(D) $16+17\ln 3$

Q8-MCQ 6-12

1. $\int \frac{8}{x^2-4}dx =$

(A) $4\tan^{-1}\left(\frac{x}{2}\right)+c$

(B) $8\ln|x^2-4|+c$

(C) $2\ln\left|\frac{x-2}{x+2}\right|+c$

(D) $2\ln\left|\frac{x+2}{x-2}\right|+c$

(E) $2\ln|x+2|+2\ln|x-2|+c$

$* \frac{8}{(x-2)(x+2)}dx = \text{linear} \frac{A}{x-2} + \frac{B}{x+2}$

$*\text{deno}=0 \quad x=2 \;,\; x=-2$

$8 = A(x+2)+B(x-2)$ #

$* x=2 \rightarrow 8 = A(2+2)+B(2-2) \rightarrow 8=4A \rightarrow A=2$

$* x=-2 \rightarrow 8 = A(-2+2)+B(-2-2) \rightarrow 8=-4B \rightarrow B=-2$

$\int \frac{2}{x-2} + \frac{-2}{x+2}dx = 2\ln|x-2| - 2\ln|x+2| + C$

$2\ln\left|\frac{x-2}{x+2}\right|+C$

$- \rightarrow \div$
$+ \rightarrow \times$
$\Rightarrow \ln$

---

2. $\int \frac{2x}{(x+2)(x+1)}dx =$

(A) $\ln|x+2|+\ln|x+1|+C$

(B) $\ln|x+2|+\ln|x+1|-3x+C$

(C) $-4\ln|x+2|+2\ln|x+1|+C$

(D) $4\ln|x+2|-2\ln|x+1|+C$

(E) $2\ln|x|+\frac{2}{3}x+\frac{1}{2}x^2+C$

$\frac{2x}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$

$2x = A(x+1)+B(x+2)$ #

$\text{deno}=0 \quad x=-2 \;,\; x=-1$

$* x=-2 \rightarrow 2(-2) = A(-2+1)+B(-2+2)$

$-4 = -A \rightarrow \boxed{A=4}$

$* x=-1 \rightarrow 2(-1) = A(-1+1)+B(-1+2)$

$\boxed{-2 = B}$

$$\int \frac{4}{x+2} + \frac{-2}{x+1}\,dx$$

$$4\ln|x+2| - 2\ln|x+1| + C$$

---

3. For $0 < P < 100$, which of the following is an antiderivative of $\frac{1}{100P - P^2}$? $= \frac{1}{P(100-P)}$

(A) $\frac{1}{100}\ln(P) - \frac{1}{100}\ln(100 - P)$

(B) $\frac{1}{100}\ln(P) + \frac{1}{100}\ln(100 - P)$

(C) $100\ln(P) - 100\ln(100 - P)$

(D) $\ln(100P - P^2)$

(E) $\frac{1}{50P^2 - \frac{P^3}{3}}$

$$\frac{1}{100P - P^2} = \frac{A}{P} + \frac{B}{100-P}$$

$$1 = A(100-P) + BP$$

$$\text{denom} = 0 \quad P = 0 \;,\; P = 100$$

$$*P = 0 \rightarrow 1 = A(100-0) + B(0) \rightarrow 1 = 100A \rightarrow A = \frac{1}{100}$$

$$*P = 100 \rightarrow 1 = A(100-100) + 100B \rightarrow B = \frac{1}{100}$$

$$\int \frac{\frac{1}{100}}{P} + \frac{\frac{1}{100}}{100-P}\,dP$$

$$\frac{1}{100}\ln|P| - \frac{1}{100}\ln|100-P| + C$$

$$\frac{1}{100}\ln\left|\frac{P}{100-P}\right| + C$$

7. $\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$ is $= 2.89$

(A) $\ln(8)$

(B) $\ln\left(\frac{27}{2}\right)$

(C) $\ln(18) \approx 2.89$

(D) $\ln(288)$

(E) divergent

$$\frac{5x+8}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$5x+8 = A(x+2) + B(x+1)$$

$den = 0 \quad x = -1 \;,\; x = -2$

* $x = -1 \rightarrow 5(-1)+8 = A(-1+2) + B(-1+1)$

$$\boxed{3 = A}$$

* $x = -2 \rightarrow 5(-2)+8 = A(-2+2) + B(-2+1)$

$$-2 = -B \rightarrow \boxed{B = 2}$$

$$\int \frac{3}{x+1} + \frac{2}{x+2}\,dx$$

$$\left(3\ln|x+1| + 2\ln|x+2|\right)\Big|_0^1$$

14. $\int \frac{-2x^2+7x-8}{(x+2)(2x-1)(1-x)}dx =$

(A) $-4\ln|x+2|+\ln|2x-1|+\ln|1-x|+C$

(B) $2\ln|x+2|-2\ln|2x-1|-\ln|1-x|+C$

(C) $2\ln|x+2|-2\ln|2x-1|+\ln|1-x|+C$

(D) $2\ln|x+2|-4\ln|2x-1|-\ln|1-x|+C$

$\frac{A}{x+2}+\frac{B}{2x-1}+\frac{C}{1-x}$

$-2x^2+7x-8 = A(2x-1)(1-x)+B(x+2)(1-x)$

$+C(x+2)(2x-1)$

* $x=-2$, $x=\frac{1}{2}$, $x=1$

* $x=-2 \Rightarrow -2(-2)^2+7(-2)-8 = A(2(-2)-1)(1+2)$

$+B(-2+2)(1+2)+C(-2+2)(2(-2)-1)$

$A=2$

$B=-4$

$C=-1$

$\int \frac{2}{x+2}-\frac{4}{2x-1}-\frac{1}{1-x}dx$

$2\ln|x+2|-\frac{4}{2}\ln|2x-1|-\ln|1-x|+C$

$2\ln|x+2|-2\ln|2x-1|+\ln|1-x|+C$

Q9-MCQ 7-1

1. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$
(B) $\frac{dP}{dt} = 200t$
(C) $\frac{dP}{dt} = 100t^2$
(D) $\frac{dP}{dt} = 200P$
(E) $\frac{dP}{dt} = 100P^2$

$P(t) = at$

$\frac{dP}{dt} = a$

---

2. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$
(B) $V(t) = k\sqrt{V}$
(C) $\frac{dV}{dt} = k\sqrt{t}$
(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$
(E) $\frac{dV}{dt} = k\sqrt{V}$

$\dot{V}(t) = k \cdot \sqrt{V}$

---

4. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A) $\frac{dp}{dt} = kp$
(B) $\frac{dp}{dt} = kp(N - p)$
(C) $\frac{dp}{dt} = kp(p - N)$
(D) $\frac{dp}{dt} = kt(N - t)$
(E) $\frac{dp}{dt} = kt(t - N)$

* heard $= P$
* N heard $= N - P$
$kP(N-P)$

5. A rectangle has length x, width y, and area A. The instantaneous rate of change of the area with respect to the width is equal to the sum of the length and twice the width. Which of the following equations describes this relationship?

(A) $A = x + 2y$

(B) $\frac{dy}{dx} = x + 2y$

(C) $\frac{dA}{dx} = x + 2y$

(D) $\frac{dA}{dy} = x + 2y$

$\ell = x$, $w = y$ A

$\frac{dA}{dy} = x + 2y$

---

6. A population described by a function P at time t decreases at a rate proportional to P. Which of the following differential equations could describe the rate of change of the population?

(A) $\frac{dP}{dt} = -0.015P$

(B) $\frac{dP}{dt} = -\frac{0.015}{P}$

(C) $\frac{dP}{dt} = -0.02t$

(D) $\frac{dP}{dt} = 6e^{-0.02t}$

$\frac{dp}{dt} = -mp \rightarrow$ direct

---

10. The weight of a population of yeast is given by a differentiable function, W, where $W(t)$ is measured in grams and t is measured in hours. The weight of the yeast population increases with respect to t at a rate that is directly proportional to the weight. At time $t = 10$ hours, the weight of the yeast is 200 grams and is increasing at the rate of 5 grams per hour. Which of the following is a differential equation that models this situation?

(A) $W = 5(t - 10) + 200$

(B) $\frac{dW}{dt} = \frac{1}{2}t$

(C) $\frac{dW}{dt} = \frac{1}{40}W$

(D) $\frac{dW}{dt} = \frac{1}{2}W$

$\frac{dw}{dt} = kw$, $w = 200$, $w' = 5$

$5 = 200k \rightarrow k = \frac{5}{200} = \frac{1}{40}$

---

11. If the pressure P applied to a gas is increased while the gas is held at a constant temperature, then the volume V of the gas will decrease. The rate of change of the volume of gas with respect to the pressure is proportional to the reciprocal of the square of the pressure. Which of the following is a differential equation that could describe this relationship?

Tip

(A) $V = \frac{k}{P^2}$, where k is a positive constant.

(B) $\frac{dV}{dP} = \frac{k}{P^2}$, where k is a positive constant.

(C) $\frac{dV}{dP} = \frac{k}{P^2}$, where k is a negative constant.

(D) $\frac{dP}{dV} = \frac{k}{P^2}$, where k is a negative constant.

$\frac{dV}{dP} = \frac{-k}{P^2}$

12. The rate at which a quantity M of a certain radioactive substance decays is proportional to the amount of the substance present at a given time. Which of the following is a differential equation that could describe this relationship?

(A) $\frac{dM}{dt} = -3.7t^2$

(B) $\frac{dM}{dt} = -0.11M$

(C) $\frac{dM}{dt} = 0.08t^2$

(D) $\frac{dM}{dt} = 1.2M$

$\frac{dM}{dt} = -kM$

Q10-MCQ 7-2

$e^y \cdot e^x = e^{y+x}$

1. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln(-e^x + 5)$

(D) $y = -\ln(e^x + 3)$

(E) $y = \ln(e^x + 3)$

$\int \frac{dy}{e^y} = \int e^x dx$ $\quad -e^{-y} = e^x + C$

$\int e^{y} = e^x + C$ $\quad \frac{-1}{e^y} = e^x + C$

$\frac{1}{e^y} = -e^x + C \rightarrow e^{-y} = -e^x + C$

$\ln e^{-y} = \ln(-e^x + C) \rightarrow -y = \ln(-e^x + C)$

$y = -\ln(-e^x + C)$

2. A curve has slope $2x + 3$ at each point (x,y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1,2)$?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

x y

$\frac{dy}{dx} = 2x + 3 \rightarrow \int dy = \int (2x + 3)dx$

$y = \frac{2x^2}{2} + 3x + C$ $\quad C = 2 - 4 = -2$

$y = x^2 + 3x + C$

$2 = 1 + 3 + C$

3. Which of the following is a solution to the differential equation $y'' - 4y = 0$?

(A) $y = e^{2x}$
(B) $y = 2e^{x}$
(C) $y = sin(2x)$
(D) $y = cos(2x)$

$y' = 2e^{2x}$ $y'' = 4e^{2x}$

$4e^{2x} - 4 \cdot e^{2x} = 0$

---

4. The function $y = e^{3x} - 5x + 7$ is a solution to which of the following differential equations?

(A) $y'' - 3y' - 15 = 0$
(B) $y'' - 3y' + 15 = 0$
(C) $y'' - y' - 5 = 0$
(D) $y'' - y' + 5 = 0$

$y' = 3e^{3x} - 5$ $y'' = 9e^{3x}$

$9e^{3x} - 9e^{3x} + 15 - 15 = 0$

---

5. If $dy/dx = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$
(B) $e^{\tan x} + 5$
(C) $5e^{\tan x}$
(D) $\tan x + 5$
(E) $\tan x + 5e^{x}$

$\frac{dy}{dx} = y \cdot \sec^2 x$

$\int \frac{dy}{y} = \int \sec^2 x \, dx$

$\ln y = \tan x + c$

$y = ce^{\tan x}$ $\rightarrow (0, 5)$

$5 = ce^{\tan 0}$ $\rightarrow c = 5$

$y = 5e^{\tan x}$

6. For what value of k, if any, will $y = k\sin(5x) + 2\cos(4x)$ be a solution to the differential equation $y'' + 16y = -27\sin(5x)$?

(A) -27

(B) $-\frac{9}{5}$

(C) 3

(D) There is no such value of k.

$y' = 5k\cos 5x - 8\sin 4x$

$y'' = -25k\sin 5x - 32\cos 4x$

$-25k\sin 5x - 32\cos 4x + 16k\sin 5x + 32\cos 4x = -27\sin 5x$

$-9k\sin 5x = -27\sin 5x$

$k = \frac{27}{9} = \boxed{3}$

---

7. For what value of k, if any, is $y = e^{2x} + ke^{-3x}$ a solution to the differential equation $4y - y'' = 10e^{-3x}$?

(A) -2

(B) $\frac{10}{3}$

(C) 10

(D) There is no such value of k.

$y' = 2e^{2x} - 3ke^{-3x}$

$y'' = 4e^{2x} + 9ke^{-3x}$

$4e^{2x} + 4ke^{-3x} - 4e^{2x} - 9ke^{-3x} = 10e^{-3x}$

$-5ke^{-3x} = 10e^{-3x}$

$k = \frac{10}{-5} = -2$

10. Of the following, which are solutions to the differential equation $y'' - 6y' + 8y = 0$?

I. $y = 2\sin(4x)$

II. $y = 3e^{2x}$

III. $y = Ce^{4x}$, where C is a constant.

(A) I only

(B) II only

(C) III only

(D) II and III only

* II $y' = 6e^{2x}$ $y'' = 12e^{2x}$

$12e^{2x} - 36e^{2x} + 24e^{2x} = 0$ ✓

* III $y' = 4Ce^{4x}$ $y'' = 16Ce^{4x}$

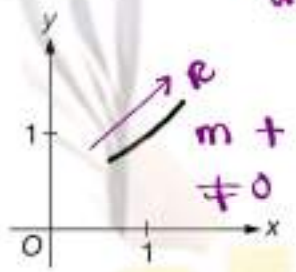
$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 0$ ✓

---

12. Which of the following could be the graph of a solution of the differential equation $\frac{dy}{dx} = (x-1)y^2$ near the point $(1, 1)$?

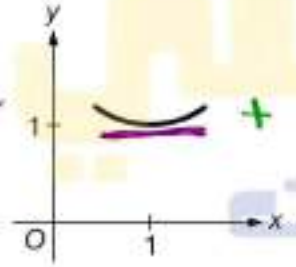
$\frac{dy}{dx}(1,1) = (1-1)1^2 = 0$ Horizontal ↔

(A)



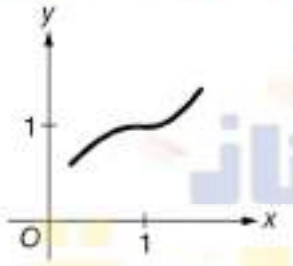
m + ≠ 0

(B)

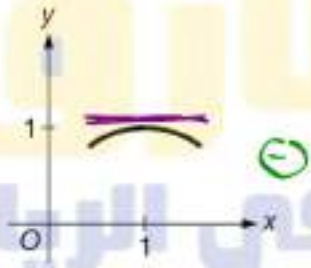


+

(C)



(D)



−

13. Which of the following is a solution to the differential equation $\frac{dy}{dx} = e^{2x} + 2y$?

(A) $y = e^{2x}$

(B) $y = -e^{2x}$

(C) $y = xe^{2x}$

(D) $y = -xe^{2x}$

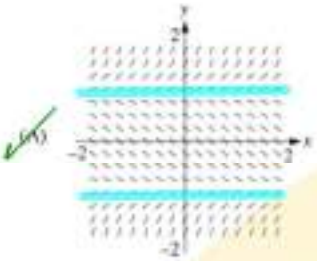
$y = x \cdot e^{2x}$

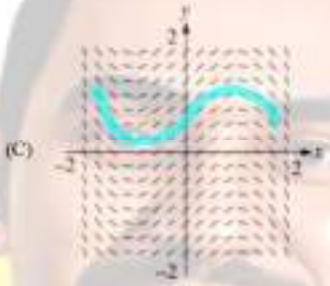
$\frac{dy}{dx} = 1 \cdot e^{2x} + x \cdot 2e^{2x}$

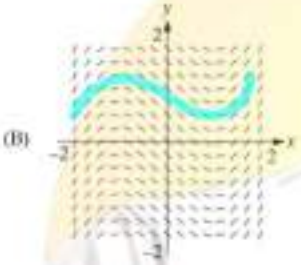
$e^{2x} + 2xe^{2x} = e^{2x} + 2 \cdot xe^{2x}$ ✓ R.S = L.S

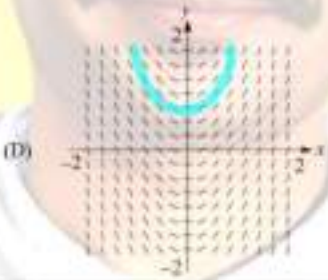
Q11-MCQ 7-3

1. Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?

(A) 

(C) 

(B) 

(D) 

$y' = 0$ Horizontal line

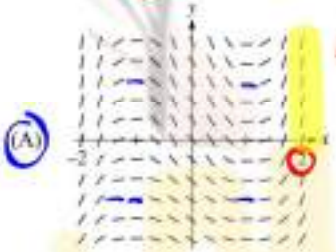
$y^2 - 1 = 0$

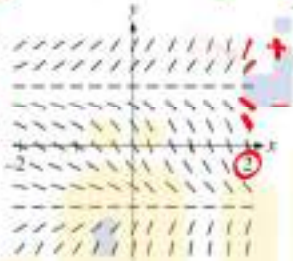
$y^2 = 1$

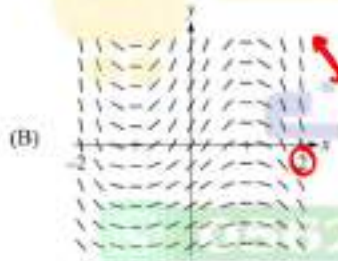
$y = \pm 1$

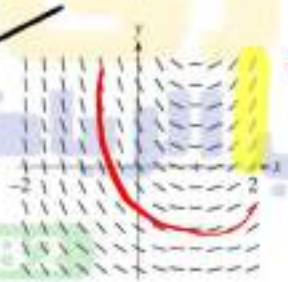
---

2. Let g be a function such that $g(y) > 0$ for all y. Which of the following could be a slope field for the differential equation $\frac{dy}{dx} = (x^2 - 1)g(y)$?

