# End of Term 3 Revision


| تم تحميل هذا الملف من موقع المناهج الإماراتية | |
|---|---|
| موقع المناهج ⇦ المناهج الإماراتية ⇦ الصف الحادي عشر المتقدم ⇦ رياضيات ⇦ الفصل الثالث ⇦ ملفات متنوعة ⇦ الملف | |
| تاريخ إضافة الملف على موقع المناهج: 15-06-2026 23:29:04 | |
| المزيد من مادة رياضيات: | ملفات ا كتب للمعلم ا كتب للطالب ا اختبارات الكترونية ا اختبارات ا حلول ا عروض بوربوينت ا أوراق عمل منهج انجليزي ا ملخصات وتقارير ا مذكرات وبنوك ا الامتحان النهائي ا للمدرس |
| | إعداد: أحمد جمعة |

| التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم | | | | | |
|---|---|---|---|---|---|
| صفحة المناهج الإماراتية على فيسبوك | | | | | |
| المواد على تلغرام | التربية الاسلامية | اللغة العربية | اللغة الانجليزية | الرياضيات | |


| المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الثالث | |
|---|---|
| 1 | End of Term 3 Revision Maths |
| 2 | كراسة تدريبية للاختبار النهائي 2026م |
| 3 | منهج ريفيل حلول مراجعة امتحانية تدريبات وأسئلة |
| 4 | منهج ريفيل مراجعة امتحانية تدريبات وأسئلة غير محلول |
| 5 | ملزمة هيكل واختياري وأسئلة |

مجمع زايد التعليمي
الخوانيج

# BE READY

# END OF TERM 3 REVISION

# MATHS

# GRADE 11 ADV

COMPILED BY
MR. AHMED GOMAA

وحدة الشؤون الأكاديمية
جلال عبدالرحمن صالح
حمدة العامري

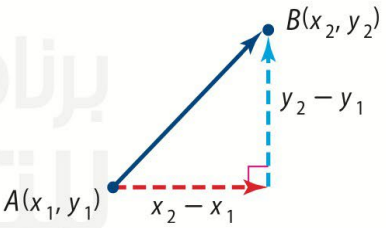
مدير المجمع
أمجد الجبر

1 Represent and operate with vectors in the coordinate plane. Example 1 & (1A,1B)

## KeyConcept Component Form of a Vector

The component form of a vector $\overrightarrow{AB}$ with initial point $A(x_1, y_1)$ and terminal point $B(x_2, y_2)$ is given by

$$\langle x_2 - x_1, y_2 - y_1 \rangle.$$




## Example 1 Express a Vector in Component Form

**Find the component form of $\overrightarrow{AB}$ with initial point $A(-4, 2)$ and terminal point $B(3, -5)$.**

$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$ Component form

$= \langle 3 - (-4), -5 - 2 \rangle$ $(x_1, y_1) = (-4, 2)$ and $(x_2, y_2) = (3, -5)$

$= \langle 7, -7 \rangle$ Subtract.

### GuidedPractice

**Find the component form of $\overrightarrow{AB}$ with the given initial and terminal points.**

**1A.** $A(-2, -7), B(6, 1)$

**1B.** $A(0, 8), B(-9, -3)$

# extra Q's to practice....

**Find the component form and magnitude of $\overrightarrow{AB}$ with the given initial and terminal points.** (Examples 1 and 2)

1. $A(-3, 1), B(4, 5)$
2. $A(2, -7), B(-6, 9)$
3. $A(10, -2), B(3, -5)$
4. $A(-2, 7), B(-9, -1)$
5. $A(-5, -4), B(8, -2)$
6. $A(-2, 6), B(1, 10)$
7. $A(2.5, -3), B(-4, 1.5)$
8. $A(-4.3, 1.8), B(9.4, -6.2)$
9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$
10. $A\left(\frac{3}{5}, -\frac{2}{5}\right), B(-1, 7)$

2 Write a vector as a linear combination of unit vectors. Example 6 & (6A,6B)

## Example 6 Find Component Form

**Find the component form of the vector v with magnitude 10 and direction angle 120°.**

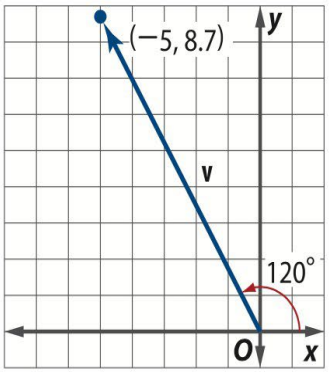
$\mathbf{v} = \langle |\mathbf{v}| \cos\theta, |\mathbf{v}| \sin\theta \rangle$ — Component form of v in terms of $|\mathbf{v}|$ and $\theta$

$= \langle 10 \cos 120°, 10 \sin 120° \rangle$ — $|\mathbf{v}| = 10$ and $\theta = 120°$

$= \left\langle 10\left(-\frac{1}{2}\right), 10\left(\frac{\sqrt{3}}{2}\right) \right\rangle$ — $\cos 120° = -\frac{1}{2}$ and $\sin 120° = \frac{\sqrt{3}}{2}$

$= \langle -5, 5\sqrt{3} \rangle$ — Simplify.

**CHECK** Graph $\mathbf{v} = \langle -5, 5\sqrt{3} \rangle \approx \langle -5, 8.7 \rangle$. The measure of the angle v makes with the positive $x$-axis is about 120° as shown, and $|\mathbf{v}| = \sqrt{(-5)^2 + (5\sqrt{3})^2}$ or 10. ✔




### GuidedPractice

**Find the component form of v with the given magnitude and direction angle.**

**6A.** $|\mathbf{v}| = 8, \theta = 45°$

**6B.** $|\mathbf{v}| = 24, \theta = 210°$

## extra Q's to practice....

**Find the component form of $v$ with the given magnitude and direction angle.** (Example 6)

**38.** $|\mathbf{v}| = 12, \theta = 60°$

**39.** $|\mathbf{v}| = 4, \theta = 135°$

**40.** $|\mathbf{v}| = 6, \theta = 240°$

**41.** $|\mathbf{v}| = 16, \theta = 330°$

**42.** $|\mathbf{v}| = 28, \theta = 273°$

**43.** $|\mathbf{v}| = 15, \theta = 125°$

| | | |
|---|---|---|
| 3 | Find the dot product of two vectors and use the dot product to find the angle between them. | Example 3 & (3A,3B) |

## Example 3 Find the Angle Between Two Vectors

**Find the angle $\theta$ between vectors u and v to the nearest tenth of a degree.**

**a.** $\mathbf{u} = \langle 6, 2\rangle$ and $\mathbf{v} = \langle -4, 3\rangle$

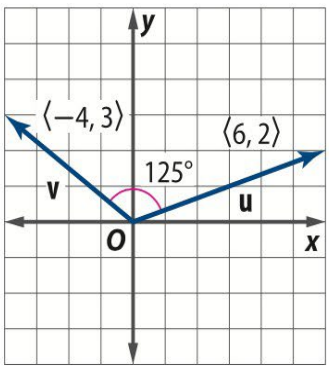
$\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|\,|\mathbf{v}|}$ — Angle between two vectors

$\cos\theta = \dfrac{\langle 6, 2\rangle \cdot \langle -4, 3\rangle}{|\langle 6, 2\rangle|\,|\langle -4, 3\rangle|}$ — $\mathbf{u} = \langle 6, 2\rangle$ and $\mathbf{v} = \langle -4, 3\rangle$

$\cos\theta = \dfrac{-24 + 6}{\sqrt{40}\sqrt{25}}$ — Evaluate.

$\cos\theta = \dfrac{-9}{5\sqrt{10}}$ — Simplify.

$\theta = \cos^{-1}\dfrac{-9}{5\sqrt{10}}$ or about $124.7°$ — Solve for $\theta$.




The measure of the angle between **u** and **v** is about $124.7°$.

