دليل التحضير لاختبار نهاية الفصل (محلول)



تم تحميل هذا الملف من موقع المناهج الإماراتية	
موقع المناهج ⇦ المناهج الإماراتية ⇦ الصف الحادي عشر المتقدم ⇦ فيزياء ⇦ الفصل الأول ⇦ ملفات متنوعة ⇦ الملف	
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ملفات ا كتب للمعلم ا كتب للطالب ا اختبارات الكترونية ا اختبارات ا حلول ا عروض بوربوينت ا أوراق عمل منهج انجليزي ا ملخصات وتقارير ا مذكرات وبنوك ا الامتحان النهائي ا للمدرس	المزيد من مادة فيزياء:
إعداد: Alwardat Mohammed	

التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم					
صفحة المناهج الإماراتية على فيسبوك					
المواد على تلغرام	التربية الاسلامية	اللغة العربية	اللغة الانجليزية	الرياضيات	

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة فيزياء في الفصل الأول	
حزمة التحضير لاختبار نهاية الفصل	1
نموذج امتحان الرسمي	2
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GRADE 11 ADVANCED · PHYSICS TERM 1 (2026 / 2027)

Teacher Copy · Complete Step-by-Step Solutions

End-of-Term Exam Preparation Pack

Organized by Topic · Multi-Color Visual Guide · Clearly Labeled MCQ & FRQ Sections · Prepared by: **Mohammed Alwardat (0568779171)**

Student Name: | Class / Section: | Date:

16	4	100	150
MCQ · Q1–Q16	**FRQ · Q17–Q20**	**Maximum Grade**	**Minutes**
2, 3 or 4 marks each	9 – 11 marks each	Calculator Allowed	Electronic & Paper

Topic Color Roadmap (All Questions Grouped by Concept)

Unit 1: Rotational Kinematics & Frequency
Q1 – Q5 (MCQ) + FRQ Q17 Pt.1 · pp. 3–6

Unit 2: Torque, CM & Static Equilibrium
Q6, Q8, Q9 (MCQ) + FRQ Q17 Pt.2 · pp. 8–24

Unit 3: Moment of Inertia & Rotational Dynamics
Q7, Q10 (MCQ) + FRQ Q18 Pt.1 · pp. 12–24

Unit 4: Linear Momentum & Impulse
Q11, Q12, Q14 (MCQ) + FRQ Q19 · pp. 30–37

Unit 5: Conservation of Linear & Angular Momentum
Q13, Q15, Q16 (MCQ) + FRQ Q18 Pt.2 & FRQ Q20 · pp. 34–47

Key Equations Quick-Reference

Topic	Core Equations
Angular / Linear	$\theta_{rad} = \theta° \times \frac{\pi}{180°}$; $v = r\omega$; $a = r\alpha$; $\omega = 2\pi f$
Torque & Equilibrium	$\tau = Fr \sin\theta$; $\Sigma F = 0$ & $\Sigma\tau = 0$
Rotational Dynamics	$I = \Sigma mr^2$; $\alpha = \frac{\tau_{net}}{I}$; $L = I\omega$; $\tau\Delta t = L_f - L_i$
Linear Momentum	$p = mv$; $F\Delta t = m(v_f - v_i) = p_f - p_i$; $p_i = p_f$

Rotational Kinematics & Angular Frequency

Q1 | MCQ OBJECTIVE

Define a radian and convert degrees to radians and vice-versa.

pp. 3–4 · S.T.B
2, 3 or 4 marks

STUDENT BOOK pp. 3–4

Measuring revolution: A degree is $\frac{1}{360}$ of a revolution. In physics, the **radian (rad)** is defined as $\frac{1}{2\pi}$ of a complete revolution. Thus, **1 complete revolution = 360° = 2π rad**.

- Degrees → Radians: $\theta_{rad} = \theta_{deg} \times \frac{\pi}{180°}$ | Radians → Degrees: $\theta_{deg} = \theta_{rad} \times \frac{180°}{\pi}$
- Counterclockwise rotation is **positive** (+); clockwise rotation is **negative** (−).

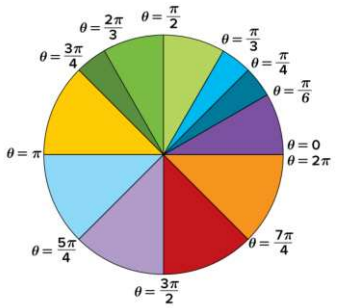


Figure 1 A fraction of a revolution can be measured in degrees or radians. Some common angles are shown below measured in radians. Each angle is measured in the counterclockwise direction from $\theta = 0$.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q1 — rising in difficulty

1 One complete revolution is equal to an angular displacement of:

A π rad	**B 2π rad**
C 180 rad	D 360 rad

Answer: B — A full circle is 360°, which equals **2π rad** (≈ 6.28 rad).

2 An angle of 135° expressed in radians is:

A 0.75 rad	B 1.18 rad
C 2.36 rad	D 4.71 rad

Answer: C — $\theta = 135° \times \frac{\pi}{180°} = \frac{3\pi}{4} \approx$ **2.36 rad**.

3 **A wheel turns through 7.5 complete revolutions. Its angular displacement is:**

A 15.0 rad	**B** **47.1 rad**
C 23.6 rad	**D** 2700 rad

Answer: B — θ = 7.5 rev × 2π rad/rev = 15π rad ≈ **47.1 rad**.

Q2 & Q3

MCQ OBJECTIVE

Relate linear velocity ($v = r\omega$) and tangential acceleration ($a = r\alpha$) to angular quantities and distance (r).

pp. 5–6 · S.T.B
P.P(3,4) · 2–4 marks

STUDENT BOOK p. 5

For any point at distance r from the axis of a rotating rigid body: $\boldsymbol{x = r\theta}$, $\boldsymbol{v = r\omega}$, $\boldsymbol{a = r\alpha}$ (with θ in rad, ω in rad/s, α in rad/s^2).

Table 1 Linear and Angular Measures

Quantity	Linear	Angular	Relationship
Displacement	x (m)	θ (rad)	$x = r\theta$
Velocity	v (m/s)	ω (rad/s)	$v = r\omega$
Acceleration	a (m/s^2)	α (rad/s^2)	$a = r\alpha$

CHOOSE THE CORRECT ANSWER 3 MCQs on Q2 (Linear vs. Angular Velocity) — rising in difficulty

1 **The linear velocity of a point on a rotating rigid object is given by:**

A $v = \frac{\omega}{r}$	**B** **$v = r\omega$**
C $v = \frac{r}{\omega}$	**D** $v = r\omega^2$

Answer: B — Linear velocity is radius times angular velocity: **$v = r\omega$**.

2 **A point 0.25 m from the axis of a disc turning at 12 rad/s has a linear speed of:**

A 0.021 m/s	**B** **3.0 m/s**
C 12 m/s	**D** 48 m/s

Answer: B — $v = r\omega$ = 0.25 × 12 = **3.0 m/s**.

3 **Two children sit 1.2 m and 2.4 m from the centre of a merry-go-round turning at 0.80 rad/s. Which statement is correct?**

A Both have the same linear speed	**B** **The outer child has twice the linear speed**
C The outer child has twice the angular speed	**D** The inner child has the greater linear speed

Answer: B — Both share $\omega = 0.80$ rad/s. Since $v = r\omega$ and 2.4 m = 2 × 1.2 m, the outer child has **twice the linear speed**.

PRACTICE PROBLEM 3 Student Book p. 6

If a truck has a linear acceleration of 1.85 m/s^2 and the wheels have an angular acceleration of 5.23 rad/s^2, what is the diameter of the wheels?

Given	$a = 1.85$ m/s^2, $\alpha = 5.23$ rad/s^2
Step 1	**Relate *a* and α:** $a = r\alpha \rightarrow r = \frac{a}{\alpha} = \frac{1.85}{5.23} = 0.3537$ m
Step 2	**Find diameter *d*:** $d = 2r = 2 \times 0.3537 =$ **0.707 m**
Answer	***d* = 0.707 m**

PRACTICE PROBLEM 4 Student Book p. 6

The truck in Problem 3 tows a trailer with wheels of diameter 48 cm. **a.** Compare the trailer's linear acceleration with the truck's. **b.** Compare their wheel angular accelerations.

Given	$d_{trailer} = 0.48$ m → $r_{trailer} = 0.24$ m; $r_{truck} = 0.354$ m
Step 1	**Part (a): Linear acceleration:** Hitched together, both move identically → **Same linear acceleration (1.85 m/s^2)**
Step 2	**Part (b): Angular acceleration:** $\alpha_{trailer} = \frac{a}{r_{trailer}} = \frac{1.85}{0.24} =$ **7.7 rad/s^2** > 5.23 rad/s^2 (trailer wheels have greater α).
Answer	**a) Same linear acceleration (1.85 m/s^2) | b) Trailer wheels have greater α (7.7 rad/s^2)**

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CHOOSE THE CORRECT ANSWER 3 MCQs on Q3 (Linear vs. Angular Acceleration) — rising in difficulty

1 **The tangential acceleration of a point on a rotating object is given by:**

A $a = \frac{\alpha}{r}$

B $a = r\alpha$

C $a = \frac{r}{\alpha}$

D $a = r\omega^2$

Answer: B — Tangential acceleration is $a = r\alpha$.

2 **A grinding wheel of radius 0.15 m has an angular acceleration of 8.0 rad/s^2. A point on its rim has a tangential acceleration of:**

A 0.019 m/s^2

B **1.2 m/s^2**

C 8.0 m/s^2

D 53 m/s^2

Answer: B — $a = r\alpha = 0.15 \times 8.0 =$ **1.2 m/s^2**.

3 **A tyre of radius 0.33 m takes a car from rest to 25 m/s in 8.0 s without slipping. The angular acceleration of the tyre is:**

A 1.03 rad/s^2

B 3.1 rad/s^2

C **9.5 rad/s^2**

D 75.8 rad/s^2

Answer: C — Step 1: $a = \frac{25 - 0}{8.0} = 3.125$ m/s^2. Step 2: $\alpha = \frac{3.125}{0.33} \approx$ **9.5 rad/s^2**.

Q4 & Q5

MCQ OBJECTIVE

1. Relate frequency (*f*) to angular velocity (ω = 2π*f*) and solve rotational problems.
2. Show that all points on a rigid body rotate through the same angle at the same rate.

pp. 4–6 · S.T.B
P.P(2) · C.Y.P(6,8,10)
2–4 marks

STUDENT BOOK pp. 4–6

Angular frequency: $f = \frac{\omega}{2\pi}$ ↔ $\omega = 2\pi f$. Convert RPM to rad/s: ω = RPM × $\frac{2\pi}{60}$.

All points on a rigid body (like Earth: $\omega_E = 7.27 \times 10^{-5}$ rad/s) turn through the **same angle Δθ** and have the **same ω**, while outer points (larger *r*) move a larger linear distance ($d = r\theta$).

Earth's angular velocity Angular velocity is measured in rad/s. You can calculate Earth's angular velocity as follows:

$$\omega_E = \frac{(2\pi \text{ rad})}{(24.0 \text{ h})(3600 \text{ s/h})} = 7.27\times10^{-5} \text{ (rad/s)}$$

In the same way that counterclockwise rotation produces positive angular displacement, it also results in positive angular velocity.