(A) 

(C) 

(B) 

(D) 

$y' = 0$

$x^2 - 1 = 0$

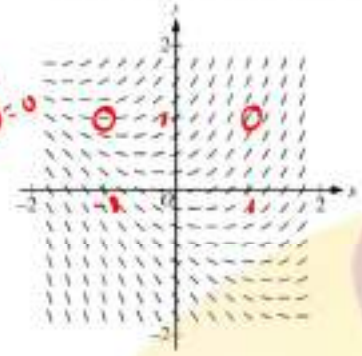
$x = \pm 1$

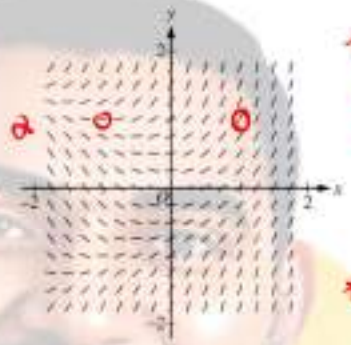
$g(y)$ (+)

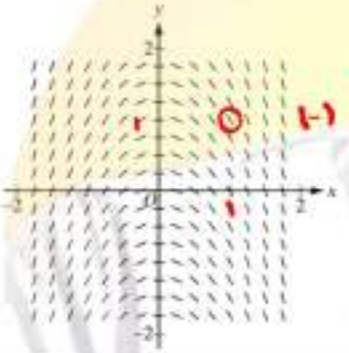
$x = 2$

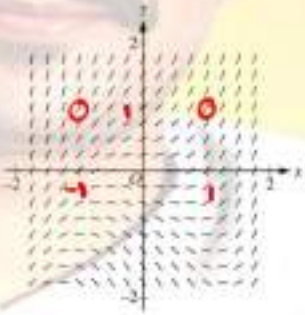
$(2^2 - 1)(+) = +3$

3. Which of the following could be a slope field for the differential equation $\frac{dy}{dx} = x^2 + y$?

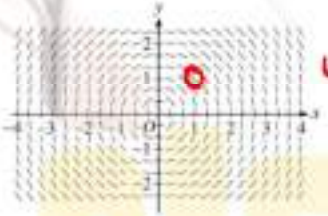
(A) 

(C) 

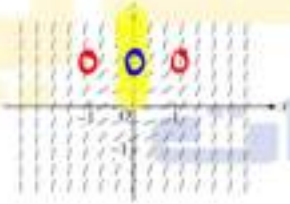
(B) 

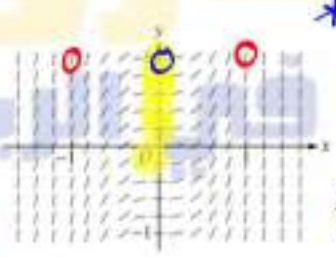
(D) 

4. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = x^2 + y^2$?

(A) 

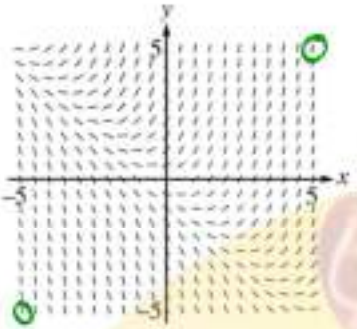
(C) 

(B) 

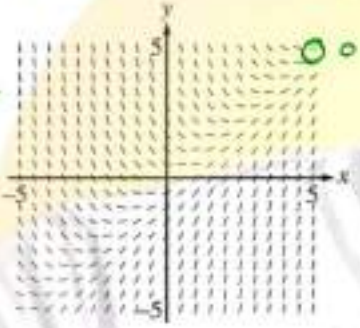
(D) 

5. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

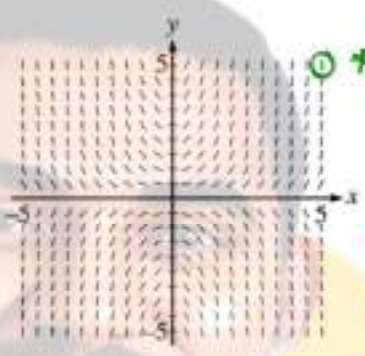
(A)



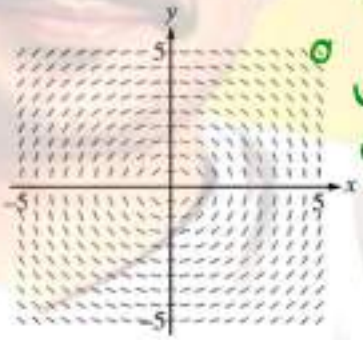
(B)



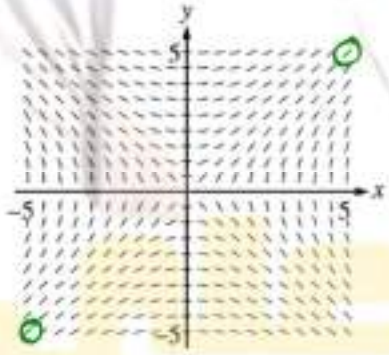
(C)



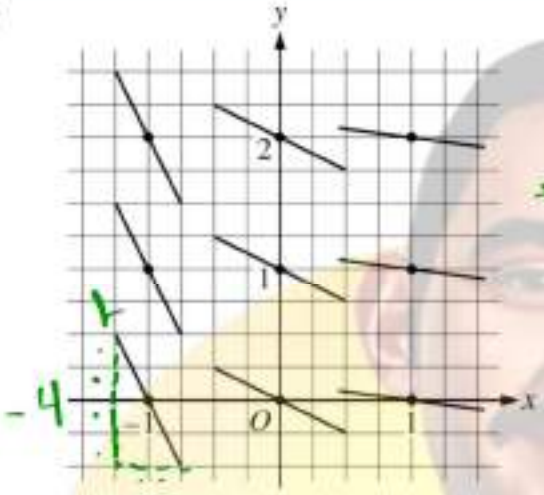
(D)



(E)



6.



$* f(-1) = m = \frac{y}{x} = \frac{\text{rise}}{\text{run}}$

$= \frac{-4}{2}$

$= \boxed{-2}$

$y = f(-1) = 2$

A slope field for a differential equation is shown in the figure above. If $y = f(x)$ is the particular solution to the differential equation through the point $(-1,2)$ and $h(x) = 3x \cdot f(x)$, then $h'(-1) =$

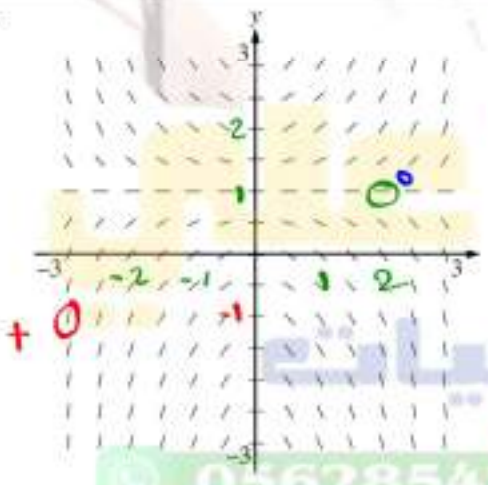
(A) -6
(B) -2
(C) 0
(D) 1
(E) 12

$h'(-1) = 3 \cdot f(x) + 3x \cdot f'(x)$

$= 3 \cdot f(-1) + 3(-1) \cdot f'(-1)$

$= 3 \cdot 2 \quad -3 \cdot (-2) = 12$

7.



$* (2,1) \Rightarrow y' = 0$

(A) $\frac{dy}{dx} = xy - x = 2(1) - 2 = 0$ ✓

(B) $\frac{dy}{dx} = xy + x = 2(1) + 2 = 4$ ✗

(C) $\frac{dy}{dx} = y - x^2 = 1 - 2^2 = -3$ ✗

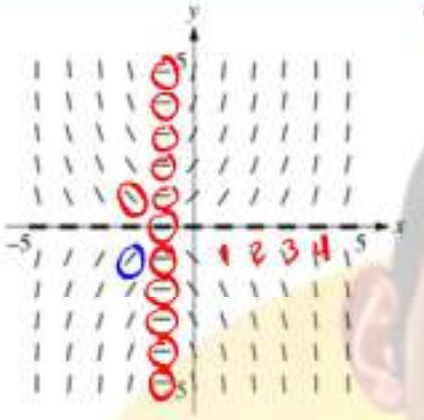
(D) $\frac{dy}{dx} = (y-1)x^2 = (1-1)(2)^2 = 0$

(E) $\frac{dy}{dx} = (y-1)^3 = (1-1)^3 = 0$

$* (-3,-1)$ A → +
D → −
E → −

Shown above is a slope field for which of the following differential equations?

8.



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$

(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$

(E) $\frac{dy}{dx} = (x+1)^3$

* $(-1,1) \rightarrow y' = 0$

$= -1(1) = -$

$= (-1)(1) - 1 = -$

$= (-1)(1) + 1 = 0$

$= (-1)(1) - 1 = -$

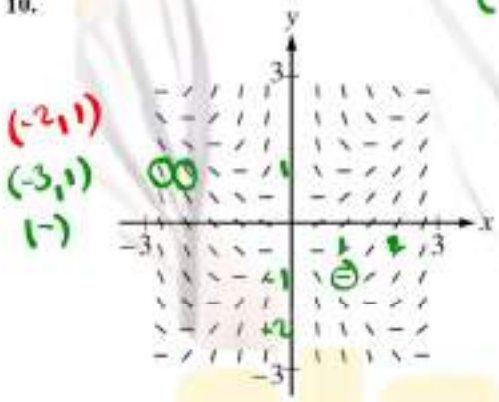
$= (-1+1)^3 = 0$

* $(-2,-1) \rightarrow y' = (+)$

C $\rightarrow -2(-1) - 1 = +$ ✓

E $\rightarrow (-2+1)^3 = (-)$

10.



Shown above is a slope field for which of the following differential equations?

$(1,-1) \rightarrow y' = 0$, $(-2,1) \rightarrow (-)$

(A) $\frac{dy}{dx} = \frac{x^2 - y^2}{x}$ $= \frac{1-(-1)^2}{1} = 0$

(B) $\frac{dy}{dx} = \frac{x^2 - y^2}{y}$ $= \frac{1-(-1)^2}{-1} = 0$

(C) $\frac{dy}{dx} = x^2 - y^2$ $= 1^2 - (-1)^2 = 0$

(D) $\frac{dy}{dx} = \frac{x^2 + y^2}{y}$ $= \frac{1+(-1)^2}{-1} = -2$ ✗

(E) $\frac{dy}{dx} = x^2 + y^2$ $= 1^2 + (-1)^2 = 2$ ✓

A person stands 30 feet from point P and watches a balloon rise vertically from the point, as shown in the figure above. The balloon is rising at a constant rate of 2 feet per second. What is the rate of change, in radians per second, of angle θ at the instant when the balloon is 40 feet above point P?

(A) $\frac{3}{100}$
(B) $\frac{3}{125}$
(C) $\frac{1}{12}$
(D) $\frac{5}{27}$

$\dot{y} = \frac{dy}{dt} = 2$ $\quad \tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\frac{y}{x}$ $\quad y \to \dot{y}$ $\quad x \to \dot{x}$

$\frac{d\theta}{dt} = \frac{u'}{1+u^2}$

$= \frac{\frac{\dot{y}\cdot x - \dot{x}\, y}{x^2}}{1 + \left(\frac{y}{x}\right)^2} = \frac{\frac{2(30) - 0(40)}{30^2}}{1 + \left(\frac{40}{30}\right)^2}$

rise
$y = 40$
opp

θ 30 P
x adj

$x = 30$
$y = 40$
$\dot{y} = 2$
$\dot{x} = 0$

A portion of the slope field for the differential equation $\frac{dy}{dx} = x - y$ is shown in the figure. If the linear function $y = f(x)$ is the solution to the differential equation such that $f(-1) = -2$, then $f(3) =$

(A) 0
(B) 1
(C) 2
(D) 3

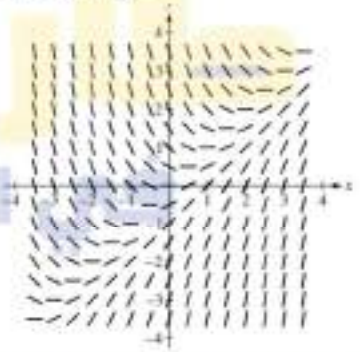
$(-1, -2)$

$\dot{y} = -1 - -2 = 1$ (+)

$1 = x - y$ $\quad f(3)$ $\quad x = 3$

$1 = 3 - y$

$y = 3 - 1 = \boxed{2}$



Q12-MCQ 7-5

1. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x + y$ with initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length, starting at $x = 1$

(A) 0
(B) 1
(C) 2.75
(D) 3
(E) 6

$\Delta x = \frac{b-a}{n} = \frac{2-1}{2} = 0.5$

x	1	1.5	2
old $y = y_1$	0	1	3
$m = \frac{dy}{dx}$	$2(1)+0 = 2$	$2(1.5)+1 = 4$	
new y $= m \cdot \Delta x + y_1$	$2(0.5)+0 = 1$	$4(0.5)+1 = 3$	

2. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

(A) $-\frac{5}{4}$
(B) 1
(C) $\frac{7}{4}$
(D) 2
(E) $\frac{21}{4}$

$\Delta x = \frac{2-1}{2} = 0.5$

x	1	1.5	2
old y y_1	3	2	$1.75 = \frac{7}{4}$
m	$1-3 = -2$	$1.5-2 = -0.5$	
new y $m\Delta x + y_1$	$-2(0.5)+3 = 2$	$-0.5(0.5)+2 = 1.75$	

3.

x	$f'(x)$
1	0.2
1.5	0.5
2	0.9

m

old $y = y_1$ | new $y = m \cdot \Delta x + y_1$

4 | $0.2(0.5) + 4 = 4.1$

4.1 | $0.5(0.5) + 4.1 = 4.35$

4.35

The table above gives values of f', the derivative of a function f. If $f(1) = 4$, what is the approximation to $f(2)$ obtained by using Euler's method with a step size of 0.5 ?

$\Delta x = 0.5$

(A) 2.35
(B) 3.65
(C) 4.35
(D) 4.70
(E) 4.80

10.

x	2	2.2	2.4
$f'(x)$	−0.5	−0.3	−0.1

$\Delta x = \frac{b-a}{2} = \frac{2.4-2}{2} = 0.2$

Let y = f(x) be the solution to the differential equation $\frac{dy}{dx} = f'(x)$ with initial condition f(2) = 3. Selected values of f' are given in the table above. What is the approximation for f(2.4) if Euler's method is used, starting at x = 2 with two steps of equal size?

(A) 2.80
(B) 2.82
(C) 2.84
(D) 2.92
(E) 3.16

old $y = y_1$ | new $y = m \cdot \Delta x + y_1$

3 | $-0.5(0.2) + 3 = 2.9$

2.9 | $-0.3(0.2) + 2.9 = 2.84$

2.84

A slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown. The point $(2, -1)$ is indicated in the figure. Let $y = f(x)$ be the solution to the differential equation with initial condition $f(2) = -1$. If $f(3)$ is approximated by using Euler's method with two steps of equal size starting at $x = 2$, which of the following statements is true?

(A) $f(3) \approx \frac{1}{2}$, and this value is an underapproximation of $f(3)$.

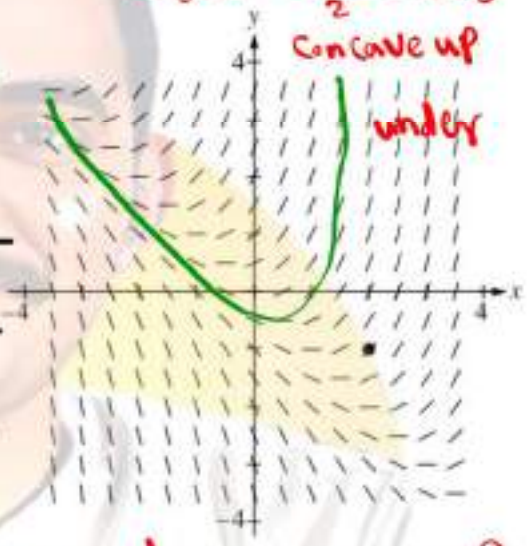
(B) $f(3) \approx \frac{1}{2}$, and this value is an overapproximation of $f(3)$.

(C) $f(3) \approx 0$, and this value is an underapproximation of $f(3)$.

(D) $f(3) \approx 0$, and this value is an overapproximation of $f(3)$.

$\Delta x = \frac{3-2}{2} = 0.5$

x	2	2.5	3
y_1	-1	-0.5	0.5
m	$2 - 1 = 1$	$2.5 - 0.5 = 2$	
y	$1(0.5) - 1 = -0.5$	$2(0.5) - 0.5 = 0.5$	



Concave up

under

Concave down → over ∩

Q13-MCQ 7-7

1. A particle moves along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the particle is at position $x = 7$ at $t = 1$ time, what is the position of the particle at time $t = 2$?

(A) -10

(B) -5

(C) -3

(D) 3

(E) 17

$x(t) = \int v(t)\,dt = \int 4 - 6t^2\,dt$

$x(t) = 4t - \frac{6t^3}{3} + C$

$x(t) = 4t - 2t^3 + C$

$x = 7$, $t = 1$

$7 = 4(1) - 2(1)^3 + C$

$C = 5$

$x(2) = 4(2) - 2(2)^3 + 5$

$= -3$

2. If $dy/dx = y\sec^2 x$ and $y=5$ when $x=0$, then $y=$

(A) $e^{\tan x}+4$
(B) $e^{\tan x}+5$
(C) $5e^{\tan x}$
(D) $\tan x+5$
(E) $\tan x+5e^x$

$\int \frac{dy}{y} = \int \sec^2 x\,dx$

$\ln y = \tan x + C$

$y = Ce^{\tan x}$ $\Rightarrow$ $5 = Ce^{\tan 0}$ $C=5$

$y = 5e^{\tan x}$

---

4. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$, where $y(2) = -2$?