**b.** $\mathbf{u} = \langle 3, 1\rangle$ and $\mathbf{v} = \langle 3, -3\rangle$

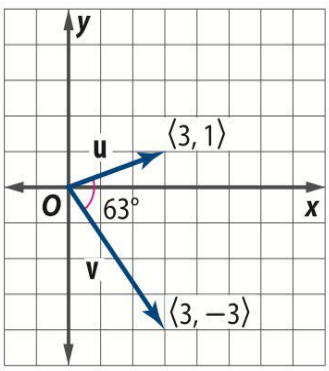
$\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|\,|\mathbf{v}|}$ — Angle between two vectors

$\cos\theta = \dfrac{\langle 3, 1\rangle \cdot \langle 3, -3\rangle}{|\langle 3, 1\rangle|\,|\langle 3, -3\rangle|}$ — $\mathbf{u} = \langle 3, 1\rangle$ and $\mathbf{v} = \langle 3, -3\rangle$

$\cos\theta = \dfrac{9 + (-3)}{\sqrt{10}\sqrt{18}}$ — Evaluate.

$\cos\theta = \dfrac{1}{\sqrt{5}}$ — Simplify.

$\theta = \cos^{-1}\dfrac{1}{\sqrt{5}}$ or about $63.4°$ — Solve for $\theta$.




The measure of the angle between **u** and **v** is about $63.4°$.

### GuidedPractice

**3A.** $\mathbf{u} = \langle -5, -2\rangle$ and $\mathbf{v} = \langle 4, 4\rangle$

**3B.** $\mathbf{u} = \langle 9, 5\rangle$ and $\mathbf{v} = \langle -6, 7\rangle$

| | | |
|---|---|---|
| 3 | Find the dot product of two vectors and use the dot product to find the angle between them. | Example 3 & (3A,3B) |

# extra Q's to practice....

**Find the angle $\theta$ between u and v to the nearest tenth of a degree.** (Example 3)

**16.** $\mathbf{u} = \langle 0, -5 \rangle, \mathbf{v} = \langle 1, -4 \rangle$

**17.** $\mathbf{u} = \langle 7, 10 \rangle, \mathbf{v} = \langle 4, -4 \rangle$

**18.** $\mathbf{u} = \langle -2, 4 \rangle, \mathbf{v} = \langle 2, -10 \rangle$

**19.** $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j}, \mathbf{v} = -4\mathbf{i} - 2\mathbf{j}$

**20.** $\mathbf{u} = \langle -9, 0 \rangle, \mathbf{v} = \langle -1, -1 \rangle$

**21.** $\mathbf{u} = -\mathbf{i} - 3\mathbf{j}, \mathbf{v} = -7\mathbf{i} - 3\mathbf{j}$

**22.** $\mathbf{u} = \langle 6, 0 \rangle, \mathbf{v} = \langle -10, 8 \rangle$

**23.** $\mathbf{u} = -10\mathbf{i} + \mathbf{j}, \mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$

**24.** **CAMPING** Omar and Ali set off from their campsite to search for firewood. The path that Omar takes can be represented by $\mathbf{u} = \langle 3, -5 \rangle$. The path that Ali takes can be represented by $\mathbf{v} = \langle -7, 6 \rangle$. Find the angle between the pair of vectors. (Example 3)

| | | |
|---|---|---|
| 4 | Find dot products of and angles between vectors in space. | Example 1 & (1A,1B) |

## Example 1 Find the Dot Product to Determine Orthogonal Vectors in Space

Find the dot product of u and v. Then determine if u and v are orthogonal.

a. $\mathbf{u} = \langle -7, 3, -3 \rangle$, $\mathbf{v} = \langle 5, 17, 5 \rangle$

$\mathbf{u} \cdot \mathbf{v} = -7(5) + 3(17) + (-3)(5)$

$= -35 + 51 + (-15)$ or 1

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

b. $\mathbf{u} = \langle 3, -3, 3 \rangle$, $\mathbf{v} = \langle 4, 7, 3 \rangle$

$\mathbf{u} \cdot \mathbf{v} = 3(4) + (-3)(7) + 3(3)$

$= 12 + (-21) + 9$ or 0

Since $\mathbf{u} \cdot \mathbf{v} = 0$, **u** and **v** are orthogonal.

### GuidedPractice

1A. $\mathbf{u} = \langle 3, -5, 4 \rangle$, $\mathbf{v} = \langle 5, 7, 5 \rangle$

1B. $\mathbf{u} = \langle 4, -2, -3 \rangle$, $\mathbf{v} = \langle 1, 3, -2 \rangle$

## extra Q's to practice....

**Find the dot product of u and v. Then determine if u and v are orthogonal.** (Example 1)

1. $\mathbf{u} = \langle 3, -9, 6 \rangle$, $\mathbf{v} = \langle -8, 2, 7 \rangle$
2. $\mathbf{u} = \langle 5, 0, -4 \rangle$, $\mathbf{v} = \langle 6, -1, 4 \rangle$
3. $\mathbf{u} = \langle 2, -8, -7 \rangle$, $\mathbf{v} = \langle 5, 9, -7 \rangle$
4. $\mathbf{u} = \langle -7, -3, 1 \rangle$, $\mathbf{v} = \langle -4, 5, -13 \rangle$
5. $\mathbf{u} = \langle 11, 4, -2 \rangle$, $\mathbf{v} = \langle -1, 3, 8 \rangle$
6. $\mathbf{u} = 6\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$
7. $\mathbf{u} = 3\mathbf{i} - 10\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 7\mathbf{i} + 2\mathbf{j} - \mathbf{k}$
8. $\mathbf{u} = 9\mathbf{i} - 9\mathbf{j} + 6\mathbf{k}$, $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

## Example 5 Area of a Parallelogram in Space

Find the area of the parallelogram with adjacent sides $\mathbf{u} = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$.

**Step 1** Find $\mathbf{u} \times \mathbf{v}$.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & -3 \\ 1 & -5 & 3 \end{vmatrix}$$ $\mathbf{u} = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

$$= \begin{vmatrix} 4 & -3 \\ -5 & 3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 2 & -3 \\ 1 & 3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 2 & 4 \\ 1 & -5 \end{vmatrix}\mathbf{k}$$ Determinant of a $3 \times 3$ matrix

$$= -3\mathbf{i} - 9\mathbf{j} - 14\mathbf{k}$$ Determinants of $2 \times 2$ matrices

**Step 2** Find the magnitude of $\mathbf{u} \times \mathbf{v}$.

$$|\mathbf{u} \times \mathbf{v}| = \sqrt{(-3)^2 + (-9)^2 + (-14)^2}$$ Magnitude of a vector in space

$$= \sqrt{286} \text{ or about } 16.9$$ Simplify.

The area of the parallelogram shown in Figure 7.5.4 is about 16.9 square units.

### GuidedPractice

5. Find the area of the parallelogram with adjacent sides $\mathbf{u} = -6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

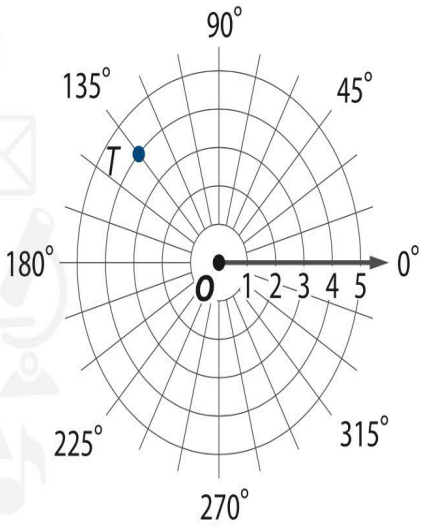
## extra Q's to practice....

**Find the area of the parallelogram with adjacent sides u and v.** (Example 5)

24. $\mathbf{u} = \langle 2, -5, 3 \rangle$, $\mathbf{v} = \langle 4, 6, -1 \rangle$
25. $\mathbf{u} = \langle -9, 1, 2 \rangle$, $\mathbf{v} = \langle 6, -5, 3 \rangle$
26. $\mathbf{u} = \langle 4, 3, -1 \rangle$, $\mathbf{v} = \langle 7, 2, -2 \rangle$
27. $\mathbf{u} = 6\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $\mathbf{v} = 5\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$
28. $\mathbf{u} = \mathbf{i} + 4\mathbf{j} - 8\mathbf{k}$, $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$
29. $\mathbf{u} = -3\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$, $\mathbf{v} = 4\mathbf{i} - \mathbf{j} + 6\mathbf{k}$

## Example 3 Multiple Representations of Polar Coordinates

**Find four different pairs of polar coordinates that name point $T$ if $-360° \le \theta \le 360°$.**




One pair of polar coordinates that name point $T$ is $(4, 135°)$. The other three representations are as follows.

