من المهارة إلى الصدارة

نحو مستقبل تصنعه المهارات

Maryam Bint Sultan School- Cycle3
Cluster AD 2.9

UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

الإمارات العربية المتحدة
وزارة التربية والتعليم

Grade 11 Advanced

Mathematics

Academic Year 2025/2026

EOT1 Exam Coverage in Term 1

Math Teacher: Noura Ali Al kaabi

Student Name:............................
Class : 11 ADV ……

Cluster Manager:
Dr. **Mariam Al Darmaki**

School Principal:
Azza Al Kaabi

Academic Year	2025-2026		Term	1		Subject	Math\Reveal
Grade\stream	11\AD		Number Of MCQ	20		Marks Of MCQ	(2-4)
Maximum over Grade	100		Number Of FRQ	5		Marks Of FRQ	(7-12)
Exam Duration	150 min		Mode of Implementation	SwiftAssess & Paper-Based			
Calculator	Allowed						

Question		Lesson	Example/Exercise	Page
الأسئلة الموضوعية - MCQ	1	Graph exponential growth functions.	(1-10) 1	221 254
	2	Analyze expressions and functions involving the natural base e.	Example 2 (10-12)	234 237
	3	Write logarithmic expressions in exponential form and write exponential expressions in logarithmic form.	(1-12)	265
	4	Evaluate logarithmic expressions by using the Change of Base Formula.	Example 5 (21-26)	279 282
	5	Simplify expressions with natural logarithms.	Example 3 (19-27)	286 291
	6	Write and solve exponential growth equations and inequalities.	Example 1,check (1,2)	(295- 297) 301
	7	Simplify rational expressions.	(1-10)	315
	8	Simplify rational expressions by adding and subtracting.	(1-12)	323
	9	Graph and analyze rational functions with oblique asymptotes.	(11-22)	344
	10	Recognize and solve direct and joint variation equations	(7-12)	351
	11	Solve rational equations in one variable.	(1-12)	361
	12	Classify sampling methods and identify bias in samples and survey questions.	(4-10)	375
	13	Compare theoretical and experimental probabilities.	(1-3) 4	383 412
	14	Describe distributions by finding their mean and standard deviation. (No need for graphing calculator)	Example 1 & (1,2) 7	388 & 391 413
	15	Convert between degrees and radian measures and find arc lengths by using central angles.	(54-65)	422
	16	Find values of trigonometric functions by using reference angles.	(40-45)	432
	17	Find values of trigonometric functions given a point on a unit circle or the measure of a special angle.	(1-6)	441
	18	Model periodic real-world situations with sine and cosine functions. (No need for graphing calculator)	Example 5 & 6 (17-20)	449 & 450 452
	19	Graph vertical translations of trigonometric functions.	(13-18)	469
	20	Find values of angle measures by using inverse trigonometric functions.	Example 1 & (1-9)	473 & 475
المقالي FRQ	21	Solve exponential equations in one variable.	(1-6)	229
	22	a) Solve logarithmic equations using properties of equality.	(7-12)	273
		b) Simplify and evaluate expressions by using the properties of logarithms.	(13-24)	
	23	Graph and write reciprocal functions by using transformations.	(17-22)	334
	24	Analyze standardized data and distributions by using z-values .(The table will be provided in the question)	Example 6, 7 & 8 (8-13) & 14	399 & 400 401 & 402
	25	Graph and analyze tangent functions. (No need for graphing calculator)	Example 1 & 2 (1-6)	455,456 & 457461

	Model 5 Exponential Function (2 MCQ +1 FRQ)	Lesson 5-1	MCQ
1	Graph exponential growth functions.	(1-10) 1	221 254

Example 1

Graph each function. Find the domain, range, *y*-intercept, asymptote, and end behavior.

1. $f(x) = 3^x$

2. $f(x) = 5^x$

3. $f(x) = 1.5^x$

4. $f(x) = \left(\frac{5}{2}\right)^x$

Example 2

Graph each function.

5. $f(x) = 2(3)^x$

6. $f(x) = -2(4)^x$

7. $f(x) = 4^{x+1} - 5$

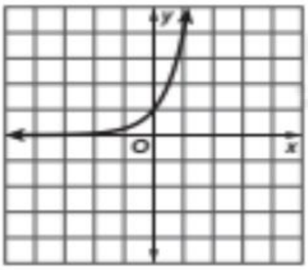
8. $f(x) = 3^{2x} + 1$

9. $f(x) = -0.4(3)^{x+2} + 4$

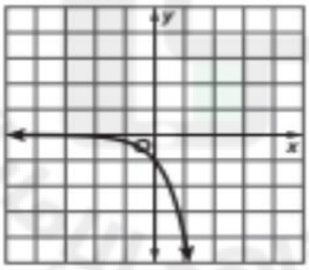
10. $f(x) = 1.5(2)^x + 6$

1. MULTIPLE CHOICE Select the graph of $f(x) = 4^x$. (Lesson 5-1)

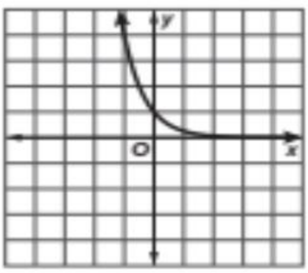
A.



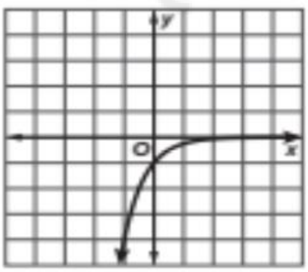
B.



C.



D.



	Model 5 Exponential Function (2 MCQ +1 FRQ)	Lesson 5-3	MCQ
2	Analyze expressions and functions involving the natural base e.	Example 2 (10-12)	234 237

Example 2 Graph Functions with Base e

Consider $g(x) = -2e^{x-3} + 2$.

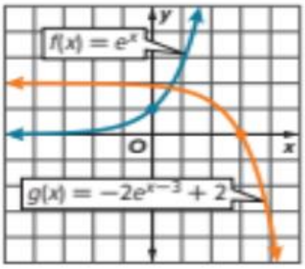
Part A Graph the function.

The function $g(x) = -2e^{x-3} + 2$ represents a transformation of the graph $f(x) = e^x$.

$a = -2$; The graph is reflected in the x-axis and stretched vertically.

$h = 3$; The graph is translated 3 units right.

$k = 2$; The graph is translated 2 units up.