If an object's angular velocity is ω, then the linear velocity of a point a distance r from the axis of rotation is given by $v = r\omega$. The speed at which an object on Earth's equator moves as a result of Earth's rotation is given by $v = r\omega$ $= (6.38\times10^6 \text{ m/rad})(7.27\times10^{-5} \text{ rad/s}) = 464 \text{ m/s}$. Earth is an example of a rotating, rigid body, as shown in **Figure 3**. Even though different points at different latitudes on Earth do not move the same distance in each revolution, all points rotate through the same angle. All parts of a rigid body rotate at the same rate. The Sun is not a rigid body. Different parts of the Sun rotate at different angular velocities. Most objects that we will consider in this chapter are rigid bodies.

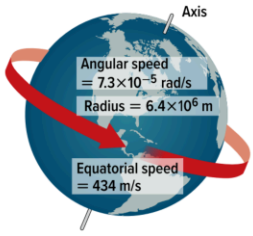


Figure 3 Earth is a rotating, rigid body and all parts rotate at the same rate.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q4 (Frequency & Angular Velocity) — rising in difficulty

1 **The angular velocity ω of an object rotating with frequency *f* is:**

A $\omega = \frac{f}{2\pi}$ **B** **$\omega = 2\pi f$**

C $\omega = \frac{2\pi}{f}$ **D** $\omega = f^2$

Answer: B — Each revolution is 2π rad, so ω = **2π*f***.

2 **A fan blade spins at 1200 rpm. Its angular velocity is about:**

A 20 rad/s **B** **126 rad/s**

C 1200 rad/s **D** 7540 rad/s

Answer: B — $f = \frac{1200}{60}$ = 20 rev/s → ω = 2π(20) ≈ **126 rad/s**.

3 **A rotor starts from rest and reaches 45 rad/s in 6.0 s at constant angular acceleration. The number of revolutions turned in that time is about:**

A 7.5	**B** **21**
C 43	**D** 135

Answer: B — $\theta = \frac{0 + 45}{2}(6.0) = 135$ rad → Revolutions = $\frac{135}{2\pi} \approx$ **21 rev**.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q5 (Rigid Body Rotation) — rising in difficulty

1 **Two points at different distances from the axis of a rotating rigid object always have the same:**

A linear speed	**B** **angular displacement**
C distance travelled	**D** tangential acceleration

Answer: B — All points on a rigid body turn through the same angle in the same time (same **angular displacement**).

2 **A point on Earth's equator and a point at latitude 60° both complete one rotation in 24 h. The point at 60° has:**

A a smaller angular velocity	**B** the same linear speed
C **a smaller linear speed**	**D** a larger linear speed

Answer: C — Both share the same ω, but latitude 60° has a smaller radius r, so $v = r\omega$ is **smaller**.

3 **Two dust specks sit 1.5 cm and 5.5 cm from the centre of a CD turning at 500 rpm. In 4.0 s:**

A both travel the same distance	**B** **both turn through the same angle, but the outer one travels further**
C the outer speck turns through the larger angle	**D** the inner speck has the larger angular velocity

Answer: B — Same angular displacement θ, larger arc length $s = r\theta$ for the outer speck.

FRQ Q17 (Pt.1)	FRQ · FREE RESPONSE **Define angular displacement and apply ω = Δθ/Δ*t* to rotational problems.**	pp. 4–6 · P.P(1,2) C.Y.P(6,7,8,9,10) 9 – 11 marks

PRACTICE PROBLEM 1 FRQ · FREE RESPONSE Student Book p. 6

Find the angular displacement of each clock hand in 1.00 h (3 sig. figs.): **a.** second hand, **b.** minute hand, **c.** hour hand.

Given	Δt = 1.00 h; Clockwise rotation is negative (−).
Step 1	**Part (a): Second hand:** 60 revolutions in 1 h → $\Delta\theta = -60 \times 2\pi =$ **−377 rad**
Step 2	**Part (b): Minute hand:** 1 revolution in 1 h → $\Delta\theta = -1 \times 2\pi =$ **−6.28 rad**
Step 3	**Part (c): Hour hand:** $\frac{1}{12}$ revolution in 1 h → $\Delta\theta = -\frac{2\pi}{12} =$ **−0.524 rad**
Answer	**a) −377 rad | b) −6.28 rad | c) −0.524 rad**

PRACTICE PROBLEM 2 FRQ · FREE RESPONSE Student Book p. 6

A crib toy makes 1 complete counterclockwise rotation in 1 min. Find: **a.** Δθ in 3 min, **b.** ω in rad/min, **c.** sign of α when turned off.

Given	1 CCW rotation = +2π rad in 1 min
Step 1	**Part (a): Angular displacement:** $\Delta\theta = 3 \times 2\pi = +6\pi$ rad ≈ **+18.8 rad**
Step 2	**Part (b): Angular velocity:** $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{1 \text{ min}} =$ **6.28 rad/min**
Step 3	**Part (c): Sign of α:** Slowing down while spinning in + direction means Δω < 0 → **Negative α**
Answer	**a) +18.8 rad | b) 6.28 rad/min | c) Negative**

CHECK YOUR PROGRESS 6 **FRQ · FREE RESPONSE** Student Book p. 6

The Moon ($r = 1.74 \times 10^6$ m) rotates once in 27.3 days. Find: **a.** period in s, **b.** angular velocity in rad/s, **c.** equatorial linear speed, **d.** compare with Earth's equator (464 m/s).

Given	$r = 1.74 \times 10^6$ m, $T = 27.3$ days
Step 1	**Part (a): Period in seconds:** $T = 27.3 \times 24 \times 3600 =$ **2.36×10^6 s**
Step 2	**Part (b): Angular velocity:** $\omega = \dfrac{2\pi}{2.359 \times 10^6} =$ **2.66×10^{-6} rad/s**
Step 3	**Part (c): Linear speed:** $v = r\omega = (1.74 \times 10^6)(2.664 \times 10^{-6}) =$ **4.64 m/s**
Step 4	**Part (d): Compare with Earth:** Ratio $= \dfrac{464}{4.64} \approx$ **100×** (Earth's equator is ~100× faster).
Answer	**a) 2.36×10^6 s | b) 2.66×10^{-6} rad/s | c) 4.64 m/s | d) Earth is ~100× faster**

CHECK YOUR PROGRESS 7 **FRQ · FREE RESPONSE** Student Book p. 6

In a 2.0-h movie, find the angular displacement $\Delta\theta$ of the hour, minute, and second hands of a clock.

9. **Angular Displacement** Do all parts of the minute hand on a watch such as the one shown in **Figure 4** have the same angular displacement? Do they move the same linear distance? Explain.
10. **Critical Thinking** A CD-ROM has a spiral track that starts 2.7 cm from the center of the disk and ends 5.5 cm from the center. The disk drive must turn



Figure 4

Given	$\Delta t = 2.0$ h; Clockwise rotation is negative (−).
Step 1	**Hour hand:** $\Delta\theta = -\dfrac{2}{12}(2\pi) =$ **−1.05 rad**
Step 2	**Minute hand:** $\Delta\theta = -2(2\pi) =$ **−12.6 rad**
Step 3	**Second hand:** $\Delta\theta = -120(2\pi) =$ **−754 rad**
Answer	**Hour: −1.05 rad | Minute: −12.6 rad | Second: −754 rad**

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CHECK YOUR PROGRESS 9 **FRQ · FREE RESPONSE** Student Book p. 6

Do all parts of a watch's minute hand have the same angular displacement Δθ? Do they travel the same linear distance?

Given	Rigid body rotation: $d = r\theta$
Step 1	**Angular displacement: Yes:** All points on a rigid hand turn through the **same angle Δθ**.
Step 2	**Linear distance: No:** Since $d = r\theta$, the tip (larger r) travels a **greater linear distance**.
Answer	**Same Δθ, different linear distance**

CHECK YOUR PROGRESS 8 **FRQ · FREE RESPONSE** Student Book p. 6

A washer drum spins at 635 rev/min and comes to a stop in 8.0 s. Find its average angular acceleration α.

Given	$f_i = 635$ rev/min, $\omega_f = 0$, $\Delta t = 8.0$ s
Step 1	**Convert rpm to rad/s:** $\omega_i = 635 \times \frac{2\pi}{60} = 66.5$ rad/s
Step 2	**Angular acceleration:** $\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 - 66.5}{8.0} =$ **-8.3 rad/s^2**
Answer	**$\alpha = -8.3$ rad/s^2**

CHECK YOUR PROGRESS 10 **FRQ · FREE RESPONSE** Student Book p. 6

A CD track starts at $r_1 = 2.7$ cm and ends at $r_2 = 5.5$ cm at constant linear speed $v = 1.4$ m/s for 76 min. Find ω_1, ω_2, and average α.

Given	$r_1 = 0.027$ m, $r_2 = 0.055$ m, $v = 1.4$ m/s, $\Delta t = 76 \times 60 = 4560$ s
Step 1	**Initial ω_1:** $\omega_1 = \frac{v}{r_1} = \frac{1.4}{0.027} =$ **52 rad/s** (495 rpm)
Step 2	**Final ω_2:** $\omega_2 = \frac{v}{r_2} = \frac{1.4}{0.055} =$ **25 rad/s** (243 rpm)
Step 3	**Average α:** $\alpha = \frac{25.45 - 51.85}{4560} =$ **-5.8×10^{-3} rad/s^2**
Answer	**$\omega_1 = 52$ rad/s, $\omega_2 = 25$ rad/s, $\alpha = -5.8 \times 10^{-3}$ rad/s^2**

PRACTICE · YOUR TURN | FRQ · FREE RESPONSE | 3 Free-Response questions on FRQ Q17 — rising in difficulty

1 RECALL

Define angular displacement, and write the equation for average angular velocity in terms of angular displacement and time interval.

Step 1	**Definition & Equation:** Angular displacement (Δθ) is the change in angular position (rad). Average angular velocity: $\boldsymbol{\omega = \frac{\Delta\theta}{\Delta t}}$.
Answer	**Δθ = change in angle (rad); $\omega = \frac{\Delta\theta}{\Delta t}$**

2 APPLY

A wheel turns through 8.0 rad in 2.5 s. Find its average angular velocity in rad/s and then in revolutions per minute.

Given	$\Delta\theta = 8.0$ rad, $\Delta t = 2.5$ s
Step 1	**In rad/s:** $\omega = \frac{8.0}{2.5}$ = **3.2 rad/s**
Step 2	**In rev/min:** $f = 3.2 \times \frac{60}{2\pi}$ = **30.6 rev/min**
Answer	**3.2 rad/s | 31 rev/min**

3 CHALLENGE

A spanner of length 0.28 m is pulled with 85 N at 55° to the handle. Find the torque, the angle for maximum torque, and that maximum torque.

Given	$r = 0.28$ m, $F = 85$ N, $\theta = 55°$
Step 1	**Torque at 55°:** $\tau = 85 \times 0.28 \times \sin 55° =$ **19.5 N·m**
Step 2	**Maximum torque:** Occurs at **90°**: $\tau_{max} = 85 \times 0.28 \times 1 =$ **23.8 N·m**
Answer	**τ = 19.5 N·m; Max at 90° = 23.8 N·m**

Torque, Center of Mass & Static Equilibrium

Q6	MCQ OBJECTIVE **Assign torque signs ("clocks are negative") and calculate net torque ($\tau = Fr \sin\theta$).**	pp. 8–9 · S.T.B E.P(1) · P.P(11–14) 2–4 marks

STUDENT BOOK p. 8

Torque: $\tau = Fr \sin\theta$ (unit: N·m), where $r \sin\theta$ is the perpendicular lever arm. Counterclockwise (CCW) is **+**; clockwise (CW) is **−**.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q6 (Torque & Sign Convention) — rising in difficulty

1 **By the usual sign convention, a torque that tends to turn a body clockwise is:**

A positive
B negative
C zero
D always larger than an anticlockwise torque

Answer: B — "Clocks are negative": clockwise torque is **negative**.

