حل تجميعة أسئلة وفق مخرجات الهيكل الوزاري (من 11 إلى 12) القسم الالكتروني

المزيد من الملفات بحسب الصف الثاني عشر المتقدم والمادة رياضيات في الفصل الثالث	
1	حل تجميعة أسئلة وفق مخرجات الهيكل الوزاري (من 6 إلى 10) القسم الالكتروني
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11	Integrate functions of the form $\sec^n x \tan^m x$ / $\sec^n x \tan^m x$ إيجاد تكاملات دوال بصيغة	Exercises (9-20)	P507

Evaluate the integral. جـــد قيمة التكامل

9) $\int \tan x \sec^3 x \, dx$

Calc

$\int \sec^2 x \cdot \sec x \tan x \, dx$

$= \int u^2 \, du$

$= \frac{1}{3} u^3 + C$

$= \frac{1}{3} \sec^3 x + C$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\int [\sec x]^2 \sec x \tan x \, dx$

$= \frac{1}{3} \sec^3 x + C$

$\frac{d}{dx}(\quad)_{x=2}$ 30.319

A) $\frac{1}{3}\sec^3 x + c$

B) $\frac{1}{2}\tan^2 x + c$

C) $\frac{2}{3}\sec^3 x + c$

D) $\sec x \tan x + c$

Radian

$\int [f(x)]^n \cdot f'(x) \, dx = \frac{[f(x)]^{n+1}}{n+1} + C$

+201003261312

Evaluate the integral. جـــد قيمة التكامل

10) $\int \cot x \csc^4 x \, dx$

$= \int \csc^3 x \cdot \cot x \csc x \, dx$

$= -\int [\csc x]^3 [-\cot x \csc x] \, dx$

$= -\frac{1}{4} \csc^4 x + C$

$\int [f(x)]^n \cdot f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C$

$\int \csc^3 x \cdot \csc x \cot x \, dx$

$= -\int u^3 \, du$

$= -\frac{1}{4} u^4 + C$

$= -\frac{1}{4} \csc^4 x + C$

$u = \csc x$

$du = -\csc x \cot x \, dx$

+201003261312

Evaluate the integral. جـــد قيمة التكامل

11) $\int x \tan^3(x^2+1)\sec(x^2+1)\,dx$

odd

$t = x^2+1 \Rightarrow dt = 2x\,dx \Rightarrow dx = \frac{dt}{2x}$

$\int x \tan^3 t \sec t \cdot \frac{dt}{2x}$

$= \frac{1}{2}\int \tan^3 t \sec t\, dt$

$= \frac{1}{2}\int \tan^2 t \sec t \tan t\, dt$

$= \frac{1}{2}\int (\sec^2 t - 1)\sec t \tan t\, dt$

$= \frac{1}{2}\int (u^2-1)\,du = \frac{1}{2}\left[\frac{1}{3}u^3 - u\right] + C$

$= \frac{1}{6}\sec^3 t - \frac{1}{2}\sec t + C$

$= \frac{1}{6}\sec^3(x^2+1) - \frac{1}{2}\sec(x^2+1) + C$

$u = \sec t$

$du = \sec t \tan t\, dx$

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

A) $\frac{1}{6}\sec^3(x^2+1) + \frac{1}{2}\sec(x^2+1) + c$

B) $\frac{1}{2}\sec^3(x^2+1) + \frac{1}{6}\sec(x^2+1) + c$

C) $\frac{1}{6}\sec^3(x^2+1) - \frac{1}{2}\sec(x^2+1) + c$

D) $-\frac{1}{6}\sec^3(x^2+1) - \frac{1}{2}\sec(x^2+1) + c$

Evaluate the integral. جـــد قيمة التكامل

12) $\int \tan(2x+1)\sec^3(2x+1)\,dx$

$\theta = 2x+1 \Rightarrow d\theta = 2\,dx \Rightarrow dx = \frac{1}{2}d\theta$

$\frac{1}{2}\int \tan\theta \sec^3\theta\, d\theta$

$= \frac{1}{2}\int [\sec\theta]^2 \sec\theta\tan\theta\, d\theta$

$= \frac{1}{2}\cdot\frac{1}{3}\sec^3\theta + C$

$= \frac{1}{6}\sec^3(2x+1) + C$

$\int [f(x)]^n \cdot f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C$

A) $\frac{1}{6}\sec^3(2x+1) + c$

B) $\frac{1}{2}\sec^3(2x+1) + c$

C) $\frac{1}{3}\sec^3(x^2+1) + c$

D) $6\sec^3(x^2+1) + c$

Evaluate the integral.

جـــد قيمة التكامل

13) $\int \cot^{2}x \csc^{4}x \, dx$ (even, even)

$\int \cot^2 x \csc^2 x \csc^2 x \, dx$

$= \int \cot^2 x (1+\cot^2 x) \csc^2 x \, dx$

$u = \cot x \Rightarrow du = -\csc^2 x \, dx$

$= \int u^2(1+u^2)(-du)$

$= \int (-u^2 - u^4) \, du$

$= -\frac{1}{3}u^3 - \frac{1}{5}u^5 + C = -\frac{1}{3}\cot^3 x - \frac{1}{5}\cot x + C$

$\tan \rightarrow \sec^2 x$

$1 + \cot^2\theta = \csc^2\theta$

A) $\frac{1}{3}\cot^3 x - \frac{1}{5}\cot^5 x + c$

B) $-\frac{1}{3}\cot^3 x - \frac{1}{5}\cot^5 x + c$

C) $\frac{1}{3}\cot^3 x + c$

D) $-\frac{1}{3}\cot^3 x + c$

Evaluate the integral.