$(4, 135°) = (4, 135° - 360°)$ Subtract 360° from $\theta$.
$= (4, -225°)$

$(4, 135°) = (-4, 135° + 180°)$ Replace $r$ with $-r$ and
$= (-4, 315°)$ add 180° to $\theta$.

$(4, 135°) = (-4, 135° - 180°)$ Replace $r$ with $-r$ and
$= (-4, -45°)$ subtract 180° from $\theta$.

### GuidedPractice

**Find three additional pairs of polar coordinates that name the given point if $-360° \le \theta \le 360°$ or $-2\pi \le \theta \le \pi$.**

**3A.** $(5, 240°)$

**3B.** $\left(2, \frac{\pi}{6}\right)$

**Find three different pairs of polar coordinates that name the given point if $-360° \le \theta \le 360°$ or $-2\pi \le \theta \le 2\pi$.** (Example 3)

**14.** $(1, 150°)$

**15.** $(-2, 300°)$

**16.** $\left(4, -\frac{7\pi}{6}\right)$

**17.** $\left(-3, \frac{2\pi}{3}\right)$

**18.** $\left(5, \frac{11\pi}{6}\right)$

**19.** $\left(-5, -\frac{4\pi}{3}\right)$

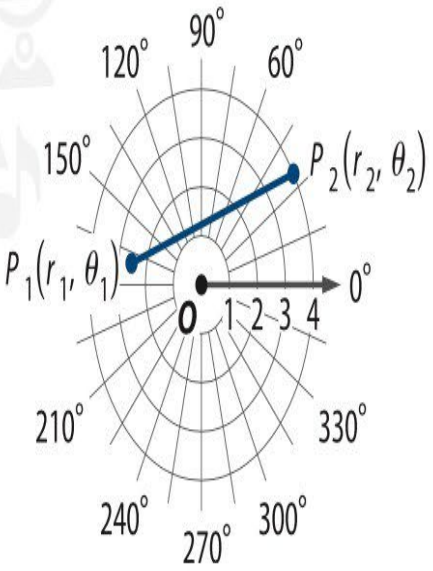
**20.** $(2, -30°)$

**21.** $(-1, -240°)$

## KeyConcept Polar Distance Formula

If $P_1(r_1, \theta_1)$ and $P_2(r_2, \theta_2)$ are two points in the polar plane, then the distance $P_1P_2$ is given by

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}.$$




## extra Q's to practice....

**Find the distance between each pair of points.** (Example 5)

**30.** $(2, 30°), (5, 120°)$

**31.** $\left(3, \frac{\pi}{2}\right), \left(8, \frac{4\pi}{3}\right)$

**32.** $(6, 45°), (-3, 300°)$

**33.** $\left(7, -\frac{\pi}{3}\right), \left(1, \frac{2\pi}{3}\right)$

**34.** $\left(-5, \frac{7\pi}{6}\right), \left(4, \frac{\pi}{6}\right)$

**35.** $(4, -315°), (1, 60°)$

**36.** $(-2, -30°), (8, 210°)$

**37.** $\left(-3, \frac{11\pi}{6}\right), \left(-2, \frac{5\pi}{6}\right)$

**38.** $\left(1, -\frac{\pi}{4}\right), \left(-5, \frac{7\pi}{6}\right)$

**39.** $(7, -90°), (-4, -330°)$

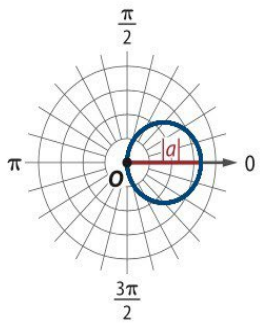
**40.** $\left(8, -\frac{2\pi}{3}\right), \left(4, -\frac{3\pi}{4}\right)$

**41.** $(-5, 135°), (-1, 240°)$

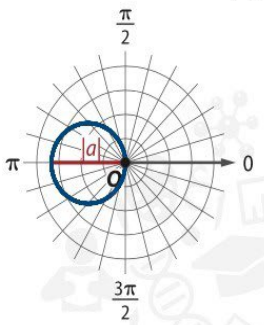
## ConceptSummary Special Types of Polar Graphs

### Circles

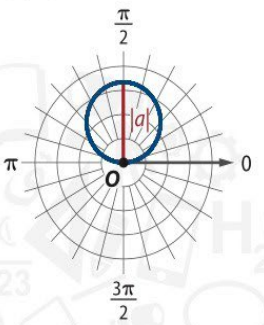
$r = a\cos\theta$ or $r = a\sin\theta$



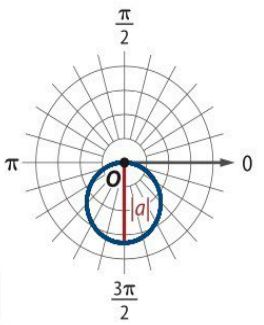

$a > 0$
$r = a\cos\theta$




$a < 0$
$r = a\cos\theta$



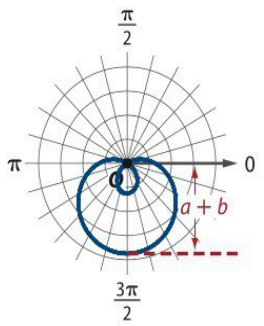

$a > 0$
$r = a\sin\theta$



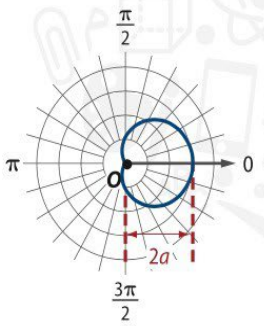

$a < 0$
$r = a\sin\theta$

### Limaçons

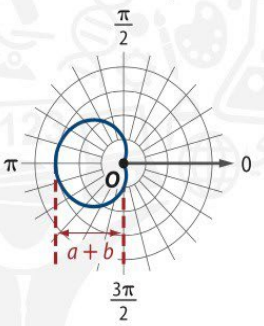
$r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$, where $a$ and $b$ are both positive



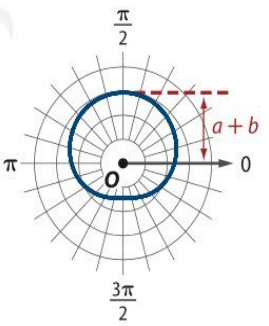

Limaçon with inner loop
$a < b$
$r = a - b\sin\theta$




Cardioid
$a = b$
$r = a + b\cos\theta$



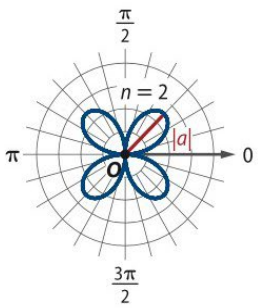

Dimpled limaçon
$b < a < 2b$
$r = a - b\cos\theta$



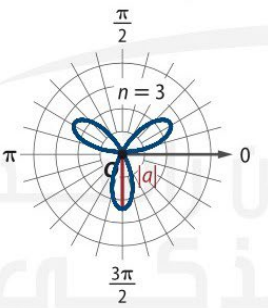

Convex limaçon
$a \geq 2b$
$r = a + b\sin\theta$

### Roses

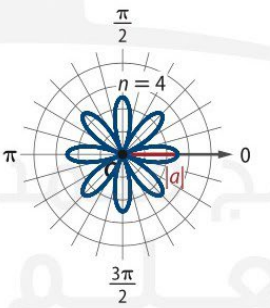
$r = a\cos n\theta$ or $r = a\sin n\theta$, where $n \geq 2$ is an integer
The rose has $n$ petals if $n$ is odd and $2n$ petals if $n$ is even.