Part B Determine the domain and range.

The domain is all real numbers. The range is all real numbers less than 2.

Part C Find the average rate of change.

Determine the average rate of change of g(x) over each interval.

a. [−4, −1]

Based on the graph, the graph from −4 to −1 appears approximately horizontal. So, the average rate of change should be close to 0.

$\frac{g(-1) - g(-4)}{-1 - (-4)} \approx \frac{1.963 - 1.998}{3}$ Evaluate $g(-1)$ and $g(-4)$.

$\approx \frac{-0.035}{3}$ or -0.012 Simplify.

b. [0, 3]

Based on the graph, the curve appears to pass through approximately (0,2) and (3,0). So, the average rate of change for the interval [0, 3] should be about $\frac{0-2}{3-0}$ or $-\frac{2}{3}$.

$\frac{g(3) - g(0)}{3 - 0} \approx \frac{0 - 1.900}{3}$ Evaluate $g(3)$ and $g(0)$.

$\approx \frac{-1.900}{3}$ or -0.633 Simplify.

c. [4, 8]

The graph is very steep for values greater than 4. So, the average rate of change from 4 to 8 should be a negative number with a large absolute value.

$\frac{g(8) - g(4)}{8 - 4} \approx \frac{-294.826 - (-3.437)}{4}$ Evaluate $g(8)$ and $g(4)$.

$\approx \frac{-291.389}{4}$ or -72.847 Simplify.

	Model 5 Exponential Function (2 MCQ +1 FRQ)	Lesson 5-3	MCQ
2	Analyze expressions and functions involving the natural base e.	Example 2 (10-12)	234 237

10. Consider the function $f(x) = 3e^{x-1} + 3$.

a. Graph the function.

b. Determine domain and range.

c. Find the average rate of change over the interval $[-5, -2]$.

11. Consider the function $f(x) = 4e^{2x} - 1$.

a. Graph the function.

b. Determine domain and range.

c. Find the average rate of change over the interval $[-3, -1]$.

12. Consider the function $f(x) = -2e^{x+3} + 2$.

a. Graph the function.

b. Determine domain and range.

c. Find the average rate of change over the interval $[-7, -4]$.

	Model 5 Exponential Function (2 MCQ +1 FRQ)	Lesson 5-2	FRQ
21	**Solve exponential equations in one variable.**	**(1-6)**	**229**

Example 1

Solve each equation.

1. $25^{2x+3} = 25^{5x-9}$
2. $9^{8x-4} = 81^{3x+6}$
3. $4^{x-5} = 16^{2x-31}$
4. $4^{3x-3} = 8^{4x-4}$
5. $9^{-x+5} = 27^{6x-10}$
6. $125^{3x-4} = 25^{4x+2}$

	Model 6 Logarithmic Function (4 MCQ +1 FRQ)	Lesson 6-1	MCQ
3	Write logarithmic expressions in exponential form and write exponential expressions in logarithmic form.	(1-12)	265

Example 1

Write each equation in exponential form.

1. $\log_{15} 225 = 2$
2. $\log_3 \frac{1}{27} = -3$
3. $\log_5 \frac{1}{25} = 2$
4. $\log_3 243 = 5$
5. $\log_4 64 = 3$
6. $\log_4 32 = \frac{5}{2}$

Example 2

Write each equation in logarithmic form.

7. $2^7 = 128$
8. $3^{-4} = \frac{1}{81}$
9. $7^{-2} = \frac{1}{49}$
10. $\left(\frac{1}{7}\right)^3 = \frac{1}{343}$
11. $2^9 = 512$
12. $64^{\frac{2}{3}} = 16$

	Model 6 Logarithmic Function (4 MCQ +1 FRQ)	Lesson 6-3	MCQ
4	Evaluate logarithmic expressions by using the Change of Base Formula.	Example 5 (21-26)	279 282

Example 5 Change of Base Formula

Evaluate $\log_2 11$ by writing it in terms of common logarithms. Round to the nearest ten-thousandth.

$\log_2 11 = \frac{\log_{10} 11}{\log_{10} 2}$ Change of Base Formula

≈ 3.4594 Use a calculator.

Example 5

Express each logarithm in terms of common logarithms. Then approximate its value to the nearest ten-thousandth.

21. $\log_4 22$
22. $\log_{12} 200$
23. $\log_2 50$
24. $\log_8 15$
25. $\log_3 2$
26. $\log_5 0.4$

	Model 6 Logarithmic Function (4 MCQ +1 FRQ)	Lesson 6-4	MCQ
5	**Simplify expressions with natural logarithms.**	Example 3 (19-27)	286 291

Example 3 Simplify Logarithmic Expressions

Write $\frac{1}{3}\ln 8 - \ln 3 + \ln 9$ as a single logarithm.

$\frac{1}{3}\ln 8 - \ln 3 + \ln 9$ Original expression

$= \ln 8^{\frac{1}{3}} - \ln 3 + \ln 9$ Power Property of Logarithms

$= \ln 2 - \ln 3 + \ln 9$ $8^{\frac{1}{3}} = \sqrt[3]{8}$ or 2

$= \ln \frac{2}{3} + \ln 9$ Quotient Property of Logarithms

$= \ln 6$ Product Property of Logarithms

Example 3

Write each expression as a single logarithm.

19. $3 \ln 3 - \ln 9$

20. $4 \ln 16 - \ln 256$

21. $2 \ln x + 2 \ln 4$

22. $3 \ln 4 + 3 \ln 3$

23. $\ln 125 - 2 \ln 5$

24. $3 \ln 10 + 2 \ln 100$

25. $4 \ln \frac{1}{3} - 6 \ln \frac{1}{9}$

26. $7 \ln \frac{1}{2} + 5 \ln 2$

27. $8 \ln x - 4 \ln 5$

	Model 6 Logarithmic Function (4 MCQ +1 FRQ)	Lesson 6-5	MCQ
6	**Write and solve exponential growth equations and inequalities.**	Example 1,check (1,2)	(295- 297) 301

Example 1 Continuous Exponential Growth

POPULATION **In 2016, the population of Florida was 20.61 million people. In 2000, it was 15.98 million.**

Part A Write an exponential growth equation.

Because the population is growing as time passes, use the year as the independent variable and the population as the dependent variable. Find the value of k, Florida's relative growth rate. Then write an equation that represents the population of Florida t years after 2000.