جـــد قيمة التكامل

14) $\int \cot^2 x \csc^2 x \, dx$

$\int \cot^2 x \csc^2 x \, dx$

$= -\int [\cot x]^2 (-\csc^2 x \, dx)$

$= -\frac{1}{3}\cot^3 x + C$

$\int [F(x)]^n \cdot F'(x) \, dx = \frac{[F(x)]^{n+1}}{n+1} + C$

A) $\frac{1}{3}\cot^3 x - \frac{1}{5}\cot^5 x + c$

B) $-\frac{1}{3}\cot^3 x - \frac{1}{5}\cot^5 x + c$

C) $\frac{1}{3}\cot^3 x + c$

D) $-\frac{1}{3}\cot^3 x + c$

Evaluate the integral. جـــد قيمة التكامل

15) $\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx$

even even

$\int_0^{\pi/4} \tan^4 x \sec^2 x \sec^2 x \, dx$

$x=\frac{\pi}{4} \Rightarrow u=\tan\frac{\pi}{4}=1$, $x=0 \Rightarrow u=\tan 0=0$

$= \int_0^{\pi/4} \tan^4 x (1+\tan^2 x) \sec^2 x \, dx$

$u = \tan x \Rightarrow du = \sec^2 x \, dx$

$= \int_0^1 u^4 (1+u^2) \, du = \int_0^1 u^4 + u^6 \, du$

$= \frac{1}{5}u^5 + \frac{1}{7}u^7 \Big|_0^1 = \left(\frac{1}{5}+\frac{1}{7}\right) - (0) = \frac{12}{35}$

$\left(\frac{1}{5}\tan^5 x + \frac{1}{7}\tan^7 x\right)\Big|_0^{\pi/4}$

A) $\left(\frac{1}{5}\tan^5 x + \frac{1}{7}\tan^7 x\right)\Big|_0^{\pi/4}$

B) $\left(\frac{1}{5}\tan^5 x - \frac{1}{7}\tan^7 x\right)\Big|_0^{\pi/4}$

C) $\frac{12}{35}$

D) $\frac{11}{35}$

Evaluate the integral. جـــد قيمة التكامل

16) $\int_{-\pi/4}^{\pi/4} \tan^4 x \sec^2 x \, dx$

even

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} [\tan x]^4 \sec^2 x \, dx$

$= \frac{1}{5}\tan^5 x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$= \frac{1}{5}\left[[\tan\frac{\pi}{4}]^5 - [\tan\frac{-\pi}{4}]^5\right]$

$= \frac{1}{5}[1-(-1)] = \frac{2}{5}$

A) $\left(\frac{1}{5}\tan^5 x\right)\Big|_{-\pi/4}^{\pi/4}$

B) $\left(\frac{1}{5}\tan^5 x - \frac{1}{7}\tan^7 x\right)\Big|_{-\pi/4}^{\pi/4}$

C) $\frac{2}{5}$

D) $\frac{1}{5}$

Determine the value of m if: $\int \tan x \sec^m x \, dx = \frac{1}{3}\sec^3 x + c$

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) 2

D) 3

$\frac{d}{dx}\left[\frac{1}{3}[\sec x]^3 + c\right]$

$= \frac{1}{3} \cdot 3[\sec x]^2 \cdot \sec x \tan x$ chain Rule

$= \sec x \sec x \tan x$

$= \sec^3 x \tan x$

$m = 3$

$\int \sec^{m-1} x \sec x \tan x \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\int u^{m-1} du = \frac{u^m}{m} + C$

$= \frac{1}{m} u^m + C$

$= \frac{1}{m} \sec^m x + C$

$m = 3$

If : $\int \tan^3 x \sec x \, dx = \frac{1}{n}\sec^n x - \sec x + c$ then the value of n is

A) $\frac{1}{3}$

B) 3

C) $\frac{1}{4}$

D) 4

$\int \tan^2 x \sec x \tan x \, dx$

$= \int (\sec^2 x - 1) \sec x \tan x \, dx$

$= \int u^2 - 1 \, du$

$= \frac{1}{3}u^3 - u + C$

$= \frac{1}{3}\sec^3 x - \sec x + C$

$\tan^2 x = \sec^2 x - 1$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\frac{1}{n} = \frac{1}{3} \Rightarrow n = 3$

$\sec^n x = \sec^3 x \quad n = 3$

If : $\int \sec^4 x\, dx = \frac{1}{3} f(x) + c$ then

even

$\sec^2 x = 1 + \tan^2 x$

A) $f(x) = \tan x + 3\tan^3 x$

B) $f(x) = 3\tan x + \tan^3 x$

C) $f(x) = 3\tan x - \tan^3 x$

D) $f(x) = \tan x - 3\tan^3 x$

$\int \sec^2 x \quad \sec^2 x\, dx$

$= \int (1 + \tan^2 x)\ \sec^2 x\, dx$

$u = \tan x \qquad du = \sec^2 x\, dx$

$= \int 1 + u^2\, du$

$= u + \frac{1}{3}u^3 + C$

$= \tan x + \frac{1}{3}\tan^3 x + C$

$= \frac{1}{3}\left[3\tan x + \tan^3 x\right] + C$

$\frac{1}{3} : 3$

Evaluate the integral.