2 **A 25 N force acts on a door 0.40 m from the hinge, at 60° to the door. The torque about the hinge is:**

A 5.0 N·m
B 8.7 N·m
C 10 N·m
D 20 N·m

Answer: B — $\tau = 25 \times 0.40 \times \sin 60° =$ **8.7 N·m**.

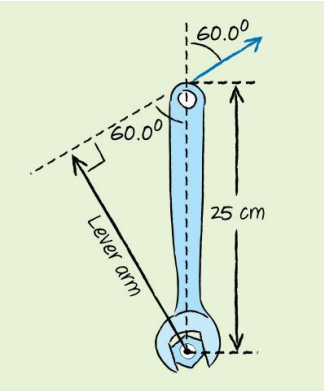
3 **A rod is pivoted at its centre. A 12 N force pushes down 0.50 m to the left and an 18 N force pushes down 0.30 m to the right. The net torque is:**

A +0.6 N·m, anticlockwise
B −0.6 N·m, clockwise
C +11.4 N·m, anticlockwise
D zero

Answer: A — $\tau_{net} = +(12)(0.50) - (18)(0.30) = +6.0 - 5.4 =$ **+0.6 N·m (anticlockwise)**.

EXAMPLE PROBLEM 1 Student Book p. 9

A bolt requires 35 N·m of torque. You pull a 25-cm wrench at 60.0° to the handle. Find the lever arm and the applied force.



Given	$\tau = 35$ N·m, $r = 0.25$ m, $\theta = 60.0°$
Step 1	**Find lever arm:** $r_\perp = r \sin\theta = 0.25 \sin 60.0° =$ **0.22 m**
Step 2	**Find force *F*:** $\tau = F r_\perp \rightarrow F = \dfrac{\tau}{r \sin\theta} = \dfrac{35}{0.2165} =$ **1.6×10^2 N**
Answer	**Lever arm = 0.22 m; Force = 1.6×10^2 N**

PRACTICE PROBLEM 11 Student Book p. 9

In Example Problem 1, what force is needed if you pull perpendicular (90°) to the 25-cm wrench?

Given	$\tau = 35$ N·m, $r = 0.25$ m, $\theta = 90°$ ($\sin 90° = 1$)
Step 1	**Apply $\tau = Fr \sin\theta$:** $F = \dfrac{\tau}{r \sin 90°} = \dfrac{35}{0.25 \times 1} =$ **1.4×10^2 N**
Answer	***F* = 1.4×10^2 N**

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PRACTICE PROBLEM 12 Student Book p. 9

A bolt requires a torque of 55.0 N·m, and your maximum force is 135 N. What is the minimum lever arm required?

14. You stand on a bicycle pedal, as shown in **Figure 7.** Your mass is 65 kg. If the pedal makes an angle of 35° above the horizontal and the pedal is 18 cm from the center of the chain ring, how much torque would you exert?



Figure 7

Given	$\tau = 55.0$ N·m, $F = 135$ N
Step 1	**Solve for lever arm:** $\tau = F \times$ (lever arm) → lever arm $= \frac{55.0}{135} =$ **0.407 m**
Answer	**Lever arm = 0.407 m**

PRACTICE PROBLEM 13 Student Book p. 9

A 0.234-m wrench must deliver 32.4 N·m of torque. Find the smallest angle θ if the applied force is: **a.** 232 N, **b.** 275 N.

Given	$r = 0.234$ m, $\tau = 32.4$ N·m
Step 1	**Part (a): F = 232 N:** $\sin\theta = \frac{\tau}{Fr} = \frac{32.4}{232 \times 0.234} = 0.5968$ → $\theta =$ **36.6°**
Step 2	**Part (b): F = 275 N:** $\sin\theta = \frac{32.4}{275 \times 0.234} = 0.5035$ → $\theta =$ **30.2°**
Answer	**a) 36.6° | b) 30.2°**

PRACTICE PROBLEM 14 Student Book p. 9

A 65-kg rider stands vertically on an 18-cm bicycle pedal angled 35° above the horizontal. Find the torque on the pedal.

Given	m = 65 kg → F_g = 65 × 9.8 = 637 N; r = 0.18 m; angle between vertical F_g and pedal = 90° − 35° = 55°
Step 1	**Calculate torque:** $\tau = Fr \sin\theta$ = 637 × 0.18 × sin 55° = **94 N·m**
Answer	**τ = 94 N·m**

Q8 & Q9

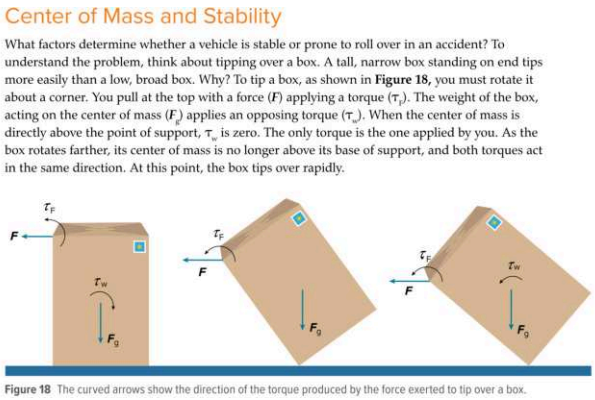
MCQ OBJECTIVE

Locate center of mass, explain stability, and state the two conditions for static equilibrium ($\Sigma F = 0$ and $\Sigma\tau = 0$).

pp. 18–20, 24 · S.T.B
C.Y.P(41,42,44,45)
2–4 marks

STUDENT BOOK pp. 18–20

Center of Mass & Stability: Suspend from two points and intersect the vertical plumb lines. An object stays stable while the vertical line from its CM stays inside its base (low CM + wide base = most stable).
Static Equilibrium: $\Sigma F = 0$ (translational) AND **$\Sigma\tau = 0$** (rotational).



CHOOSE THE CORRECT ANSWER 3 MCQs on Q8 (Center of Mass & Stability) — rising in difficulty

1 **To locate the centre of mass of a flat, irregular sheet you suspend it from two or more points and:**

A measure its weight each time

B mark the vertical line below each point and find where the lines cross

C balance it on a knife edge once

D measure its area

Answer: B — The CM lies on each vertical plumb line, so their intersection locates it.

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2 **An object is most stable when its centre of mass is:**

A high, with a narrow base	**B low, with a wide base**
C high, with a wide base	D outside its base

Answer: B — A **low CM** and **wide base** maximize stability.

3 **A vehicle begins to topple at the moment when:**

A its centre of mass is above the centre of its base	**B the vertical line through its centre of mass falls outside its base**
C its weight exceeds the normal force	D its base is wider than its height

Answer: B — Once the CM's vertical line falls **outside the base**, gravity tips the vehicle over.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q9 (Static Equilibrium) — rising in difficulty

1 **For an object to be in static equilibrium:**

A $\Sigma F = 0$ only	B $\Sigma\tau = 0$ only
C both $\Sigma F = 0$ and $\Sigma\tau = 0$	D its velocity must be constant but not zero

Answer: C — Both net force and net torque must equal zero.

2 **A uniform plank 4.0 m long weighing 120 N rests on a support at each end. Each support pushes up with:**

A 30 N	**B 60 N**
C 120 N	D 240 N

Answer: B — By symmetry, each support carries $\frac{120\ \text{N}}{2}$ = **60 N**.

3 **A uniform 6.0 m beam of weight 200 N rests on a pivot 2.0 m from its left end. The downward force needed at the left end to keep it horizontal is:**

A 50 N	**B 100 N**
C 150 N	D 200 N

Answer: B — $F_{\text{left}}(2.0) = 200(1.0) \rightarrow F_{\text{left}} = \frac{200}{2.0}$ = **100 N**.

CHECK YOUR PROGRESS 41, 42, 44 & 45 Student Book p. 24

41. Example of (a) rotational but not translational equilibrium; (b) translational but not rotational. **42.** Can CM be where there is no mass? **44.** Where is CM on a roll of tape? **45.** How to find CM of a textbook?

Step 1	**CYP 41:** (a) $F_{net} \neq 0$, $\tau_{net} = 0$: pushing a box across a floor (or falling non-rotating ball). (b) $F_{net} = 0$, $\tau_{net} \neq 0$: ceiling fan speeding up.
Step 2	**CYP 42 & 44: Yes** (e.g. ring, hollow sphere, boomerang); on a roll of tape it is in the **center of the hollow hole**.
Step 3	**CYP 45:** Suspend from two corners with plumb lines and find where the vertical lines intersect.
Answer	**Conceptual equilibrium & center-of-mass answers complete.**

FRQ Q17 (Pt.2)

FRQ · FREE RESPONSE

Solve multi-step torque and static equilibrium problems.

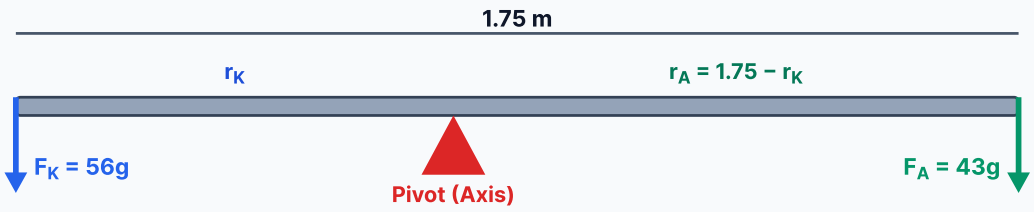
pp. 11–12, 21–22
E.P(2,5) · P.P(18,38–40)
9 – 11 marks

EXAMPLE PROBLEM 2 **FRQ · FREE RESPONSE** Student Book p. 11

Kariann (56 kg) and Aysha (43 kg) balance on a 1.75-m massless seesaw. Where should the pivot be placed?

FREE-BODY / VECTOR DIAGRAM

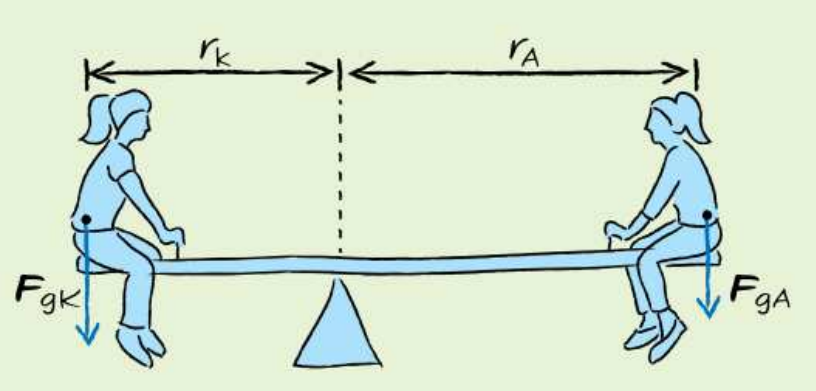
Forces: ↑+ , ↓− | Torques: ↺+ , ↻−



Given	$m_K = 56$ kg, $m_A = 43$ kg, $L = 1.75$ m, $r_K + r_A = 1.75$ m
Step 1	**Rotational Equilibrium ($\tau_{net} = 0$ about pivot):** $+F_K r_K - F_A r_A = 0 \rightarrow (56g)r_K = (43g)(1.75 - r_K)$
Step 2	**Solve for r_K:** $56r_K = 75.25 - 43r_K \rightarrow 99r_K = 75.25 \rightarrow r_K = \frac{75.25}{99} =$ **0.76 m** from Kariann (0.99 m from Aysha).
Answer	**0.76 m from Kariann (0.99 m from Aysha)**

PRACTICE PROBLEM 18 **FRQ · FREE RESPONSE** Student Book p. 12

Two baskets hang from two joined pulleys of radii 4.5 cm (A) and 1.1 cm (B). Basket B has mass 0.23 kg. Find mass m_A for equilibrium.