(A) $y = 2x$ for $x > 0$
(B) $y = 2x - 6$ for $x \neq 3$
(C) $y = -\sqrt{4x^2 - 12}$ for $x > \sqrt{3}$
(D) $y = \sqrt{4x^2 - 12}$ for $x > \sqrt{3}$
(E) $y = -\sqrt{4x^2 - 6}$ for $x > \sqrt{1.5}$

$\int y\,dy = \int 4x\,dx$

$\frac{y^2}{2} = \frac{4x^2}{2} + C$

$y^2 = 4x^2 + C$ $\rightarrow$ $y = \pm\sqrt{4x^2 + C}$

Negative

$y = -\sqrt{4x^2 + C}$

$(2,-2)$

$+2 = +\sqrt{4(2)^2 + C}$ $\rightarrow$ $2 = \sqrt{16 + C}$

$C = -12$

$y = -\sqrt{4x^2 - 12}$

5. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

(A) $y = 2e^{-9+x^3/3}$

(B) $y = -2e^{-9+x^3/3}$

(C) $y = \sqrt{\frac{2x^3}{3}}$

(D) $y = \sqrt{\frac{2x^3}{3} - 14}$

(E) $y = -\sqrt{\frac{2x^3}{3} - 14}$

$\int y\,dy = \int x^2\,dx$ (−)

$\frac{y^2}{2} = \frac{x^3}{3} + C$

$y^2 = \frac{2}{3}x^3 + C \rightarrow y = \pm\sqrt{\frac{2}{3}x^3 + C}$

$y = -\sqrt{\frac{2}{3}x^3 + C}$ (3, −2)

$-2 = -\sqrt{\frac{2}{3}(27) + C}$ $C = -14$

$y = -\sqrt{\frac{2}{3}x^3 - 14}$

---

7. What is the particular solution to the differential equation $\frac{dy}{dx} = xy^2$ with the initial condition $y(2) = 1$?

(A) $y = e^{\frac{x^2}{2}-2}$

(B) $y = e^{\frac{x^2}{2}}$

(C) $y = -\frac{2}{x^2}$

(D) $y = \frac{2}{6-x^2}$

(E) $y = \frac{6-x^2}{2}$

$\int \frac{dy}{y^2} = \int x\,dx$

$\int y^{-2}\,dy = \frac{x^2}{2} + C$

$\frac{y^{-1}}{-1} = \frac{x^2}{2} + C \rightarrow \frac{1}{y} = -\frac{x^2}{2} + C$

$y = \frac{1}{-\frac{x^2}{2} + C}$ $(2,1)$

$1 = \frac{1}{-\frac{4}{2} + C}$ $\boxed{C = 3}$

$y = \frac{1}{-\frac{x^2}{2} + 3} = \frac{1}{\frac{-x^2+6}{2}}$

$y = \frac{2}{-x^2+6}$

---

6. A student attempted to solve the differential equation $\frac{dy}{dx} = xy$ with initial condition $y = 2$ when $x = 0$. In which step, if any, does an error first appear?

Step 1: $\int \frac{1}{y} dy = \int x dx$ ✓

Step 2: $\ln|y| = \frac{x^2}{2} + C$ ✓

Step 3: $|y| = e^{x^2/2} + C$ ✗

Step 4: Since $y = 2$ when $x = 0$, $2 = e^0 + C$.

Step 5: $y = e^{x^2/2} + 1$

$\int \frac{dy}{y} = \int x\,dx$

$\ln y = \frac{x^2}{2} + C$

$y = C\, e^{\frac{x^2}{2}}$

(A) Step 2

(B) Step 3

(C) Step 4

(D) Step 5

(E) There is no error in the solution.

9. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$
(B) $y = x - \ln 4$
(C) $y = -\ln(-e^x + 5)$
(D) $y = -\ln(e^x + 3)$
(E) $y = \ln(e^x + 3)$

$\frac{dy}{dx} = e^y \cdot e^x$

$\int \frac{dy}{e^y} = \int e^x \, dx$

$\int e^{-y} dy = e^x + C$

$\frac{e^{-y}}{-1} = e^x + C \rightarrow e^{-y} = -e^x + C$

$-y = \ln(-e^x + C)$

$y = -\ln(-e^x + C)$ $(0, -\ln 4)$

$+\ln 4 = +\ln(-e^0 + C)$

$4 = -1 + C$ $C = 4 + 1 = 5$

$y = -\ln(-e^x + 5)$

25. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2y+1}$ with the initial condition $y(0) = 0$. Which of the following gives an expression for $f(x)$ and the domain for which the solution is valid?

(A) $y = \frac{e^{2x}-1}{2}$ for all x

(B) $y = \tan x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(C) $y = \sqrt{x}$ for $x \geq 0$

(D) $y = \frac{-1+\sqrt{1+4x}}{2}$ for $x > -\frac{1}{4}$

$\int (2y+1)\,dy = \int 1\,dx$

$y^2 + y = x + C$

$(0,0) \rightarrow C = 0$

$y^2 + y = x$

$y^2 + y - x = 0$

$a = 1, b = 1, c = -x$

$y = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1^2-4(1)(-x)}}{2(1)}$

$y = \frac{-1 \pm \sqrt{1+4x}}{2}$

$y(0) = 0$ +

$y = \frac{-1+\sqrt{1+4x}}{2}$

domain $1+4x > 0$

$4x > -1$

$x > -\frac{1}{4}$

Q14-MCQ 8-1

1. The rate at which water is sprayed on a field of vegetables is given by $R(t) = 2\sqrt{1+5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \le t \le 4$, what is the average rate of water flow, in gallons per minute?

(A) 8.458
(B) 13.395
(C) 14.691
(D) 18.916
(E) 35.833

$\frac{1}{b-a}\int_a^b f(t)\,dt$

$\frac{1}{4-0}\int_0^4 2\sqrt{1+5t^3}\,dt$

2. If f is continuous on the interval $[a,b]$, then there exists c such that a and $\int_a^b f(x)dx =$

(A) $\frac{f(c)}{b-a}$
(B) $\frac{f(b)-f(a)}{b-a}$
(C) $f(b)-f(a)$
(D) $f'(c)(b-a)$
(E) $f(c)(b-a)$

avg $= \bar{F}(c)$

$\frac{F(b)-F(a)}{b-a} = \bar{F}(c) \rightarrow \frac{\int_a^b f(x)dx}{b-a} = F(c)$

$\int_a^b F(t)\,dx = F(c)\cdot(b-a)$

3. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

(A) $A_1 - A_2 = 0$

(B) $A = 0$

(C)

(D)

(E) $A = -$ not 0

4. If the average value of the function f over the closed interval $[2, 4]$ is 3 and if $f(x) \geq 0$ for all x in $[2, 4]$, what is the area of the region enclosed by the graph of $y = f(x)$, the lines $x = 2$ and $x = 4$, and the x-axis?

(A) 12
(B) 6
(C) 3
(D) 3/2

$$\text{avg value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$3 = \frac{1}{4-2}\int_2^4 f(x)\,dx$$

$$3 = \frac{1}{2}\cdot \int_2^4 f(x)\,dx = 6$$

---

5. The average value of 1/x on the closed interval [1,3] is

(A) 12
(B) 23
(C) ln2/2
(D) ln3/2
(E) ln3

$$\frac{1}{3-1}\int_1^3 \frac{1}{x}\,dx \quad \# \quad \text{دكتور}$$

$$\frac{1}{2}\cdot \ln x \Big|_1^3 = \frac{1}{2}(F(3) - F(1))$$

$$\frac{1}{2}(\ln 3 - \ln 1) = \frac{1}{2}\ln 3$$

$$= \frac{\ln 3}{2}$$

---

6. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval [0,2] ?

(A) $\frac{26}{9}$
(B) $\frac{52}{9}$
(C) $\frac{26}{3}$
(D) $\frac{52}{3}$
(E) 24

$$\frac{1}{2-0}\int_0^2 x^2\sqrt{x^3+1}\,dx$$

7. What is the average value of y for the part of the curve $y = 3x - x^2$, which is the *first quadrant*?

(A) −6
(B) −2
(C) $\frac{3}{2}$
(D) $\frac{9}{4}$
(E) $\frac{9}{2}$

$* a, b \rightarrow y = 0 \quad -x^2 + 3x = 0$

$x = 0 \quad , \quad x = 3$

$x(-x + 3) = 0$

$x = 0 \quad , \quad x = 3$

$* \frac{1}{3-0} \int_0^3 3x - x^2 \, dx$

---

8. Let f be the function defined by $f(x) = \frac{1}{x}$. What is the average value of f on the interval [4, 6] ?

(A) -1/24
(B) 5/24
(C) (1/2) ln (3/2)
(D) ln (3/2)
(E) (1/2) ln 2

$\frac{1}{6-4} \int_4^6 \frac{1}{x} \, dx$

$\frac{1}{2} \ln x \Big|_4^6 = \frac{1}{2} [\ln 6 - \ln 4]$

$= \frac{1}{2} \cdot (\ln \frac{6}{4}) = \frac{1}{2} \ln \frac{3}{2}$

---

10. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \le t \le 2$?

(A) 11/4
(B) 7/2
(C) 8
(D) 33/4
(E) 16

$\frac{1}{2 - -1} \int_{-1}^{2} 3t^3 - t^2 \, dt$

$\frac{1}{3} \int_{-1}^{2} 3t^3 - t^2 \, dt$

12. If the average value of a continuous function f on the interval $[-2, 4]$ is 12, what is $\int_{-2}^{4} \frac{f(x)}{8}\,dx$?

(A) $\frac{3}{2}$
(B) 3
(C) 9
(D) 72

$12 = \frac{1}{4-(-2)} \int_{-2}^{4} f(x)\,dx$

$12 = \frac{1}{6} \int_{-2}^{4} f(x)\,dx$

$* \int_{-2}^{4} f(x)\,dx = 72$

$\frac{72}{8} = 9$

---

15. Let f be a continuous function for all real numbers. Let g be the function defined by $g(x) = \int_{1}^{x} f(t)dt$. If the average rate of change of g on the interval $2 \le x \le 5$ is 6, which of the following statements must be true?

(A) The average value of f on the interval $2 \le x \le 5$ is 6.
(B) $g'(2) = 6$
(C) $\frac{g'(5)+g'(2)}{2} = 6$
(D) $\int_{2}^{5} g(x)dx = 6$

$\text{rate} = \frac{F(b)-F(a)}{b-a} = \frac{g(5)-g(2)}{5-2} = 6$

$= \frac{1}{b-a} \int_{a}^{b} f(x)\,dx = 6$

---

20. over 10

$0 \to 10$

Time (weeks)	0	2	6	10
Level	210	200	190	180

$h = x_2 - x_1$

$A = \int_{a}^{b} = \frac{1}{2}h(f_{x_1} + f_{x_2})$

The table above gives the level of a person's cholesterol at different times during a 10-week treatment period. What is the average level over this 10-week period obtained by using a trapezoidal approximation with the subintervals [0, 2], [2, 6], and [6, 10] ?

(A) 188
(B) 193
(C) 195
(D) 198

$\frac{1}{10-0} \int_{0}^{10} f(t)\,dt = \frac{1}{10} \cdot \text{Area}$

$\frac{1}{10} \cdot \frac{1}{2} \cdot \left[2(210+200) + 4(200+190) + 4(190+180)\right]$

23. The velocity of a particle moving along the x-axis is given by $v(t) = 2 - t^2$ for time $t > 0$. What is the average velocity of the particle from time $t = 1$ to time $t = 3$?

(A) -4

(B) -3

(C) $-\frac{7}{3}$

(D) $\frac{7}{3}$

$$\frac{1}{3-1}\int_1^3 2 - t^2\,dt$$

26.

If the function f given by $f(x) = x^3$ has an average value of 9 on the closed interval $[0, k]$, then $k =$

(A) 3

(B) $3^{\frac{1}{2}}$

(C) $18^{\frac{1}{3}}$

(D) $36^{\frac{1}{4}}$

(E) $36^{\frac{1}{3}}$

$$\frac{1}{k-0}\int_0^k x^3\,dx = 9$$

$$\frac{1}{k}\cdot\frac{x^4}{4}\Big|_0^k = 9$$

$$x^4\Big|_0^k = 36k \rightarrow k^4 - 0^4 = 36k$$

$$k^4 = 36k \rightarrow k(k^3 - 36) = 0$$

$$k = 0 \quad , \quad k = \sqrt[3]{36} = 36^{\frac{1}{3}}$$

29. A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t)$, and the acceleration of the particle at time t is given by $a(t)$. Which of the following gives the average velocity of the particle from time $t = 0$ to time $t = 8$?

(A) $\frac{a(8)-a(0)}{8}$

(B) $\frac{1}{8}\int_0^8 v(t)dt$

(C) $\frac{1}{8}\int_0^8 |v(t)|dt$

(D) $\frac{1}{2}\int_0^8 v(t)dt$

(E) $\frac{v(0)+v(8)}{2}$

$$\frac{1}{8-0}\int_0^8 v(t)\,dt$$

$$= \frac{1}{8}\int_0^8 v(t)\,dt$$

---

34. The intensity of light at a distance x meters from a source is modeled by the function R given by $R(x) = \frac{k}{x^2}$, where k is a positive constant. Which of the following gives the average intensity of the light between 4 meters and 6 meters from the source?

(A) $\frac{k}{25}$

(B) $-\frac{k}{6}+\frac{k}{4}$

(C) $\frac{1}{2}\left(-\frac{k}{6}+\frac{k}{4}\right)$

(D) $\frac{1}{2}\left(\frac{k}{16}+\frac{k}{36}\right)$

$$\frac{1}{6-4}\int_4^6 \frac{k}{x^2}\,dx$$

$$\frac{1}{2}\int_4^6 kx^{-2}\,dx = \frac{1}{2}\cdot\frac{kx^{-1}}{-1}\Big|_4^6$$

$$-\frac{1}{2}\cdot\frac{k}{x}\Big|_4^6 = -\frac{1}{2}\left[\frac{k}{6}-\frac{k}{4}\right]$$

$$= \frac{1}{2}\left[-\frac{k}{6}+\frac{k}{4}\right]$$

35. The average value of a function f over the interval $[-2,3]$ is -6, and the average value of f over the interval $[3,5]$ is 20. What is the average value of f over the interval $[-2,5]$?

(A) 2
(B) 7
(C) $\frac{10}{7}$
(D) 5

$* \frac{1}{3-(-2)} \int_{-2}^{3} f(x)\,dx = -6 \rightarrow \int_{-2}^{3} f = -30$

$* \frac{1}{5-3} \int_{3}^{5} f\,dx = 20 \rightarrow \int_{3}^{5} f = 40$

$\rightarrow \frac{1}{5-(-2)} \left[\int_{-2}^{5} f\,dx \right] = \frac{1}{7} \cdot \left(\int_{-2}^{3} + \int_{3}^{5} \right)$

$= \frac{1}{7} \cdot (-30 + 40) = \frac{10}{7}$

---

37. Let f be a continuous function defined on the closed interval $2 \le x \le k$, where $k > 2$. If the average value of f on this interval is 6 and $\int_2^k f(x)dx = 42$, then $k =$

(A) 9
(B) 8
(C) 7
(D) 5

$* \frac{1}{k-2} \int_{2}^{k} f(x)\,dx = 6$

$\frac{1}{k-2} \cdot 42 = 6$

Shift → solve

$k = 9$

Q15-MCQ 8-2

$x = 4$

1. If the velocity of a particle moving along the x-axis is $v(t) = 2t - 4$ and if at $t=0$ its position is 4, then at any time t its position $x(t)$ is

$x(t) = \int 2t - 4\, dt$

$= \frac{2t^2}{2} - 4t + C = t^2 - 4t + C$

$4 = 0^2 - 4(0) + C \rightarrow C = 4$

$t^2 - 4t + 4$

(A) t^2-4t
(B) t^2-4t-4
(C) t^2-4t+4
(D) $2t^2-4t$
(E) $2t^2-4t+4$

---

3. A particle moves along the x-axis so that at any time $t \geq 0$ the acceleration of the particle is $a(t) = e^{-2t}$. If at $t=0$ the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$, then its position at any time $t>0$ is $x(t)=$

(A) $-\frac{e^{-2t}}{2} + 3$
(B) $\frac{e^{-2t}}{4} + 4$
(C) $4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$
(D) $\frac{e^{-2t}}{2} + 3t + \frac{15}{4}$
(E) $\frac{e^{-2t}}{4} + 3t + 4$

$v(t) = \frac{e^{-2t}}{-2} + C$

$\frac{5}{2} = \frac{e^0}{-2} + C \rightarrow C = \frac{5}{2} + \frac{1}{2} = 3$

$v(t) = -\frac{e^{-2t}}{2} + 3$

$* x(t) = -\frac{1}{2}\frac{e^{-2t}}{-2} + 3t + C = \frac{1}{4}e^{-2t} + 3t + C$

$\frac{17}{4} = \frac{1}{4}e^0 + 3(0) + C \rightarrow C = \frac{17}{4} - \frac{1}{4} = 4$

$x(t) = \frac{1}{4}e^{-2t} + 3t + 4$

6. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \sin(2t)$. If the position of the particle at time $t = \frac{\pi}{2}$ is $x = 4$, what is the particle's position at time $t = 0$?

(A) $-\frac{1}{2}$
(B) 2
(C) 3
(D) 5
(E) 8

$x(t) = \frac{-\cos 2t}{2} + C$ Radian

$4 = -\frac{\cos \frac{2\pi}{2}}{2} + C \rightarrow C = \frac{1}{2} = 3.5$

$4 = \frac{1}{2} + C \rightarrow C = 4 - \frac{1}{2} = \frac{7}{2} = 3.5$

$x(0) = \frac{-\cos 2(0)}{2} + \frac{7}{2} = 3$

---

8. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 - 4$ for time $t \geq 0$. If the particle is at position $x = -2$ at time $t = 0$, what is the position of the particle at the time $t = 3$?

(A) 13
(B) 15
(C) 16
(D) 17
(E) 25

$x(t) = \frac{3t^3}{3} - 4t + C$

$= t^3 - 4t + C$

$-2 = 0^3 - 4(0) + C \rightarrow \boxed{C = -2}$

$x(t) = t^3 - 4t - 2 \quad = 3^3 - 4(3) - 2 = 13$

---

25. An object moves along a straight line so that at any time t its acceleration is given by $a(t) = 6t$. At time $t = 0$, the object's velocity is 10 and the object's position is 7. What is the object's position at time $t = 2$?

(A) 22
(B) 27
(C) 28
(D) 35

* $v(t) = \frac{6t^2}{2} + C = 3t^2 + C$

$10 = 3(0)^2 + C \rightarrow C = 10 \rightarrow v = 3t^2 + 10$

* $x(t) = \frac{3t^3}{3} + 10t + C = t^3 + 10t + C$

$C = 7$

$x(2) = (2)^3 + 10(2) + 7 = 35$

27. An object moves along a straight line so that at any time $t \geq 0$ its velocity is given by $v(t) = 2\cos(3t)$. What is the distance traveled by the object from $t = 0$ to the first time that it stops?