جـــد قيمة التكامل

17) $\int \cos^2 x \sin^2 x\, dx$

$\cos^2 x = \left(\frac{1+\cos 2x}{2}\right)$

$\sin^2 x = \left(\frac{1-\cos 2x}{2}\right)$

$= \int \frac{1+\cos 2x}{2} \cdot \frac{1-\cos 2x}{2}\, dx$

$= \frac{1}{4}\int 1 - \cos^2 2x\, dx$

$= \frac{1}{4}\int 1 - \frac{1}{2}(1+\cos 4x)\, dx$

$= \frac{1}{4}\int 1 - \frac{1}{2} - \frac{1}{2}\cos 4x\, dx$

$= \int \frac{1}{8} - \frac{1}{8}\cos 4x\, dx$

$= \frac{1}{8}x - \frac{1}{8}\cdot\frac{1}{4}\sin 4x + C$

$= \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

18) $\int (\cos^2 x + \sin^2 x)\, dx$

$= \int 1\, dx$

$= x + C$

$\int 4\cos^2 x + 4\sin^2 x\, dx$

1 STEP

$\int (\cos x \sin x)^2$

$\sin 2A = 2\sin A \cos A$

$\int \left(\frac{\sin 2x}{2}\right)^2$

$= \frac{1}{4}\int \frac{1}{2}(1-\cos 4x)\, dx$

A) $\frac{1}{4}x - \frac{1}{32}\sin 4x + c$

B) $\frac{1}{4}x + \frac{1}{32}\sin 4x + c$

C) $\frac{1}{8}x - \frac{1}{32}\sin 4x + c$

D) $\frac{1}{8}x + \frac{1}{32}\sin 4x$

Evaluate the integral. جـــد قيمة التكامل

19) $\int_{-\pi/3}^{0} \sqrt{\cos x}\sin^3 x\ dx$

$= \int_{-\pi/3}^{0} \sqrt{\cos x}\,\sin^2 x \sin x\,dx$ $\quad \sin^2 x = 1-\cos^2 x$

Let $u = \cos x, du = -\sin x dx$

$\int_{-\pi/3}^{0} \sqrt{\cos x}\sin^3 x dx = \int_{-\pi/3}^{0} \sqrt{\cos x}(1-\cos^2 x)\sin x dx$

$u = \cos x$

$du = -\sin x dx$

$= \int_{1/2}^{1} \sqrt{u}(1-u^2)(-du) = \int_{1/2}^{1} (u^{5/2} - u^{1/2})du$

$= \left[\frac{2}{7}u^{7/2} - \frac{2}{3}u^{3/2}\right]\Big|_{1/2}^{1} = \frac{25}{168}\sqrt{2} - \frac{8}{21}$

$\frac{2}{7}\cos^{7/2}x - \frac{2}{3}\cos^{3/2}x \Big|_{-\pi/3}^{0}$

A) $\left(\frac{2}{7}\cos^{7/2}x - \frac{2}{3}\cos^{3/2}x\right)\Big|_{-\pi/3}^{0}$

B) $\left(\frac{2}{7}\cos^{7/2}x + \frac{2}{3}\cos^{3/2}x\right)\Big|_{-\pi/3}^{0}$

C) $\left(-\frac{2}{7}\cos^{7/2}x - \frac{2}{3}\cos^{3/2}x\right)\Big|_{-\pi/3}^{0}$

D) $\left(-\frac{2}{7}\cos^{7/2}x + \frac{2}{3}\cos^{3/2}x\right)\Big|_{-\pi/3}^{0}$

Evaluate the integral. جـــد قيمة التكامل

20) $\int_{\pi/4}^{\pi/2} \cot^2 x \csc^4 x\ dx$ (even, even)

$= \int \cot^2 x \csc^2 x \csc^2 x\,dx$ $\quad \csc^2 x\,dx$

Let $u = \cot x, \quad du = -\csc^2 x dx$

$\int_{\pi/4}^{\pi/2} \cot^2 x \csc^4 x dx = \int_{\pi/4}^{\pi/2} \cot^2 x \csc^2 x \csc^2 x dx$

$= \int_{\pi/4}^{\pi/2} \cot^2 x(1+\cot^2 x)\csc^2 x dx = -\int_{1}^{0} u^2(1+u^2)du$

$= -\left[\frac{u^3}{3} + \frac{u^5}{5}\right]\Big|_{1}^{0} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

$-\frac{1}{3}\cot^3 x - \frac{1}{5}\cot^5 x + C$

A) $\frac{8}{17}$

B) $\frac{8}{15}$

C) $\frac{15}{8}$

D) $\frac{8\pi}{15}$

12	Integrate trigonometric functions using the substitution: $x = a\tan(\theta)$ إيجاد تكاملات دوال مثلثية باستخدام التعويض بـ $x = a\tan(\theta)$	Exercises (25-40)	P507