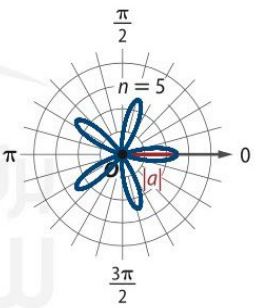

$r = a\sin n\theta$




$r = a\sin n\theta$



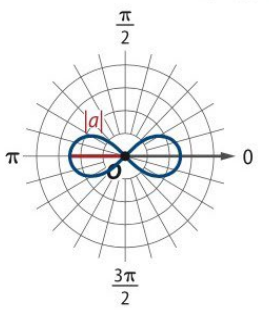

$r = a\cos n\theta$



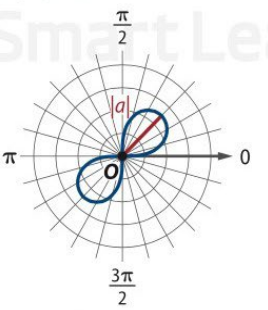

$r = a\cos n\theta$

### Lemniscates (LEM-nis-keyts)

$r^2 = a^2\cos 2\theta$ or $r^2 = a^2\sin 2\theta$



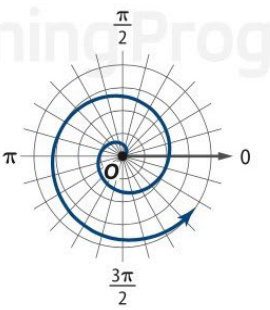

$r^2 = a^2\cos 2\theta$



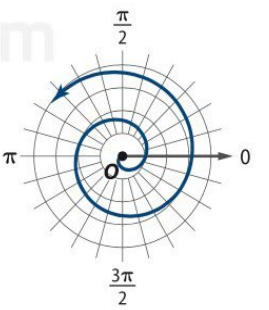

$r^2 = a^2\sin 2\theta$

### Spirals of Archimedes (ahr-kuh-MEE-deez)

$r = a\theta + b$



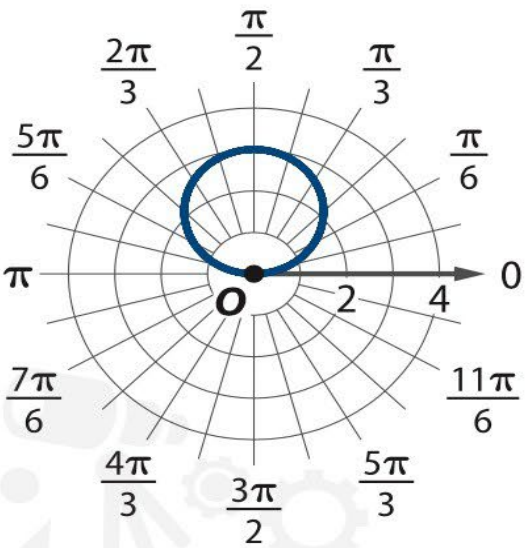

$\theta > 0, r = a\theta + b$



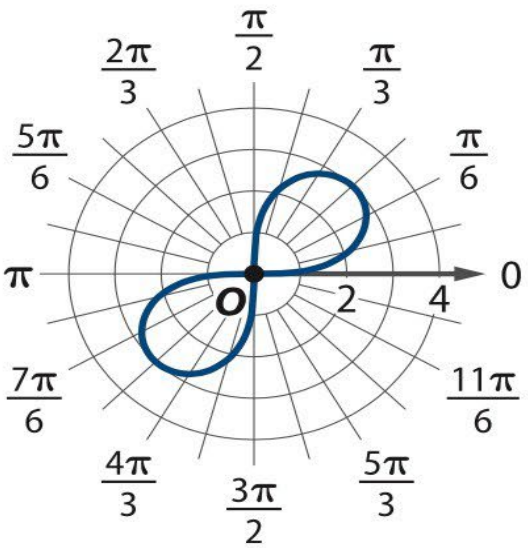

$\theta < 0, r = a\theta + b$

## Write an equation for each graph.

**35.**




**36.**

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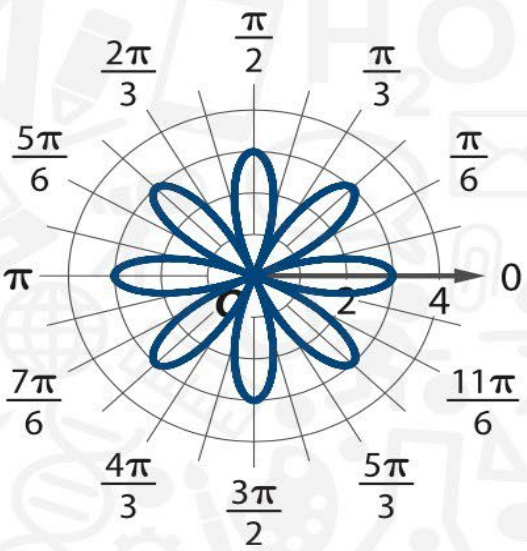
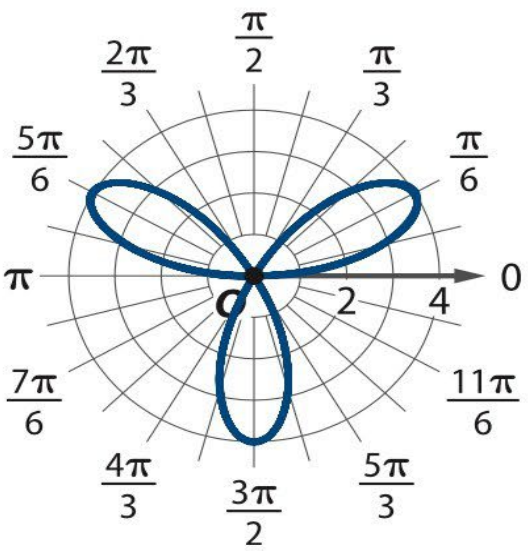
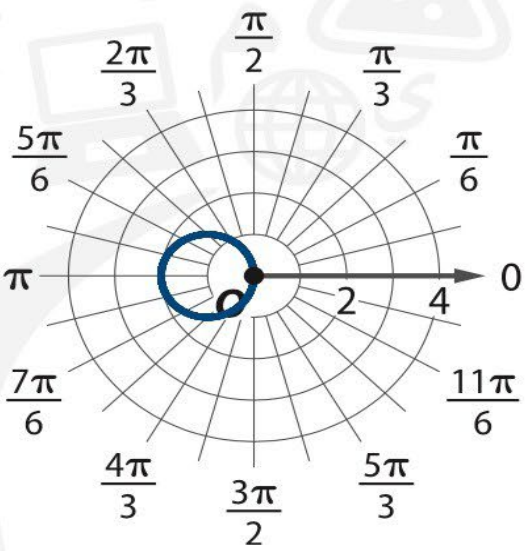


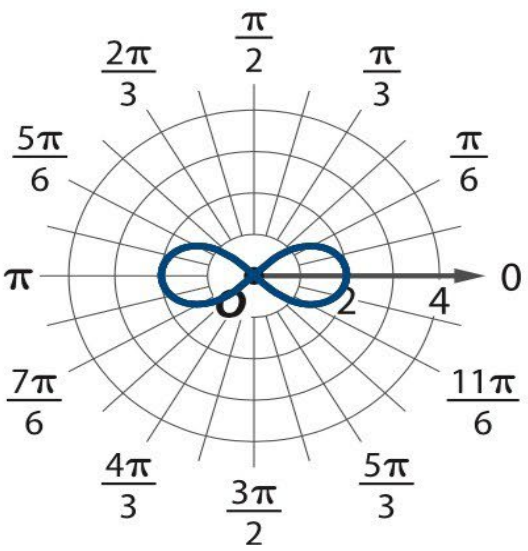

**38.**




**39.**




**40.**



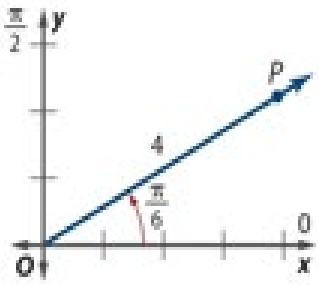
## Example 1 Polar Coordinates to Rectangular Coordinates

**Find the rectangular coordinates for each point with the given polar coordinates.**

**a.** $P\left(4, \frac{\pi}{6}\right)$

For $P\left(4, \frac{\pi}{6}\right)$, $r = 4$ and $\theta = \frac{\pi}{6}$.

| | | |
|---|---|---|
| $x = r\cos\theta$ | Conversion formula | $y = r\sin\theta$ |
| $= 4\cos\frac{\pi}{6}$ | $r = 4$ and $\theta = \frac{\pi}{6}$ | $= 4\sin\frac{\pi}{6}$ |
| $= 4\left(\frac{\sqrt{3}}{2}\right)$ | Simplify. | $= 4\left(\frac{1}{2}\right)$ |
| $= 2\sqrt{3}$ | | $= 2$ |

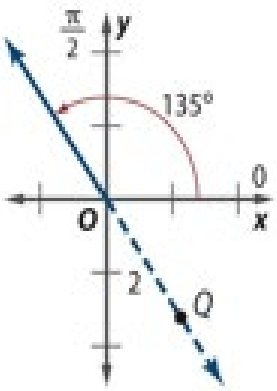



The rectangular coordinates of $P$ are $(2\sqrt{3}, 2)$ or approximately $(3.46, 2)$ as shown.

