

تجميعية هيكل - حادي عشر متقدم - YourPhysicsCompass



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف الحادي عشر المتقدم ← فيزياء ← الفصل الثالث ← ملفات متنوعة ← الملف

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المزيد من مادة
فيزياء:

التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم



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المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة فيزياء في الفصل الثالث

حل نموذج تدريبي للاختبار النهائي وفق الهيكل الوزاري

1

نموذج تدريبي للاختبار النهائي وفق الهيكل الوزاري

2

المراجعة النهائية القسم الالكتروني وفق الهيكل الوزاري متبوعة بالإجابات

3

حل تجميعية شاملة المنهج وفق الهيكل الوزاري

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تجميعية شاملة المنهج وفق الهيكل الوزاري بدون الحل

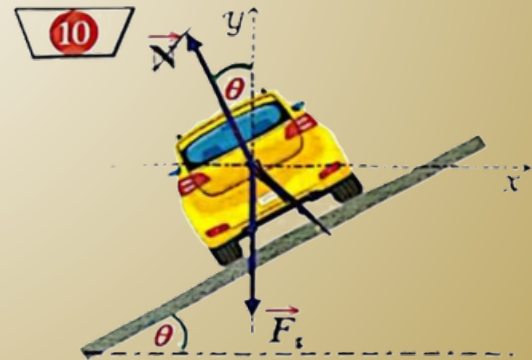
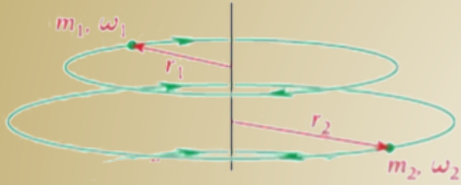
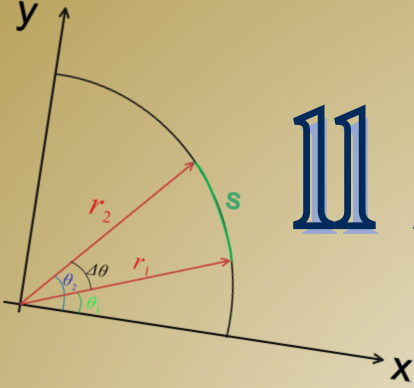
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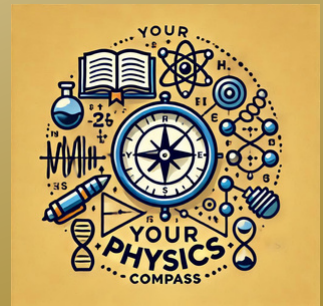
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الفصل الثامن

1	<ul style="list-style-type: none"> Locate the center of mass of an extended, symmetric object of uniform mass distribution by using the symmetry. Recall that center of gravity is equivalent to center of mass in situations where the gravitational force is constant everywhere throughout the object. 	Student Book	226
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Where do you think is the center of mass of our solar system? Is it fixed or does it change its position?

8.1 Center of Mass and Center of Gravity

So far, we have represented the location of an object by coordinates of a single point. However, a statement such as "a car is located at $x = 3.2 \text{ m}$ " surely does not mean that the entire car is located at that point. So, what does it mean to give the coordinate of one particular point to represent an extended object? Answers to this question depend on the particular application. In auto racing, for example, a car's location is represented by the coordinate of the frontmost part of the car. When this point crosses the finish line, the race is decided. On the other hand, in soccer, a goal is counted only if the entire ball has crossed the goal line; in this case, it makes sense to represent the soccer ball's location by the coordinates of the rearmost part of the ball. However, these examples are exceptions. In almost all situations, there is a natural choice of a point to represent the location of an extended object. This point is called the *center of mass*.

Definition

The **center of mass** is the point at which we can imagine all the mass of an object to be concentrated.

Thus, the center of mass is also the point at which we can imagine the force of gravity acting on the entire object to be concentrated. If we can imagine all of the mass to be concentrated at this point when calculating the force due to gravity, it is legitimate to call this point the *center of gravity*, a term that can often be used interchangeably with *center of mass*. (To be precise, we should note that these two terms are only equivalent in situations where the gravitational force is constant everywhere throughout the object. In Chapter 12, we will see that this is not the case for very large objects.)

It is appropriate to mention here that if an object's mass density is constant, the center of mass (center of gravity) is located in the geometrical center of the object. Thus, for most objects in everyday experience, it is a reasonable first guess that the center of gravity is the middle of the object. The derivations in this chapter will bear out this conjecture.

Describe that the center of mass of two-point masses (or two objects each of which can be replaced by a particle having its mass and located at its center) always lies on the connecting line between the two masses.