Evaluate the integral. جـــد قيمة التكامل

25. $\int_0^2 \sqrt{4-x^2}\,dx$

This is the area of a quarter of a circle of radius 2,

$= \frac{1}{4}$ Circle Area

$= \frac{1}{4}\cdot \pi r^2$

$= \frac{1}{4}\pi(2)^2$

$= \pi$

$\int_0^2 \sqrt{4-x^2}dx = \pi$

$\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta$

$x = 2\sin\theta \quad dx = 2\cos\theta\,d\theta$

$\sqrt{4-x^2} = 2\cos\theta$

$\int 2\cos\theta \cdot 2\cos\theta\,d\theta$

$= 4\int \cos^2\theta\,d\theta$

$= 4\int \frac{1}{2}(1+\cos 2\theta)\,d\theta$

$y = \sqrt{4-x^2}$

$y^2 = 4-x^2$

$y^2 + x^2 = 4$

Circle $r = \sqrt{4} = 2$

Center $(0,0)$

A) π

B) $\frac{\pi}{3}$

C) $2-\sqrt{3}$

D) $2+\sqrt{3}$

A) $4\int \cos^2\theta\,d\theta$

Evaluate the integral. جـــد قيمة التكامل

26. $-\frac{1}{2}\int_0^1 \frac{-2x}{\sqrt{4-x^2}}\,dx$

Let $u = 4-x^2$, $du = -2x\,dx$

$\int_0^1 \frac{x}{\sqrt{4-x^2}}dx = -\int_4^3 \frac{du}{2\sqrt{u}}$

$= -u^{1/2}\Big|_4^3$

$= 2-\sqrt{3}$

$= -\frac{1}{2}\cdot 2\sqrt{4-x^2}\Big|_0^1$

$= -\sqrt{4-x^2}\Big|_0^1$

$= -[\sqrt{4-1} - \sqrt{4}]$

$= 2-\sqrt{3}$

$x = 2\sin\theta \quad \sin\theta = \frac{x}{2}$

$dx = 2\cos\theta\,d\theta$

$\sqrt{4-x^2} = 2\cos\theta$

$\int \frac{2\sin\theta}{2\cos\theta}\cdot 2\cos\theta\,d\theta$

$= -2\cos\theta + c$

$= -2\left(\frac{\sqrt{4-x^2}}{2}\right)\Big|_0^1$

A) π

B) $\frac{\pi}{3}$

C) $2-\sqrt{3}$

D) $2+\sqrt{3}$

Evaluate the integral.

27. $\int \frac{x^2}{\sqrt{x^2-9}}\,dx$

جـــد قيمة التكامل

$\sqrt{x^2-a^2} \Rightarrow x = a\sec\theta$

$x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

A) $\frac{1}{2}x\sqrt{x^2-9} + \frac{9}{2}\ln\left|\frac{x+\sqrt{x^2-9}}{3}\right| + c$

B) $\frac{1}{2}x\sqrt{x^2-9} - \frac{9}{2}\ln\left|\frac{x+\sqrt{x^2-9}}{3}\right| + c$

C) $\frac{1}{2}\sqrt{x^2-9} + \frac{9}{2}\ln\left|\frac{x+\sqrt{x^2-9}}{3}\right| + c$

D) $\frac{3}{2}\sqrt{x^2-9} + \frac{9}{2}\ln\left|\frac{x+\sqrt{x^2-9}}{3}\right| + c$

$\int 9\sec^3\theta\,d\theta$

Let $x = 3\sec\theta$, $dx = 3\sec\theta\tan\theta d\theta$.

$I = \int \frac{x^2}{\sqrt{x^2-9}}dx = \int \frac{27\sec^2\theta\sec\theta\tan\theta}{\sqrt{9\sec^2\theta-9}}d\theta$

$= \int 9\sec^3\theta d\theta$

Use integration by parts.

Let $u = \sec\theta$ and $dv = \sec^2\theta d\theta$. This gives

$\int \sec^3\theta d\theta = \sec\theta\tan\theta - \int \sec\theta\tan^2\theta d\theta$

$= \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)d\theta$

$= \sec\theta\tan\theta + \int \sec\theta d\theta - \int \sec^3\theta d\theta$

$2\int \sec^3\theta d\theta = \sec\theta\tan\theta + \int \sec\theta d\theta$

$\int \sec^3\theta d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\int \sec\theta\,d\theta$

This leaves us to compute $\int \sec\theta d\theta$.

For this notice if $u = \sec\theta + \tan\theta$ then $du = \sec\theta\tan\theta + \sec^2\theta$.

$\int \sec\theta d\theta = \int \frac{\sec\theta(\sec\theta+\tan\theta)}{\sec\theta+\tan\theta}d\theta$

$= \int \frac{1}{u}du = \ln|u| + c$

$= \ln|\sec\theta + \tan\theta| + c$

Putting all these together and using $\sec\theta = \frac{x}{3}$, $\tan\theta = \frac{\sqrt{x^2-9}}{3}$:

$\int \frac{x^2}{\sqrt{x^2-9}}dx = \int 9\sec^3\theta\,d\theta$

$= \frac{9}{2}\sec\theta\tan\theta + \frac{9}{2}\int \sec\theta\,d\theta$

$= \frac{9}{2}\sec\theta\tan\theta + \frac{9}{2}\ln|\sec\theta+\tan\theta| + c$

$= \frac{9}{2}\left(\frac{x}{3}\right)\left(\frac{\sqrt{x^2-9}}{3}\right) + \frac{9}{2}\ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right| + c$

$= \frac{x\sqrt{x^2-9}}{2} + \frac{9}{2}\ln\left|\frac{x+\sqrt{x^2-9}}{3}\right| + c$

$\int \sec^3\theta\,d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta| + C$

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Evaluate the integral.