**b.** $Q(-2, 135°)$

For $Q(-2, 135°)$, $r = -2$ and $\theta = 135°$.

| | | |
|---|---|---|
| $x = r\cos\theta$ | Conversion formula | $y = r\sin\theta$ |
| $= -2\cos 135°$ | $r = -2$ and $\theta = 135°$ | $= -2\sin 135°$ |
| $= -2\left(-\frac{\sqrt{2}}{2}\right)$ | Simplify. | $= -2\left(\frac{\sqrt{2}}{2}\right)$ |
| $= \sqrt{2}$ | | $= -\sqrt{2}$ |

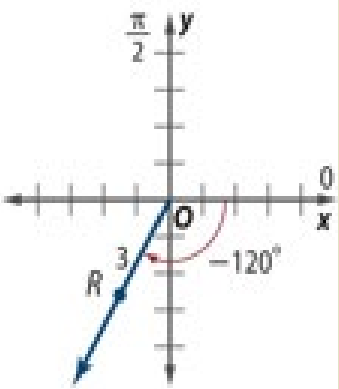



The rectangular coordinates of $Q$ are $(\sqrt{2}, -\sqrt{2})$ or approximately $(1.41, -1.41)$ as shown.

**c.** $V(3, -120°)$

For $V(3, -120°)$, $r = 3$ and $\theta = -120°$.

| | | |
|---|---|---|
| $x = r\cos\theta$ | Conversion formula | $y = r\sin\theta$ |
| $= 3\cos -120°$ | $r = 3$ and $\theta = -120°$ | $= 3\sin -120°$ |
| $= 3\left(-\frac{1}{2}\right)$ | Simplify. | $= 3\left(-\frac{\sqrt{3}}{2}\right)$ |
| $= -\frac{3}{2}$ | | $= -\frac{3\sqrt{3}}{2}$ |




The rectangular coordinates of $V$ are $\left(-\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$ or approximately $(-1.5, -2.6)$ as shown.

## GuidedPractice

**1A.** $R(-6, -120°)$ **1B.** $S\left(5, \frac{\pi}{3}\right)$ **1C.** $T(-3, 45°)$

## extra Q's to practice....

**Find the rectangular coordinates for each point with the given polar coordinates. Round to the nearest hundredth, if necessary.** (Example 1)

1. $\left(2, \frac{\pi}{4}\right)$
2. $\left(\frac{1}{4}, \frac{\pi}{2}\right)$
3. $(5, 240°)$
4. $(2.5, 250°)$
5. $\left(-2, \frac{4\pi}{3}\right)$
6. $(-13, -70°)$
7. $\left(3, \frac{\pi}{2}\right)$
8. $\left(\frac{1}{2}, \frac{3\pi}{4}\right)$
9. $(-2, 270°)$
10. $(4, 210°)$
11. $\left(-1, -\frac{\pi}{6}\right)$
12. $\left(5, \frac{\pi}{3}\right)$

10 | Convert between polar and rectangular equations. | Example 4 & (4A,4B)

## Example 4 Rectangular Equations to Polar Equations

**Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.**

**a.** $(x-4)^2+y^2=16$

The graph of $(x-4)^2+y^2=16$ is a circle with radius 4 centered at (4, 0). To find the polar form of this equation, replace $x$ with $r\cos\theta$ and $y$ with $r\sin\theta$. Then simplify.

| | |
|---|---|
| $(x-4)^2+y^2=16$ | Original equation |
| $(r\cos\theta-4)^2+(r\sin\theta)^2=16$ | $x=r\cos\theta$ and $y=r\sin\theta$ |
| $r^2\cos^2\theta-8r\cos\theta+16+r^2\sin^2\theta=16$ | Multiply. |
| $r^2\cos^2\theta-8r\cos\theta+r^2\sin^2\theta=0$ | Subtract 16 from each side. |
| $r^2\cos^2\theta+r^2\sin^2\theta=8r\cos\theta$ | Isolate the squared terms. |
| $r^2(\cos^2\theta+\sin^2\theta)=8r\cos\theta$ | Factor. |
| $r^2(1)=8r\cos\theta$ | Pythagorean Identity |
| $r=8\cos\theta$ | Divide each side by $r$. |

The graph of this polar equation (Figure 8.3.3) is a circle with radius 4 centered at (4, 0).

**b.** $y=x^2$

The graph of $y=x^2$ is a parabola with vertex at the origin that opens up.

| | |
|---|---|
| $y=x^2$ | Original equation |
| $r\sin\theta=(r\cos\theta)^2$ | $x=r\cos\theta$ and $y=r\sin\theta$ |
| $r\sin\theta=r^2\cos^2\theta$ | Multiply. |
| $\dfrac{\sin\theta}{\cos^2\theta}=r$ | Divide each side by $r\cos^2\theta$. |
| $\dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\cos\theta}=r$ | Rewrite. |
| $\tan\theta\sec\theta=r$ | Quotient and Reciprocal Identities |

10 | Convert between polar and rectangular equations. | Example 4 & (4A,4B)

The graph of the polar equation $r = \tan\theta \sec\theta$ (Figure 8.3.4) is a parabola with vertex at the pole that opens up.

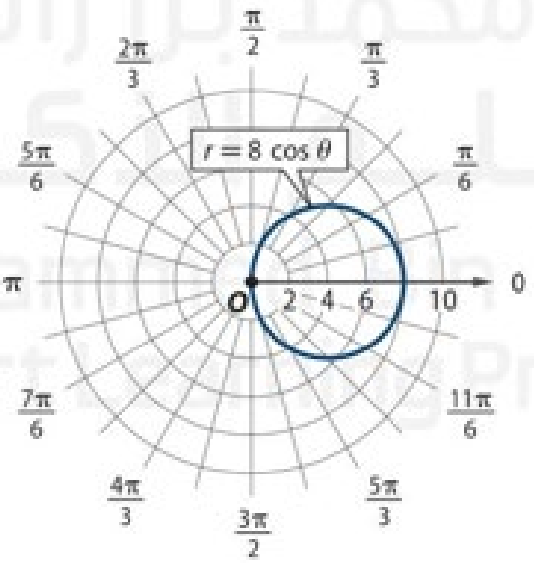



Figure 8.3.3

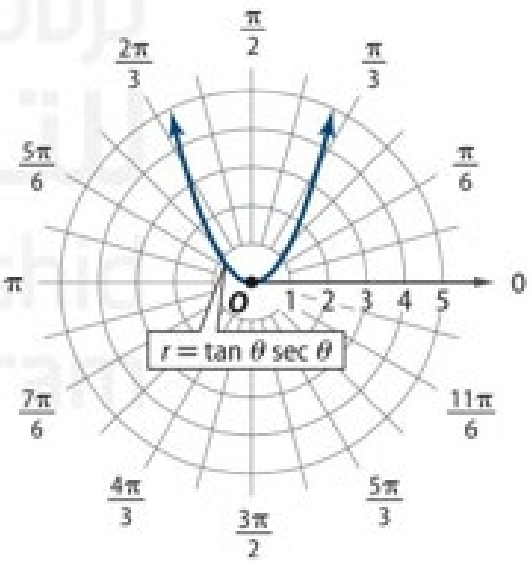



Figure 8.3.4

## GuidedPractice

**4A.** $x^2 + (y-3)^2 = 9$

**4B.** $x^2 - y^2 = 1$

## extra Q's to practice....

**Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.** (Example 4)

**26.** $x = -2$

**27.** $(x+5)^2 + y^2 = 25$

**28.** $y = -3$

**29.** $x = y^2$

30. $(x-2)^2 + y^2 = 4$

**31.** $(x-1)^2 - y^2 = 1$

**32.** $x^2 + (y+3)^2 = 9$

**33.** $y = \sqrt{3}x$

**34.** $x^2 + (y+1)^2 = 1$

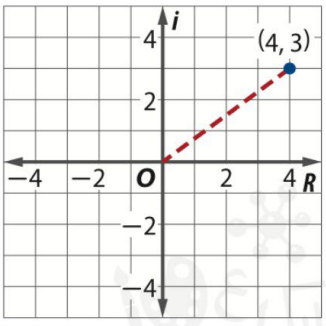
**35.** $x^2 + (y-8)^2 = 64$

## Example 1 Graphs and Absolute Values of Complex Numbers

**Graph each number in the complex plane, and find its absolute value.**

**a.** $z = 4 + 3i$

$(a, b) = (4, 3)$




$|z| = \sqrt{a^2 + b^2}$ Absolute value formula

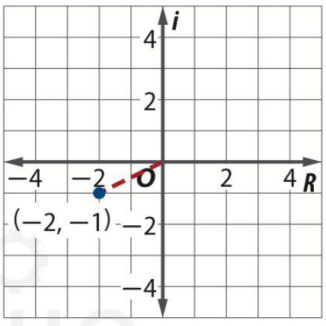
$= \sqrt{4^2 + 3^2}$ $a = 4$ and $b = 3$

$= \sqrt{25}$ or 5 Simplify.