Student Book
Figure 8.2
Solved problem 8.1

Combined Center of Mass for Two Objects

If we have two identical objects of equal mass and want to find the center of mass for the combination of the two, it is reasonable to assume from considerations of symmetry that the combined center of mass of this system lies exactly midway between the individual centers of mass of the two objects. If one of the two objects is more massive, then it is

equally reasonable to assume that the center of mass for the combination is closer to that of the more massive one. Thus, we have a general formula for calculating the location of the center of mass, \vec{R} , for two masses m_1 and m_2 located at positions \vec{r}_1 and \vec{r}_2 in an arbitrary coordinate system (Figure 8.2):

$$\vec{R} = \frac{\vec{r}_1 m_1 + \vec{r}_2 m_2}{m_1 + m_2} \quad (8.1)$$

This equation says that the center-of-mass position vector is an average of the position vectors of the individual objects, weighted by their mass. Such a definition is consistent with the empirical evidence we have just cited. For now, we will use this equation as an operating definition and gradually work out its consequences. Later in this chapter and in the following chapters, we will see additional reasons why this definition makes sense.

Note that we can immediately write vector equation 8.1 in Cartesian coordinates as follows:

$$X = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}, \quad Y = \frac{y_1 m_1 + y_2 m_2}{m_1 + m_2}, \quad Z = \frac{z_1 m_1 + z_2 m_2}{m_1 + m_2} \quad (8.2)$$

In Figure 8.2, the location of the center of mass lies exactly on the straight (dashed black) line that connects the two masses. Is this a general result—does the center of mass always lie on this line? If yes, why? If no, what is the special condition that is needed for this to be the case? The answer is that this is a general result for all two-body systems: The center of mass of such a system always lies on the connecting line between the two objects. To see this, we can place the origin of the coordinate system at one of the two masses in Figure 8.2, say m_1 . (As we know, we can always shift the origin of a coordinate system without changing the physics results.) Using equation 8.1, we then see that $\vec{R} = \vec{r}_2 m_2 / (m_1 + m_2)$, because with this choice of coordinate system, we define \vec{r}_1 as zero. Thus, the two vectors \vec{R} and \vec{r}_2 point in the same direction, but \vec{R} is shorter by a factor of $m_2 / (m_1 + m_2) < 1$. This shows that \vec{R} always lies on the straight line that connects the two masses.

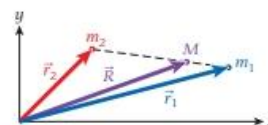


FIGURE 8.2 Location of the center of mass for a system of two masses m_1 and m_2 , where $M = m_1 + m_2$.

Concept Check 8.1

In the case shown in Figure 8.2, what are the relative magnitudes of the two masses m_1 and m_2 ?

- $m_1 < m_2$
- $m_1 > m_2$
- $m_1 = m_2$
- Based solely on the information given in the figure, it is not possible to decide which of the two masses is larger.

Concept Check 8.2

A cylindrical bottle of oil-and-vinegar salad dressing whose volume is 1/2 vinegar (mass density of 1.01 g/cm^3) and 1/2 oil (mass density of 0.910 g/cm^3) rests on a table. Initially, the oil and the vinegar are separated, with the oil floating on top of the vinegar. The bottle is shaken so that the oil and vinegar mix uniformly and then returned to the table. How has the height of the center of mass of the salad dressing changed as a result of the mixing?

- It is higher.
- It is lower.
- It is the same.
- There is not enough information to answer this question.

SOLVED PROBLEM 8.1 Center of Mass of Earth and Moon

The Earth has a mass of $5.97 \times 10^{24} \text{ kg}$, and the Moon has a mass of $7.36 \times 10^{22} \text{ kg}$. The Moon orbits the Earth at a distance of 384,000 km; that is, the center of the Moon is a distance of 384,000 km from the center of Earth, as shown in Figure 8.3a.

PROBLEM

How far from the center of the Earth is the center of mass of the Earth-Moon system?

SOLUTION

THINK The center of mass of the Earth-Moon system can be calculated by taking the center of the Earth to be located at $x = 0$ and the center of the Moon to be located at $x = 384,000 \text{ km}$. The center of mass of the Earth-Moon system will lie along a line connecting the center of the Earth and the center of the Moon (as in Figure 8.3a).

SKETCH A sketch showing Earth and Moon to scale is presented in Figure 8.3b.

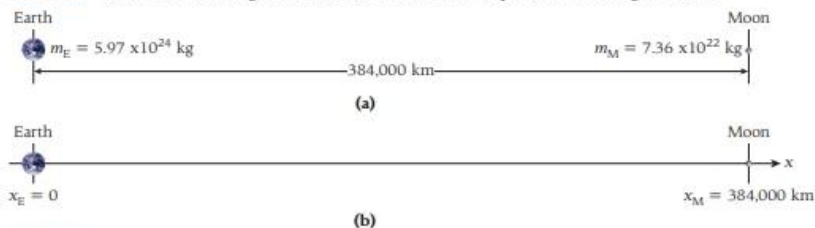


FIGURE 8.3 (a) The Moon orbits the Earth at a distance of 384,000 km (drawing to scale). (b) A sketch showing the Earth at $x_E = 0$ and the Moon at $x_M = 384,000 \text{ km}$.