Since t is the number of years after 2000, 15.98 represents the initial population, and 20.61 represents the population 16 years after 2000, or at $t = 16$.

(continued on the next page)

$y = ae^{kt}$ Formula for continuous exponential growth

$20.61 = 15.98e^{k(16)}$ $y = 20.61$, $a = 15.98$, and $t = 2016 - 2000$ or 16

$\frac{20.61}{15.98} = e^{16k}$ Divide each side by 15.98.

$\ln \frac{20.61}{15.98} = \ln e^{16k}$ Property of Equality for Logarithmic Equations

$\ln \frac{20.61}{15.98} = 16k$ $\ln e^x = x$

$\frac{\ln \frac{20.61}{15.98}}{16} = k$ Divide each side by 16.

$0.0159 \approx k$ Use a calculator.

Florida's relative rate of growth is about, 0.0159, or about 1.59%.

$y = ae^{kt}$ Formula for continuous exponential growth

$y = 15.98e^{0.0159t}$ $a = 15.98$ and $k = 0.0159$

So, the population of Florida t years after 2000 can be modeled by the equation $y = 15.98e^{0.0159t}$.

	Model 6 Logarithmic Function (4 MCQ +1 FRQ)	**Lesson 6-5**	**MCQ**
6	**Write and solve exponential growth equations and inequalities.**	**Example 1,check (1,2)**	**(295- 297) 301**

Part B Predict when the population will reach 25 million people.

Use your equation from **Part A** to predict the population.

$y = 15.98e^{0.0159t}$	Original equation
$25 = 15.98e^{0.0159t}$	$y = 25$
$1.564 = e^{0.0159t}$	Divide each side by 15.98.
$\ln 1.564 = \ln e^{0.0159t}$	Property of Equality for Logarithmic Equations
$\ln 1.564 = 0.0159t$	$\ln e^x = x$
$\frac{\ln 1.564}{0.0159} = t$	Divide each side by 0.0159.
$28.129 \approx t$	Use a calculator.

Florida's population will reach 25 million people approximately 28.129 years after 2000, or in 2028.

Part C Compare the populations of Florida and California.

California's population in 2000 was 33.9 million, and its population growth can be modeled by $y = 33.9e^{0.0092t}$. Determine when Florida's population will surpass California's.

$15.98e^{0.0159t} > 33.9e^{0.0092t}$	Formula for exponential growth
$\ln 15.98e^{0.0159t} > \ln 33.9e^{0.0092t}$	Prop. of Ineq. for Logarithms
$\ln 15.98 + \ln e^{0.0159t} > \ln 33.9 + \ln e^{0.0092t}$	Product Property of Logarithms
$\ln 15.98 + 0.0159t > \ln 33.9 + 0.0092t$	$\ln e^x = x$
$\ln 15.98 + 0.0067t > \ln 33.9$	Subtract $0.0092t$ from each side.
$0.0067t > \ln 33.9 - \ln 15.98$	Subtract ln 15.98 from each side.
$t > \frac{\ln 33.9 - \ln 15.98}{0.0067}$	Divide each side by 0.0067.
$t > 112.25$	Use a calculator.

Assuming that this trend continues, Florida's population is on track to surpass California's in 2112.

Check

SCIENCE An experiment starts with 20 bacteria A cells. After 45 minutes, there are 710 bacteria A cells.

Part A

Write the equation that models the number of bacteria A cells y after t minutes. Round the value of k to the nearest thousandth.

Part B

After how many minutes will there be 1000 bacteria A cells? Round to the nearest tenth if necessary.

___?___ minutes

Part C

Bacteria B grows exponentially according to the model $y = 52e^{0.064t}$. After how many minutes will there be more bacteria A cells than bacteria B cells? Round to the nearest tenth if necessary.

___?___ minutes

Example 1

1. **POPULATION** In 2000, the world population was estimated to be 6.124 billion people. In 2005, it was 6.515 billion.
   a. Write an exponential growth equation to represent the population y in billions t years after 2000.
   b. Use the equation to predict the year in which the world population reached 7.5 billion people.

	Model 6 Logarithmic Function (4 MCQ +1 FRQ)	Lesson 6-5	MCQ
6	Write and solve exponential growth equations and inequalities.	Example 1,check (1,2)	(295- 297) 301

2. CONSUMER AWARENESS Jason wants to buy a new HD television but he thinks that if he waits, the quality of HD televisions will improve. The television he wants to buy costs \$2500 now, and based on pricing trends, Jason thinks that the price will increase by 4% each year.

a. Write an exponential growth equation to represent the price y of a new HD television t years from now.

b. Use the equation to predict when a new HD television will cost \$3000.

c. Jason decides to wait to buy a new television and saves his money. He puts \$2200 in a savings account with 4.7% annual interest compounded continuously. Determine when the amount in his savings will exceed the cost of a new television.

	Model 6 Logarithmic Function (4 MCQ +1 FRQ)	Lesson 6- 2	FRQ
22	a) Solve logarithmic equations using properties of equality.	(7-12)	273
	b) Simplify and evaluate expressions by using the properties of logarithms.	(13-24)	

Example 2

Solve each equation.

7. $\log_4 (2x^2 - 4) = \log_4 2x$ 8. $\log_5 (x^2 - 6) = \log_5 x$ 9. $\log_3 (x^2 - 8) = \log_3 2x$

	Model 6 Logarithmic Function (4 MCQ +1 FRQ)	**Lesson 6- 2**	**FRQ**
22	**a) Solve logarithmic equations using properties of equality.**	**(7-12)**	**273**
	b) Simplify and evaluate expressions by using the properties of logarithms.	**(13-24)**	

Example 2

Solve each equation.

10. $\log_4 (2x^2 - 20) = \log_4 6x$ **11.** $\log_2 (6x^2 + 1) = \log_2 5x$ **12.** $\log_6 (6x^2 - 3) = \log_6 7x$

Examples 3 and 4

Use $\log_4 2 = 0.5$, $\log_4 3 \approx 0.7925$, and $\log_4 5 \approx 1.1610$ to approximate the value of each expression.

13. $\log_4 30$ **14.** $\log_4 20$ **15.** $\log_4 \frac{2}{3}$

16. $\log_4 \frac{4}{3}$ **17.** $\log_4 9$ **18.** $\log_4 8$

Example 5

Use $\log_2 3 \approx 1.5850$ and $\log_2 5 \approx 2.3219$ to approximate the value of each expression.