28. $\int x^3\sqrt{x^2-1}\,dx$

جـــد قيمة التكامل

$x = \sec\theta$ $\quad dx = \sec\theta\tan\theta\,d\theta$ $\quad \sqrt{x^2-1} = \tan\theta$

$I = \int \sec^3\theta \cdot \tan\theta \cdot \sec\theta\tan\theta\,d\theta$

$= \int \tan^{\text{even}(2)}\theta\,\sec^{\text{even}(4)}\theta\,d\theta$

$= \int \tan^2\theta\,\sec^2\theta\,\sec^2\theta\,d\theta \quad | \sec^2\theta = 1+\tan^2\theta$

$= \int \tan^2\theta(1+\tan^2\theta)\sec^2\theta\,d\theta \quad u = \tan\theta,\ du = \sec^2\theta\,d\theta$

$= \int u^2(1+u^2)du = \int u^2+u^4\,du = \frac{1}{3}u^3 + \frac{1}{5}u^5 + C$

$= \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta + C = \frac{1}{3}(x^2-1)^{3/2} + \frac{1}{5}(x^2-1)^{5/2} + C$

A) $\frac{1}{5}(x^2-1)^{5/2} - \frac{1}{3}(x^2-1)^{3/2} + c$

B) $\frac{1}{3}(x^2-1)^{3/2} - \frac{1}{5}(x^2-1)^{5/2} + c$

C) $\frac{1}{5}(x^2-1)^{5/2} + \frac{1}{3}(x^2-1)^{3/2} + c$

D) $\frac{2}{5}(x^2-1)^{5/2} + \frac{2}{3}(x^2-1)^{3/2} + c$

+201003261312

Evaluate the integral. **29.** $\int \frac{2}{\sqrt{x^2-4}}\,dx$ جـــد قيمة التكامل

$x = 2\sec\theta$ $\left[\sec\theta = \frac{x}{2}\right]$

$dx = 2\sec\theta\tan\theta\,d\theta$, $\sqrt{x^2-4} = 2\tan\theta$

$$I = \int \frac{2}{2\tan\theta}\cdot 2\sec\theta\tan\theta\,d\theta$$

$$= \int 2\sec\theta\,d\theta = 2\ln|\sec\theta+\tan\theta| + C$$

$$= 2\ln\left|\frac{x}{2}+\frac{\sqrt{x^2-4}}{2}\right| + C = 2\ln\left|\frac{x+\sqrt{x^2-4}}{2}\right| + C$$

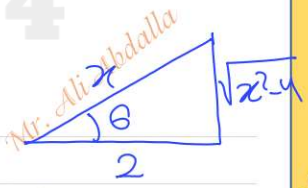
$$= 2\ln|x+\sqrt{x^2-4}| - 2\ln 2 + C = 2\ln|x+\sqrt{x^2-4}| + C$$

A) $2\ln|x+\sqrt{x^2-4}|+c$
B) $2\ln|x-\sqrt{x^2-4}|+c$
C) $\sqrt{x^2-4}+c$
D) $\frac{1}{2}x+2\sqrt{x^2-4}$

Evaluate the integral. **30.** $\int \frac{x}{\sqrt{x^2-4}}\,dx$ جـــد قيمة التكامل

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$, $\sqrt{x^2-4} = 2\tan\theta$



$$I = \int \frac{2\sec\theta}{2\tan\theta}\cdot 2\sec\theta\tan\theta\,d\theta$$

$$= 2\int \sec^2\theta\,d\theta$$

$$= 2\tan\theta + C$$

$$= 2\left(\frac{\sqrt{4-x^2}}{2}\right) + C$$

$$= \sqrt{4-x^2} + C$$

بالتعويض

$$\frac{1}{2}\int \frac{2x}{\sqrt{x^2-4}}\,dx$$

$$= \frac{1}{2}\cdot 2\sqrt{x^2-4} + C$$

$$= \sqrt{x^2-4} + C$$

A) $2\ln|x+\sqrt{x^2-4}|+c$
B) $2\ln|x-\sqrt{x^2-4}|+c$
C) $\sqrt{x^2-4}+c$
D) $\frac{1}{2}x+2\sqrt{x^2-4}$

Evaluate the integral.