(A) 0
(B) $\frac{\pi}{6}$
(C) $\frac{2}{3}$
(D) $\frac{\pi}{3}$
(E) $\frac{4}{3}$

$v = 0$

$* \; 2\cos 3t = 0 \rightarrow \cos 3t = 0$

$3t = \cos^{-1}(0) \rightarrow 3t = \frac{\pi}{2}$

$\rightarrow t = \frac{\pi}{6}$

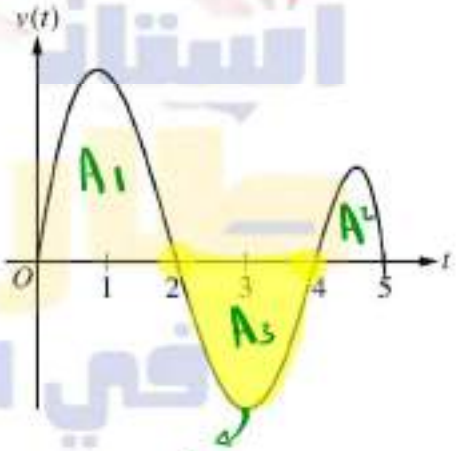
$$d = \int_0^{\frac{\pi}{6}} |v(t)|\,dt = \int_0^{\frac{\pi}{6}} |2\cos 3t|\,dt$$

$$d = \frac{2\sin 3t}{3}\Big|_0^{\frac{\pi}{6}} = F\left(\frac{\pi}{6}\right) - F(0)$$

$$\frac{2}{3}\left[\sin\frac{3\pi}{6} - \sin 0\right] = \frac{2}{3}$$

Over the time interval $0 \leq t \leq 5$, a particle moves along the x-axis. The graph of the particle's velocity, v, is shown above. Over the time interval $0 \leq t \leq 5$, the particle's displacement is 3 and the particle travels a total distance of 13. What is the value of $\int_2^4 v(t)\,dt$?

(A) -10
(B) -5
(C) 5
(D) 10



Total Area $= A_1 + A_2 + A_3$

let $A_1 + A_2 = A$

$* \; A + A_3 = 13$ ✓

$*$ displacement $A - A_3 = 3$ $*$ ✓

mode → equation → 1 → 2

$1 = 1 \quad 1 = 1 \quad 13 = \} \; A = 8$

$1 = 1 \quad -1 = 1 \quad 3 = \} \; A_3 = 5$

A particle moves along a straight line for 6 seconds so that its velocity, in centimeters per second, is modeled by the graph shown. During the time interval $0 \le t \le 6$, what is the total distance the particle travels?

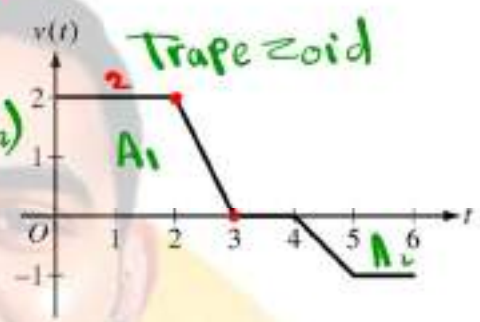
(A) 2 cm
(B) 3.5 cm
(C) 4 cm
(D) 6.5 cm
(E) 8.5 cm

$A_1 + A_2$

$\frac{1}{2}h(b_1+b_2) + \frac{1}{2}h(b_1+b_2)$

$\frac{1}{2}(2)(2+3) + \frac{1}{2}(1)(1+2)$

$= 6.5$



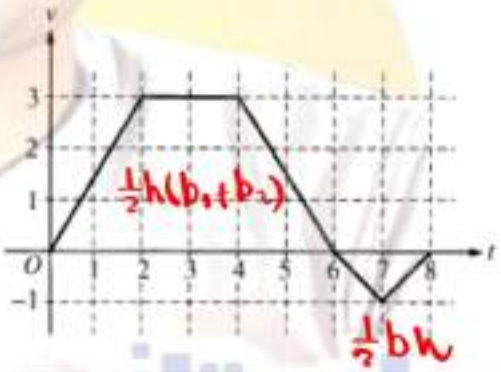
A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

What is the total distance the bug traveled from $t = 0$ to $t = 8$

(A) 14
(B) 13
(C) 11
(D) 8
(E) 6

$\frac{1}{2}(3)(2+6) + \frac{1}{2}(1)(2)$

$= 13$



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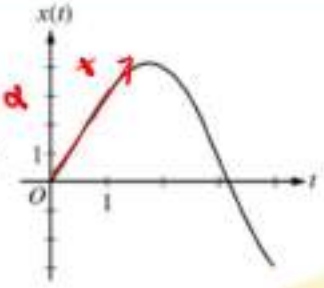
t	0	1	2	3	4
$v(t)$	−1	2	3	0	−4

The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

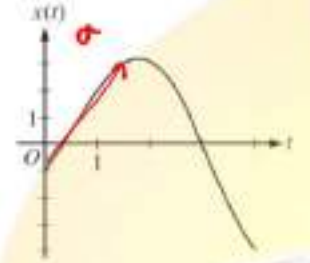
(0,0)

at $x = 0$

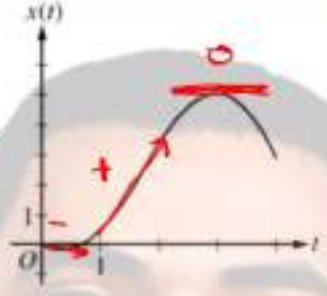
(A)



(B)

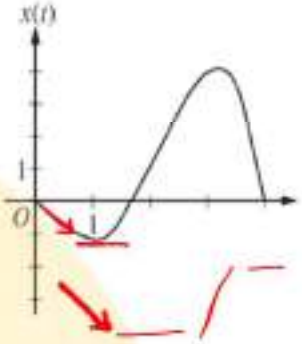


(C)



(D)

(E)



37. For time $t \geq 0$, the position of a particle traveling along a line is given by a differentiable function s. If s is increasing for $0 \leq t < 2$ and s is decreasing for $t > 2$, which of the following is the total distance the particle travels for $0 \leq t \leq 5$?

(A) $s(0) + \int_0^2 s'(t)dt - \int_2^5 s'(t)dt$

(B) $s(0) + \int_2^5 s'(t)dt - \int_0^2 s'(t)dt$

(C) $\int_2^5 s'(t)dt - \int_0^2 s'(t)dt$

(D) $\left|\int_0^5 s'(t)dt\right|$

(E) $\int_0^5 |s'(t)|dt$

$\int_0^5 |v(t)|\,dt = \int_0^5 |s'(t)|\,dt$

38. A particle travels in a straight line with a constant acceleration of 3 meters per second per second. If the velocity of the particle is 10 meters per second at time 2 seconds, how far does the particle travel during the time interval when its velocity increases from 4 meters per second to 10 meters per second?

(A) 20 m
(B) 14 m
(C) 7 m
(D) 6 m
(E) 3 m

$a = 3$, $v = 10$, $t = 2$ → Total distance

$v = \int 3 = 3t + C$

$10 = 3(2) + C \rightarrow C = 4$

$v = 3t + 4$ → Total $d = \int_0^2 |3t + 4|\, dt$

$= 14$

---

40. For time $t \geq 0$, the velocity of a particle moving along the x-axis is given by $v(t) = x^3 - 3x^2 + 2x$. The initial position of the particle at time $t = 0$ is $x = 4$. Which of the following gives the total distance the particle travels from time $t = 0$ to time $t = 3$?

(A) $\int_0^3 (x^3 - 3x^2 + 2x)\,dx$

(B) $4 + \int_0^3 |x^3 - 3x^2 + 2x|\,dx$

(C) $\int_0^3 |x^3 - 3x^2 + 2x|\,dx$

(D) $4 + \int_0^3 (x^3 - 3x^2 + 2x)\,dx$

For time $t \geq 0$, a particle moves along a straight line with velocity $v(t)$ meters per second. The graph of v is shown. Of the following, which is the best estimate for the total distance, in meters, traveled by the particle from time $t = 0$ to $t = 5$ seconds?

(A) 24
(B) 80
(C) 104
(D) 136

b x h

40x3 + 8x2

120 + 16 = 136

Q16-MCQ 8-3

1. A music group expects to sell a new compact disc (CD) at the rate $R(t) = 20,000e^{-0.12t}$ CDs per week, where t denotes the number of weeks since the CD was first released. To the nearest thousand, how many CDs are expected to be sold during the first 12 weeks after the release?

(A) 5,000
(B) 11,000
(C) 57,000
(D) 127,000
(E) 240,000

$$\int_0^{12} 20\,000\, e^{-0.12t}\, dt$$

---

3. Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil leaks out of the tanker from time $t = 0$ to $t = 10$?

(A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons

$$\int_0^{10} 20\,00\, e^{-0.2t}\, dt$$

---

5. Snow is falling at a rate of $r(t) = 2e^{-0.1t}$ inches per hour, where t is the time in hours since the beginning of the snowfall. Which of the following expressions gives the amount of snow, in inches, that falls from time $t = 0$ to time $t = 5$ hours?

(A) $2e^{-0.5} - 2$
(B) $0.2 - 0.2e^{-0.5}$
(C) $4 - 4e^{-0.5}$
(D) $20 - 20e^{-0.5}$

$$\int_0^5 2e^{-0.1t}\, dt = \frac{2e^{-0.1t}}{-0.1}\Big|_0^5$$

$$= -20e^{-0.1t}\Big|_0^5 = F(5) - F(0)$$

$$-20\left[e^{-0.1(5)} - e^{-0.1(0)}\right] = -20e^{-0.5} + 20$$

6. Water is pumped into a tank at a rate of $r(t) = 30\left(1 - e^{-0.16t}\right)$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

(A) 380 gallons
(B) 420 gallons
(C) 829 gallons
(D) 1220 gallons
(E) 1376 gallons

$$800 + \int_0^{20} 30\left(1 - e^{-0.16t}\right) dt \approx 1220$$

---

10. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F) is taken out of an oven and placed in a (75°F) room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F
(B) 119°F
(C) 147°F
(D) 238°F
(E) 335°F

$$350 + \int_0^5 -110e^{-0.4t}\, dt$$

---

12. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

distance $\int |v|$

(A) $e^2 - 1$
(B) $e - 1$
(C) $2e$
(D) e^2
(E) $\frac{e^3}{3}$

$$\int_0^2 |e^t|\, dt = e^t \Big|_0^2 = F(2) - F(0)$$

$$e^2 - e^0 = e^2 - 1$$

15. A city is built around a circular lake that has a radius of 1 mile. The population density of the city is $f(r)$ people per square mile, where r is the distance from the center of the lake, in miles. Which of the following expressions gives the number of people who live within 1 mile of the lake?

(A) $2\pi \int_0^1 rf(r)dr$

(B) $2\pi \int_0^1 r(1+f(r))dr$

(C) $2\pi \int_0^2 r(1+f(r))dr$

(D) $2\pi \int_1^2 rf(r)dr$

(E) $2\pi \int_1^2 r(1+f(r))dr$

$r: 1 \to 1, 2, 3$

$C = 2\pi r$

$\int_1^2 C \cdot f(r)\,dr = \int_1^2 2\pi r \cdot F(r)\,dr$

$= 2\pi \int_1^2 r \cdot F(r)\,dr$

---

23. The number of bacteria in a container increases at the rate of $R(t)$ bacteria per hour. If there are 1000 bacteria at time $t = 0$, which of the following expressions gives the number of bacteria in the container at time $t = 3$ hours?

(A) $R(3)$

(B) $1000 + R(3)$

(C) $\int_0^3 R(t)\,dt$

(D) $1000 + \int_0^3 R(t)\,dt$

$1000 + \int_0^3 R(t)\,dt$

The rate at which ants arrive at a picnic is modeled by the function A, where $A(t)$ is measured in ants per minute and t is measured in minutes. The graph of A for $0 \le t \le 30$ is shown in the figure above. How many ants arrive at the picnic during the time interval $0 \le t \le 30$?

(A) 8

(B) 70

(C) 120

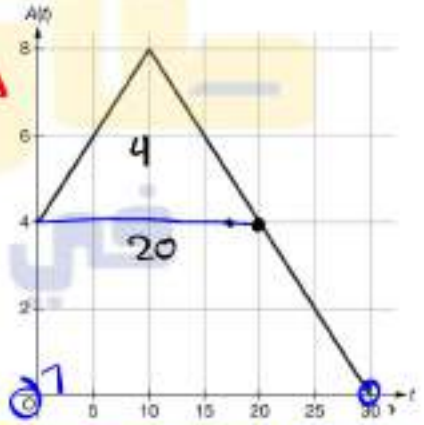
(D) 140

$\int_0^{30} A(t)\,dt = A_\triangle + A_{\text{trapezoid}}$

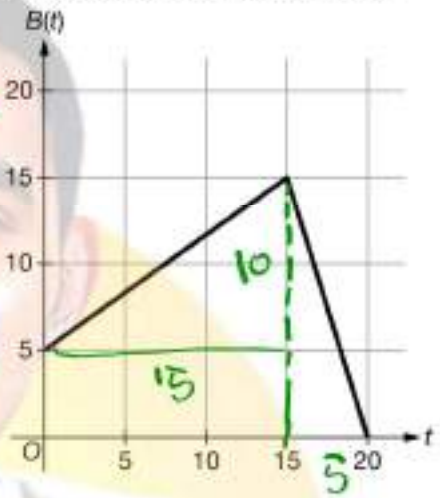
$\frac{1}{2}bh + \frac{1}{2}h(b_1 + b_2)$

$\frac{1}{2}(20)(4) + \frac{1}{2}(4)(20 + 30)$

$40 + 100 = 140$



The rate at which people arrive at a theater box office is modeled by the function B, where $B(t)$ is measured in people per minute and t is measured in minutes. The graph of B for $0 \le t \le 20$ is shown in the figure above. Which of the following is closest to the number of people that arrive at the box office during the time interval $0 \le t \le 20$?



(A) 15

(B) 38

(C) 150

(D) ~~188~~

$\int_0^{20} B(t)\,dt = A_\Delta + A_\Delta + A_\square$

$\frac{1}{2}bh + \frac{1}{2}bh + \ell \times w$

$\frac{1}{2}(15)(10) + \frac{1}{2}(5)(15) + 5(15) = 75 + 37.5 + 75$

$= 187.5 \approx 188$

Q17-MCQ 8-4

1. The area of the region between the graph of $y = 4x^3 + 2$ and the x-axis from $x = 1$ to $x = 2$ is

(A) 36

(B) 23

(C) 20

(D) ~~17~~

(E) 9

$\int_1^2 (4x^3 + 2)\,dx$

Big

$6(1.5)^2 - 18(1.5) = -13.5$ $\quad -6(1.5) = -9$

2. What is the total area of the regions between the curves $y = 6x^2 - 18x$ and $y = -6x$ from $x = 1$ to $x = 3$?

(A) 4

(B) ~~12~~

(C) 16

(D) 20

① $6x^2 - 18x = -6x$

$6x^2 - 18x + 6x = 0$

$6x^2 - 12x = 0$

$x = 0$ $\quad x = 2$ $\notin (1,3)$

$1 \to 2 \to 3$

$\int_1^2 up - down + \int_2^3$

$\int_1^2 (-6x) - (6x^2 - 18x)\,dx$

$\int_2^3 (6x^2 - 18x) - (-6x)\,dx$

3. The area of the region enclosed by the graphs of y=x and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$
(B) 1
(C) $\frac{4}{3}$
(D) 2
(E) $\frac{14}{3}$

$x = 2 \quad y = 2$ $\quad y = 4 - 6 + 3 = 1$

$* x^2 - 3x + 3 = x$

$x^2 - 4x + 3 = 0$

$x = 1, 3$

$\int_1^3 x - (x^2 - 3x + 3)\,dx$

4. If $0 < c < 1$, what is the area of the region enclosed by the graphs of $y = 0$, $y = 1/x$, $x = c$ and $x = 1$?

(A) ln (1-c)
(B) ln (1/c)
(C) ln c
(D) $(1/c^2)$-1
(E) $1-(1/c^2)$

lower

$\int_c^1 \frac{1}{x}\,dx = \ln x \Big|_c^1$

$= \ln 1 - \ln c = 0 - \ln c = \ln c^{-1} = \ln \frac{1}{c}$

5. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$
(B) $\frac{8}{3}$
(C) 4
(D) $\frac{14}{3}$
(E) $\frac{16}{3}$

$0 \longrightarrow 1 \longrightarrow 2$

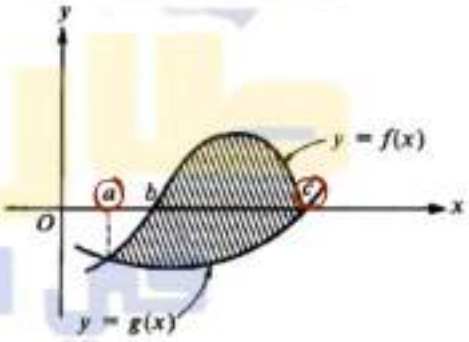
$\int_0^2 x^2 - (-x)\,dx$

$y = 1^2 = 1$ Big

$y = -1$

The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c (|f(x)| - |g(x)|)dx$
(B) $\int_b^c f(x)dx - \int_a^c g(x)dx$
(C) $\int_a^c (g(x) - f(x))dx$
(D) $\int_a^c (f(x) - g(x))dx$
(E) $\int_a^b (g(x) - f(x))dx + \int_b^c (f(x) - g(x))dx$



8. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\pi/2$ and $x = \pi/2$ is separated into two regions by the line $x = k$. If the area of the region for $-\pi/2 \le x \le k$ is three times the area of the region for $k \le x$, then $k =$

(A) $\arcsin(1/4)$
(B) $\arcsin(1/3)$
(C) $\pi/6$
(D) $\pi/4$
(E) $\pi/3$

$$\int_{-\frac{\pi}{2}}^{k} \cos x\,dx = 3\int_{k}^{\frac{\pi}{2}} \cos x\,dx$$

$$\sin x\Big|_{-\frac{\pi}{2}}^{k} = 3\sin x\Big|_{k}^{\frac{\pi}{2}}$$

$$\sin k - \sin\left(-\frac{\pi}{2}\right) = 3\left(\sin\frac{\pi}{2} - \sin k\right)$$

$$\sin k + 1 = 3(1 - \sin k)$$

$$\sin k + 1 = 3 - 3\sin k$$

$$4\sin k = 2 \rightarrow \sin k = \frac{2}{4} = \frac{1}{2}$$

$$k = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

10. The area of the region in the first quadrant that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

(A) 1/4
(B) 1/2
(C) 3/4
(D) 1
(E) 65/4

$x^3 + 8 = x + 8$

$x^3 - x = 0$

$x = 0, -1, 1$

$\int_0^1 (x+8) - (x^3+8)\,dx$

$= \frac{1}{4}$

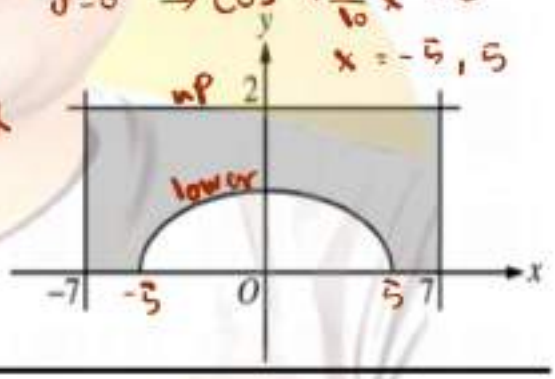
The shaded region in the figure above is bounded by the graph of $y = \sqrt{\cos\left(\frac{\pi x}{10}\right)}$ and the lines $x = -7, x = 7$, and $y = 2$. What is the area of this region?