19. $\log_2 25$ **20.** $\log_2 27$ **21.** $\log_2 125$

22. $\log_2 625$ **23.** $\log_2 81$ **24.** $\log_2 243$

	Model 7 Rational Function (5 MCQ +1 FRQ)	Lesson 7-1	MCQ
7	Simplify rational expressions.	(1-10)	315

Example 1

Simplify each expression, and state when the original expression is undefined.

1. $\dfrac{x(x-3)(x+6)}{x^2+x-12}$

2. $\dfrac{y^2(y^2+3y+2)}{2y(y-4)(y+2)}$

3. $\dfrac{(x^2-9)(x^2-z^2)}{4(x+z)(x-3)}$

4. $\dfrac{(x^2-16x+64)(x+2)}{(x^2-64)(x^2-6x-16)}$

5. $\dfrac{x^2(x+2)(x-4)}{6x(x^2+x-20)}$

6. $\dfrac{3y(y-8)(y^2+2y-24)}{15y^2(y^2-12y+32)}$

Example 2

Simplify each expression.

7. $\dfrac{x^2-5x-14}{28+3x-x^2}$

8. $\dfrac{9x^2-x^3}{x^2-3x-54}$

9. $\dfrac{(x-4)(x^2+2x-48)}{(36-x^2)(x^2+4x-32)}$

10. $\dfrac{16-c^2}{c^2+c-20}$

	Model 7 Rational Function (5 MCQ +1 FRQ)	Lesson 7-2	MCQ
8	**Simplify rational expressions by adding and subtracting.**	**(1-12)**	**323**

Examples 1 and 2

Simplify each expression.

1. $\frac{3}{x} + \frac{5}{y}$

2. $\frac{3}{8p^2r} + \frac{5}{4p^2r}$

3. $\frac{2c-7}{3} + 4$

4. $\frac{2}{m^2p} + \frac{5}{p}$

5. $\frac{12}{5y^2} - \frac{2}{5yz}$

6. $\frac{7}{4gh} + \frac{3}{4h^2}$

7. $\frac{3}{w-3} - \frac{2}{w^2-9}$

8. $\frac{3t}{2-x} + \frac{5}{x-2}$

9. $\frac{k}{k-n} - \frac{k}{n-k}$

10. $\frac{4z}{z-4} + \frac{z+4}{z+1}$

11. $\frac{n}{n-3} + \frac{2n+2}{n^2-2n-3}$

12. $\frac{3}{y^2+y-12} - \frac{2}{y^2+6y+8}$

	Model 7 Rational Function (5 MCQ +1 FRQ)	Lesson 7-4	MCQ
9	**Graph and analyze rational functions with oblique asymptotes.**	**(11-22)**	**344**

Example 4

Find the zeros and asymptotes of each function. Then graph each function.

11. $f(x) = \frac{(x-4)^2}{x+2}$

12. $f(x) = \frac{(x+3)^2}{x-5}$

13. $f(x) = \frac{6x^2+4x+2}{x+2}$

14. $f(x) = \frac{2x^2+7x}{x-2}$

15. $f(x) = \frac{3x^2+8}{2x-1}$

16. $f(x) = \frac{2x^2+5}{3x+4}$

Example 5

Graph each function. Find the point discontinuity.

17. $f(x) = \frac{x^2-2x-8}{x-4}$

18. $f(x) = \frac{x^2+4x-12}{x-2}$

19. $f(x) = \frac{x^2-25}{x+5}$

20. $f(x) = \frac{x^2-64}{x-8}$

21. $f(x) = \frac{(x-4)(x^2-4)}{x^2-6x+8}$

22. $f(x) = \frac{(x+5)(x^2+2x-3)}{x^2+8x+15}$

	Model 7 Rational Function (5 MCQ +1 FRQ)	Lesson 7-5	MCQ
10	**Recognize and solve direct and joint variation equations**	**(7-12)**	**351**

Example 2

If a varies jointly as b and c, find a when $b = 4$ and $c = -3$.

7. $a = -96$ when $b = 3$ and $c = -8$

8. $a = -60$ when $b = -5$ and $c = 4$

9. $a = -108$ when $b = 2$ and $c = 9$

10. $a = 24$ when $b = 8$ and $c = 12$

11. If y varies jointly as x and z, and $y = 18$ when $x = 2$ and $z = 3$, find y when x is 5 and z is 6.

12. If y varies jointly as x and z, and $y = -16$ when $x = 4$ and $z = 2$, find y when x is -1 and z is 7.

	Model 7 Rational Function (5 MCQ +1 FRQ)	Lesson 7-6	MCQ
11	**Solve rational equations in one variable.**	**(1-12)**	**361**

Example 1

Solve each equation. Check your solutions.

1. $\frac{2x+3}{x+1} = \frac{3}{2}$

2. $\frac{-12}{y} = y - 7$

3. $\frac{9}{x-7} - \frac{7}{x-6} = \frac{13}{x^2 - 13x + 42}$

4. $\frac{13}{y+3} - \frac{12}{y+4} = \frac{18}{y^2 + 7y + 12}$

5. $\frac{14}{x-2} - \frac{18}{x+1} = \frac{22}{x^2 - x - 2}$

6. $\frac{2}{a+2} + \frac{10}{a+5} = \frac{36}{a^2 + 7a + 10}$

Example 2

7. $\frac{x}{2x-1} + \frac{3}{x+4} = \frac{21}{2x^2 + 7x - 4}$

8. $\frac{2}{y-5} + \frac{y-1}{2y+1} = \frac{2}{2y^2 - 9y - 5}$

9. $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$

10. $\frac{12p+19}{p^2 + 7p + 12} - \frac{3}{p+3} = \frac{5}{p+4}$

11. $\frac{2f}{f^2 - 4} + \frac{1}{f-2} = \frac{2}{f+2}$

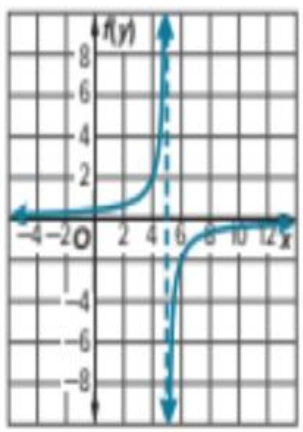
12. $\frac{8}{t^2 - 9} + \frac{4}{t+3} = 1$

	Model 7 Rational Function (5 MCQ +1 FRQ)	Lesson 7-3	FRQ
23	**Graph and write reciprocal functions by using transformations.**	**(17-22)**	**334**

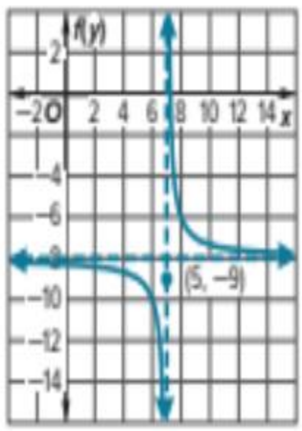
Example 5

Identify the values of a, h, and k. Then write a function for the graph $g(x) = \frac{a}{x-h} + k$.