— Continued



RESEARCH We define an x -axis and place the Earth at $x_E = 0$ and the Moon at $x_M = 384,000$ km. We can use equation 8.2 to obtain an expression for the x -coordinate of the center of mass of the Earth-Moon system:

$$X = \frac{x_E m_E + x_M m_M}{m_E + m_M}.$$

SIMPLIFY Since we have put the origin of our coordinate system at the center of Earth, we set $x_E = 0$. This results in

$$X = \frac{x_M m_M}{m_E + m_M}.$$

CALCULATE Inserting the numerical values, we get the x -coordinate of the center of mass of the Earth-Moon system:

$$X = \frac{x_M m_M}{m_E + m_M} = \frac{(384,000 \text{ km})(7.36 \times 10^{22} \text{ kg})}{5.97 \times 10^{24} \text{ kg} + 7.36 \times 10^{22} \text{ kg}} = 4676.418 \text{ km}.$$

ROUND All of the numerical values were given to three significant figures, so we report our result as

$$X = 4680 \text{ km}.$$

DOUBLE-CHECK Our result is in kilometers, which is the correct unit for a position. The center of mass of the Earth-Moon system is close to the center of the Earth. This distance is small compared to the distance between the Earth and the Moon, which makes sense because the mass of the Earth is much larger than the mass of the Moon. In fact, this distance is less than the radius of the Earth, $R_E = 6370$ km. Both Earth and Moon actually orbit the common center of mass. Thus, the Earth seems to wobble as the Moon orbits it.

الفصل التاسع

3

- Express the Cartesian coordinates (x, y) in terms of the polar coordinates (r, θ) and vice versa.
- Convert polar coordinates to Cartesian coordinates and vice versa.

Student Book
Example 9.1

255~256
256

9.1 Polar Coordinates

In Chapter 3, we discussed motion in two dimensions. In this chapter, we examine a special case of motion in a two-dimensional plane: motion of an object along the circumference of a circle. To be precise, we will only study circular motion of objects that we can consider to be point particles. In Chapter 10 on rotation, we will relax this condition and also examine extended bodies.

Circular motion is surprisingly common. Riding on a carousel or many other amusement park rides, such as those shown in Figure 9.2, qualifies as circular motion. Indy-style car racing also involves circular motion, as the cars alternate between moving along straight sections and half-circle segments of the track. CD, DVD, and Blu-ray players also operate with circular motion, although this motion is usually hidden from the eye.

During an object's **circular motion**, its x - and y -coordinates change continuously, but the distance from the object to the center of the circular path stays the same. We can take advantage of this fact by using **polar coordinates** to study circular motion. Shown in Figure 9.3 is the position vector, \vec{r} , of an object in circular motion. This vector changes as a function of time, but its tip always moves on the circumference of a circle. We can specify \vec{r} by giving its x - and y -components. However, we can specify the same vector by giving two other numbers: the angle of \vec{r} relative to the x -axis, θ , and the length of \vec{r} , $r = |\vec{r}|$ (Figure 9.3).

Trigonometry provides the relationship between the Cartesian coordinates x and y and the polar coordinates θ and r :

$$r = \sqrt{x^2 + y^2} \quad (9.1)$$

$$\theta = \tan^{-1}(y/x). \quad (9.2)$$

The inverse transformation from polar to Cartesian coordinates is given by

$$x = r \cos \theta \quad (9.3)$$

$$y = r \sin \theta. \quad (9.4)$$

The major advantage of using polar coordinates for analyzing circular motion is that r never changes. It remains the same as long as the tip of the vector \vec{r} moves along the circular path. Thus, we can reduce the description of two-dimensional motion on the circumference of a circle to a one-dimensional problem involving the angle θ .



FIGURE 9.2 Circular motion in the horizontal and in the vertical plane.

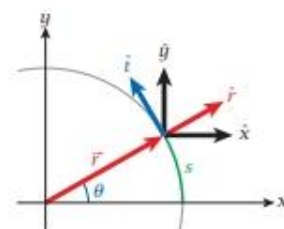
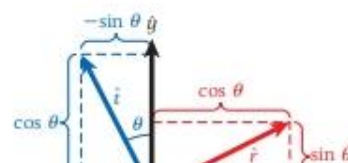


FIGURE 9.3 Polar coordinate system for circular motion.