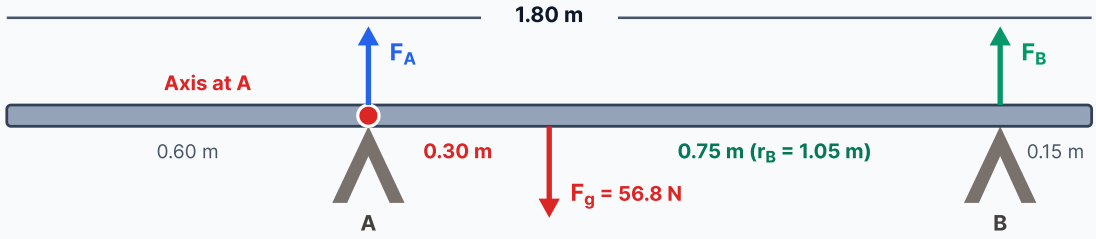
Given	r_A = 4.5 cm, r_B = 1.1 cm, m_B = 0.23 kg
Step 1	**Rotational Equilibrium (τ_{net} = 0):** $+m_A g\, r_A - m_B g\, r_B = 0 \rightarrow m_A(4.5) = 0.23(1.1)$
Step 2	**Solve for m_A:** $m_A = \dfrac{0.23 \times 1.1}{4.5}$ = **0.056 kg** (56 g)
Answer	**m_A = 0.056 kg**

EXAMPLE PROBLEM 5 **FRQ · FREE RESPONSE** Student Book p. 21

A 5.8-kg ladder, 1.80 m long, rests on two sawhorses. Sawhorse A is 0.60 m from the left end, and sawhorse B is 0.15 m from the right end. Find the force exerted by each sawhorse.

FREE-BODY / VECTOR DIAGRAM

Forces: ↑+ , ↓− | Torques: ↻+ , ↺−



Given

m = 5.8 kg → $\boldsymbol{F_g = 5.8 \times 9.8 = 56.8\ N}$ (at CM = 0.90 m); Choose axis of rotation at **A** → r_g = 0.30 m, r_B = 0.30 + 0.75 = 1.05 m

Rotational Equilibrium ($\tau_{net} = 0$) ②

~~τ_A~~ + τ_g + τ_B = 0

$-F_g(0.30) + F_B(1.05) = 0$

$-56.8(0.30) + 1.05\,F_B = 0$

Translational Equilibrium ($F_{net} = 0$) ①

$F_A + F_B - F_g = 0$

$F_A + F_B = 56.8$ N

$F_A = 56.8 - F_B$

Step 1 **Solve Eq. 2 for F_B:** $-56.8(0.30) + F_B(1.05) = 0 \rightarrow F_B = \dfrac{56.8 \times 0.30}{1.05} =$ **16.2 N**

Step 2 **Substitute into Eq. 1 for F_A:** $F_A = 56.8 - 16.2 =$ **40.6 N**

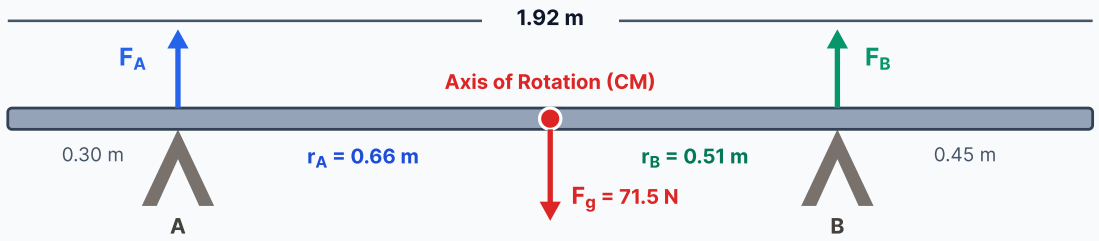
Answer $\boldsymbol{F_A = 41}$ **N upward;** $\boldsymbol{F_B = 16}$ **N upward**

PRACTICE PROBLEM 38 **FRQ · FREE RESPONSE** Student Book p. 22

A 7.3-kg ladder, 1.92 m long, rests on two sawhorses. Sawhorse A (left) is 0.30 m from the end, and B (right) is 0.45 m from the other end. Choose the axis of rotation at the center of mass (CM). **a.** What are the torques acting on the ladder? **b.** Write the equation for rotational equilibrium. **c.** Solve for F_A in terms of F_g (and find F_A, F_B). **d.** How do forces change if A moves very close to CM?

FREE-BODY / VECTOR DIAGRAM

Forces: ↑+ , ↓− | Torques: ↺+ , ↻−



Given m = 7.3 kg → $\boldsymbol{F_g = 7.3 \times 9.8 = 71.5}$ **N**; r_A = 0.96 − 0.30 = **0.66 m**; r_B = 0.96 − 0.45 = **0.51 m**

(b) Rotational Equilibrium ($\tau_{net} = 0$) ②

$\tau_A + \tau_B + \cancel{\tau_g} = 0$

$-F_A(0.66) + F_B(0.51) = 0$

$\mathbf{-0.66\,F_A + 0.51\,F_B = 0}$

Translational Equilibrium ($F_{net} = 0$) ①

$F_A + F_B - F_g = 0$

$F_A + F_B = F_g = 71.5$ N

$\therefore F_B = F_g - F_A$

Step 1 **Part (a): Torques about CM:** τ_A = $\mathbf{-F_A(0.66\ m)}$ (CW −); τ_B = $\mathbf{+F_B(0.51\ m)}$ (CCW +); τ_g = **0** (at axis)

Step 2 **Part (b) & (c): Substitute Eq. 1 into Eq. 2:** $-0.66\,F_A + 0.51(F_g - F_A) = 0$ → $1.17\,F_A = 0.51\,F_g$ → $F_A = \frac{0.51}{1.17} F_g$ = **$0.436\,F_g$**

Numerical: F_A = 0.436 × 71.5 = **31.2 N**; F_B = 71.5 − 31.2 = **40.3 N**

Step 3 **Part (d): Moving A close to CM:** As $r_A \to 0$, **F_A increases** (approaches F_g) and **F_B decreases** (approaches 0).

Answer **a) $-0.66F_A$, $+0.51F_B$, 0 | b) $-0.66F_A + 0.51F_B = 0$ | c) $0.436F_g$ (31.2 N), F_B = 40.3 N | d) F_A increases, F_B decreases**

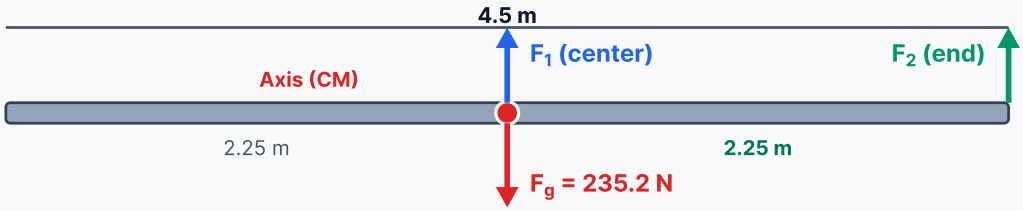
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PRACTICE PROBLEM 39 **FRQ · FREE RESPONSE** Student Book p. 22

A 4.5-m-long wooden plank with a 24-kg mass is supported in two places: one directly under the center (F_1) and the other at one end (F_2). Find the forces exerted by both supports.

FREE-BODY / VECTOR DIAGRAM

Forces: ↑+ , ↓− | Torques: ↺+ , ↻−



Given

m = 24 kg → $\boldsymbol{F_g = 24 \times 9.8 = 235.2\ N}$; Choose axis at **center (CM)** → $r_1 = 0$, $r_g = 0$, $r_2 = 2.25$ m

Rotational Equilibrium ($\tau_{net} = 0$) ②

~~τ_1~~ + τ_2 + ~~τ_g~~ = 0

$\tau_2 = 0$ → $F_2(2.25) = 0$

$\boldsymbol{F_2 = 0\ N}$

Translational Equilibrium ($F_{net} = 0$) ①

$F_1 + F_2 - F_g = 0$

$F_1 + 0 = 235.2$ N

$\boldsymbol{F_1 = 235.2\ N}$

Step 1 **Solve Eq. 2 for F_2:** $\tau_2 = 0$ → $F_2(2.25) = 0$ → **$F_2 = 0$ N**

Step 2 **Substitute into Eq. 1 for F_1:** $F_1 + 0 = 235.2$ N → **$F_1 = 235.2$ N** (2.4×10^2 N)

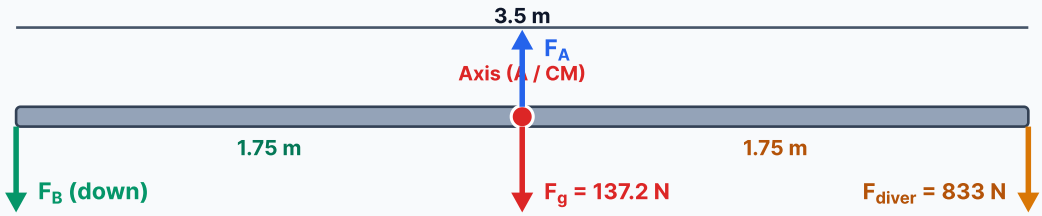
Answer $\boldsymbol{F_{center} = 2.4 \times 10^2\ N}$ **upward;** $\boldsymbol{F_{end} = 0\ N}$

PRACTICE PROBLEM 40 (CHALLENGE) **FRQ · FREE RESPONSE** Student Book p. 22

An 85-kg diver walks to the end of a 3.5-m diving board (mass 14 kg), supported at its center of mass (A) and at the opposite end (B). Find the forces on both supports.

FREE-BODY / VECTOR DIAGRAM

Forces: ↑+ , ↓− | Torques: ↺+ , ↻−



Given $F_{diver} = 85 \times 9.8 =$ **833 N** (at $r = 1.75$ m from A); $F_g = 14 \times 9.8 =$ **137.2 N** (at A); Axis at **A (CM)**

Rotational Equilibrium ($\tau_{net} = 0$) ②

$\tau_{diver} + \cancel{\tau_A} + \tau_B + \cancel{\tau_g} = 0$

$-833(1.75) - F_B(1.75) = 0$

$F_B = -833$ N (down)

Translational Equilibrium ($F_{net} = 0$) ①

$F_A + F_B - F_{diver} - F_g = 0$

$F_A - 833 - 833 - 137.2 = 0$

$F_A = 1803$ N (up)

Step 1 **Solve Eq. 2 for F_B:** $+833(1.75) + F_B(1.75) = 0 \rightarrow F_B =$ **−833 N** (833 N downward)

Step 2 **Substitute into Eq. 1 for F_A:** $F_A - 833 - 137.2 - 833 = 0 \rightarrow F_A =$ **1803 N** (1.8×10^3 N upward)

Answer **Center support A = 1.8×10^3 N upward; End support B = 8.3×10^2 N downward**

Moment of Inertia, Rotational Dynamics & Coriolis

Q7 & Q10	MCQ OBJECTIVE	pp. 12, 14–15, 23–24 E.P(3) · P.P(21,24) 2–4 marks
	Calculate moment of inertia of point masses ($I = \Sigma mr^2$) and describe the apparent Coriolis force in rotating frames.	