17.

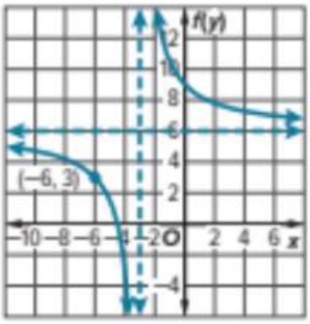


18.

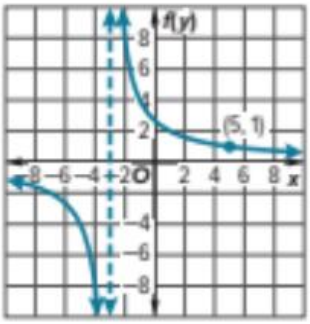


	Model 7 Rational Function (5 MCQ +1 FRQ)	Lesson 7-3	FRQ
23	**Graph and write reciprocal functions by using transformations.**	**(17-22)**	**334**

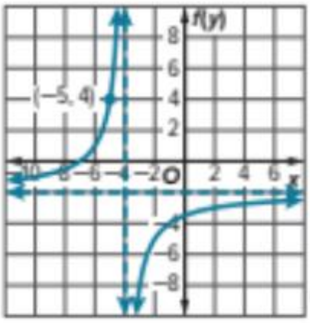
19.



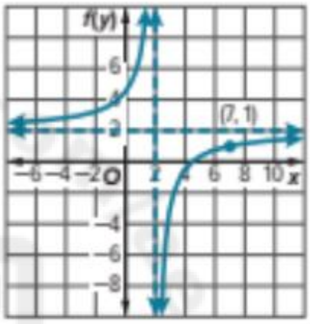
20.



21.



22.



	Model 8 Inferential Statistics (3 MCQ +1 FRQ)	Lesson 8-1	MCQ
12	**Classify sampling methods and identify bias in samples and survey questions.**	**(4-10)**	**375**

Examples 2 and 3

Identify each sample or question as *biased* or *unbiased*. Explain your reasoning.

4. A random sample of eight people is asked to select their favorite food for a survey about Americans' food preferences.

5. Every tenth student at band camp is asked to name his or her favorite band for a survey about the campers.

6. Every fifth person entering a museum is asked to name his or her favorite type of book to read for a survey about reading interests of people in the city.

7. Do you think that the workout facility needs a new treadmill and racquetball court?

8. Which is your favorite type of music, pop, or country?

9. Are you a member of any after-school clubs?

10. Don't you agree that employees should pack their lunch?

	Model 8 Inferential Statistics (3 MCQ +1 FRQ)	Lesson 8-2	MCQ
13	**Compare theoretical and experimental probabilities.**	(1-3) 4	383 412

Example 1

1. A student spun a spinner with 4 equal sections 100 times and recorded the results.

Spinner Section	Frequency
Red	35
Blue	38
Green	13
Yellow	14

a. Find the theoretical probability of spinning blue. Write your answer as a percentage rounded to the nearest tenth, if necessary.

b. Find the experimental probability of spinning blue. Write your answer as a percentage rounded to the nearest tenth, if necessary.

2. A student flipped a coin 125 times and recorded the results.

Coin Result	Frequency
Heads	73
Tails	52

a. Find the theoretical probability of the coin landing on heads. Write your answer as a percentage rounded to the nearest tenth, if necessary.

b. Find the experimental probability of the coin landing on heads. Write your answer as a percentage rounded to the nearest tenth, if necessary.

3. A fair 6-sided die is rolled 150 times.

Number on Die	Frequency
1	32
2	18
3	27
4	16
5	33
6	24

a. Find the theoretical probability of rolling a 3. Write your answer as a percentage rounded to the nearest tenth, if necessary.

b. Find the experimental probability of rolling a 3. Write your answer as a percentage rounded to the nearest tenth, if necessary.

4. **MULTIPLE CHOICE** Leonard randomly selected a card from a standard deck of playing cards, recorded the suit, and returned the card. He followed this set of steps 120 times. The results are shown.

Suit	Frequency
Heart	28
Diamond	37
Spade	34
Club	21

Which statement about the results is true? (Lesson 8-2)

A. The theoretical probability of selecting a heart is less than the experimental probability of selecting a heart.

B. The theoretical probability of selecting a diamond is less than the experimental probability of selecting a diamond.

C. The experimental probability of selecting a club is greater than the theoretical probability of selecting a club.

D. The experimental probability of selecting a heart is equal to the experimental probability of selecting a diamond.

	Model 8 Inferential Statistics (3 MCQ +1 FRQ)	Lesson 8-3	MCQ
14	Describe distributions by finding their mean and standard deviation. (No need for graphing calculator)	Example 1 & (1,2) 7	388 & 391 413

Example 1 Find a Standard Deviation

TRACK A coach recorded the times of each track member for a 400-meter race. Find and interpret the standard deviation of the data.

400m Race Times (seconds)	
57.1	55.9
59.3	54.9
54.6	50.3
55.2	53.5

Step 1 Find the mean, μ.

$$\mu = \frac{57.1 + 59.3 + 54.6 + 55.2 + 55.9 + 54.9 + 50.3 + 53.5}{8}$$
$$= 55.1$$

The mean running time for the team is 55.1 seconds.

Step 2 Find the squares of the differences, $(\mu - x_n)^2$.

$(55.1 - 57.1)^2 = 4.00$ $\quad (55.1 - 55.9)^2 = 0.64$
$(55.1 - 59.3)^2 = 17.64$ $\quad (55.1 - 54.9)^2 = 0.04$
$(55.1 - 54.6)^2 = 0.25$ $\quad (55.1 - 50.3)^2 = 23.04$
$(55.1 - 55.2)^2 = 0.01$ $\quad (55.1 - 53.5)^2 = 2.56$

Step 3 Find the sum.