(A) 6.372
(B) 7.628
(C) 20.372
(D) 21.634
(E) 24.923

$\int_{-7}^{7} 2\,dx - \int_{-5}^{5} \sqrt{\cos\frac{\pi}{10}x}\,dx$

$y = 0 \rightarrow \cos\frac{\pi}{10}x = 0$

$x = -5, 5$



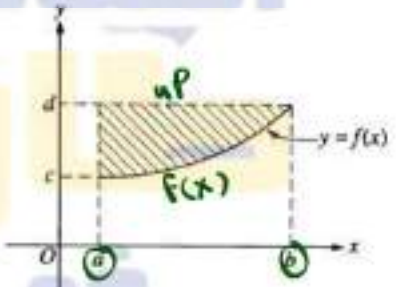
Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)dy$
(B) $\int_a^b (d - f(x))dx$
(C) $f'(b) - f'(a)$
(D) $(b-a)[f(b) - f(a)]$
(E) $(d-c)[f(b) - f(a)]$

$dx \rightarrow$

$dy \uparrow$

$\int_a^b d - f(x)\,dx$



15. Let R be the region between the graph of $y = e^{-2x}$ and the x-axis for $x \geq 3$. The area of R is

(A) $\frac{1}{2e^6}$
(B) $\frac{1}{e^6}$
(C) $\frac{2}{e^6}$
(D) $\frac{\pi}{2e^6}$
(E) infinite

exponential $[3,\infty)$

$$\int_3^\infty e^{-2x}\,dx = \frac{e^{-2x}}{-2}\Big|_3^\infty = \frac{e^{-\infty}}{-2} - \frac{e^{-2(3)}}{-2}$$

$$0 + \frac{e^{-6}}{2} = \frac{1}{2e^6}$$

---

17. What is the area of the region enclosed by the graphs of $f(x) = x - 2x^2$ and $g(x) = -5x$?

(A) $\frac{7}{3}$
(B) $\frac{16}{3}$
(C) $\frac{20}{3}$
(D) 9
(E) 36

$x - 2x^2 = -5x$

$2x^2 - x - 5x = 0$

$2x^2 - 6x = 0$

$x = 0 \quad , x = 3$

$0 \longrightarrow 3$

Big $f(x) = 1 - 2 = -1$

$g(x) = -5(1) = -5$

$$\int_0^3 (x - 2x^2) - (-5x)\,dx = 9$$

---

18. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

$0 \longrightarrow 2$

(A) $2e - 2$
(B) $2e$
(C) $\frac{e}{2} - 1$
(D) $\frac{e-1}{2}$
(E) $e - 1$

$$\int_0^2 e^{\frac{x}{2}}\,dx = 3.4365$$

$$= 2e - 2$$

23. The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

(A) $e^4/2-e$
(B) $e^4/2-1$
(C) $e^4/2-1/2$
(D) $2e^4-e$
(E) $2e^4-2$

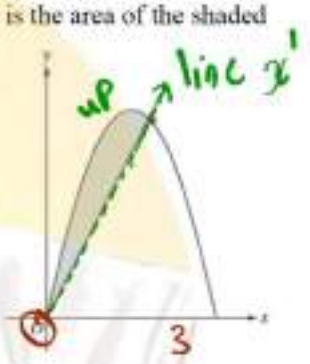
$x = 0$

$\int_0^2 e^{2x}\,dx = 26.799$

$= \frac{e^4}{2} - \frac{1}{2}$

The figure above shows the graph of $y = 5x - x^2$ and the graph of the line $y = 2x$. What is the area of the shaded region?

(A) $\frac{25}{6}$
(B) $\frac{9}{2}$
(C) 9
(D) $\frac{27}{2}$
(E) $\frac{45}{2}$



$5x - x^2 = 2x$

$x^2 - 5x + 2x = 0$

$x^2 - 3x = 0$

$x = 0 \quad , \quad x = 3$

$\int_0^3 \text{up - down}\,dx = \int_0^3 (5x - x^2) - 2x\,dx$

---

27. What is the area of the region in the first quadrant bounded by the lines $x = 1$ and $x = 3$ and the graphs of $y = e^x$ and $y = x - 1$?

(A) $e^3 - e - 6$
(B) $e^3 - e - 2$
(C) $e^3 - 3$
(D) $e^3 - e$

$1 \xrightarrow{2} 3$

$y = e^2 = 7.38$ Big $\quad , y = 2-1 = 1$

$\int_1^3 e^x - (x-1)\,dx = 15.367$

30. $\int_{-1}^{1} \frac{x^2-x}{x} dx$ is

(A) -2

(B) 0

(C) $\frac{4}{3}$

(D) nonexistent

$= \int_{-1}^{1} \frac{x^2}{x} - \frac{x}{x} dx = \int_{-1}^{1} x - 1 \, dx$

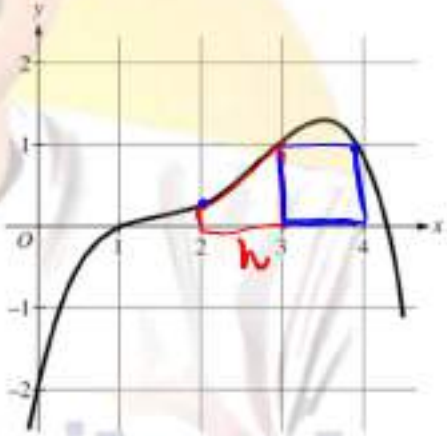
The graph of f', the derivative of the function f, is shown. Which of the following differences is greatest?

(A) $f(1) - f(0)$

(B) $f(2) - f(1)$

(C) $f(3) - f(2)$

(D) $f(4) - f(3)$



Graph of f'

A. $\int_0^1 f' \, dx$ down $(-)$

B. $\int_1^2 f' \, dx = \frac{1}{2}(1)(\frac{1}{2}) = \frac{1}{4}$

C. $\int_1^3 f' \, dx = \frac{1}{2}(1)(1 + \frac{1}{2}) = \frac{3}{4}$

D. $\int_2^4 f' \, dx = 1 \times 1 + A \bigcirc > 1$

Q18-MCQ 8-5

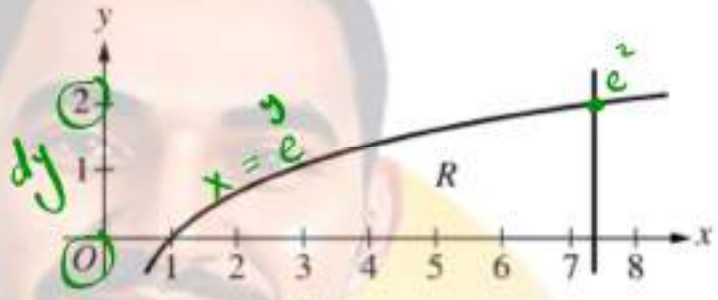
Let R be the region bounded by the graph of $y = \ln x$, the vertical line $x = e^2$, and the x-axis, as shown in the figure. Which of the following integrals are equal to the area of R?

I. $\int_1^{e^2} \ln x\,dx$

II. $\int_0^2 e^y dy$

III. $\int_0^2 \left(e^2 - e^y\right) dy$

(A) I only
(B) III only
(C) I and II only
(D) I and III only



$y = \ln x \rightarrow e^y = x$

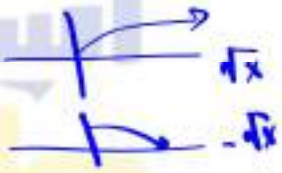
$\int_0^2$ Right - left $dy = \int_0^2 e^2 - e^y\,dy$

---

2. What is the area of the region in the first quadrant bounded on the left by the graph of $x = y^2$ and on the right by the graph of $x = 4y - 3$ for $1 \le y \le 3$?

(A) $\frac{4}{3}$
(B) $\frac{56}{3}$
(C) 54
(D) $\frac{320}{3}$

$\int_1^3 (y^2) - (4y - 3)\,dy$

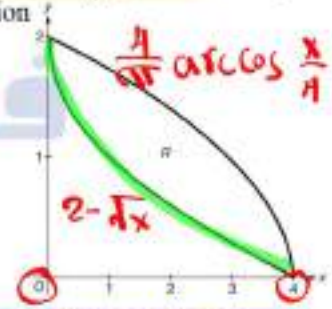


Let R be the region in the first quadrant bounded above by the graph of $y = \frac{4}{\pi}\arccos\left(\frac{x}{4}\right)$ and below by the graph of $y = 2 - \sqrt{x}$, as shown in the figure above. What is the area of the region R?

(A) $\frac{4}{3}$
(B) $\frac{16}{\pi} + \frac{8}{3}$
(C) $\frac{16}{\pi} - \frac{8}{3}$
(D) $\frac{16}{3}$

$\int_0^4 \left(\frac{4}{\pi}\arccos\frac{x}{4}\right) - (2 - \sqrt{x})\,dx$

≈ 2.426



4. Let R be the region in the first quadrant bounded above by the graph of $y = 1 + 4x$ and below by the graph of $y = 3^x$ for $0 \le x \le 2$. Which of the following definite integrals gives the area of region R?

I. $\int_0^2 (1 + 4x - 3^x)dx$

II. $\int_0^9 \left(\frac{\ln y}{\ln 3} - \frac{y-1}{4}\right)dy$

III. $\int_1^9 \left(\frac{\ln y}{\ln 3} - \frac{y-1}{4}\right)dy$

(A) None

(B) I only

(C) I and II only

(D) I and III only

$* \int_0^2 (1+4x) - (3^x)\,dx$

$4x = y - 1$

x alone

$* x = \left|\frac{y-1}{4}\right|$

$* y = 3^x \rightarrow \ln y = x \ln 3$

$x = \left|\frac{\ln y}{\ln 3}\right|$

$* y = 1 + 4x$ at $x = 0 \rightarrow y = 1 + 4(0) = 1$

at $x = 2 \rightarrow y = 1 + 8 = 9$

$\int_1^9$, $1 \rightarrow 5 \rightarrow 9$

$* x = \frac{5-1}{4} = 1$

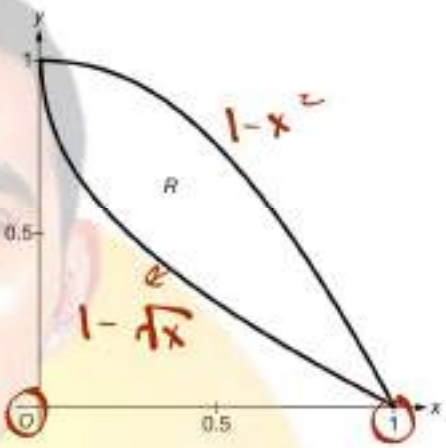
$* x = \frac{\ln 5}{\ln 3} = 1.46$

$\int_1^9 \left(\frac{\ln y}{\ln 3} - \left(\frac{y-1}{4}\right)\right)dy$

Let R be the region in the first quadrant bounded above by the graph of $y = 1 - x^2$ and below by the graph of $y = 1 - \sqrt{x}$, as shown in the figure above. What is the area of the region?

(A) -1
(B) $\frac{1}{6}$
(C) $\frac{1}{3}$
(D) 1

$$\int_0^1 (1 - x^2) - (1 - \sqrt{x})\,dx$$



7. Let R be the region in the first quadrant bounded above by the graph of $y = \frac{7}{3}x + 1$ and bounded below by the graph of $y = 2^x$ for $0 \le x \le 3$. Which of the following definite integrals gives the area of region R?

I. $\int_0^3 \left(\left(\frac{7}{3}x + 1\right) - 2^x\right)dx$

II. $\int_1^8 \left(\frac{\ln y}{\ln 2} - \frac{3}{7}(y-1)\right)dy$

III. $\int_1^8 \left(\frac{\ln y}{\ln 2} - \frac{3}{7}(y-1)\right)dy$

x alone $\quad y - 1 = \frac{7}{3}x$

$$x = \frac{3}{7}(y-1)$$

$y = 2^x \quad \ln y = x \ln 2$

$$x = \frac{\ln y}{\ln 2}$$

(A) None
(B) I only
(C) I and II only
(D) I and III only

$x = 0 \to y = \frac{7}{3}(0) + 1 = 1$

$x = 3 \to y = \frac{7}{3}(3) + 1 = 8$

$$\int_1^8 \left(\frac{\ln y}{\ln 2}\right) - \left(\frac{3}{7}(y-1)\right)dy$$

$\frac{\ln 2}{\ln 2} = 1 \quad \frac{3}{7}$

8. Let R be the region bounded by the graph of $x = e^y$, the vertical line $x = 10$, and the horizontal lines $y = 1$ and $y = 2$. Which of the following gives the area of R?

(A) $\int_1^2 e^y dy$

(B) $\int_e^{e^2} \ln x dx$

(C) $\int_1^2 (10 - e^y) dy$

(D) $\int_e^{10} (\ln x - 1) dx$

$\int_1^2 (10 - e^y)\, dy \rightarrow x$ alone

$1 \xrightarrow{1.5} 2$

$e^{1.5} \simeq 4.48$

$x = 10$ Big

Q19-MCQ 8-10

1. What is the volume of the solid generated when the region bounded by the graph of $y = x^2$, the vertical line $x = 3$, and the horizontal line $y = 4$ is revolved about the horizontal line $y = 4$?

(A) $\pi \int_2^3 (x^2 - 4) dx$

(B) $\pi \int_2^3 (x^4 - 16) dx$

(C) $\pi \int_2^3 (x^2 - 4)^2 dx$

(D) $\pi \int_2^3 (x^2 + 4)^2 dx$

$x^2 = 4$

$x = \pm\sqrt{4} = \pm 2$

$\pi \int_2^3 (x^2 - 4)^2 dx$

$V = \pi \int R^2$

2. What is the volume of the solid generated when the region bounded by the graph of $y = 2x$, the horizontal line $y = 2$, and the y-axis is revolved about the horizontal line $y = 2$?

(A) π

(B) $\frac{4\pi}{3}$

(C) $\frac{8\pi}{3}$

(D) $\frac{16\pi}{3}$

$\pi \int_0^1 (2 - 2x)^2 dx$

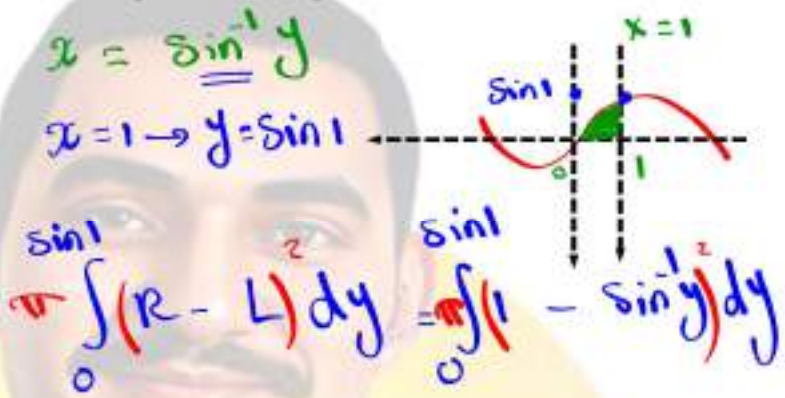
3. Let R be the region in the first quadrant bounded by the graph of $y = \sin x$, the x-axis, and the vertical line $x = 1$. Which of the following gives the volume of the solid generated when region R is revolved about the vertical line $x = 1$?

(A) $\pi \int_0^{\sin 1} (1 - \arcsin y)^2 dy$

(B) $\pi \int_0^{1} (1 - \arcsin y)^2 dy$

(C) $\pi \int_0^{\sin 1} (1 - \arcsin y) dy$

(D) $\pi \int_0^{\sin 1} \left(1 - (\arcsin y)^2\right) dy$



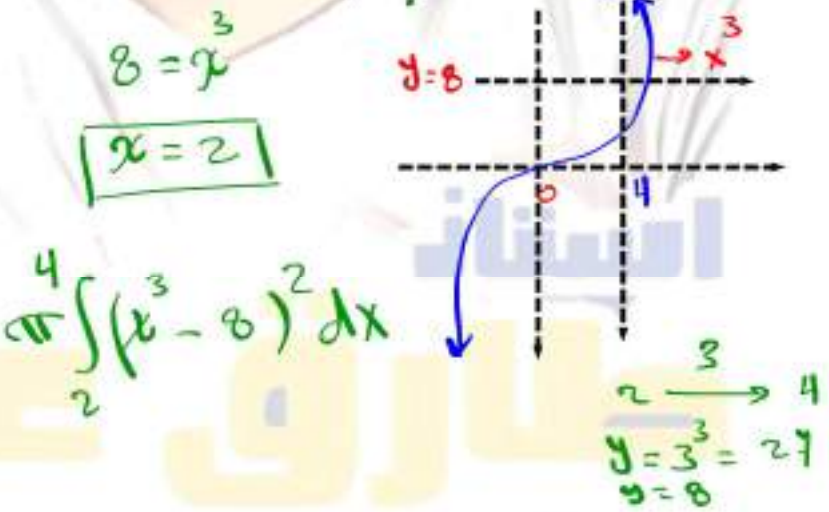
4. What is the volume of the solid generated when the region bounded by the graph of $y = x^3$, the vertical line $x = 4$, and the horizontal line $y = 8$ is revolved about the horizontal line $y = 8$?

(A) $\pi \int_2^4 (x^3 - 8) dx$

(B) $\pi \int_2^4 (x^6 - 64) dx$

(C) $\pi \int_2^4 (x^3 - 8)^2 dx$

(D) $\pi \int_2^4 (x^3 + 8)^2 dx$



5. What is the volume of the solid generated when the region bounded by the graph of $y = 4x$, the horizontal line $y = 8$, and the y-axis is revolved about the horizontal line $y = 8$?