31. $\int \frac{\sqrt{4x^2-9}}{x}\,dx$

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$\frac{\sqrt{(2x)^2-9}}{x}$

$\int \frac{3\tan\theta}{\frac{3}{2}\sec\theta} \cdot \frac{3}{2}\sec\theta\tan\theta\,d\theta$

$= 3\int \tan^2\theta\,d\theta$

$= 3\int (\sec^2\theta - 1)\,d\theta$

$= 3\tan\theta - 3\theta + C$

$= 3\left(\frac{\sqrt{4x^2-9}}{3}\right) - 3\cos^{-1}\left(\frac{3}{2x}\right)$

$\sec^{-1}\left(\frac{2x}{3}\right)$

$\theta = \sec^{-1}\left(\frac{2x}{3}\right)$

$\theta = \cos^{-1}\left(\frac{3}{2x}\right)$

$\cos\theta = \frac{3}{2x}$

$\sec\theta = \frac{2x}{3}$

$2x = 3\sec\theta$

$2\,dx = 3\sec\theta\tan\theta\,d\theta$

$\sqrt{4x^2-9} = 3\tan\theta$

A) $\sqrt{4x^2-9} - 3\cos^{-1}\left(\frac{2x}{3}\right) + c$

B) $\sqrt{4x^2-9} + 3\cos^{-1}\left(\frac{2x}{3}\right) + c$

C) $\sqrt{4x^2-9} - 3\sec^{-1}\left(\frac{2x}{3}\right) + c$

D) $3\sqrt{4x^2-9} - 3\sec^{-1}\left(\frac{2x}{3}\right) + c$

Evaluate the integral.

32. $\int \frac{\sqrt{x^2-4}}{x^2}\,dx$

جـــد قيمة التكامل

$x = 2\sec\theta$, $\sqrt{x^2-4} = 2\tan\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$I = \int \frac{2\tan\theta}{4\sec^2\theta} \cdot 2\sec\theta\tan\theta\,d\theta$

$= \int \frac{\tan^2\theta}{\sec\theta}\,d\theta = \int \frac{\sec^2\theta - 1}{\sec\theta}\,d\theta$

$= \int \sec\theta - \cos\theta\,d\theta$

$= \ln|\sec\theta + \tan\theta| - \sin\theta + C$

$= \ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| - \frac{\sqrt{x^2-4}}{x} + C$

$x = 2\sec\theta \Rightarrow \sec\theta = \frac{x}{2}$ (Adj / Hyp)

A) $\ln|x + \sqrt{x^2-4}| - \frac{\sqrt{x^2-4}}{x} + c$

B) $\ln|x + \sqrt{x^2-4}| + \frac{\sqrt{x^2-4}}{x} + c$

C) $\ln\left|\frac{x}{2} + \sqrt{x^2-4}\right| - \frac{\sqrt{x^2-4}}{4} + c$

D) $\ln|x + \sqrt{x^2-4}| - \frac{\sqrt{x^2-4}}{4} + c$

Evaluate the integral.

جـــد قيمة التكامل

$$\int \sec^n x \ dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x \ dx$$

33) $\int \frac{x^2}{\sqrt{9+x^2}}\,dx$

$x = 3\tan\theta$

$\sqrt{9+x^2} = 3\sec\theta$

$dx = 3\sec^2\theta\, d\theta$

Let $x = 3\tan\theta, dx = 3\sec^2\theta d\theta$

$$\int \frac{x^2}{\sqrt{9+x^2}}dx = \int \frac{27\tan^2\theta\sec^2\theta}{\sqrt{9+9\tan^2\theta}}d\theta = \int 9\tan^2\theta\sec\theta d\theta = 9\int(\sec^2\theta - 1)\sec\theta d\theta$$

$$= 9\int \sec^3\theta d\theta - 9\int \sec\theta d\theta = \frac{9}{2}\sec\theta\tan\theta - \frac{9}{2}\ln|\sec\theta+\tan\theta| + c$$

$$= \frac{9}{2}\left(\frac{\sqrt{9+x^2}}{3}\right)\left(\frac{x}{3}\right) - \frac{9}{2}\ln\left|\frac{\sqrt{9+x^2}}{3}+\frac{x}{3}\right| + c$$

$$= \frac{x\sqrt{9+x^2}}{2} - \frac{9}{2}\ln\left|\frac{x+\sqrt{9+x^2}}{3}\right| + c$$

even (2) odd (1)

$9\int \tan^2\theta \sec\theta$

$9\int(\sec^2\theta - 1)\sec\theta\, d\theta$

$9\left[\int \sec^3\theta d\theta - \int \sec\theta d\theta\right]$

$\int \sec\theta d\theta = \ln|\sec\theta+\tan\theta| + C_1$

$\int \sec^3\theta d\theta = \frac{1}{2}\ln|\sec\theta+\tan\theta| + \frac{1}{2}\sec\theta\tan\theta + C_2$

A) $\frac{x\sqrt{9+x^2}}{2} - \frac{9}{2}\ln\left|\frac{x+\sqrt{9+x^2}}{3}\right| + c$

B) $\frac{x\sqrt{9+x^2}}{2} + \frac{9}{2}\ln\left|\frac{x+\sqrt{9+x^2}}{3}\right| + c$

C) $\frac{x\sqrt{9+x^2}}{2} + \frac{5}{2}\ln\left|\frac{x+\sqrt{9+x^2}}{3}\right| + c$

D) $\frac{x\sqrt{9+x^2}}{2} - \frac{5}{2}\ln\left|\frac{x+\sqrt{9+x^2}}{3}\right| + c$

The appropriate change of variables for the integral $\int \frac{x^2}{\sqrt{9+x^2}}dx$ is

$x = 3\tan x$

A) $x = 9\tan\theta$
B) $x = 3\sin\theta$
C) $x = 3\sec\theta$
D) $x = 3\tan\theta$

The appropriate change of variables for the integral $\int \sqrt{16+x^2}\,dx$ is

$x = 4\tan x$

A) $x = 16\tan\theta$
B) $x = 4\sin\theta$
C) $x = 4\sec\theta$
D) $x = 4\tan\theta$

Evaluate the integral.