Find the sum of the values from **Step 2**.

$4.00 + 17.64 + 0.25 + 0.01 + 0.64 + 0.04 + 23.04 + 2.56$
$= 48.18$

Step 4 Divide by the number of values.

Divide the sum from **Step 3** by the number of running times.

$\frac{48.18}{8} = 6.0225$ This is the variance.

Step 5 Take the square root of the variance.

$\sqrt{6.0225} \approx 2.45$ This is the standard deviation.

The standard deviation is about 2.45. This is small compared with the run times, which means that the majority of the team members' times are close to the mean of 55.1 seconds, and almost all of the times will likely fall within 2 standard deviations of the mean, or between 50.2 and 60.0 seconds.

Example 1

1. **BARBER** A barber wants to purchase new professional shears from a Web site. The prices of all of the shears are shown in the table. Use the standard deviation formula to find and interpret the standard deviation of the data. Round your answers to the nearest cent.

Cost of Shears ($)			
50	165	55	79
84	68	38	42

2. **READING** Ms. Sanchez keeps track of the total number of books each student in the book club reads during the school year. Use the standard deviation formula to find and interpret the standard deviation of the data. Round your answers to the nearest tenth.

Books Read		
9	6	12
8	9	14
10	13	8

7. **MULTI-SELECT** Zain is analyzing two sets of data. (Lesson 8-3)

Set A				
35	28	35	33	32
32	31	30	36	49
36	33	29	34	37

Set B				
22	26	26	24	27
38	35	31	22	30
24	33	30	25	38

Select all the true statements about each data set.

A. Data set A has a higher average value.
B. Data set B has a higher average value.
C. C. Data set A is more varied.
D. D. Data set B is more varied.

	Model 8 Inferential Statistics (3 MCQ +1 FRQ)	Lesson 8-4	FRQ
24	**Analyze standardized data and distributions by using z-values .(The table will be provided in the question)**	**Example 6, 7 & 8 (8-13) & 14**	**399 & 400 401 & 402**

Example 6 Use z-Values to Locate Position

Find z if $X = 24$, $\mu = 19$, and $\sigma = 3.8$.

$z = \frac{X - \mu}{\sigma}$ Formula for z-value

$z = \frac{24 - 19}{3.8}$ $X = 24$, $\mu = 19$, and $\sigma = 3.8$

$z = \frac{5}{3.8}$ Subtract.

$z \approx 1.316$ Divide.

Check

Find z if $X = 106.3$, $\mu = 88.8$, and $\sigma = 9.6$. Round your answer to the nearest thousandth. $z =$ __?__

Example 7 Find Area Under the Standard Normal Curve by Using a Table

Use a Standard Normal Distribution Table to find the area under the normal curve within the interval $z < -1.18$.

Step 1 Identify the probability for the left endpoint, $z = -1.18$.

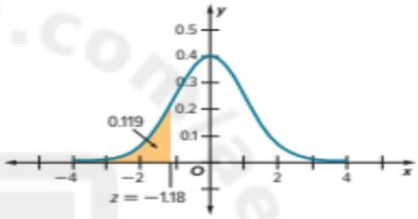
Using a Standard Normal Distribution Table, move down the left column until you reach the correct tenths place, −1.1. Then move across the row until you reach the hundredths place, 0.08.

z	0.00	···	0.08	0.09
−3.4	0.0003	···	0.0003	0.0002
⋮	···	···	···	···
−1.1	0.1357	···	0.1190	0.1170

The probability associated with $z = -1.18$ is 0.1190.

Step 2 Identify the area under the curve within the interval.

As shown in the graph, 0.119 represents the area under the curve to the left of $z = -1.18$.



Example 8 Find Area Under the Standard Normal Curve by Using a Calculator

INTERNET TRAFFIC **The number of daily hits to a local news Web site is normally distributed with $\mu = 98{,}452$ hits and $\sigma = 10{,}325$ hits. Find the probability that the Web site will get at least 100,000 hits on a given day, $P(X > 100{,}000)$.**

Step 1 Find the corresponding z-value for $X = 100{,}000$.

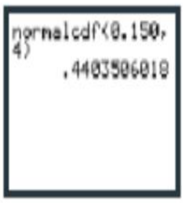
$z = \frac{X - \mu}{\sigma}$ Formula for z-value

$z = \frac{100{,}00 - 98{,}452}{10{,}325}$ $X = 100{,}000$, $\mu = 98{,}452$, and $\sigma = 10{,}325$

$z \approx 0.150$ Simplify.

Step 2 Find the probability.

The area under the curve when $z > 4$ is negligible. So $z = 4$ can be used as an upper bound for finding area. You can use a graphing calculator to find the area between $z = 0.150$ and $z = 4$.



Press **2nd** [DISTR] and select **normalcdf**. Enter the interval and press **enter** to display the area.

The area is 0.44. Therefore, the probability of the Web site getting at least 100,000 hits in one day is 44.0%.

	Model 8 Inferential Statistics (3 MCQ +1 FRQ)	Lesson 8-4	FRQ
24	Analyze standardized data and distributions by using z-values .(The table will be provided in the question)	Example 6, 7 & 8 (8-13) & 14	399 & 400 401 & 402

Example 6

Find the *z*-value for each standard normal distribution.

8. $\sigma = 9.8$, $X = 55.4$, and $\mu = 68.34$

9. $\sigma = 11.6$, $X = 42.80$, and $\mu = 68.2$

10. $\sigma = 11.9$, $X = 119.2$, and $\mu = 112.4$

Example 7

Use a table to find the area under the normal curve for each interval.

11. $z > 0.58$ **12.** $z < -1.56$ **13.** $-2.29 < z < 2.76$

Example 8

14. TESTING The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 12.3.

a. What percent of the scores are between 70 and 80?

b. What percent of the scores are over 115?

c. If 75 people take the test, how many would you expect to score lower than 75?