(A) 8π

(B) $\frac{128\pi}{3}$

(C) $\frac{256\pi}{3}$

(D) $\frac{512\pi}{3}$

$g(1) = 4$ — dx — $8 = 4x \rightarrow x = 2$ — $0 \rightarrow 1 \rightarrow 2$

$$\pi \int_0^2 (8 - 4x)^2 dx = \frac{128}{3} \pi$$

---

6. Let R be the region in the first quadrant bounded by the graph of $y = \tan x$, the x-axis, and the vertical line $x = 1$. Which of the following gives the volume of the solid generated when region R is revolved about the vertical line $x = 1$?

(A) $\pi \int_0^{\tan 1} (1 - \arctan y)^2 dy$

(B) $\pi \int_0^1 (1 - \arctan y)^2 dy$

(C) $\pi \int_0^{\tan 1} (1 - \arctan y) dy$

(D) $\pi \int_0^1 \left(1^2 - (\arctan y)^2\right) dy$

dy — about x alone — $x = \tan^{-1} y$

about: $x = $ ثابت $\rightarrow dy$, $y = $ ثابت $\rightarrow dx$, x-axis $\rightarrow dy$, y-axis $\rightarrow dx$

$\rightarrow x = \tan^{-1} 1 \approx 0.78$ — $1 \quad x = 1$

$$\pi \int_0^{\tan 1} \left(1 - \tan^{-1} y\right)^2 dy$$

Q20-MCQ 8-13

1.

x	1	3	5	7
$f(x)$	4	6	7	5
$f'(x)$	2	1	0	−1

$\Delta x = \frac{b-a}{n} = \frac{7-1}{3} = 2$

$3-1=2,\quad 5-3=2,\quad 7-5=2$

The table above gives selected values for a differentiable function f and its first derivative. Using a left Riemann sum with 3 subintervals of equal length, which of the following is an approximation of the length of the graph of f on the interval $[1, 7]$?

(A) 6
(B) 34
(C) $2\sqrt{3}+2\sqrt{2}+2$
(D) $2\sqrt{5}+2\sqrt{2}+2$
(E) $2\sqrt{5}+4\sqrt{2}+2$

$L = \sum \sqrt{1+f'^2}\cdot \Delta x$

$2\left[\sqrt{1+2^2} + \sqrt{1+1^2} + \sqrt{1+0^2}\right]$

$2(\sqrt{5}+\sqrt{2}+1)$

$= 2\sqrt{5}+2\sqrt{2}+2$

---

2. The length of the curve $y = \sin x$ from $x = 0$ to $x = 3\pi/4$ is given by

(A) $\int_0^{3\pi/4} \sin x\,dx$
(B) $\int_0^{3\pi/4} \sqrt{1+\sin^2 x}\,dx$
(C) $\int_0^{3\pi/4} \sqrt{1-\cos^2 x}\,dx$
(D) $\int_0^{3\pi/4} \sqrt{1+\cos^2 x}\,dx$

$y' = \cos x$

$L = \int_0^{3\pi/4} \sqrt{1+(y')^2}\,dx$

3. Which of the following gives the length of the curve $y = \sqrt{x}$ over the closed interval $[1,4]$?

(A) $\int_1^4 \sqrt{1 + \frac{1}{2\sqrt{x}}}dx$

(B) $\int_1^4 \sqrt{1 + \frac{1}{2x}}dx$

(C) $\int_1^4 \sqrt{1 - \frac{1}{4x}}dx$

(D) $\int_1^4 \sqrt{1 + \frac{1}{4x}}dx$

(E) $\int_1^4 \sqrt{1 + \frac{1}{4}x^2}dx$

$y' = \frac{1}{2\sqrt{x}}$

$\int_1^4 \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2}\,dx$

$\int_1^4 \sqrt{1 + \frac{1}{4x}}\,dx$

---

4. The length of the curve $y = \ln \sec x$ from $x = 0$ to $x = b$, where $0 < b < \frac{\pi}{2}$, may be expressed by which of the following integrals?

(A) $\int_0^b \sec x\,dx$

(B) $\int_0^b \sec^2 x\,dx$

(C) $\int_0^b (\sec x \tan x)\,dx$

(D) $\int_0^b \sqrt{1 + (\ln \sec x)^2}\,dx$

(E) $\int_0^b \sqrt{1 + (\sec^2 x \tan^2 x)}\,dx$

$y' = \frac{F}{F} = \frac{\sec x \cdot \tan x}{\sec x} = \tan x$

$\int_0^b \sqrt{1 + \tan^2 x}\,dx$

$\int_0^b \sqrt{\sec^2 x}\,dx$

$\int_0^b \sec x\,dx$

5. The length of the curve $y = \sin(3x)$ from $x = 0$ to $x = \frac{\pi}{6}$ is given by

(A) $\int_0^{\pi/6} (1 + 9\cos^2(3x))dx$

(B) $\int_0^{\pi/6} \sqrt{1 + \sin^2(3x)}dx$

(C) $\int_0^{\pi/6} \sqrt{1 + 3\cos(3x)}dx$

(D) $\int_0^{\pi/6} \sqrt{1 + 9\cos^2(3x)}dx$

$y' = 3\cos 3x$

$\int_0^{\pi/6} \sqrt{1+(3\cos 3x)^2}\,dx$

$\int_0^{\pi/6} \sqrt{1+9\cos^2 3x}\,dx$

---

6. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

(A) $\int_1^2 \sqrt{1 + \frac{1}{x^2}}\,dx$

(B) $\int_1^2 \left(1 + \frac{1}{x^2}\right)dx$

(C) $\int_1^2 \sqrt{1 + e^{2x}}\,dx$

(D) $\int_1^2 \sqrt{1 + (\ln x)^2}\,dx$

(E) $\int_1^2 \left(1 + (\ln x)^2\right)dx$

$y' = \frac{1}{x}$

$\int_1^2 \sqrt{1 + \frac{1}{x^2}}\,dx$

7. The length of the curve $y = x^4$ from $x=1$ to $x=5$ is given by

(A) $\int_1^5 \sqrt{1+4x^3}\,dx$

(B) $\int_1^5 \sqrt{1+x^4}\,dx$

(C) $\int_1^5 \sqrt{1+4x^6}\,dx$

(D) $\int_1^5 \sqrt{1+16x^6}\,dx$

(E) $\int_1^5 \sqrt{1+x^8}\,dx$

$$\int_1^5 \sqrt{1+(4x^3)^2}\,dx$$

$$\int_1^5 \sqrt{1+16x^6}\,dx$$

---

8. The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1+9x^4}dx$. If the curve contains the point $(1, 6)$, which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$

(B) $y = 5 + x^3$

(C) $y = 6 + x^3$

(D) $y = 6 - x^3$

(E) $y = \frac{10}{5} + x + \frac{9}{6}x^5$

$x_1 = 1 \quad , \quad y_1 = 6$

$y = \sqrt{9x^4} = 3x^2$

$y = \int 3x^2\,dx = \frac{3x^3}{3} + c = x^3 + c \quad (1,6)$

$6 = 1^3 + c \rightarrow c = 6 - 1 = 5$

$y = x^3 + 5$

9. Which of the following integrals gives the length of the graph $y = \sin(\sqrt{x})$ between $x = a$ and $x = b$, where $0 < a < b$?

(A) $\int_a^b \sqrt{x + \cos^2(\sqrt{x})}\,dx$

(B) $\int_a^b \sqrt{1 + \cos^2(\sqrt{x})}\,dx$

(C) $\int_a^b \sqrt{\sin^2(\sqrt{x}) + \frac{1}{4x}\cos^2(\sqrt{x})}\,dx$

(D) $\int_a^b \sqrt{1 + \frac{1}{4x}\cos^2(\sqrt{x})}\,dx$

(E) $\int_a^b \sqrt{\frac{1 + \cos^2(\sqrt{x})}{4x}}\,dx$

---

14. The length of the graph of $y = f(x)$ from $x = 0$ to $x = 1$ is given by $\int_0^1 \sqrt{1 + e^{2x}}dx$. Of the following, which could be $f(x)$?

(A) e^{-x}

(B) e^{x}

(C) e^{-2x}

(D) e^{2x}

---

15. Which of the following gives the length of the curve $y = \tan x$ from $x = -\frac{\pi}{4}$ to $x = \frac{\pi}{4}$?

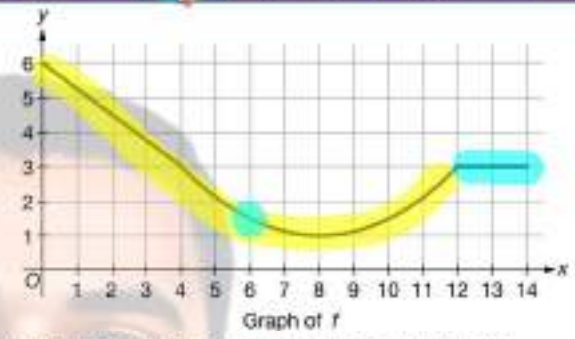
(A) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + \tan x}dx$

(B) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x}dx$

(C) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + \sec^2 x}dx$

(D) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + \sec^4 x}dx$

$$f(x) = \begin{cases} 6 - \frac{3}{4}x & \text{for } 0 \le x < 4 \\ 1 + \frac{1}{8}(x-8)^2 & \text{for } 4 \le x \le 12 \\ 3 & \text{for } 12 < x \le 14 \end{cases}$$



Graph of f

A skateboard track consists of a straight ramp followed by a curved section and a horizontal ledge. The track is modeled by the piecewise-defined function f above, and the graph of f is shown in the figure above. Which of the following expressions gives the total length of the track from $x = 0$ to $x = 14$?

(A) $2 + \int_0^{12} \sqrt{1 + \left(-\frac{3}{4} + \frac{1}{4}(x-8)\right)^2}\,dx$

(B) $2 + \int_0^{12} \left(\sqrt{1 + \left(-\frac{3}{4}\right)^2} + \sqrt{1 + \frac{1}{16}(x-8)^2}\right) dx$

(C) $7 + \int_4^{12} \sqrt{1 + \left(1 + \frac{1}{8}(x-8)^2\right)^2}\,dx$

(D) $7 + \int_4^{12} \sqrt{1 + \frac{1}{16}(x-8)^2}\,dx$

$\int_4^{12} \sqrt{1 + y'^2}\,dx$

$y' = \frac{1}{8} \cdot 2(x-8)$

$= \frac{1}{4}(x-8)$

$y'^2 = \frac{1}{16}(x-8)^2$

الجزء الكتابي Q21-FRQ

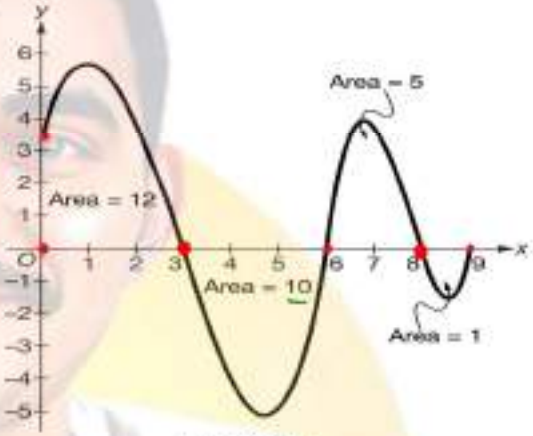
Lessons 6-6, 6-7, 6-8, 6-9

The figure above shows the graph of f', the derivative of a differentiable function f, on the closed interval $0 \le x \le 9$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(6) = 7$.

Let g be the function defined by $g(x) = x^2 - 1$.

(a) Find the value of $\int_0^9 f'(x)dx$. $= A$

$A = 12 - 10 + 5 - 1 = 6$



(b) Given that $f(6) = 7$, write an expression for $f(x)$ that involves an integral. Use this expression to find the absolute minimum value of f and the absolute maximum value of f on the closed interval $0 \le x \le 9$. Justify your answers.

$f(x) = 7 + \int_6^x \acute{f}(t)dt$

$\rightarrow \acute{f}(x) = 0$ at $x = 3, 6, 8, 9, 0$

$* f(0) = 7 + \int_6^0 \acute{f}(t) = 7 - \int_0^6 \acute{f}(t)dt$

$= 7 - (12 - 10) = \boxed{5}$

$* f(3) = 7 + \int_6^3 \acute{f}(t)dt = 7 - \int_3^6 \acute{f}(t)dt$

$= 7 - 10 = \boxed{-3}$

$* f(6) = 7 + \int_6^6 f(t)\,dt = 7 + 0 = \boxed{7}$

$* f(9) = 7 + \int_6^9 f(t)\,dt = 7 + (5-1) = \boxed{11}$

$* f(8) = 7 + \int_6^8 f(t)\,dt = 7 + 5 = \boxed{12}$

minimum = 5 → f(0) = 5

maximum = 17 → F(3) = 17

(c) Find $\int g(x)dx$. $= \int x^2 - 1\,dx$

$= \frac{x^3}{3} - x + C$

(d) Find the value of $\int_2^3 x\,f'(g(x))dx$. $= \int_2^3 x \cdot f'(x^2-1)\,dx$

$u = x^2 - 1 \rightarrow du = 2x\,dx \rightarrow dx = \frac{du}{2x}$

$x = 2 \rightarrow u = 4 - 1 = 3$

$x = 3 \rightarrow u = 9 - 1 = 8$

$\int_3^8 x \cdot f(u) \cdot \frac{du}{2x} = \frac{1}{2}\int_3^8 f(u)\,du$

$\frac{1}{2}\int_3^8 f'(u)\,du = \frac{1}{2}(5-10) = \frac{-5}{2}$

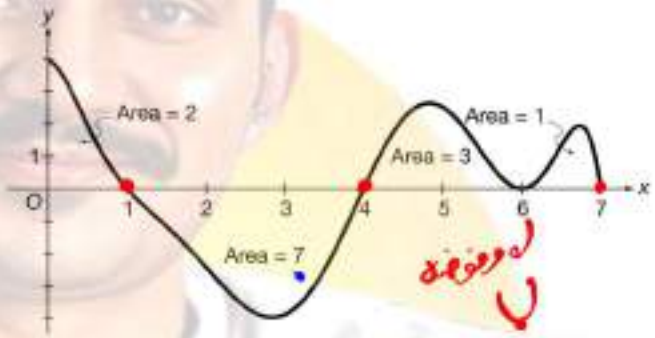
The figure above shows the graph of f', the derivative of a differentiable function f, on the closed interval $0 \le x \le 7$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(4) = 10$.

Let g be the function defined by $g(x) = 5 - x^2$.

(a) Find the value of $\int_0^7 f'(x)dx$. $= A$

$A = 2 - 7 + 3 + 1$

$A = -1$



Graph of f'

سالب تحت الـ x-axis

Curve

(b) Given that $f(4) = 10$, write an expression for $f(x)$ that involves an integral. Use this expression to find the absolute minimum value of f and the absolute maximum value of f on the closed interval $0 \le x \le 7$. Justify your answers.

$f(x) = 10 + \int_4^x f'(x)\,dx$, $f'(x) = 0 \Rightarrow x = 0, 1, 4, 6, 7$

* $f(0) = 10 + \int_4^0 f'(x)\,dx = 10 - \int_0^4 f'(x)\,dx$

$= 10 - (2 - 7) = \boxed{15}$

* $f(1) = 10 + \int_4^1 f'(x)\,dx = 10 - \int_1^4 f'(x)\,dx$

$= 10 - 2 - 7 = \boxed{1}$

$* f(11) = 10 + \int_{11}^{11} f(x)\,dx = 10 + 0 = \boxed{10}$

$* f(7) = 10 + \int_{11}^{7} f(x)\,dx = 10 + 3 + 1$

$= \boxed{14}$

→ maximum = 17 → $f(1) = 17$

→ minimum = 10 → $f(11) = 10$

(c) Find $\int g(x)\,dx$. $= \int 5 - x^2\,dx$

$= 5x - \frac{x^3}{3} + C$

(d) Find the value of $\int_1^2 x\, f'(g(x))\,dx$. $= \int^{2} x \cdot f(5-x^2)\,dx$

$u = 5 - x^2 \rightarrow du = -2x\,dx \rightarrow dx = \frac{du}{-2x}$

$x = 1 \rightarrow u = 5 - 1^2 = 4$

$x = 2 \rightarrow u = 5 - 2^2 = 1$

$$\int_4^1 x \cdot \bar{f}(u) \cdot \frac{dx}{-2x} = -\frac{1}{2}\int_4^1 \bar{f}(u)\,du$$

$$= -\frac{1}{2} \cdot -\int_1^4 \bar{f}(u)\,du = \frac{1}{2}\int_1^4 \bar{f}(u)\,du$$

$$= \frac{1}{2}(-7) = \frac{-7}{2}$$

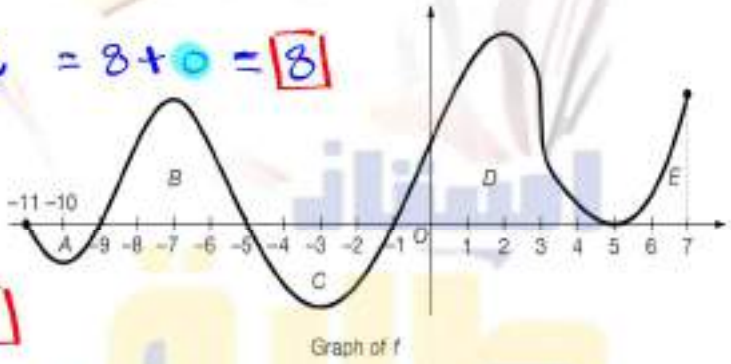
The figure above shows the graph of the continuous function f. The regions A, B, C, D, and E have areas 2, 14, 10, 27, and 4, respectively. For $-11 \le x \le 7$, the function g is defined by $g(x) = 8 + \int_{-5}^{x} f(t)dt$.