The absolute value of $4 + 3i = 5$.

**b.** $z = -2 - i$

$(a, b) = (-2, -1)$




$|z| = \sqrt{a^2 + b^2}$ Absolute value formula

$= \sqrt{(-2)^2 + (-1)^2}$ $a = -2$ and $b = -1$

$= \sqrt{5}$ or 2.24 Simplify.

The absolute value of $-2 - i \approx 2.24$.

### GuidedPractice

**1A.** $5 + 2i$

**1B.** $-3 + 4i$

## extra Q's to practice....

**Graph each number in the complex plane, and find its absolute value.** (Example 1)

**1.** $z = 4 + 4i$

**2.** $z = -3 + i$

**3.** $z = -4 - 6i$

**4.** $z = 2 - 5i$

**5.** $z = 3 + 4i$

**6.** $z = -7 + 5i$

**7.** $z = -3 - 7i$

**8.** $z = 8 - 2i$

**9.** **VECTORS** The force on an object is given by $z = 10 + 15i$, where the components are measured in newtons (N). (Example 1)

**a.** Represent $z$ as a vector in the complex plane.

**b.** Find the magnitude and direction angle of the vector.

## Example 4 Product of Complex Numbers in Polar Form

**Find $2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) \cdot 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ in polar form. Then express the product in rectangular form.**

$2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) \cdot 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ — Original expression

$= 2(4)\left[\cos\left(\frac{5\pi}{3} + \frac{\pi}{6}\right) + i\sin\left(\frac{5\pi}{3} + \frac{\pi}{6}\right)\right]$ — Product Formula

$= 8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$ — Simplify.

Now find the rectangular form of the product.

$8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$ — Polar form

$= 8\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)$ — Evaluate.

$= 4\sqrt{3} - 4i$ — Distributive Property

The polar form of the product is $8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$. The rectangular form of the product is $4\sqrt{3} - 4i$.

### GuidedPractice

**Find each product in polar form. Then express the product in rectangular form.**

**4A.** $3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \cdot 5\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

**4B.** $-6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \cdot 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

extra Q's to practice....

**Find each product or quotient, and express it in rectangular form.** (Examples 4 and 5)

**26.** $6\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \cdot 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

**27.** $5(\cos 135° + i\sin 135°) \cdot 2(\cos 45° + i\sin 45°)$

**28.** $3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \div \frac{1}{2}(\cos\pi + i\sin\pi)$

**29.** $2(\cos 90° + i\sin 90°) \cdot 2(\cos 270° + i\sin 270°)$

**30.** $3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \div 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

**31.** $4\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right) \div 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

**32.** $\frac{1}{2}(\cos 60° + i\sin 60°) \cdot 6(\cos 150° + i\sin 150°)$

**33.** $6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \div 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

**34.** $5(\cos 180° + i\sin 180°) \cdot 2(\cos 135° + i\sin 135°)$

**35.** $\frac{1}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \div 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

## Example 1 Identify Arithmetic Sequences

**Determine whether each sequence is arithmetic.**

**a.** 5, −6, −17, −28, ...

5 −6 −17 −28

−11 −11 −11

The common difference is −11.
The sequence is arithmetic.

**b.** −4, 12, 28, 42, ...

−4 12 28 42

+16 +16 +14

There is no common difference.
This is not an arithmetic sequence.

### GuidedPractice

**1A.** 7, 12, 16, 20, ...

**1B.** −6, 3, 12, 21, ...

## extra Q's to practice....

**Determine whether each sequence is arithmetic. Write *yes* or *no*.**

**1.** 8, −2, −12, −22,

**2.** −19, −12, −5, 2, 9

**3.** 1, 2, 4, 8, 16

**4.** 0.6, 0.9, 1.2, 1.8, ...

**21.** $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

**22.** −9, −3, 0, 3, 9

**23.** 14, −5, −19, ...

**24.** $\frac{2}{9}, \frac{5}{9}, \frac{8}{9}, \frac{11}{9}, \ldots$

| | | |
|---|---|---|
| 14 | Relate geometric sequences to exponential functions. | (14-17) |

**Find the next three terms of each geometric sequence. Then graph the sequence.**

**14.** 8, 12, 18, 27, ...

**15.** 8, 16, 32, 64, ...

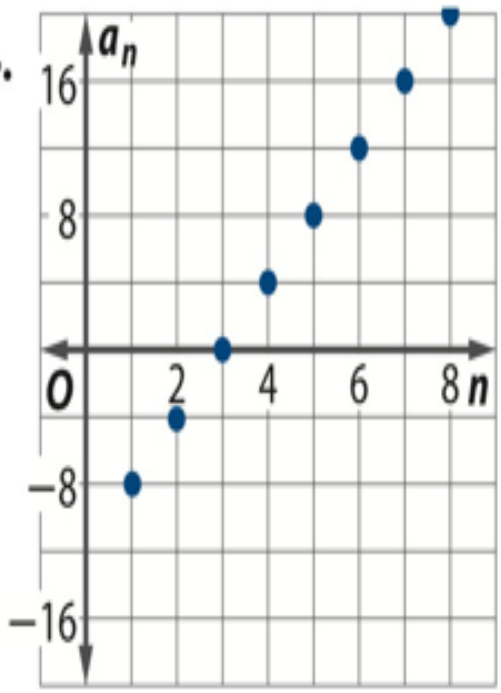
**16.** 250, 50, 10, 2, ...

**17.** $9, -3, 1, -\frac{1}{3}, \ldots$

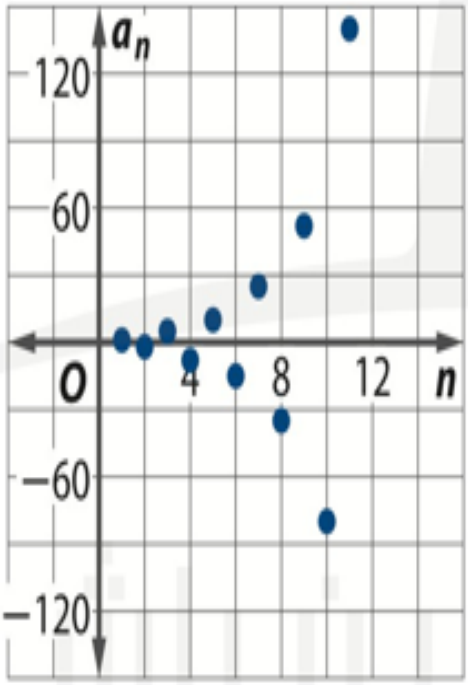
15 Investigate several different types of sequences. (74-77)

Determine whether each sequence is *convergent* or *divergent*.