STUDENT BOOK pp. 12, 23–24

Point-mass moment of inertia: $I = \Sigma mr^2$ (kg·m^2).
Coriolis "Force": An apparent force observed only in a rotating frame of reference; in the Northern Hemisphere, moving objects appear deflected **to the right**.



Figure 22 An observer on Earth sees the Coriolis "force" cause a projectile fired due north to deflect to the right of the intended target.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q7 (Moment of Inertia) — rising in difficulty

1 **The moment of inertia of a single point mass m at a distance r from the axis is:**

A $I = mr$
B $I = mr^2$
C $I = \frac{1}{2}mr^2$
D $I = \frac{m}{r^2}$

Answer: B — For a point mass, $I = mr^2$.

2 **A 2.0 kg mass sits 0.60 m from the axis. If its distance is doubled, the moment of inertia becomes:**

A twice as large
B four times as large
C half as large
D unchanged

Answer: B — Since $I \propto r^2$, doubling r gives **4 times** the moment of inertia.

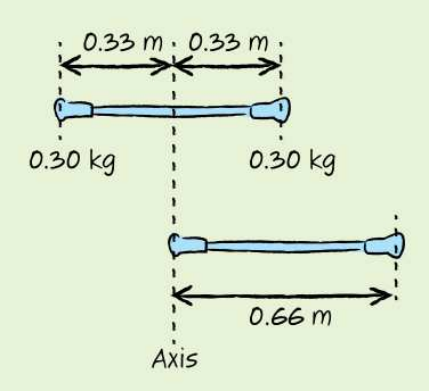
3 **Masses of 1.5 kg at 0.20 m, 2.0 kg at 0.40 m and 3.0 kg at 0.60 m are fixed on a light rod. The total moment of inertia is:**

A 0.38 kg·m^2	**B** **1.46 kg·m^2**
C 2.60 kg·m^2	**D** 6.5 kg·m^2

Answer: B — $I = 1.5(0.20)^2 + 2.0(0.40)^2 + 3.0(0.60)^2 =$ **1.46 kg·m^2**.

EXAMPLE PROBLEM 3 Student Book p. 14

A twirling baton (L = 0.66 m) has two 0.30-kg end masses on a massless rod. Find I about: **a.** the midpoint, and **b.** one end. Which is harder to spin?



Given	$m_1 = m_2 = 0.30$ kg, $L = 0.66$ m
Step 1	**About midpoint (r = 0.33 m each):** $I_{mid} = m_1r_1^2 + m_2r_2^2 = 2(0.30)(0.33)^2 =$ **0.065 kg·m^2**
Step 2	**About one end ($r_1 = 0, r_2 = 0.66$ m):** $I_{end} = 0.30(0)^2 + 0.30(0.66)^2 =$ **0.13 kg·m^2** (twice as hard to spin)
Answer	**I_{mid} = 0.065 kg·m^2; I_{end} = 0.13 kg·m^2 (greater)**

PRACTICE PROBLEM 21 Student Book p. 15

Two children of equal mass sit on a seesaw 0.3 m from the pivot, then move out to 0.6 m. How does the moment of inertia change?

Given	$r_2 = 2r_1$, $I = \Sigma mr^2$
Step 1	**Compare I_2 to I_1:** Since $I \propto r^2$, doubling r gives $(2)^2 =$ **4 times as large**
Answer	**I becomes 4 times as large**

PRACTICE PROBLEM 24 Student Book p. 15

Three 0.10-kg spheres are spaced 0.20 m apart on a light rod (A at left, C in middle, B at right). Find I about end sphere A and about middle sphere C.

Given	m = 0.10 kg each; spacing = 0.20 m
Step 1	**About end sphere A:** $r_A = 0$, $r_C = 0.20$ m, $r_B = 0.40$ m → $I_A = 0.10(0.20^2 + 0.40^2) =$ **0.020 kg·m²**
Step 2	**About middle sphere C:** $r_A = 0.20$ m, $r_C = 0$, $r_B = 0.20$ m → $I_C = 2(0.10)(0.20)^2 =$ **0.0080 kg·m²**
Answer	**I_A = 0.020 kg·m² (greater); I_C = 0.0080 kg·m²**

CHOOSE THE CORRECT ANSWER 3 MCQs on Q10 (Coriolis Force) — rising in difficulty

1 **The Coriolis force is called an apparent force because it:**

A acts only on moving fluids

B appears only in a rotating frame of reference

C is always smaller than gravity

D acts only at the equator

Answer: B — It appears only when motion is observed from a **rotating frame of reference**.

2 **In the Northern Hemisphere a moving object appears to be deflected:**

A to the left

B to the right

C downward

D not at all

Answer: B — Deflection is **to the right** in the Northern Hemisphere.

3 **A ball is rolled from the centre of a rotating platform straight towards the edge. Relative to the platform its path appears:**

A straight

B curved

C circular

D to stop at the edge

Answer: B — Because the platform rotates underneath, its path appears **curved**.

FRQ Q18 (Pt.1)

FRQ · FREE RESPONSE

Apply Newton's second law for rotation ($\alpha = \tau_{net}/I$) and moments of inertia of extended objects.

pp. 13–17 · S.T.B
E.P(4) · P.P(23,30)
C.Y.P(33–35) · 9–11 marks

STUDENT BOOK pp. 13–15

Newton's 2nd Law for Rotation: $\alpha = \dfrac{\tau_{net}}{I} \leftrightarrow \tau_{net} = I\alpha$.

Table 2 Moments of Inertia for Various Objects

Object	Location of Axis	Diagram	Moment of Inertia
Thin hoop of radius r	through central diameter		mr^2
Solid, uniform cylinder of radius r	through center		$\left(\frac{1}{2}\right)mr^2$
Uniform sphere of radius r	through center		$\left(\frac{2}{5}\right)mr^2$
Long, uniform rod of length l	through center		$\left(\frac{1}{12}\right)ml^2$
Long, uniform rod of length l	through end		$\left(\frac{1}{3}\right)ml^2$
Thin, rectangular plate of length l and width w	through center		$\left(\frac{1}{12}\right)m\,(l^2+w^2)$

PRACTICE PROBLEM 23 **FRQ · FREE RESPONSE** Student Book p. 15

Calculate I for each object (r = 2.0 m, m = 1.0 kg): **a.** thin hoop, **b.** solid cylinder, **c.** uniform sphere.

Given m = 1.0 kg, r = 2.0 m

Step 1 **a) Thin hoop:** $I = mr^2 = (1.0)(2.0)^2 =$ **4.0 kg·m²**

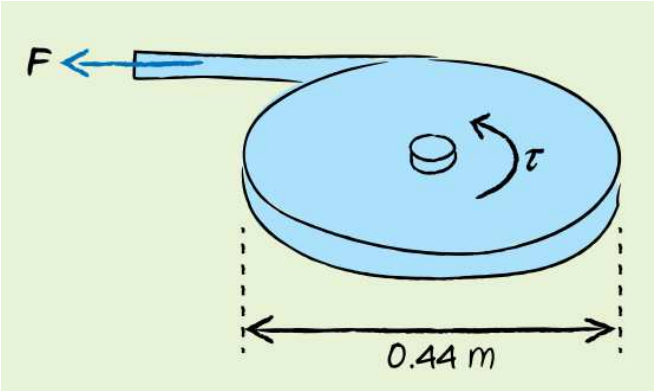
Step 2 **b) Solid cylinder:** $I = \dfrac{1}{2}mr^2 = \dfrac{1}{2}(1.0)(2.0)^2 =$ **2.0 kg·m²**

Step 3 **c) Uniform sphere:** $I = \dfrac{2}{5}mr^2 = \dfrac{2}{5}(1.0)(2.0)^2 =$ **1.6 kg·m²**

Answer **a) 4.0 kg·m² | b) 2.0 kg·m² | c) 1.6 kg·m²**

EXAMPLE PROBLEM 4 **FRQ · FREE RESPONSE** Student Book p. 16

A solid steel wheel (m = 15 kg, d = 0.44 m) reaches 8.0 rev/s from rest in 15 s. Find the torque τ and the strap force F.



Given	m = 15 kg, r = 0.22 m, f_f = 8.0 rev/s, Δt = 15 s
Step 1	**Moment of inertia (solid cylinder):** $I = \frac{1}{2}mr^2 = \frac{1}{2}(15)(0.22)^2 = 0.363$ kg·m^2
Step 2	**Angular acceleration:** $\omega_f = 2\pi(8.0) = 50.27$ rad/s → $\alpha = \frac{50.27 - 0}{15} = 3.35$ rad/s^2
Step 3	**Torque & Strap force:** $\tau = I\alpha = 0.363 \times 3.35 =$ **1.2 N·m** → $F = \frac{\tau}{r} = \frac{1.216}{0.22} =$ **5.5 N**
Answer	**τ = 1.2 N·m; F = 5.5 N**

CHECK YOUR PROGRESS 33 **FRQ · FREE RESPONSE** Student Book p. 17

A solid wheel (m = 5.2 kg, d = 0.55 m) reaches 12 rev/s from rest in 35 s. Find the torque τ and the strap force F.

Given	m = 5.2 kg, r = 0.275 m, f_f = 12 rev/s, Δt = 35 s
Step 1	**Moment of inertia:** $I = \frac{1}{2}(5.2)(0.275)^2 = 0.1966$ kg·m^2
Step 2	**Angular acceleration:** $\alpha = \frac{2\pi(12)}{35} = 2.15$ rad/s^2
Step 3	**Torque & Force:** $\tau = 0.1966 \times 2.154 =$ **0.42 N·m** → $F = \frac{0.4235}{0.275} =$ **1.5 N**
Answer	**τ = 0.42 N·m; F = 1.5 N**

PRACTICE PROBLEM 30 (CHALLENGE) **FRQ · FREE RESPONSE** Student Book p. 17

A disk (I_1 = 0.26 kg·m^2) is mounted with a smaller disk (d = 0.180 m, m = 2.5 kg). Find the strap force on the smaller disk for α = 2.57 rad/s^2.

30. CHALLENGE A disk with a moment of inertia of 0.26 kg·m² is attached to a smaller disk mounted on the same axle. The smaller disk has a diameter of 0.180 m and a mass of 2.5 kg. A strap is wrapped around the smaller disk, as shown in **Figure 14.** Find the force needed to give this system an angular acceleration of 2.57 rad/s².

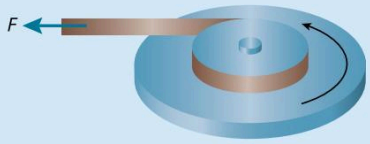


Figure 14

Given	I_1 = 0.26 kg·m^2, r_2 = 0.090 m, m_2 = 2.5 kg, α = 2.57 rad/s^2
Step 1	**Total moment of inertia:** $I_{total} = 0.26 + \frac{1}{2}(2.5)(0.090)^2 = 0.2701$ kg·m^2
Step 2	**Torque & Force:** $\tau = I_{total}\alpha = 0.2701 \times 2.57 = 0.694$ N·m → $F = \frac{0.694}{0.090} =$ **7.7 N**
Answer	***F* = 7.7 N**

CHECK YOUR PROGRESS 34 **FRQ · FREE RESPONSE** Student Book p. 17

A wheel (d = 2.4 m) is pulled clockwise with 43 N and counterclockwise with 67 N. Find the net torque.