(a) Is there a value of x, for $-5 \le x \le -1$, such that $g(x) = 0$? Justify your answer.

$$g(-5) = 8 + \int_{-5}^{-5} f(t)\,dt = 8 + 0 = 8$$

$$g(-1) = 8 + \int_{-5}^{-1} f(t)\,dt$$

$$= 8 - 10 = -2$$

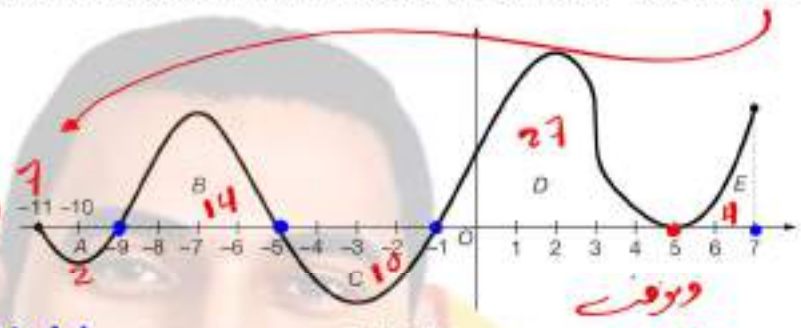


Graph of f

$-2 \longrightarrow 0 \longrightarrow 8$

by IVT yes there point that $g(x) = 0$ on $[-5, -1]$

(b) Find the absolute minimum value of g and the absolute maximum value of g on the interval $-11 \leq x \leq 7$. Justify your answer.



$\rightarrow g'(x) = f(x)$

$x = -11, -9, -5, -1, 7$

وبوف curve

$$* g(-11) = 8 + \int_{-5}^{-11} f(t)\,dt$$

$$= 8 - \int_{-11}^{-5} f(t)\,dt = 8 - (-2 + 14) = \boxed{-4}$$

$$* g(-9) = 8 + \int_{-5}^{-9} f(t)\,dt = 8 - \int_{-9}^{-5} f(t)\,dt$$

$$= 8 - 14 = \boxed{-6}$$

$$* g(-5) = 8 + \int_{-5}^{-5} f(t)\,dt = 8 + 0 = \boxed{8}$$

$$* g(-1) = 8 + \int_{-5}^{-1} f(t)\,dt = 8 - 10 = \boxed{-2}$$

$$* g(7) = 8 + \int_{-5}^{7} f(t)\,dt = 8 + (-10 + 27 + 4)$$

$$= \boxed{29}$$

* maximum = 29 * minimum = −6

(c)

(i) Find $\int (2x+1)dx$. $= \frac{2x^2}{2} + x + C = x^2 + x + C$

(ii) Find the value of $\int_{-5}^{-1} f(2x+1)dx$.

$u = 2x + 1 \rightarrow du = 2\,dx \rightarrow dx = \frac{du}{2}$

$x = -5 \rightarrow u = 2(-5) + 1 = -9$

$x = -1 \rightarrow u = 2(-1) + 1 = -1$

$\int_{-9}^{-1} f(u) \cdot \frac{du}{2} = \frac{1}{2} \int_{-9}^{-1} f(u) = \frac{1}{2}[14 - 10]$

$= \boxed{2}$

Q22-FRQ

Lessons 6-2, 6-7, 6-11, 6-13

Let f be the function defined by $f(x) = \frac{4}{x^2}$.

(a) Approximate the value of $\int_1^{12} f(x)dx$ using a left Riemann sum with the subintervals $[1, 4]$, $[4, 8]$, and $[8, 12]$.

غير متصل

$1 \rightarrow 4 \rightarrow 8 \rightarrow 12$ x

$\Delta x = 4-1 = 3$, $8-4 = 4$, $12-8 = 4$

$$\int_1^{12} f(x)dx = 3 \cdot f(1) + 4 \cdot f(4) + 4 \cdot f(8)$$

$$= 3 \cdot \frac{4}{1^2} + 4 \cdot \frac{4}{4^2} + 4 \cdot \frac{4}{8^2}$$

$$= 12 + 1 + \frac{1}{4} = 13\frac{1}{4} = \frac{53}{4}$$

(b) Find the value of $\int_1^{12} f(x)dx$. Show the work that leads to your answer.

$$\int_1^{12} \frac{4}{x^2} dx = \int_1^{12} 4x^{-2} dx = \frac{4x^{-1}}{-1} \Big|_1^{12}$$

$$= \frac{-4}{x} \Big|_1^{12} = F(12) - F(1)$$

$$= \frac{-4}{12} - \frac{-4}{1} = -\frac{1}{3} + 4 = 3\frac{2}{3} = \frac{11}{3}$$

(c) Find the value of $\int_1^{\infty} f(x)dx$ or explain why it does not exist.

$$\int_1^{\infty} \frac{4}{x^2}\,dx = \lim_{n\to\infty} \int_1^{n} \frac{4}{x^2}\,dx$$

$$= \lim_{n\to\infty} \int_1^{n} 4x^{-2}\,dx = \lim_{n\to\infty} \frac{-4}{x}\Big|_1^{n}$$

$$= \lim_{n\to\infty} f(n) - f(1) = \lim_{n\to\infty} \frac{-4}{n} - \frac{-4}{1}$$

$$= \lim_{n\to\infty} -\frac{4}{n} + 4 = -\frac{4}{\infty} + 4 \quad \frac{\text{عدد}}{\infty} = 0$$

$$= 0 + 4 = \boxed{4}$$

(d) Find the value of $\int_1^{12} f(x)\cdot \ln x\,dx$. Show the work that leads to your answer.

$$\int_1^{12} \frac{4}{x^2}\cdot \ln x\,dx \quad \text{by parts}$$

$u = \ln x$ ① $\quad dv = \frac{4}{x^2}$

$du = \frac{1}{x}dx$ $\quad -\int$ $\quad v = -\frac{4}{x}$ ②

$uv - \int v\,du$

$$-\frac{4\ln x}{x}\Big|_1^{12} - \int_1^{12} -\frac{4}{x}\cdot\frac{1}{x}\,dx$$

$$-\frac{4\ln x}{x}\Big|_1^{12} + \int_1^{12} \frac{4}{x^2}\,dx$$

$$= -\frac{4\ln x}{x}\Big|_1^{12} + \frac{-4}{x}\Big|_1^{12} = \left(-\frac{4\ln x}{x} - \frac{4}{x}\right)\Big|_1^{12}$$

$$= F(12) - F(1)$$

$$= \left(-\frac{4\ln 12}{12} - \frac{4}{12}\right) - \left(-\frac{4\ln 1}{1} - \frac{4}{1}\right)$$

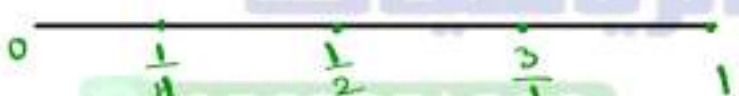
$$= -\frac{1}{3}\ln 12 - \frac{1}{3} + 4 = -\frac{1}{3}\ln 12 + \frac{11}{3}$$

---

2. Let $F(x) = \int_0^x \sin(t^2)\,dt$ for $0 \le x \le 3$. $F(1) = \int_0^1 \sin t^2\,dt$

* $\Delta x = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

(a) Use the trapezoidal rule with four equal subdivisions of the closed interval [0,1] to approximate $F(1)$.



$$A = \frac{1}{2}h(f_{x_1} + f_{x_2}) = \frac{\Delta x}{2}(f_{x_1} + f_{x_2})$$

$$A = \frac{\frac{1}{8}}{2}\left(f(0) + 2f\left(\frac{1}{8}\right) + 2f\left(\frac{1}{2}\right) + 2f\left(\frac{3}{8}\right) + f(1)\right)$$

$$A = \frac{1}{8}\left(\sin 0^2 + 2\sin\frac{1}{16} + 2\sin\frac{1}{4} + 2\sin\frac{9}{16} + \sin 1\right)$$

$$A \simeq 0.316$$

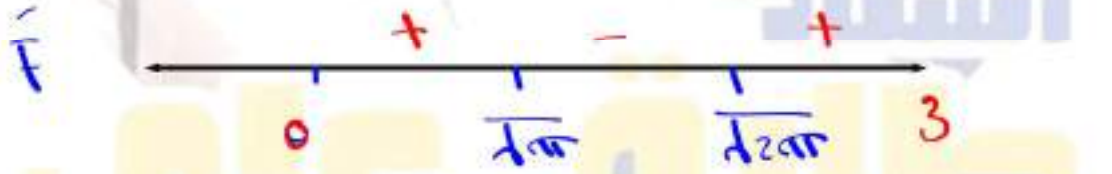
(b) On what intervals is F increasing?

$F' = 0$

$F'(x) = \sin x^2 \longrightarrow F'(x) = 0$

$x^2 = \sin^{-1}(0)$ $\quad x^2 = 0, \pi, 2\pi$

$0 \le x \le 3$ $\quad +\pi \Rightarrow x = 0, \sqrt{\pi}, \sqrt{2\pi}$



increasing $[0, \sqrt{\pi}] \cup [\sqrt{2\pi}, 3]$

(c) If the average rate of change of F on the closed interval $[1,3]$ is k, find $\int_1^3 \sin(t^2)\,dt$ in terms of k.

$$k = \frac{F(b) - F(a)}{b - a} \rightarrow k = \frac{F(3) - F(1)}{3 - 1}$$

$$k = \frac{\int_1^3 \sin t^2\,dt}{2} \rightarrow \int_1^3 \sin t^2\,dt = 2k$$

Consider the function f given by $f(x) = xe^{-x^2}$ for all real numbers x.

Find the value of $\int_0^\infty x f(x)\,dx$ given the fact that $\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$.

(c) If the average rate of change of F on the closed interval $[1,3]$ is k, find $\int_1^3 \sin(t^2)\,dt$ in terms of k.

$$k = \frac{F(b)-F(a)}{b-a} \rightarrow k = \frac{F(3)-F(1)}{3-1}$$

$$k = \frac{\int_1^3 \sin t^2\,dt}{2} \rightarrow \int_1^3 \sin t^2\,dt = 2k$$

Consider the function f given by $f(x) = xe^{-x^2}$ for all real numbers x.

Find the value of $\int_0^\infty xf(x)\,dx$ given the fact that $\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$.

by parts

$u = x$ $\quad dv = f(x)\,dx$

$du = 1\,dx$ $\quad v = \int x \cdot e^{-x^2}\,dx$

$\rightarrow v = \int x \cdot e^{-x^2}\,dx$ $= -\frac{1}{2}e^{-x^2}$

$u = -x^2 \rightarrow du = -2x\,dx \rightarrow dx = \frac{du}{-2x}$

$$\int x \cdot e^{u} \cdot \frac{du}{-2x} = -\frac{1}{2}\int e^{u}\,du = -\frac{1}{2}e^{u} = -\frac{1}{2}e^{-x^2}$$

$$-\frac{1}{2}x e^{-x^2}\Big|_0^\infty - \int_0^\infty 1 \cdot -\frac{1}{2}e^{-x^2}\,dx$$

$$-\frac{1}{2}x e^{-x^2}\Big|_0^\infty + \frac{1}{2}\int_0^\infty e^{-x^2}\,dx$$

$$-\frac{1}{2}x e^{-x^2}\Big|_0^\infty + \frac{1}{2}\cdot\frac{\sqrt{\pi}}{2}$$

$$F(\infty) - F(0) + \frac{\sqrt{\pi}}{4}$$

$$-\frac{1}{2}(\infty)\cdot e^{-\infty} - -\frac{1}{2}(0)e^{0} + \frac{\sqrt{\pi}}{4}$$

$$= \frac{\sqrt{\pi}}{4}$$

Q23-FRQ Lessons 4-6, 7-2, 7-7

A large vat is initially filled with a saltwater solution. A solution with a higher concentration of salt flows into the vat, and solution flows out of the vat at the same rate. The number of pounds of salt in the vat at time t minutes is modeled by the function A that satisfies the differential equation $\frac{dA}{dt} = 6 - 0.02A$. At time $t = 10$ minutes, the vat contains 50 pounds of salt.

$t_1 = 10$, $y_1 = 50 = A$

(a) Write an equation for the line tangent to the graph of A at $t = 10$. Use the tangent line to approximate the number of pounds of salt in the vat at time $t = 12$ minutes.

$y = m(t - t_1) + y_1$

$m = \frac{dA}{dt}\Big|_{t=10} = 6 - 0.02(50) = 5$

$* \quad y = 5(t - 10) + 50$

$* y(12) = 5(12 - 10) + 50 = 60$

$A(12) = 60$

(b) Show that $A(t) = 300 - 250e^{0.2 - 0.02t}$ satisfies the differential equation $\frac{dA}{dt} = 6 - 0.02A$ with initial condition $A(10) = 50$.

$* A(10) = 300 - 250\, e^{0.2 - 0.02(10)} = 50$

$* \frac{dA}{dt} = 0 - 250\, e^{0.2 - 0.02t} \cdot (-0.02)$

$= \boxed{5\, e^{0.2 - 0.02t}}$ #

$$* 6 - 0.02A = 6 - 0.02\left[300 - 295e^{0.2 - 0.02t}\right]$$

$$*6 - 0.02A = 6 - 6 + 5e^{0.2 - 0.02t}$$

$$= 5e^{0.2 - 0.02t}$$

$$L.S = R.S$$

(c) The flow of solution into the vat is stopped, and the solution is drained. The depth of solution in the vat is modeled by the function h that satisfies the differential equation $\frac{dh}{dt} = -k\sqrt{h}$, where $h(t)$ is measured in meters, t is the number of minutes since draining began, and k is a constant. If the depth of the solution is 16 meters at time $t = 0$ minutes and 4 meters at time $t = 30$ minutes, what is $h(t)$ in terms of t ?

$$\frac{dh}{dt} = -k\sqrt{h}$$

$$\int \frac{dh}{\sqrt{h}} = \int -k\,dt \rightarrow \int h^{-\frac{1}{2}}\,dh = \int -k\,dt$$

$$\frac{h^{\frac{1}{2}}}{\frac{1}{2}} = -kt + c$$

$$2\sqrt{h} = -kt + c$$

$t = 0$, $h = 16$

$$2\sqrt{16} = -k(0) + c \rightarrow \boxed{c = 8}$$

$$\frac{2}{2}\sqrt{h} = \frac{-kt}{2} + \frac{8}{2}$$

$$\sqrt{h} = -\frac{k}{2}t + 4 \Rightarrow \begin{matrix} h = 4 \\ t = 30 \end{matrix}$$

$$\sqrt{4} = \frac{-k}{2}(30) + 4$$

$$2 = -15k + 4 \rightarrow -2 = -15k$$

$$k = \frac{2}{15}$$

$$\sqrt{h} = \frac{-\frac{2}{15}}{2}t + 4$$

$$\sqrt{h} = -\frac{t}{15} + 4$$

$$h = \left(\frac{-t}{15} + 4\right)^2$$

3. Consider the differential equation $dy/dx = (y - 1)^2\cos(\pi x)$. Lessons 7-2, 7-3, 7-7

$m = 0$ ⟷ $m = +$ ↗ $m = -$ ↘

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

$\frac{dy}{dx} = m = (y-1)^2 \cos \pi x$

$*\ m(-1,1) = (1-1)^2 \cos(-\pi) = 0$ ⟷

$*\ m(0,1) = (1-1)^2 \cos(0) = 0$ ⟷

$*\ m(1,1) = (1-1)^2 \cos \pi = 0$ ⟷

$*\ m(-1,0) = (0-1)^2 \cos(-\pi) = -1$ ↘

$*\ m(0,0) = (0-1)^2 \cos(0) = 1$ ↗

وتكمل باقي النقاط

(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

$m = 0 = \frac{dy}{dx}$ $\quad (y-1)^2 \cos \pi x = 0$

$y - 1 = 0 \rightarrow y = 1 \rightarrow \boxed{y = C = 1}$

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

$\frac{dy}{dx} = (y-1)^2 \cos \pi x$ $\quad x = 1 \quad | \quad y = 0$

$\int \frac{dy}{(y-1)^2} = \int \cos \pi(x)\, dx$

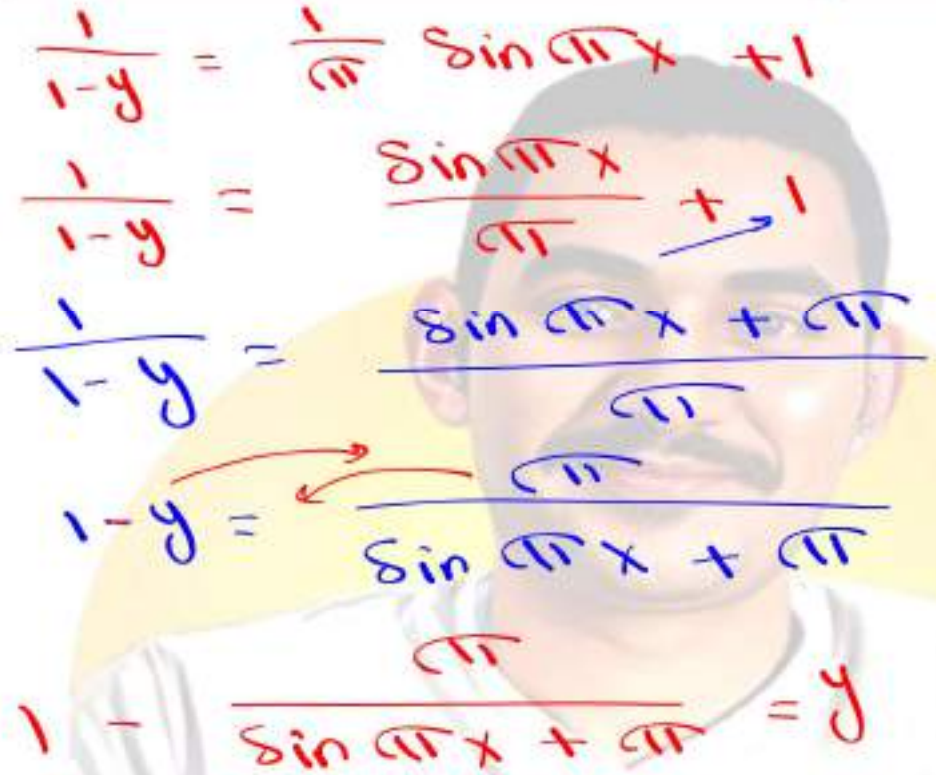
$\int (y-1)^{-2} dy = \frac{\sin \pi x}{\pi} + C$

$\frac{(y-1)^{-1}}{-1} = \frac{1}{\pi} \sin \pi x + C$

$-(y-1)^{-1} = \frac{1}{\pi} \sin \pi x + C$

$\frac{1}{1-y} = \frac{1}{\pi} \sin \pi x + C$ $\quad x = 1, y = 0$

$\frac{1}{1-0} = \frac{1}{\pi} \sin \pi + C$ $\quad \boxed{C = 1}$



Q24-FRQ Lessons 8-4, 8-7 , 8-10

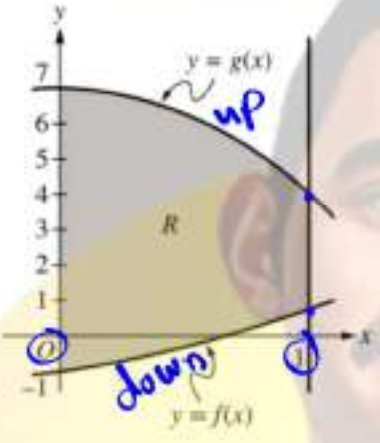


Figure 1

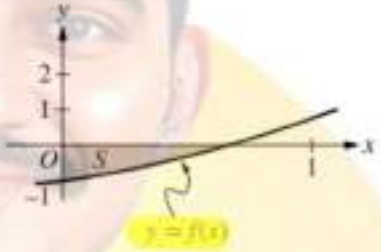


Figure 2

Let f and g be the functions given by $f(x) = e^x - 2$ and $g(x) = -3x^2 + 7$. Let R be the shaded region bounded by the graphs of f and g, the y-axis, and the vertical line $x = 1$, as shown in Figure 1. Let S be the shaded region in Quadrant IV bounded by the graph of f, the x-axis, and the y-axis, as shown in Figure 2.