	Model 9 Trigonometric Functions (6 MCQ +1 FRQ)	Lesson 9-1	MCQ
15	**Convert between degrees and radian measures and find arc lengths by using central angles.**	(54-65)	422

Mixed Exercises

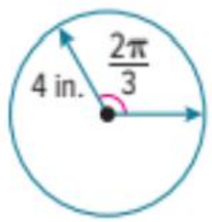
REGULARITY **Rewrite each degree measure in radians and each radian measure in degrees.**

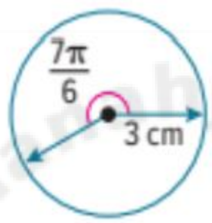
54. $18°$ **55.** $6°$ **56.** $-72°$

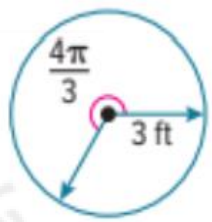
57. $-820°$ **58.** 4π **59.** $\frac{5\pi}{2}$

60. $-\frac{9\pi}{2}$ **61.** $-\frac{7\pi}{12}$ **62.** $-270°$

Find the length of each arc. Round to the nearest tenth.

63.


64.


65.


	Model 9 Trigonometric Functions (6 MCQ +1 FRQ)	Lesson 9- 2	MCQ
16	**Find values of trigonometric functions by using reference angles.**	(40-45)	432

Example 6

PRECISION **Find the exact value of each trigonometric function.**

40. $\tan 330°$ **41.** $\cos\left(-\frac{11\pi}{4}\right)$ **42.** $\cot 30°$

43. $\csc \frac{\pi}{4}$ **44.** $\sin(-150°)$ **45.** $\tan\left(-\frac{\pi}{4}\right)$

	Model 9 Trigonometric Functions (6 MCQ +1 FRQ)	Lesson 9-3	MCQ
17	**Find values of trigonometric functions given a point on a unit circle or the measure of a special angle.**	(1-6)	441

Example 1

The terminal side of angle θ in standard position intersects the unit circle at each point P. Find $\cos\theta$ and $\sin\theta$.

1. $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **2.** $P(0, -1)$ **3.** $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

4. $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$ **5.** $P\left(\frac{1}{6}, -\frac{\sqrt{35}}{6}\right)$ **6.** $P\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$

	Model 9 Trigonometric Functions (6 MCQ +1 FRQ)	Lesson 9- 4	MCQ
18	**Model periodic real-world situations with sine and cosine functions. (No need for graphing calculator)**	**Example 5 & 6 (17-20)**	**449 & 450 452**

Example 5 Characteristics of the Sine and Cosine Functions

SPRINGS An object on a spring oscillates according to the function $y = 40 \cos \pi t$, where y is the distance in centimeters above its equilibrium position at time t in seconds.

Part A Find the period and frequency, and describe them in the context of the situation.

The period is $\frac{2\pi}{|b|}$. Since $b = \pi$, the period is $\frac{2\pi}{\pi}$ or 2.

The frequency is $\frac{1}{\text{period}}$ or $\frac{1}{2}$.

Therefore, the object completes $\frac{1}{2}$ of a cycle per second, and it will reach the maximum distance above the equilibrium point every 2 seconds.

Part B Identify the domain and range in the context of the situation.

The domain of $y = 40 \cos \pi t$ is all real numbers. Because time cannot be negative, the relevant domain in the context of the situation is $[0, \infty)$.

The range is $[-40, 40]$. This references the farthest away that the object can get from its point of equilibrium. With the equilibrium at the center, the object can be as much as 40 centimeters from the center in either direction. This is verified by the amplitude.

Example 6 Model Periodic Situations

ELECTRICITY The voltage supplied by an electrical outlet can be represented by a periodic function. The voltage oscillates between −120 and 120 volts, with a frequency of 50 cycles per second. Write and graph a function for the voltage v as a function of time t.

Step 1 The voltage oscillates from −120 to 120.

The maximum is 120. The minimum is −120.

The amplitude is 120. $a = 120$

Step 2 The frequency is 50 cycles per second.

The period is $\frac{1}{\text{frequency}}$ or $\frac{1}{50}$.

$\text{period} = \frac{2\pi}{b}$ — Period in terms of b

$\frac{1}{50} = \frac{2\pi}{b}$ — Substitute.

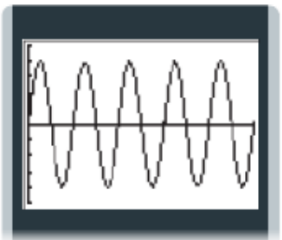
$b = 100\pi$ — Solve.

Step 3 Use t instead of x to write the function as a function of time.

$y = a \sin bt$ — General equation

$y = 120 \sin 100\pi t$ — Substitute.

Step 4 Use a graphing calculator to graph the function.



[0, 0.1] scl: 0.01 by [−150, 150] scl: 30

Example 5

17. USE A MODEL An object on a spring oscillates using the function $y = 25 \cos \pi t$, where y is the distance in inches from its equilibrium position in t seconds.

a. Find and describe the period and frequency.

b. Identify the domain and range in the context of the situation.

18. SWINGS Suppose a tire swing is rotated $\frac{\pi}{5}$ radians and released. The function $y = \frac{\pi}{5} \cos 2t$ represents the displacement of the swing at time t for a frequency of radians per second.

a. Find the period and frequency, and describe them in the context of the situation.

b. Identify the domain and range in the context of the situation.

	Model 9 Trigonometric Functions (6 MCQ +1 FRQ)	Lesson 9- 4	MCQ
18	**Model periodic real-world situations with sine and cosine functions.** **(No need for graphing calculator)**	**Example 5 & 6 (17-20)**	**449 & 450 452**

Example 6

19. **REASONING** A boat that is tied to a dock moves vertically up and down with the waves. Delray watches the boat for 30 seconds and notes that the boat moves up and down a total of 6 times. The difference between the boat's highest point and lowest point is 3 feet. Write and graph a trigonometric function that models the boat's vertical position x seconds after she began watching. Assume that when Delray began watching the boat, it was at its highest point and that its average vertical position was 0 feet.

20. **FERRIS WHEELS** A Ferris wheel at a state fair has a diameter of 65 feet and makes 4 complete revolutions each minute. Santiago boards a car of the Ferris wheel at the car's lowest point, and he rides for 2 minutes. Write and graph a trigonometric function that models his height above or below the axle of the Ferris wheel θ minutes after the ride starts.

	Model 9 Trigonometric Functions (6 MCQ +1 FRQ)	Lesson 9- 6	MCQ
19	**Graph vertical translations of trigonometric functions.**	**(13-18)**	**469**

Example 3

State the amplitude, period, phase shift, vertical shift, and midline equation of each function. Then graph the function and state the domain and range.