Given	r = 1.2 m, F_{CW} = 43 N (−), F_{CCW} = 67 N (+)
Step 1	**Net torque:** $\tau_{net} = (+67 - 43)(1.2) = 24 \times 1.2 =$ **+29 N·m** (CCW)
Answer	**τ_{net} = +29 N·m (counterclockwise)**

CHECK YOUR PROGRESS 35 **FRQ · FREE RESPONSE** Student Book p. 17

A pulley (r = 0.150 m) pulled with 13.0 N speeds up from 0 to 14.0 rev/min in 4.50 s. Find its moment of inertia I.

Given	F = 13.0 N, r = 0.150 m, f_f = 14.0 rpm, Δt = 4.50 s
Step 1	**Torque & Angular acceleration:** $\tau = 13.0 \times 0.150 = 1.95$ N·m; $\alpha = \frac{14.0 \times 2\pi/60}{4.50} = 0.3258$ rad/s^2
Step 2	**Moment of inertia:** $I = \frac{\tau}{\alpha} = \frac{1.95}{0.3258} =$ **5.99 kg·m^2**
Answer	I = **5.99 kg·m^2**

PRACTICE · YOUR TURN **FRQ · FREE RESPONSE** 3 Free-Response questions on FRQ Q18 — rising in difficulty

1 RECALL

Write Newton's second law for rotation, and give the moment of inertia of a hoop and a solid uniform cylinder (mass m, radius r).

Step 1	**Equations:** $\alpha = \frac{\tau_{net}}{I}$; $I_{hoop} = mr^2$; $I_{cylinder} = \frac{1}{2}mr^2$
Answer	$\tau_{net} = I\alpha$; $I_{hoop} = mr^2$; $I_{cyl} = \frac{1}{2}mr^2$

2 APPLY

A solid disc (m = 4.0 kg, r = 0.25 m) experiences τ = 1.5 N·m. Find α and L after 4.0 s from rest.

Given	m = 4.0 kg, r = 0.25 m, τ = 1.5 N·m, Δt = 4.0 s
Step 1	**Inertia & Acceleration:** $I = \frac{1}{2}(4.0)(0.25)^2 = 0.125$ kg·m^2 → $\alpha = \frac{1.5}{0.125} =$ **12 rad/s^2**
Step 2	**Angular Momentum:** $L = \tau\Delta t = 1.5 \times 4.0 =$ **6.0 kg·m^2/s**
Answer	α = 12 rad/s^2; L = 6.0 kg·m^2/s

3 CHALLENGE

A uniform rod (m = 2.4 kg, l = 1.8 m) rotates about one end at 5.0 rad/s. Find I, L, and the torque (with direction) to stop it in 3.0 s.

Given	m = 2.4 kg, l = 1.8 m, ω_i = 5.0 rad/s, Δt = 3.0 s
Step 1	**Inertia & Angular Momentum:** $I = \frac{1}{3}(2.4)(1.8)^2$ = **2.592 kg·m²**; L = 2.592 × 5.0 = **12.96 kg·m²/s**
Step 2	**Stopping Torque:** $\tau = \frac{0 - 12.96}{3.0}$ = **−4.32 N·m** (opposite to rotation)
Answer	**I = 2.6 kg·m²; L = 13 kg·m²/s; τ = 4.3 N·m (opposite)**

Linear Momentum & Impulse-Momentum Theorem

Q11, Q12 & Q14

MCQ OBJECTIVE

1. Derive impulse ($F\Delta t = m\Delta v = p_f - p_i$) and calculate impulse for constant & varying forces (area under F–t graph).
2. Explain everyday safety applications (air bags, bending knees).

pp. 30–33, 37 · S.T.B P.P(3) · C.Y.P(11)
2–4 marks

STUDENT BOOK pp. 30–33

Momentum: $p = mv$ (kg·m/s). **Impulse–momentum theorem:** $F = m(\frac{\Delta v}{\Delta t}) \rightarrow F\Delta t = m(v_f - v_i) = p_f - p_i$ (unit: N·s).

For varying forces, Impulse = **area under the F–t graph**. Safety devices like air bags **increase Δt** for a fixed Δp, reducing impact force $F = \frac{\Delta p}{\Delta t}$.

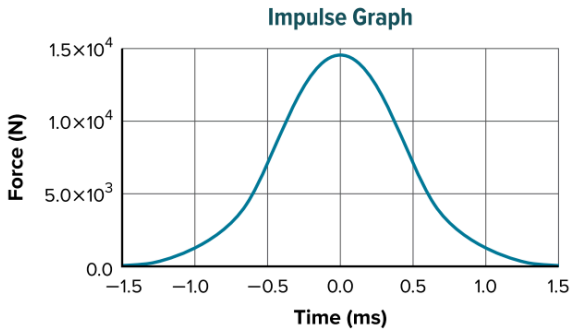


Figure 1 The force on a baseball increases then decreases during a collision.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q11 (Impulse & Newton's 2nd Law) — rising in difficulty

1 **The SI unit of impulse is:**

A N
B N·s
C N/s
D J

Answer: B — Impulse = $F\Delta t$, so its SI unit is **N·s**.

2 **Impulse is equal to:**

A mv
B $F\Delta t$
C $\frac{F}{\Delta t}$
D $\frac{1}{2}mv^2$

Answer: B — By definition, Impulse = $F\Delta t$.

3 During the collision between a bat and a ball, the force on the ball compared with the force on the bat is:

A larger	B smaller
C equal in size and opposite in direction	D zero

Answer: C — By Newton's third law, the forces are **equal in size and opposite in direction**.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q12 (Constant & Varying Force Impulse) — rising in difficulty

1 When the force is not constant, the impulse is best found from:

A the gradient of the force–time graph	**B the area under the force–time graph**
C the force at the midpoint only	D the maximum force × the total time

Answer: B — Impulse equals the **area under the *F–t* graph**.

2 A constant force of 45 N acts on a trolley for 0.25 s. The impulse delivered is:

A 0.18 N·s	**B 11 N·s**
C 45 N·s	D 180 N·s

Answer: B — Impulse = 45 × 0.25 = 11.25 ≈ **11 N·s**.

3 A force rises linearly from 0 to 600 N in 0.040 s, then falls linearly back to 0 in the next 0.020 s. The impulse is:

A 9.0 N·s	**B 18 N·s**
C 24 N·s	D 36 N·s

Answer: B — Triangle area = $\frac{1}{2}(0.060\ \text{s})(600\ \text{N})$ = **18 N·s**.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q14 (Air Bags & Safety Applications) — rising in difficulty

1 The impulse–momentum theorem states that:

A $F\Delta t = \Delta p$	B $F\Delta t = \Delta v$
C $F = ma$ only	D $p = mv$

Answer: A — Impulse equals the change in momentum: **$F\Delta t = \Delta p$**.

2 **An air bag reduces the force on a passenger during a crash because it:**

A reduces the change in momentum	**B** **increases the time over which the momentum changes**
C increases the passenger's mass	**D** reverses the momentum

Answer: B — Increasing Δt for a fixed Δp reduces the average force.

3 **A 70 kg driver travelling at 20 m/s is stopped in 0.010 s. The average force is about:**

A 1.4×10^3 N	**B** 1.4×10^4 N
C **1.4×10^5 N**	**D** 1.4×10^6 N

Answer: C — $F = \dfrac{70 \times 20}{0.010} = \mathbf{1.4 \times 10^5\ N}$.

PRACTICE PROBLEM 3 Student Book p. 33

A 7.0-kg object moving at 2.0 m/s receives two impulses shown in the graph (+5.0 N for 1.0 s, then −5.0 N for 1.0 s). Find its speed and direction after each impulse.

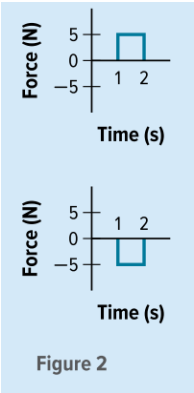


Figure 2

Given	$m = 7.0$ kg, $v_i = +2.0$ m/s; $J_1 = +5.0$ N·s, $J_2 = -5.0$ N·s
Step 1	**After 1st impulse ($t = 1.0$ s):** $m(v_1 - v_i) = +5.0 \rightarrow v_1 = 2.0 + \dfrac{5.0}{7.0} =$ **2.7 m/s** (same direction)
Step 2	**After 2nd impulse ($t = 2.0$ s):** $m(v_2 - v_1) = -5.0 \rightarrow v_2 = 2.71 - \dfrac{5.0}{7.0} =$ **2.0 m/s** (same direction)
Answer	**After 1st: 2.7 m/s (original dir.) | After 2nd: 2.0 m/s (original dir.)**

CHECK YOUR PROGRESS 11 Student Book p. 37

Why do you bend your knees when landing from a jump?

Given	Impulse–Momentum Theorem: $F = \frac{\Delta p}{\Delta t}$
Step 1	**Physical explanation:** Bending your knees **increases the stopping time Δt** while Δp stays the same, which **reduces the average force** exerted on your legs.
Answer	**Increases stopping time Δt, reducing the impact force on your legs.**

FRQ Q19

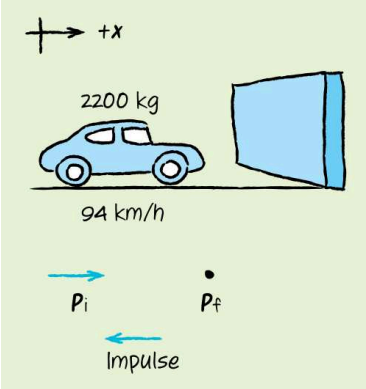
FRQ · FREE RESPONSE

Apply $p = mv$ and the impulse–momentum theorem ($F\Delta t = p_f - p_i$) to multi-step problems.

pp. 32, 33, 37 · E.P(1) P.P(1,2,4) · C.Y.P(12) 9 – 11 marks

EXAMPLE PROBLEM 1 **FRQ · FREE RESPONSE** Student Book p. 32

A 2200-kg vehicle moving at 94 km/h (26 m/s) is stopped in: (1) 21 s by gently applying brakes, (2) 3.8 s by slamming the brakes, or (3) 0.22 s by hitting a concrete wall. Find the impulse and average force in each stop.



Given	m = 2200 kg, v_i = +26 m/s, v_f = 0
Step 1	**Impulse (same for all three stops):** $F\Delta t = m(v_f - v_i) = 2200(0 - 26) =$ **-5.7×10^4 N·s**
Step 2	**Average Force (1) Gentle braking (Δt = 21 s):** $F = \frac{-5.72 \times 10^4}{21} =$ **-2.7×10^3 N**
Step 3	**Average Force (2) Slamming brakes (Δt = 3.8 s):** $F = \frac{-5.72 \times 10^4}{3.8} =$ **-1.5×10^4 N**
Step 4	**Average Force (3) Hitting wall (Δt = 0.22 s):** $F = \frac{-5.72 \times 10^4}{0.22} =$ **-2.6×10^5 N**
Answer	**Impulse = -5.7×10^4 N·s | Forces: -2.7×10^3 N, -1.5×10^4 N, -2.6×10^5 N**

PRACTICE PROBLEM 1 **FRQ · FREE RESPONSE** Student Book p. 33

A 725-kg compact car is moving at 115 km/h toward the east. **a.** Find the magnitude and direction of its momentum. **b.** A second car of mass 2175 kg has the same momentum; what is its velocity?