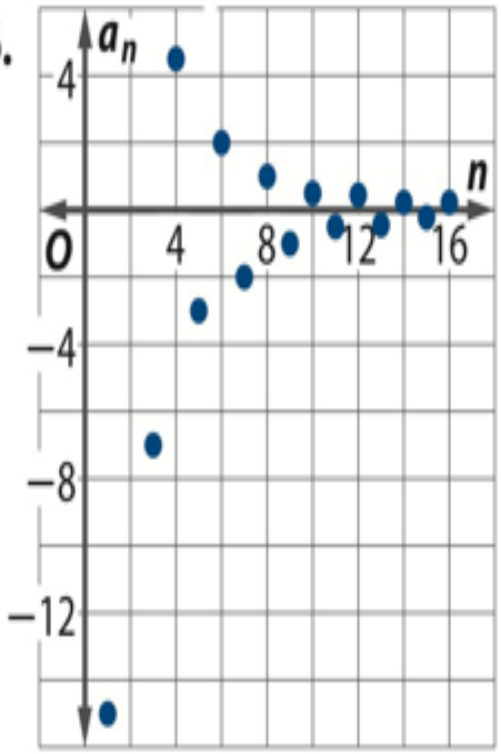
**74.**



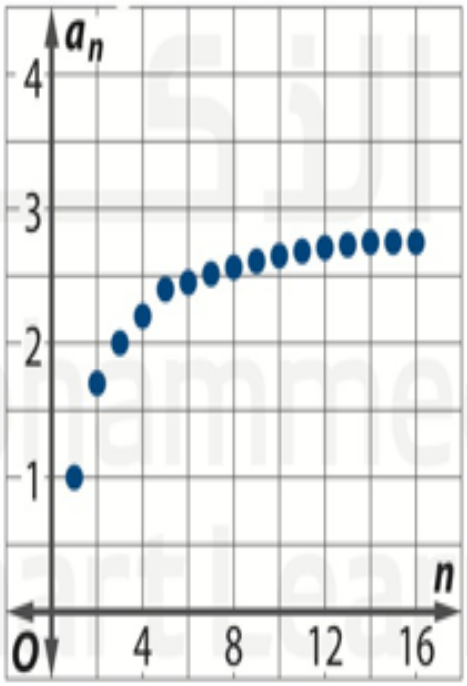

**75.**




**76.**




**77.**



16 Use sigma notation to represent and calculate sums of series.

Key Concept • Sigma Notation

(57-62)

## KeyConcept Sigma Notation

For any sequence $a_1, a_2, a_3, a_4, \ldots$, the sum of the first $k$ terms is denoted

$$\sum_{n=1}^{k} a_n = a_1 + a_2 + a_3 + \ldots + a_k,$$

where $n$ is the index of summation, $k$ is the upper bound of summation, and 1 is the lower bound of summation.

**ReadingMath**

**Sigma Notation** $\sum_{n=1}^{k} a_n$ is read *the summation from $n = 1$ to $k$ of a sub $n$.*

Write each series in sigma notation. The lower bound is given.

**57.** $-2 - 1 + 0 + 1 + 2 + 3 + 4 + 5; n = 1$

**58.** $\frac{1}{20} + \frac{1}{25} + \frac{1}{30} + \frac{1}{35} + \frac{1}{40} + \frac{1}{45}; n = 4$

**59.** $8 + 27 + 64 + \cdots + 1000; n = 2$

**60.** $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{128}; n = 1$

**61.** $-8 + 16 - 32 + 64 - 128 + 256 - 512; n = 3$

**62.** $8\left(-\frac{1}{3}\right) + 8\left(\frac{1}{9}\right) + 8\left(-\frac{1}{27}\right) + \cdots + 8\left(-\frac{1}{243}\right); n = 1$

## KeyConcept *n*th Term of an Arithmetic Sequence

The $n$th term $a_n$ of an arithmetic sequence in which the first term is $a_1$ and the common difference is $d$ is given by the following formula, where $n$ is any natural number.

$$a_n = a_1 + (n - 1)d$$

## Example 1 Find the *n*th Term

**Find the 12th term of the arithmetic sequence 9, 16, 23, 30, … .**

**Step 1** Find the common difference.

$16 - 9 = 7 \qquad 23 - 16 = 7 \qquad 30 - 23 = 7$

So, $d = 7$.

**Step 2** Find the 12th term.

$a_n = a_1 + (n - 1)d$ — $n$th term of an arithmetic sequence

$a_{12} = 9 + (12 - 1)(7)$ — $a_1 = 9$, $d = 7$, and $n = 12$

$= 9 + 77$ or $86$ — Simplify.

### GuidedPractice

**Find the indicated term of each arithmetic sequence.**

**1A.** $a_1 = -4, d = 6, n = 9$

**1B.** $a_{20}$ for $a_1 = 15, d = -8$

## extra Q's to practice....

**Find the indicated term of each arithmetic sequence.**

**14.** $a_1 = -18, d = 12, n = 16$

**15.** $a_1 = -12, n = 66, d = 4$

**16.** $a_1 = 9, n = 24, d = -6$

**17.** $a_{15}$ for $-5, -12, -19, \ldots$

**18.** $a_{10}$ for $-1, 1, 3, \ldots$

**19.** $a_{24}$ for $8.25, 8.5, 8.75, \ldots$

18 | Find sums of arithmetic series. | (57-60)

## extra Q's to practice...

**Standardized Test Example 6** Use Sigma Notation

Find $\sum_{k=4}^{18}(6k-1)$.

A 846 B 910 C 975 D 1008

**Read the Test Item**

You need to find the sum of the series. Find $a_1$, $a_n$, and $n$.

**Solve the Test Item**

There are $18 - 4 + 1$ or 15 terms, so $n = 15$.

$a_1 = 6(4) - 1$ or 23 $a_n = 6(18) - 1$ or 107

Find the sum.

$S_n = n\left(\frac{a_1 + a_n}{2}\right)$ Sum formula

$S_{15} = 15\left(\frac{23 + 107}{2}\right)$ $n = 15$, $a_1 = 23$, and $a_n = 107$

$S_{15} = 15(65)$ or 975 The correct answer is C.

**Guided Practice**

6. Find $\sum_{m=9}^{21}(5m+6)$.

F 972 G 1053 H 1281 J 1701

**Find the sum of each arithmetic series.**

57. $\sum_{k=1}^{16}(4k-2)$

58. $\sum_{k=4}^{13}(4k+1)$

59. $\sum_{k=5}^{16}(2k+6)$

60. $\sum_{k=0}^{12}(-3k+2)$

## Example 3 Find Geometric Means

**Find three geometric means between 2 and 1250.**

**Step 1** Since there are three terms between the first and last term, there are 3 + 2 or 5 total terms, so $n = 5$.

**Step 2** Find $r$.

| | |
|---|---|
| $a_n = a_1 r^{n-1}$ | $n$th term of a geometric sequence |
| $1250 = 2r^{5-1}$ | $a_n = 1250$, $a_1 = 2$, and $n = 5$ |
| $625 = r^4$ | Divide each side by 2. |
| $\pm 5 = r$ | Take the 4th root of each side. |

**Step 3** Use $r$ to find the four arithmetic means.

$$2 \xrightarrow{\times 5} 10 \xrightarrow{\times 5} 50 \xrightarrow{\times 5} 250 \xrightarrow{\times 5} 1250 \quad \text{or} \quad 2 \xrightarrow{\times -5} -10 \xrightarrow{\times -5} 50 \xrightarrow{\times -5} -250 \xrightarrow{\times -5} 1250$$

The geometric means are 10, 50, and 250 or −10, 50, and −250.

### GuidedPractice

**3.** Find four geometric means between 0.5 and 512.

## extra Q's to practice....

**Find the geometric means of each sequence.**

**35.** 810, ?, ?, ?, 10

**36.** 640, ?, ?, ?, 2.5

**37.** $\frac{7}{2}$, ?, ?, ?, $\frac{56}{81}$

**38.** $\frac{729}{64}$, ?, ?, ?, $\frac{324}{9}$

**39.** Find two geometric means between 3 and 375.

**40.** Find two geometric means between 16 and −2.

## Example 6 Find the First Term of a Series

**Find $a_1$ in a geometric series for which $S_n = 13{,}116$, $n = 7$, and $r = 3$.**

| | |
|---|---|
| $S_n = \dfrac{a_1 - a_1r^n}{1 - r}$ | Sum formula |
| $13{,}116 = \dfrac{a_1 - a_1(3^7)}{1 - 3}$ | $S_n = 13{,}116$, $r = 3$, and $n = 7$ |
| $13{,}116 = \dfrac{a_1(1 - 3^7)}{1 - 3}$ | Distributive Property |
| $13{,}116 = \dfrac{-2186a_1}{-2}$ | Subtract. |
| $13{,}116 = 1093a_1$ | Simplify. |
| $12 = a_1$ | Divide each side by 1093. |

$$S_n = \frac{a_1 - a_1r^n}{1 - r}, r \neq 1$$

$$S_n = \frac{a_1 - a_nr}{1 - r}, r \neq 1$$

### Guided Practice

**6.** Find $a_1$ in a geometric series for which $S_n = -26{,}240$, $n = 8$, and $r = -3$.

## extra Q's to practice....

**Find $a_1$ for each geometric series described.**

**51.** $S_n = -2912, r = 3, n = 6$

**52.** $S_n = -10{,}922, r = 4, n = 7$

**53.** $S_n = 1330, a_n = 486, r = \frac{3}{2}$

**54.** $S_n = 4118, a_n = 128, r = \frac{2}{3}$

**55.** $a_n = 1024, r = 8, n = 5$

**56.** $a_n = 1875, r = 5, n = 7$

## Example 3 Infinite Series in Sigma Notation

Find $\sum_{k=1}^{\infty} 18\left(\frac{4}{5}\right)^{k-1}$.