13. $y = \cos \theta + 3$

14. $y = \tan \theta - 1$

15. $y = \tan \theta + \frac{1}{2}$

16. $y = 2 \cos \theta - 5$

17. $y = 2 \sin \theta - 4$

18. $y = \frac{1}{3} \sin \theta + 7$

	Model 9 Trigonometric Functions (6 MCQ +1 FRQ)	Lesson 9- 7	MCQ
20	**Find values of angle measures by using inverse trigonometric functions.**	**Example 1 & (1-9)**	**473 & 475**

Example 1 Evaluate Inverse Trigonometric Functions

Find $\text{Tan}^{-1}\sqrt{3}$. Write angle measures in degrees and radians.

Find the angle θ for $-90° \le \theta \le 90°$ with a tangent value of $\sqrt{3}$.

Because $\tan\theta = \frac{\sin\theta}{\cos\theta}$, use the unit circle to find a point where the ratio of sine to cosine is $\sqrt{3}$ to 1. Every special angle on the unit circle can be written as a value over 2. A point on the unit circle at which $\sin\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta = \frac{1}{2}$ has a tangent value of $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ or $\sqrt{3}$.

When $\theta = 60°$, $\tan\theta = \sqrt{3}$. So, $\text{Tan}^{-1}\sqrt{3} = 60°$ or $\frac{\pi}{3}$.

Example 1

Find each value. Write angle measures in degrees and radians.

1. $\text{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right)$
2. $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
3. $\text{Arccos}\left(-\frac{1}{2}\right)$
4. $\text{Arctan}\sqrt{3}$
5. $\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$
6. $\text{Tan}^{-1}(-1)$
7. $\text{Sin}^{-1}\frac{\sqrt{2}}{2}$
8. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
9. $\text{Arcsin } 1$

	Model 9 Trigonometric Functions (6 MCQ +1 FRQ)	Lesson 9-5	FRQ
25	**Graph and analyze tangent functions.** **(No need for graphing calculator)**	**Example 1 & 2 (1-6)**	**455,456 & 457461**

Example 1 Graph a Tangent Function with a Dilation

Find the period, asymptotes, x-intercepts, midline, and transformations of $y = \tan 3x$. Then graph the function.

Step 1 Analyze the function.

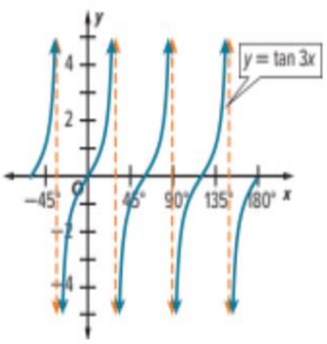
- For a tangent function in the form $y = a\tan bx$, $\frac{180°}{|b|}$ represents the period. Because $b = 3$, the period is $\frac{180°}{|3|}$ or 60°.
- The asymptotes occur at $x = \frac{(90 + 180n)°}{|3|}$ or $(60n + 30)°$, where n is an odd integer.

(continued on the next page)

- x-intercepts occur at integer multiples of the period. The period of this function is 60°, so the x-intercepts are 0°, 60°, 120°, 180°, ...
- The midline of $y = \tan 3x$ is $y = 0$.
- Because $a = 1$, the function is not vertically dilated in relation to the parent function. Because $b = 3$, the function is compressed horizontally in relation to the parent function.

Step 2 Graph the function.

Use the period, asymptotes, and x-intercepts to graph the function. Notice that $y = \tan 3x$ is compressed horizontally in relation to the parent function.



	Model 9 Trigonometric Functions (6 MCQ +1 FRQ)	Lesson 9-5	FRQ
25	**Graph and analyze tangent functions.** **(No need for graphing calculator)**	Example 1 & 2 (1-6)	455,456 & 457461

Example 2 Graph a Tangent Function with a Dilation and a Reflection

Find the period, asymptotes, x-intercepts, midline, and transformations of $y = -\frac{1}{3}\tan 2x$. Then graph the function.

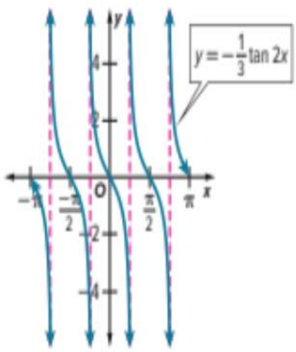
Step 1 Analyze the function.

- The period is $\frac{\pi}{|2|}$ or $\frac{\pi}{2}$.
- The asymptotes occur at $\frac{\frac{\pi}{2} + \pi n}{|b|}$, where n is an odd integer. The asymptotes are at $x = \frac{\frac{\pi}{2} + \pi n}{|2|}$ or $\frac{\pi}{4} + \frac{\pi n}{2}$, where n is an odd integer.
- x-intercepts occur at integer multiples of the period. The period of this function is $\frac{\pi}{2}$, so the x-intercepts are $-\frac{\pi}{2}$, 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, ...
- The midline of $y = -\frac{1}{3}\tan 2x$ is $y = 0$.
- Because $a = -\frac{1}{3}$, the function is compressed vertically and reflected in the x-axis in relation to the parent function. Because $b = 2$, the function is compressed horizontally in relation to the parent function.

Step 2 Graph the function.

Use the period, asymptotes, and x-intercepts to graph the function.

Notice that $y = -\frac{1}{3}\tan 2x$ is compressed vertically, compressed horizontally, and reflected in the x-axis in relation to the parent function.



Examples 1 and 2

Find the period, asymptotes, x-intercepts, midline, and transformations of each tangent function. Then graph the function.

1. $y = \tan 5x$
2. $y = \tan 4x$
3. $y = \tan 2x$
4. $y = \frac{1}{2}\tan x$
5. $y = 2\tan \frac{1}{2}x$
6. $y = -\frac{1}{2}\tan 2x$

	Model 9 Trigonometric Functions (6 MCQ +1 FRQ)	Lesson 9-5	FRQ
25	**Graph and analyze tangent functions. (No need for graphing calculator)**	**Example 1 & 2 (1-6)**	**455,456 & 457461**

Examples 1 and 2

Find the period, asymptotes, *x*-intercepts, midline, and transformations of each tangent function. Then graph the function.

4. $y = \frac{1}{2} \tan x$

5. $y = 2 \tan \frac{1}{2}x$

6. $y = -\frac{1}{2} \tan 2x$