جـــد قيمة التكامل

34) $\int x^3\sqrt{8+x^2}\,dx$

$\sqrt{8} = 2\sqrt{2}$

$x = 2\sqrt{2}\tan\theta$

$dx = 2\sqrt{2}\sec^2\theta\,d\theta$

$\sqrt{8+x^2} = 2\sqrt{2}\sec\theta$

$16\sqrt{2}\tan^3\theta$

$\int (2\sqrt{2}\tan\theta)^3 \cdot 2\sqrt{2}\sec\theta \cdot 2\sqrt{2}\sec^2\theta\,d\theta$

$= 128\sqrt{2}\int \tan^3\theta\sec^3\theta\,d\theta$

$= 128\sqrt{2}\int \tan^2\theta\sec^2\theta\sec\theta\tan\theta\,d\theta$

$= 128\sqrt{2}\int (\sec^2\theta - 1)\sec^2\theta\sec\theta\tan\theta\,d\theta$

$= 128\sqrt{2}\int (u^4 - u^2)\,du$

$u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

A) $\frac{2}{3}(8+x^2)^{\frac{3}{2}} - 16\sqrt{2}(8+x^2)^{\frac{1}{2}} + c$

B) $\frac{2\sqrt{2}}{3}(8+x^2)^{\frac{3}{2}} + 16\sqrt{2}(8+x^2)^{\frac{1}{2}} + c$

C) $\frac{4}{3}(8+x^2)^{\frac{3}{2}} - 32(8+x^2)^{\frac{1}{2}} + c$

D) $\frac{2\sqrt{2}}{3}(8+x^2)^{\frac{3}{2}} - 16\sqrt{2}(8+x^2)^{\frac{1}{2}} + c$

Evaluate the integral.

جـــد قيمة التكامل

35) $\int \sqrt{16+x^2}\,dx$

$x = 4\tan\theta$

$dx = 4\sec^2\theta\,d\theta$

$\sqrt{16+x^2} = 4\sec\theta$

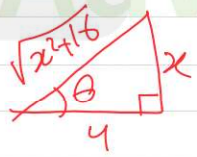
$\int 4\sec\theta \cdot 4\sec^2\theta\,d\theta$

$= 16\int \sec^3\theta\,d\theta$

$= 16\left[\frac{1}{2}\ln|\sec\theta+\tan\theta| + \frac{1}{2}\sec\theta\tan\theta\right]$

$= 8\ln|\sec\theta+\tan\theta| + 8\sec\theta\tan\theta + C$

$= 8\ln\left|\frac{\sqrt{x^2+16}}{4} + \frac{x}{4}\right| + 8\left(\frac{\sqrt{x^2+16}}{4}\right)\left(\frac{x}{4}\right) + C$



$\int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta| + c$

$\int \sec^3\theta\,d\theta = \left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\int \sec\theta\,d\theta\right)$

A) $x\sqrt{16+x^2} + 8\ln\left|\frac{\sqrt{16+x^2}+x}{4}\right| + c$

B) $2x\sqrt{16+x^2} + 8\ln\left|\frac{\sqrt{16+x^2}+x}{4}\right| + c$

C) $\frac{1}{2}x\sqrt{16+x^2} + 8\ln\left|\frac{\sqrt{16+x^2}+x}{4}\right| + c$

D) $\frac{1}{2}x\sqrt{16+x^2} - 8\ln\left|\frac{\sqrt{16+x^2}+x}{4}\right| + c$

Evaluate the integral.

جـــد قيمة التكامل

36) $\int \frac{1}{\sqrt{4+x^2}}\,dx$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$\sqrt{4+x^2} = 2\sec\theta$

$\int \frac{1}{2\sec\theta} \cdot 2\sec^2\theta\,d\theta$

$= \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$

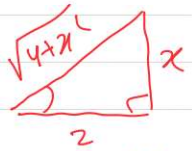
$= \ln\left|\frac{\sqrt{4+x^2}+x}{2}\right| + C = \ln|\sqrt{4+x^2}+x| - \ln 2 + C$

A) $\ln|x+\sqrt{4+x^2}|+c$

B) $\ln|x-\sqrt{4+x^2}|+c$

C) $\frac{1}{2}\ln|x+\sqrt{4+x^2}|+c$

D) $\frac{1}{3}\ln|x+\sqrt{4+x^2}|+c$



Evaluate the integral.

جـــد قيمة التكامل

37. $\int_0^1 x\sqrt{x^2+8}\,dx$

$= \frac{1}{2}\int_0^1 (x^2+8)^{\frac{1}{2}} \cdot (2x)\,dx$

$= \frac{1}{2}\left.\frac{(x^2+8)^{3/2}}{3/2}\right|_0^1$

$= \frac{1}{3}\left.(x^2+8)^{3/2}\right|_0^1$

$= \frac{1}{3}\left[9^{3/2} - 8^{3/2}\right] =$

$\int [f(x)]^n f'(x)dx = \frac{1}{n+1}[f(x)]^{n+1} + c$

Evaluate the integral.