Given	$m_1 = 725$ kg, $v_1 = \frac{115 \text{ km/h}}{3.6} = 31.94$ m/s east; $m_2 = 2175$ kg
Step 1	**Part (a): Momentum of compact car:** $p_1 = m_1 v_1 = 725 \times 31.94 =$ **2.32×10^4 kg·m/s east**
Step 2	**Part (b): Velocity of 2175-kg car:** $v_2 = \frac{p_1}{m_2} = \frac{23\,160}{2175} =$ **10.6 m/s east**
Answer	**a) 2.32×10^4 kg·m/s east | b) 10.6 m/s east**

PRACTICE PROBLEM 2 **FRQ · FREE RESPONSE** Student Book p. 33

The driver of the 725-kg compact car in Problem 1 suddenly applies the brakes hard for 2.0 s, applying an average braking force of 5.0×10^3 N. Find: **a.** the impulse on the car, and **b.** the magnitude and direction of its final momentum and velocity.

Given	$m = 725$ kg, $p_i = +2.316 \times 10^4$ kg·m/s (east +), $F = -5.0 \times 10^3$ N (west −), $\Delta t = 2.0$ s
Step 1	**Part (a): Impulse:** $J = F\Delta t = (-5.0 \times 10^3)(2.0) = -1.0 \times 10^4$ N·s = **1.0×10^4 N·s west**
Step 2	**Part (b): Final momentum & velocity:** $p_f = p_i + F\Delta t = 2.316 \times 10^4 - 1.0 \times 10^4 =$ **1.3×10^4 kg·m/s east** $v_f = \frac{p_f}{m} = \frac{13\,160}{725} =$ **18 m/s east**
Answer	**a) 1.0×10^4 N·s west | b) 1.3×10^4 kg·m/s east, 18 m/s east**

PRACTICE PROBLEM 4 **FRQ · FREE RESPONSE** Student Book p. 33

A 240.0-kg snowmobile travels at 6.00 m/s, then accelerates for 60.0 s until it reaches 28.0 m/s. Find: **a.** the change in momentum, and **b.** the magnitude of the average force exerted on it.

Given	$m = 240.0$ kg, $v_i = 6.00$ m/s, $v_f = 28.0$ m/s, $\Delta t = 60.0$ s
Step 1	**Part (a): Change in momentum:** $\Delta p = m(v_f - v_i) = 240.0(28.0 - 6.00) =$ **5.28×10^3 N·s**
Step 2	**Part (b): Average force:** $F = \frac{\Delta p}{\Delta t} = \frac{5280}{60.0} =$ **88.0 N**
Answer	**a) 5.28×10^3 N·s | b) 88.0 N**

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CHECK YOUR PROGRESS 12 **FRQ · FREE RESPONSE** Student Book p. 37

A 0.174-kg softball is pitched horizontally at 26.0 m/s. The ball is hit back in the opposite direction at 38.0 m/s. Contact with the bat lasts 0.80 ms. Find: **a.** change in momentum, **b.** impulse on the ball, and **c.** average force.

FREE-BODY / VECTOR DIAGRAM

Velocity: →+ , ←−



Given	m = 0.174 kg, v_i = −26.0 m/s, v_f = +38.0 m/s, $\Delta t = 0.80 \times 10^{-3}$ s
Step 1	**Part (a) & (b): Δ*p* and Impulse:** Impulse = $\Delta p = m(v_f - v_i)$ = 0.174[38.0 − (−26.0)] = 0.174(64.0) = **11.1 kg·m/s**
Step 2	**Part (c): Average force:** $F = \frac{\text{Impulse}}{\Delta t} = \frac{11.14}{0.80 \times 10^{-3}}$ = **1.4×10^4 N**
Answer	**a, b) 11.1 kg·m/s (or N·s) | c) 1.4×10^4 N**

PRACTICE · YOUR TURN **FRQ · FREE RESPONSE** 3 Free-Response questions on FRQ Q19 — rising in difficulty

1 RECALL

Write the equation for linear momentum and state its SI unit.

Step 1	**Equation & Unit: *p* = *mv***; SI unit: **kg·m/s** (or N·s).
Answer	***p* = *mv* (kg·m/s)**

2 APPLY

A 0.50 kg trolley at 6.0 m/s is stopped in 0.75 s. Find initial momentum and stopping force.

Given	m = 0.50 kg, v_i = 6.0 m/s, Δt = 0.75 s
Step 1	**Momentum & Force:** p_i = 0.50 × 6.0 = **3.0 kg·m/s**; $\lvert F \rvert = \frac{3.0}{0.75}$ = **4.0 N**
Answer	**p_i = 3.0 kg·m/s; *F* = 4.0 N**

3 CHALLENGE

A 0.057 kg tennis ball arrives at 25 m/s and returns at 35 m/s in 5.0 ms. Find Δp, impulse, average force, and explain why sign matters.

Given	$m = 0.057$ kg, $v_i = -25$ m/s, $v_f = +35$ m/s, $\Delta t = 5.0 \times 10^{-3}$ s
Step 1	**Impulse & Force:** Δp = Impulse = $0.057[35 - (-25)]$ = **+3.42 kg·m/s**; $F = \frac{3.42}{5.0 \times 10^{-3}} =$ **684 N**
Step 2	**Sign explanation:** Momentum is a vector; reversing direction means speeds add (35 + 25 = 60 m/s).
Answer	**Δp = Impulse = 3.4 kg·m/s; $F = 6.8 \times 10^2$ N**

Conservation of Linear & Angular Momentum

Q13, Q15 & Q16

MCQ OBJECTIVE

1. Apply conservation of linear momentum in 1D (collisions & recoil: $p_{Ci} + p_{Di} = p_{Cf} + p_{Df}$).
2. Define angular momentum ($L = I\omega$) and apply conservation of angular momentum ($I_i\omega_i = I_f\omega_f$).

pp. 34–47 · S.T.B C.Y.P(15,28) · Get it? 2–4 marks

STUDENT BOOK pp. 34–47

Linear momentum conservation (closed, isolated system): $p_{Ci} + p_{Di} = p_{Cf} + p_{Df}$.

Angular momentum: $L = I\omega$ (kg·m^2/s); **Angular impulse:** $\tau\Delta t = L_f - L_i$; **Conservation (no net external torque):** $I_i\omega_i = I_f\omega_f$ ($\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}$).

Momentum How did the momenta of the two balls the girl and the boy rolled toward each other in **Figure 9** change as a result of the collision? Compare the changes shown in **Figure 10.**

For ball C: $P_{Cf} - P_{Ci} = F_{D\text{ on }C}\Delta t$

For ball D: $P_{Df} - P_{Di} = F_{C\text{ on }D}\Delta t$

According to the impulse-momentum theorem, the change in momentum is equal to the impulse. Because the impulses are equal in magnitude but opposite in direction, we know that:

$$P_{Cf} - P_{Ci} = -(P_{Df} - P_{Di})$$

$$P_{Cf} + P_{Df} = P_{Ci} + P_{Di}$$

This equation states that the sum of the momenta of the balls in **Figure 10** is the same before and after the collision. That is, the momentum gained by ball D is equal to the momentum lost by ball C. If the system is defined as the two balls, the sum of the gain and loss in momentum is zero, and therefore, momentum is conserved for the system.

CHOOSE THE CORRECT ANSWER

3 MCQs on Q13 (Conservation of Linear Momentum) — rising in difficulty

1 **The linear momentum of a system is conserved when:**

A kinetic energy is also conserved

B no net external force acts on the system

C the objects stick together

D the collision is elastic

Answer: B — Momentum is conserved when **no net external force acts**.

2 **A 60 kg skater at rest on frictionless ice throws a 3.0 kg ball horizontally at 8.0 m/s. The skater's recoil speed is:**

A 0.40 m/s

B 0.50 m/s

C 2.7 m/s

D 8.0 m/s

Answer: A — $v_{skater} = \frac{3.0 \times 8.0}{60} = \mathbf{0.40\ m/s}$.

3 **A 1200 kg car at 15 m/s collides head-on with an 1800 kg truck at 10 m/s and they lock together. Their velocity just after impact is:**

- **A** **zero — they stop**
- **B** 0.50 m/s in the car's direction
- **C** 2.5 m/s in the car's direction
- **D** 2.5 m/s in the truck's direction

Answer: A — 1200(15) − 1800(10) = 0 → **zero — they stop**.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q15 (Angular Momentum & Shape Change) — rising in difficulty

1 **The angular momentum of a rotating object is given by:**

- **A** $L = I\omega$
- **B** $L = mv$
- **C** $L = \tau \Delta t$ only
- **D** $L = \frac{1}{2} I\omega^2$

Answer: A — $L = I\omega$.

2 **A spinning skater pulls her arms in, reducing her moment of inertia. Her angular velocity:**

- **A** decreases
- **B** **increases**
- **C** stays the same
- **D** becomes zero

Answer: B — Smaller I means larger ω (**increases**) to conserve L.

3 **A diver leaves the board with I = 12 kg·m^2 and ω = 2.0 rad/s, then tucks so that I = 3.5 kg·m^2. Her new angular velocity is:**

- **A** 0.58 rad/s
- **B** 2.0 rad/s
- **C** **6.9 rad/s**
- **D** 24 rad/s

Answer: C — $\omega_f = \frac{12 \times 2.0}{3.5} \approx$ **6.9 rad/s**.

CHOOSE THE CORRECT ANSWER 3 MCQs on Q16 (Conservation of Angular Momentum) — rising in difficulty

1 **Angular momentum is conserved when:**

- **A** **no net external torque acts**
- **B** no net external force acts
- **C** the moment of inertia is constant
- **D** the object is symmetrical

Answer: A — Conserved when **no net external torque acts**.

2 A turntable with $I = 0.80\ \text{kg·m}^2$ turns freely at 3.0 rad/s. A 0.50 kg lump of clay is dropped onto it 0.40 m from the centre and sticks. The new angular velocity is:

A 1.9 rad/s	**B 2.7 rad/s**
C 3.0 rad/s	D 3.3 rad/s

Answer: B — $I_f = 0.80 + 0.50(0.40)^2 = 0.88\ \text{kg·m}^2 \rightarrow \omega_f = \dfrac{0.80 \times 3.0}{0.88} \approx$ **2.7 rad/s**.

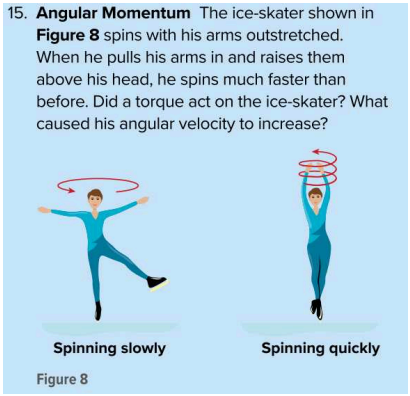
3 A student on a freely rotating stool has $I = 6.0\ \text{kg·m}^2$ at 1.5 rad/s, then pulls the masses in until $I = 2.0\ \text{kg·m}^2$. His new angular velocity is:

A 0.50 rad/s	B 1.5 rad/s
C 3.0 rad/s	**D 4.5 rad/s**

Answer: D — $\omega_f = \dfrac{6.0 \times 1.5}{2.0} =$ **4.5 rad/s**.