$S = \frac{a_1}{1-r}$ Sum formula

$= \frac{18}{1-\frac{4}{5}}$ $a_1 = 18$ and $r = \frac{4}{5}$

$= \frac{18}{\frac{1}{5}}$ or 90 Simplify.

### Guided Practice

3. Find $\sum_{k=1}^{\infty} 12\left(\frac{3}{4}\right)^{k-1}$.

## extra Q's to practice....

Find the sum of each infinite series, if it exists.

29. $\sum_{k=1}^{\infty} \frac{4}{3} \cdot \left(\frac{5}{4}\right)^{k-1}$

30. $\sum_{k=1}^{\infty} \frac{1}{4} \cdot 3^{k-1}$

31. $\sum_{k=1}^{\infty} \frac{5}{3} \cdot \left(\frac{3}{7}\right)^{k-1}$

32. $\sum_{k=1}^{\infty} \frac{2}{3} \cdot \left(\frac{4}{3}\right)^{k-1}$

33. $\sum_{k=1}^{\infty} \frac{8}{3} \cdot \left(\frac{5}{6}\right)^{k-1}$

34. $\sum_{k=1}^{\infty} \frac{1}{8} \cdot \left(\frac{1}{12}\right)^{k-1}$

## Example 2 Write Recursive Formulas

**Write a recursive formula for each sequence.**

**a. 2, 10, 18, 26, 34, ...**

**Step 1** Determine whether the sequence is arithmetic or geometric.
The sequence is arithmetic because each term after the first can be found by adding a common difference.

**Step 2** Find the common difference.

$d = 10 - 2$ or $8$

**Step 3** Write the recursive formula.

$a_n = a_{n-1} + d$ Recursive formula for arithmetic sequence

$a_n = a_{n-1} + 8$ $d = 8$

A recursive formula for the sequence is $a_n = a_{n-1} + 8$, $a_1 = 2$.

**b. 16, 56, 196, 686, 2401, ...**

**Step 1** Determine whether the sequence is arithmetic or geometric.
The sequence is geometric because each term after the first can be found after multiplying by a common ratio.

**Step 2** Find the common ratio.

$r = \frac{56}{16}$ or $3.5$

**Step 3** Write the recursive formula.

$a_n = r \cdot a_{n-1}$ Recursive formula for geometric sequence

$a_n = 3.5a_{n-1}$ $r = 3.5$

A recursive formula for the sequence is $a_n = 3.5a_{n-1}$, $a_1 = 16$.

**c. $a_4 = 108$ and $r = 3$**

**Step 1** Determine whether the sequence is arithmetic or geometric.
Because $r$ is given, the sequence is geometric.

**Step 2** Write the recursive formula.

$a_n = r \cdot a_{n-1}$ Recursive formula for geometric sequence

$a_n = 3a_{n-1}$ $r = 3$

A recursive formula for the sequence is $a_n = 3a_{n-1}$, $a_1 = 4$.

## GuidedPractice

Write a recursive formula for each sequence.

**2A.** 8, 20, 50, 125, 312.5, ...

**2B.** 8, 17, 26, 35, 44, ...

**2C.** $a_3 = 16$ and $r = 4$

## extra Q's to practice....

Write a recursive formula for each sequence.

**24.** 16, 10, 7, 5.5, 4.75, ...

**25.** 32, 12, 7, 5.75, ...

**26.** 4, 15, 224, 50,175, ...

**27.** 1, 2, 9, 730, ...

**28.** 9, 33, 129, 513, ...

**29.** 480, 128, 40, 18, ...

**30.** 393, 132, 45, 16, ...

**31.** 68, 104, 176, 320, ...

## Example 4 Iterate a Function

**Find the first three iterates $x_1$, $x_2$, and $x_3$ of $f(x) = 5x + 4$ for an initial value of $x_0 = 2$.**

| | |
|---|---|
| $x_1 = f(x_0)$ | Iterate the function. |
| $= 5(2) + 4$ or $14$ | $x_0 = 2$ |
| $x_2 = f(x_1)$ | Iterate the function. |
| $= 5(14) + 4$ or $74$ | $x_1 = 14$ |
| $x_3 = f(x_2)$ | Iterate the function. |
| $= 5(74) + 4$ or $374$ | $x_2 = 74$ |

The first three iterates are 14, 74, and 374.

### GuidedPractice

**4.** Find the first three iterates $x_1$, $x_2$, and $x_3$ of $f(x) = -3x + 8$ for an initial value of $x_0 = 6$.

## extra Q's to practice....

**Find the first three iterates of each function for the given initial value.**

**33.** $f(x) = 12x + 8, x_0 = 4$

**34.** $f(x) = -9x + 1, x_0 = -6$

**35.** $f(x) = -6x + 3, x_0 = 8$

**36.** $f(x) = 8x + 3, x_0 = -4$

**37.** $f(x) = -3x^2 + 9, x_0 = 2$

**38.** $f(x) = 4x^2 + 5, x_0 = -2$

**39.** $f(x) = 2x^2 - 5x + 1, x_0 = 6$

**40.** $f(x) = -0.25x^2 + x + 6, x_0 = 8$

**41.** $f(x) = x^2 + 2x + 3, x_0 = \frac{1}{2}$

**42.** $f(x) = 2x^2 + x + 1, x_0 = -\frac{1}{2}$

24 Use the Binomial Theorem to expand powers of binomials. Example 4 & (4)

## Example 4 Determine a Single Term

**Find the fifth term of $(y + z)^{11}$.**

**Step 1** Use the Binomial Theorem to write the expansion in sigma notation.

$$(y + z)^{11} = \sum_{k=0}^{11} \frac{11!}{k!(11-k)!} y^{11-k} z^k$$

**Step 2**

$$\frac{11!}{k!(11-k)!} y^{11-k} z^k = \frac{11!}{4!(11-4)!} y^{11-4} z^4$$

For the fifth term, $k = 4$.

$$= 330y^7z^4$$

$C(11, 4) = 330$

### GuidedPractice

**4.** Find the sixth term of $(c + d)^{10}$.

## extra Q's to practice....

**Find the indicated term of each expression.**

**23.** third term of $(x + 2z)^7$

**24.** fourth term of $(y - 3x)^6$

**25.** seventh term of $(2a - 2b)^8$

**26.** sixth term of $(4x + 5y)^6$

**27.** fifth term of $(x - 4)^9$

**28.** fourth term of $(c + 6)^8$

25 Disprove statements by finding a counter-example. (6,7,21-24)

## Example 3 Use a Counterexample to Disprove

**Find a counterexample to disprove the statement that $2^n + 2n^2$ is divisible by 4 for any natural number $n$.**

Test different values of $n$.

| $n$ | $2^n + 2n^2$ | Divisible by 4? |
|---|---|---|
| 1 | $2^1 + 2(1)^2 = 2 + 2$ or 4 | yes |
| 2 | $2^2 + 2(2)^2 = 4 + 8$ or 12 | yes |
| 3 | $2^3 + 2(3)^2 = 8 + 18$ or 26 | no |

The value $n = 3$ is a counterexample for the statement.

### GuidedPractice

**3.** Find a counterexample to disprove $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(3n-1)}{2}$.

extra Q's to practice....

**Find a counterexample to disprove each statement.**

**6.** $3^n + 1$ is divisible by 4.

**7.** $2^n + 3^n$ is divisible by 4.

**21.** $1 + 2 + 3 + \cdots + n = n^2$

**22.** $1 + 8 + 27 + \cdots + n^3 = (2n + 2)^2$

**23.** $n^2 - n + 15$ is prime.

**24.** $n^2 + n + 23$ is prime.

اللهم إني استودعك ما قرأت و ما حفظت و ما تعلمت فرده عند حاجتي إليه انك على كل شيء قدير