جـــد قيمة التكامل

38) $\int_0^2 x^2\sqrt{x^2+9}\,dx$

$x = 3\tan\theta$

Let $x = 3\tan\theta, dx = 3\sec^2\theta\,d\theta$

$I = \int x^2\sqrt{x^2+9}dx$

$= \int 27\tan^2\theta\sec^2\theta\sqrt{9\tan^2\theta+9}dx$

$= 81\int \tan^2\theta\sec^3\theta dx$

$= 81\int(\sec^2\theta-1)\sec^3\theta dx$

$= 81\int(\sec^5\theta-\sec^3\theta)dx$

To compute $\int \sec^5\theta\,d\theta$, we use integration by parts with $u = \sec^3\theta$ and $dv = \sec^2\theta d\theta$.

$\int \sec^5\theta\,d\theta$

$= \sec^3\theta\tan\theta - \int 3\sec^3\theta\tan^2\theta d\theta$

$= \sec^3\theta\tan\theta - 3\int \sec^3\theta(\sec^2\theta-1)d\theta$

$= \sec^3\theta\tan\theta - 3\int(\sec^5\theta-\sec^3\theta)d\theta$

$4\int \sec^5\theta d\theta$

$= \sec^3\theta\tan\theta + 3\int \sec^3\theta d\theta \int \sec^5\theta d\theta$

$= \frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{4}\int \sec^3\theta d\theta$

Putting all this together gives:

$I = 81\int(\sec^5\theta-\sec^3\theta)dx$

$= \frac{81}{4}\sec^3\theta\tan\theta + \frac{243}{4}\int \sec^3\theta d\theta$

$- 81\int \sec^3\theta d\theta$

$= \frac{81}{4}\sec^3\theta\tan\theta - \frac{81}{4}\int \sec^3\theta d\theta$

$= \frac{81}{4}\sec^3\theta\tan\theta - \frac{81}{8}\sec\theta\tan\theta$

$- \frac{81}{8}\ln|\sec\theta+\tan\theta| + c$

$\int\sec^5\theta$

$\int\sec^3\theta$

$\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|$

A) 8.99399

B) 9.99399

C) $2-\sqrt{13}$

D) $2+\sqrt{13}$

Evaluate the integral.

جـــد قيمة التكامل

39) $\int \frac{x^3}{\sqrt{1+x^2}}\,dx$

$x = \tan\theta \rightarrow \theta = \tan^{-1}x$

$dx = \sec^2\theta\,d\theta$

$\sqrt{1+x^2} = \sec\theta$

$\int \frac{\tan^3\theta}{\sec\theta}\cdot\sec^2\theta\,d\theta$

$= \int \tan^3\theta\sec\theta\,d\theta = \int \tan^2\theta\,\tan\theta\sec\theta\,d\theta$

$= \int(\sec^2\theta-1)\sec\theta\tan\theta\,d\theta$

$u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

$= \int u^2-1\,du = \frac{1}{3}u^3 - u + C$

$= \frac{1}{3}\sec^3(\tan^{-1}x) - \sec(\tan^{-1}x) + C$

A) $\frac{1}{2}\sec^3(\tan^{-1}x) - \sec(\tan^{-1}x) + c$

B) $\sec^3(\tan^{-1}x) - \frac{1}{3}\sec(\tan^{-1}x) + c$

C) $\frac{1}{3}\sec^3(\tan^{-1}x) - \sec(\tan^{-1}x) + c$

D) $\frac{1}{3}\sec^3(\tan^{-1}x) + \sec(\tan^{-1}x) + c$

Evaluate the integral.

جـــد قيمة التكامل

40) $\int \frac{x+1}{\sqrt{4+x^2}}\,dx$

$x = 2\tan\theta$

A) $\sqrt{4+x^2} - \ln\left|\sqrt{4+x^2}+x\right| + c$

B) $\sqrt{4+x^2} + \ln\left|\sqrt{4+x^2}+x\right| + c$

C) $2\sqrt{4+x^2} + \ln\left|\sqrt{4+x^2}+x\right| + c$

Use a trigonometric substitution to evaluate $\int \frac{dx}{\sqrt{9+x^2}}$

A) $\ln\left|\frac{9+x^2}{3}+\frac{x}{3}\right| + C$

B) $\ln\left|\frac{\sqrt{9+x^2}}{9}\right| + C$

C) $\ln\left|\frac{\sqrt{9+x^2}}{9}+\frac{x}{9}\right| + C$

D) $\ln\left|\frac{\sqrt{9+x^2}}{3}+\frac{x}{3}\right| + C$

$x = 3\tan\theta \Rightarrow dx = 3\sec^2\theta\,d\theta$

$\Rightarrow \sqrt{9+x^2} = 3\sec\theta$

$\int \frac{1}{3\sec\theta}\cdot 3\sec^2\theta\,d\theta$

$= \int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta| + C$

$= \ln\left|\frac{\sqrt{9+x^2}}{3}+\frac{x}{3}\right| + C$