C.Y.P 15, GET IT? & C.Y.P 28 Student Book pp. 37, 46–47

CYP 15: Ice-skater pulls arms in and spins faster: did torque act? Why did ω increase?
Get It?: Why does Earth precess? **CYP 28:** How does a heavy outer rim affect a flying plastic disk?



Step 1	**CYP 15:** No external torque acted; pulling arms in reduced I, so ω increased to conserve L.
Step 2	**Get It? & CYP 28:** Sun/Moon gravity exerts torque on Earth's tilted bulge causing precession. Heavy rim increases I and L, stabilizing the disk in flight.
Answer	**Conceptual angular momentum explanations complete.**

FRQ Q18 (Pt.2)

FRQ · FREE RESPONSE

Solve problems using angular momentum ($L = I\omega$) and the angular impulse–angular momentum theorem ($\tau\Delta t = L_f - L_i$).

pp. 34, 36–37
E.P(2) · P.P(6,7)
9 – 11 marks

EXAMPLE PROBLEM 2

FRQ · FREE RESPONSE Student Book p. 36

An ice-skater (I = 0.50 kg·m^2) spins at 2.5 rev/s and stops in 3.0 s. Find: **a.** initial angular momentum L_i, **b.** angular impulse, and **c.** average stopping torque.



Given	I = 0.50 kg·m^2, f_i = 2.5 rev/s, ω_f = 0, Δt = 3.0 s
Step 1	**Convert ω_i & find L_i:** $\omega_i = 2\pi(2.5) = 15.71$ rad/s → $L_i = I\omega_i = 0.50 \times 15.71 =$ **7.9 kg·m^2/s**
Step 2	**Angular impulse:** $\tau\Delta t = L_f - L_i = 0 - 7.85 =$ **−7.9 kg·m^2/s**
Step 3	**Average torque:** $\tau = \frac{\tau\Delta t}{\Delta t} = \frac{-7.85}{3.0} =$ **−2.6 N·m**
Answer	**L_i = 7.9 kg·m^2/s; $\tau\Delta t$ = −7.9 kg·m^2/s; τ = −2.6 N·m**

PRACTICE PROBLEM 6

FRQ · FREE RESPONSE Student Book p. 37

A saw blade ($I = 8.0 \times 10^{-3}$ kg·m^2) spins at 5000 rev/min and slows under a friction torque of 0.23 N·m. How long does it take to stop?

Given	$I = 8.0 \times 10^{-3}$ kg·m^2, $\omega_i = 5000 \times \frac{2\pi}{60} = 523.6$ rad/s, $\tau = -0.23$ N·m
Step 1	**Angular impulse–momentum:** $\tau\Delta t = 0 - I\omega_i \rightarrow \Delta t = \frac{-(8.0 \times 10^{-3})(523.6)}{-0.23} =$ **18 s**
Answer	**Δt = 18 s**

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PRACTICE PROBLEM 7 **FRQ · FREE RESPONSE** Student Book p. 37

A baseball ($I = 8.0 \times 10^{-5}$ kg·m^2) reaches 600 rev/min in 0.15 s from rest. Find its final angular velocity and average torque.

Given	$I = 8.0 \times 10^{-5}$ kg·m^2, $f_f = 600$ rpm, $\Delta t = 0.15$ s
Step 1	**Convert 600 rpm to rad/s:** $\omega_f = 600 \times \frac{2\pi}{60} = 62.83$ rad/s $\approx$ **63 rad/s**
Step 2	**Average torque:** $\tau = \frac{I\omega_f - 0}{\Delta t} = \frac{(8.0 \times 10^{-5})(62.83)}{0.15} =$ **3.4×10^{-2} N·m**
Answer	**$\omega_f = 63$ rad/s; $\tau = 3.4 \times 10^{-2}$ N·m**

FRQ Q20

FRQ · FREE RESPONSE

Apply conservation of linear momentum in 1D (collisions, recoil, propulsion) and impulse–momentum theorem.

pp. 38–43 · E.P(3,4) P.P(17,18,20,21,22)
9 – 11 marks

EXAMPLE PROBLEM 3 **FRQ · FREE RESPONSE** Student Book p. 40

A 1875-kg car moving at 23 m/s rear-ends a 1025-kg car moving at 17 m/s in the same direction, and they lock together. Find their common velocity v_f after the collision.

FREE-BODY / VECTOR DIAGRAM Velocity: →+



Given	$m_C = 1875$ kg, $v_{Ci} = 23$ m/s; $m_D = 1025$ kg, $v_{Di} = 17$ m/s

Initial Momentum (p_i) ①

$p_i = m_C v_{Ci} + m_D v_{Di}$

$p_i = 1875(23) + 1025(17)$

Final Momentum (p_f) ②

$p_f = (m_C + m_D)v_f$

$p_f = (1875 + 1025)v_f$

Step 1	**Solve for v_f:** $v_f = \frac{1875(23) + 1025(17)}{1875 + 1025} = \frac{43\,125 + 17\,425}{2900} = \frac{60\,550}{2900} =$ **21 m/s**
Answer	**$v_f = 21$ m/s (in the initial direction)**

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EXAMPLE PROBLEM 4 **FRQ · FREE RESPONSE** Student Book p. 42

An astronaut at rest in space (total mass 84 kg) expels 35 g of hot gas at 875 m/s. Find the astronaut's recoil velocity.

FREE-BODY / VECTOR DIAGRAM

Velocity: →+ , ←−



Given	$m_{gas} = 0.035$ kg, $v_{gas} = -875$ m/s; $m_{astro} = 84$ kg, $p_i = 0$
Step 1	**Conservation of momentum ($p_i = p_f$):** $0 = m_{gas}v_{gas} + m_{astro}v_{astro} \rightarrow v_{astro} = -\frac{0.035(-875)}{84} =$ **+0.36 m/s**
Answer	**$v_{astro} = 0.36$ m/s (opposite to gas)**

PRACTICE PROBLEM 17 **FRQ · FREE RESPONSE** Student Book p. 40

Two freight cars, each of mass 3.0×10^5 kg, collide and stick together. One was initially moving at 2.2 m/s and the other was at rest. Find their final speed.

Given	$m_1 = m_2 = 3.0 \times 10^5$ kg, $v_{1i} = 2.2$ m/s, $v_{2i} = 0$
Step 1	**Conservation of momentum:** $m_1v_{1i} + \cancel{m_2v_{2i}} = (m_1 + m_2)v_f \rightarrow v_f = \frac{2.2}{2} =$ **1.1 m/s**
Answer	**$v_f = 1.1$ m/s**

PRACTICE PROBLEM 18 **FRQ · FREE RESPONSE** Student Book p. 40

A 0.105-kg hockey puck moving at 24 m/s is caught and held by a 75-kg goalie at rest. With what speed does the goalie slide across the ice?

Given	$m_{puck} = 0.105$ kg, $v_{puck} = 24$ m/s, $m_{goalie} = 75$ kg, $v_{goalie,i} = 0$
Step 1	**Conservation of momentum:** $0.105(24) + \cancel{75(0)} = (75 + 0.105)v_f \rightarrow v_f = \frac{2.52}{75.105} =$ **0.034 m/s**
Answer	**$v_f = 0.034$ m/s**

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PRACTICE PROBLEM 20 **FRQ · FREE RESPONSE** Student Book p. 40

A 0.50-kg ball at +6.0 m/s collides head-on with a 1.00-kg ball at −12.0 m/s. After collision, the 0.50-kg ball rebounds at −14 m/s. Find the velocity of the 1.00-kg ball.

Given $m_1 = 0.50$ kg, $v_{1i} = +6.0$ m/s, $v_{1f} = -14$ m/s; $m_2 = 1.00$ kg, $v_{2i} = -12.0$ m/s

Initial Momentum (p_i) ①

$p_i = 0.50(6.0) + 1.00(-12.0)$

$p_i = 3.0 - 12.0 = \mathbf{-9.0\ kg{\cdot}m/s}$

Final Momentum (p_f) ②

$p_f = 0.50(-14) + 1.00v_{2f}$

$p_f = -7.0 + 1.00v_{2f}$

Step 1 **Solve $p_i = p_f$ for v_{2f}:** $-9.0 = -7.0 + 1.00v_{2f} \rightarrow v_{2f} =$ **−2.0 m/s** (2.0 m/s in its original direction)

Answer **$v_{2f} = -2.0$ m/s (2.0 m/s in original direction)**

PRACTICE PROBLEM 21 **FRQ · FREE RESPONSE** Student Book p. 43

A 4.00-kg model rocket expels 50.0 g of burned fuel at 625 m/s downward. What is the rocket's velocity?

Given $m_{rocket} = 4.00$ kg, $m_{fuel} = 0.0500$ kg, $v_{fuel} = -625$ m/s, $p_i = 0$

Step 1 **Conservation of momentum:** $0 = m_{fuel}v_{fuel} + m_{rocket}v_{rocket} \rightarrow v_r = \dfrac{0.0500 \times 625}{4.00} =$ **+7.81 m/s upward**

Answer **$v_{rocket} = 7.81$ m/s upward**

PRACTICE PROBLEM 22 **FRQ · FREE RESPONSE** Student Book p. 43

A compressed spring pushes a 1.5-kg cart and a 4.5-kg cart apart from rest. The 1.5-kg cart moves left at 27 cm/s. Find the velocity of the 4.5-kg cart.

Given $m_1 = 1.5$ kg, $v_{1f} = -27$ cm/s (left); $m_2 = 4.5$ kg; $p_i = 0$

Step 1 **Conservation of momentum:** $0 = 1.5(-27) + 4.5v_{2f} \rightarrow v_{2f} = -\dfrac{1.5(-27)}{4.5} =$ **+9.0 cm/s right** (0.090 m/s)

Answer **$v_{2f} = 9.0$ cm/s (0.090 m/s) right**

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1 RECALL

State the law of conservation of linear momentum and write it in equation form for a two-object collision.

Step 1	**Law & Equation:** In a closed, isolated system, total momentum is constant: $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$.
Answer	$p_{1i} + p_{2i} = p_{1f} + p_{2f}$

2 APPLY

A 4.0 kg rifle fires a 0.012 kg bullet at 520 m/s. Find the recoil velocity of the rifle.

Given	m_{rifle} = 4.0 kg, m_{bullet} = 0.012 kg, v_{bullet} = +520 m/s
Step 1	**Recoil velocity:** $v_{rifle} = -\dfrac{0.012 \times 520}{4.0}$ = **−1.56 m/s**
Answer	**−1.6 m/s (opposite to bullet)**

3 CHALLENGE

A 0.80 kg trolley at 3.0 m/s hits a stationary 1.2 kg trolley and they stick together. Find their common velocity, then find the impulse on each and show they are equal and opposite.

Given	m_1 = 0.80 kg, v_{1i} = 3.0 m/s; m_2 = 1.2 kg, v_{2i} = 0
Step 1	**Common velocity:** $v_f = \dfrac{0.80 \times 3.0}{0.80 + 1.2}$ = **1.2 m/s**
Step 2	**Impulses:** J_1 = 0.80(1.2 − 3.0) = **−1.44 N·s**; J_2 = 1.2(1.2 − 0) = **+1.44 N·s** (equal and opposite).
Answer	v_f = 1.2 m/s; J_1 = −1.44 N·s, J_2 = +1.44 N·s