(a) Find the area of R.

$$A = \int_0^1 g(x) - f(x)\, dx$$

$$A = \int_0^1 (-3x^2 + 7) - (e^x - 2)\, dx$$

$$= \int_0^1 (-3x^2 + 7 - e^x + 2)\, dx$$

$$= \int_0^1 -3x^2 - e^x + 9\, dx$$

$$= \int_0^1 -3x^2 - e^x + 9 \, dx$$

$$= \left(-\frac{3x^3}{3} - e^x + 9x \right) \Big|_0^1 = F(1) - F(0)$$

$$= \left[-(1)^3 - e^1 + 9(1) \right] - \left[-(0)^3 - e^0 + 9(0) \right]$$

$$= 8 - e + 1$$

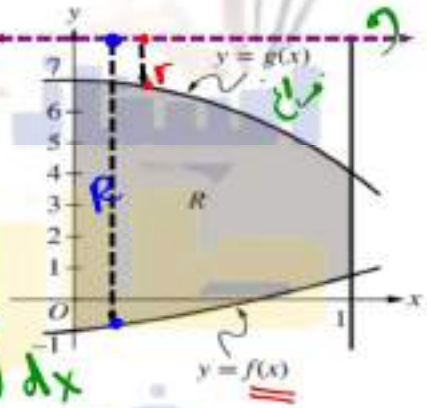
$$= \boxed{9 - e}$$

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 8$.

$y = 8$ → dx empty

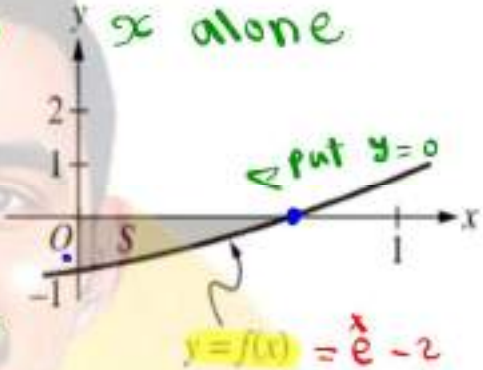
$$V = \pi \int_0^1 R^2 - r^2 \, dx$$

$$V = \pi \int_0^1 (8 - f(x))^2 - (8 - g(x))^2 \, dx$$



(c) The region S is the base of a solid. For this solid, each cross section perpendicular to the y-axis is a rectangle with height 5 and base in region S. Write, but do not evaluate, an integral expression that gives the volume of this solid.

Perpen → y-axis → dy → x alone



$y = e^x - 2$

$y + 2 = e^x$

نخرج ln مع الطرف الأيمن

$\ln(y+2) = x$

* $x = 0 \rightarrow y = e^0 - 2 = 1 - 2 = -1$

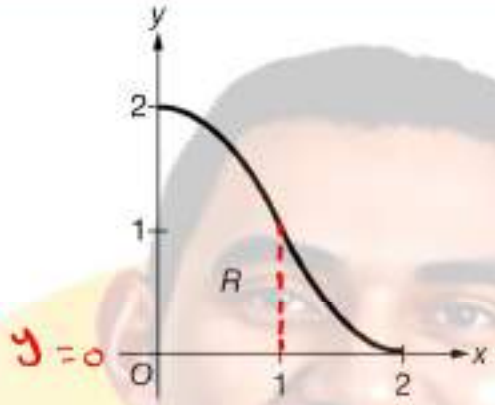
put $y = 0 \rightarrow 0 = e^x - 2 \rightarrow e^x = 2$

$x = \ln 2$

* $x = \ln 2 \rightarrow y = e^{\ln 2} - 2 = 2 - 2 = 0$

$$V = \int_{-1}^{0} h \cdot x \cdot dy$$

$$V = \int_{-1}^{0} 5 \ln(y+2)\, dy$$



Let f be the function defined by $f(x) = \begin{cases} 2 - x^3 & \text{for } 0 \le x < 1 \\ (x-2)^2 & \text{for } 1 \le x \le 2 \end{cases}$. Let R be the region in the first quadrant bounded by the graph of f and the x- and y-axes, as shown in the figure above.

(a) Find the area of R.

$$A = \int_0^1 (2 - x^2) - 0 \, dx + \int_1^2 (x-2)^2 - 0 \, dx$$

$$= 2x - \frac{x^3}{3}\Big|_0^1 + \frac{(x-2)^3}{3}\Big|_1^2$$

$$(F(1) - F(0)) + (F(2) - F(1))$$

$$\left(2(1) - \frac{1^3}{3}\right) - \left(2(0) - \frac{0^3}{3}\right) + \left(\frac{(2-2)^3}{3}\right) - \left(\frac{(1-2)^2}{3}\right) = \boxed{21}$$

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

perpen x-axis → dx → y alone

$$V = \int_0^1 A^2\,dx + \int_1^2 A^2\,dx$$

$$= \int_0^1 (2-x^2)^2\,dx + \int_1^2 (x-2)^4\,dx$$

(c) The portion of the region R for $1 \le x \le 2$ is revolved about the y-axis to form a solid. Find the volume of the solid.

y-axis → dy → x alone

$$*\sqrt{y} = \sqrt{(x-2)^2}$$

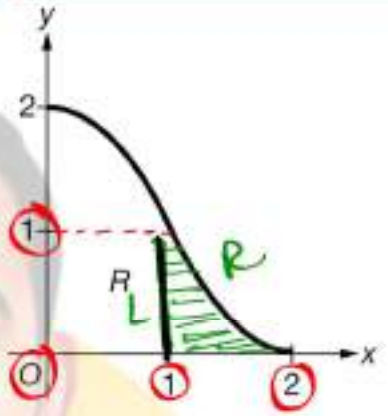
$$\sqrt{y} = x - 2 \rightarrow \boxed{x = \sqrt{y} + 2}$$

$x = 1 \rightarrow y = (1-2)^2 = 1$

$x = 2 \rightarrow y = (2-2)^2 = 0$

$$V = \pi \int_0^1 R^2 - L^2 \, dy$$

$$= \pi \int_0^1 (\sqrt{y}+2)^2 - 1^2 \, dy$$



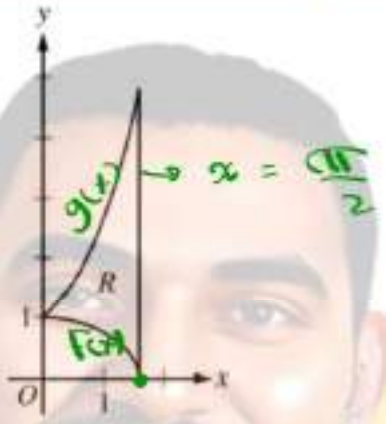
$$\pi \int_0^1 y + 4\sqrt{y} + 4 - 1 \, dy$$

$$\pi \int_0^1 y + 4y^{\frac{1}{2}} + 3 \, dy$$

$$\pi \left[\frac{y^2}{2} + \left(\frac{4y^{\frac{3}{2}}}{\frac{3}{2}} \right) + 3y \right]_0^1 = F(1) - F(0)$$

$$\pi \left[\left(\frac{1^2}{2} + \frac{2}{3} \cdot 4(1)^{\frac{3}{2}} + 3(1) \right) - \left(\frac{0^2}{2} + \frac{2}{3} \cdot 4(0)^{\frac{3}{2}} + 3(0) \right) \right]$$

$$V = \frac{37\pi}{6} \text{ cubic units}$$



Let R be the region in the first quadrant enclosed by the graph of $f(x) = \sqrt{\cos x}$, the graph of $g(x) = e^x$, and the vertical line $x = \frac{\pi}{2}$, as shown in the figure above.

6. (a) Write, but do not evaluate, an integral expression that gives the area of R.

$$A = \int_0^{\frac{\pi}{2}} g(x) - F(x)\, dx$$

$$A = \int_0^{\frac{\pi}{2}} e^x - \sqrt{\cos x}\, dx$$

(b) Find the volume of the solid generated when R is revolved about the x-axis.

$$V = \pi \int_0^{\frac{\pi}{2}} (g(x))^2 - (F(x))^2\, dx$$

$$= \pi \int_0^{\frac{\pi}{2}} e^{2x} - \cos x\, dx$$

$$V = \pi \left(\frac{e^{2x}}{2} - \sin x \right) \Big|_0^{\frac{\pi}{2}}$$

$$= F\left(\frac{\pi}{2}\right) - F(0)$$

$$= \pi \left[\left(\frac{e^{2 \cdot \frac{\pi}{2}}}{2} - \sin\frac{\pi}{2} \right) - \left(\frac{e^{2(0)}}{2} - \sin 0 \right) \right]$$

$$= \pi \left(\frac{e^{\pi}}{2} - 1 \right) - \left(\frac{1}{2} \right)$$

$$= \pi \left(\frac{e^{\pi}}{2} - \frac{3}{2} \right)$$

(c) Region R is the base of a solid whose cross sections perpendicular to the x-axis are semicircles with diameters on the xy-plane. Write, but do not evaluate, an integral expression that gives the volume of this solid.

dx

نصف دائرة

$$V = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \left(\frac{g(x) - f(x)}{2} \right)^2 dx$$

$$V = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \left(\frac{e^{x} - \sqrt{\cos x}}{2} \right)^2 dx$$

Q25-FRQ

1. Mighty Cable Company manufactures cable that sells for $120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

(a) Find Mighty's profit on the sale of a 25-meter cable.

$$P = 120(25) - \int_0^{25} 6\sqrt{x}\,dx = 2500\,\$$$

$$= 3000 - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^{25}$$

$$= 3000 - 4x^{\frac{3}{2}}\Big|_0^{25}$$

$$= 3000 - \left[4(25)^{\frac{3}{2}} - 4(0)^{\frac{3}{2}}\right]$$

$$= 3000 - 500 = 2500\,\$$$

(b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\,dx$ in the context of this problem.

This means cost in $ for 30 m cable and 25 m cable.

(c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

$$P = 120(k) - \int_0^k 6\sqrt{x}\,dx$$

(d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

$$\dot{P}(k) = 0$$

$$P(k) = 120k - \int_0^k 6\sqrt{x}\,dx$$

$$\dot{P}(k) = 120 - 6\sqrt{k} = 0$$

$$\frac{6\sqrt{k}}{6} = \frac{120}{6}$$

$$(\sqrt{k})^2 = (20)^2 \rightarrow \boxed{k = 400}$$

maximum $P(x) = P(400)$

$= 120(400) - \int_0^{400} 6\sqrt{x}\,dx$

$= 16\,000\$$

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

2. How many gallons of water leak out of the tank from $t = 0$ to $t = 3$ minutes?

③ $\int_0^3 \sqrt{t+1}\,dt = \int_0^3 (t+1)^{\frac{1}{2}}\,dt$ ⓪

$\left.\frac{(t+1)^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^3 = \left.\frac{2}{3}(t+1)^{\frac{3}{2}}\right|_0^3$

$F(3) - F(0)$

$= \frac{2}{3}\left[(3+1)^{\frac{3}{2}} - (0+1)^{\frac{3}{2}}\right] = \frac{14}{3}$

A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \frac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

4. Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

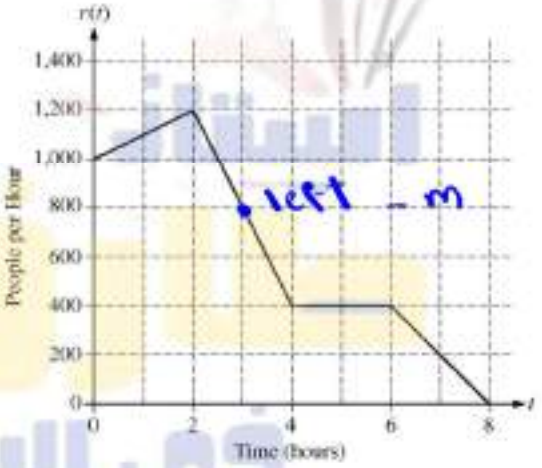
$$12000 - \int_0^A r(t)\,dt = 9000$$

There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

5. Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

* rate = slope $= mt$
$= 800t$

$$700 - 800t + \int_0^t r(t)\,dt$$



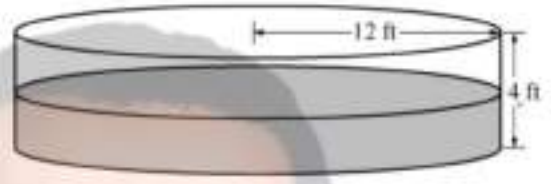
t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55° F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

8. Use the data in the table to evaluate $\int_0^{20} W'(t)dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)dt$ in the context of this problem.

$$\int_0^{20} \acute{w}(t)\,dt = w(\text{upper}) - w(\text{lower})$$
$$= w(20) - w(0)$$
$$= 71 - 55$$
$$= 16$$

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an above ground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t=0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t)=25e^{-0.05t}$.

(Note: The volume V of a cylinder with radius r and height h is given by $V=\pi r^2 h$.)

14. Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t=12$ hours. Round your answer to the nearest cubic foot.

$$1000 + \int_0^{12} P(t)\,dt - \int_0^{12} R(t)\,dt$$

$$1000 + \frac{1}{2}h\left[F_{x_1} + F_{x_n} + 2(F_{x_2} + \dots)\right] - \int_0^{12} R(t)\,dt$$

$$h = \Delta x = \frac{b-a}{n} = \frac{12-0}{6} = 2$$

$$1000 + \frac{1}{2}(2)\left[0 + 63 + 2[46+53+57+60+62]\right] - \int_0^{12} 25e^{-0.05t}\,dt = 1393.4 \approx 1400\ Ft^3$$

A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.

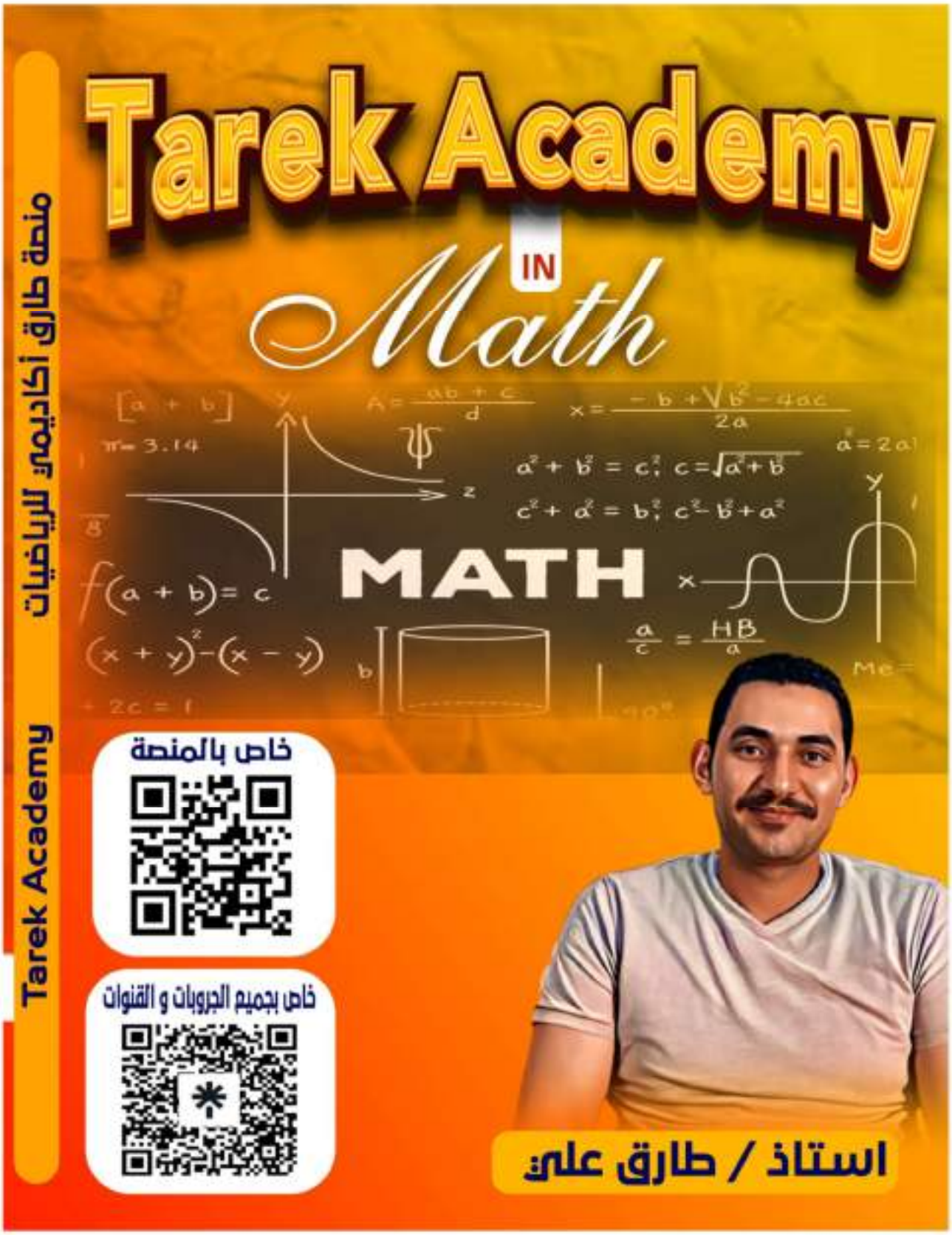
15. After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

$$\int_0^x g(p)\,dp = -\int_0^5 f(t)\,dt$$

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time.(Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

31. By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found previously.

$$V = 60\,000 + \int_0^t 2000 - f(t)\,dt \qquad t = ?$$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