9.2 Angular Coordinates and Angular Displacement

Polar coordinates allow us to describe and analyze circular motion, where the distance to the origin, r , of the object in motion stays constant and the angle θ varies as a function of time, $\theta(t)$. As was already pointed out, the angle θ is measured relative to the positive x -axis. Any point on the positive x -axis has an angle $\theta = 0$. As the definition in equation 9.2 implies, a move in the counterclockwise direction away from the positive x -axis toward the positive y -axis results in positive values for the angle θ . Conversely, a clockwise move away from the positive x -axis toward the negative y -axis results in negative values of θ .

The two most commonly used units for angles are degrees ($^\circ$) and radians (rad). These units are defined such that the angle measured by one complete circle is 360° , which corresponds to 2π rad. Thus, the unit conversion between the two angular measures is

$$\theta \text{ (degrees)} \frac{\pi}{180} = \theta \text{ (radians)} \Leftrightarrow \theta \text{ (radians)} \frac{180}{\pi} = \theta \text{ (degrees)}$$

$$1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ.$$

Like the linear position x , the angle θ can have positive and negative values. However, θ is periodic; a complete turn around the circle (2π rad, or 360°) returns the coordinate for θ to the same point in space. Just as the linear displacement, Δx , is defined to be the difference between two positions, x_2 and x_1 , the angular displacement, $\Delta\theta$, is the difference between two angles:

$$\Delta\theta = \theta_2 - \theta_1.$$

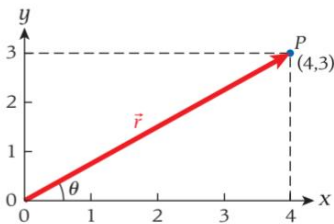


FIGURE 9.5 A point located at (4,3) in a Cartesian coordinate system.

EXAMPLE 9.1 Locating a Point with Cartesian and Polar Coordinates

A point has a location given in Cartesian coordinates as (4,3), as shown in Figure 9.5.

PROBLEM

How do we represent the position of this point in polar coordinates?

SOLUTION

Using equation 9.1, we can calculate the radial coordinate:

$$r = \sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2} = 5.$$

Using equation 9.2, we can calculate the angular coordinate:

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(3/4) = 0.64 \text{ rad} = 37^\circ.$$

Therefore, we can express the position of point P in polar coordinates as $(r, \theta) = (5, 0.64 \text{ rad}) = (5, 37^\circ)$. Note that we can specify the same position by adding (any integral multiple of) 2π rad, or 360° , to θ :

$$(r, \theta) = (5, 0.64 \text{ rad}) = (5, 37^\circ) = (5, 2\pi \text{ rad} + 0.64 \text{ rad}) = (5, 360^\circ + 37^\circ).$$

4	Recall that the common unit for angular velocity is radian per second (rad/s).	Student Book	256,258
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9.3 Angular Velocity, Angular Frequency, and Period

We have seen that the change of an object's linear coordinates in time is its velocity. Similarly, the change of an object's angular coordinate in time is its **angular velocity**. The average magnitude of the angular velocity is defined as

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}.$$

This definition uses the notation $\theta_1 \equiv \theta(t_1)$ and $\theta_2 \equiv \theta(t_2)$. The horizontal bar above the symbol ω for angular velocity again indicates a time average. By taking the limit of this expression as the time interval approaches zero, we find the instantaneous value of the magnitude of the angular velocity:

$$\omega = \lim_{\Delta t \rightarrow 0} \bar{\omega} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} \equiv \frac{d\theta}{dt}. \quad (9.8)$$

The most common unit of angular velocity is radians per second (rad/s); degrees per second is not generally used.

Angular velocity is a vector. Its direction is that of an axis through the center of the circular path and perpendicular to the plane of the circle. (This axis is a rotation axis, as we will discuss in more detail in Chapter 10.) This definition allows for two possibilities for the direction in which the vector $\vec{\omega}$ can point: up or down, parallel or antiparallel to the axis of rotation. A right-hand rule helps us decide which is the correct direction: When the fingers point in the direction of rotation along the circle's circumference, the thumb points in the direction of $\vec{\omega}$, as shown in Figure 9.8.

Angular velocity measures how fast the angle θ changes in time. Another quantity also specifies how fast this angle changes in time—the **frequency**, f . For example, the rpm number on the tachometer in your car indicates how many times per minute the engine cycles and thus specifies the frequency of engine revolution. Figure 9.9 shows a tachometer, with the units specified as "1/min \times 1000"; the engine hits the red line at 6000 revolutions per minute. Thus, the frequency, f , measures cycles per unit time, instead of radians per unit time as the angular velocity does. The frequency is related to the magnitude of the angular velocity, ω , by

$$f = \frac{\omega}{2\pi} \Leftrightarrow \omega = 2\pi f. \quad (9.9)$$

This relationship makes sense because one complete turn around a circle requires an angle change of 2π rad. (Be careful—both frequency and angular velocity have the same unit of inverse seconds and can be easily confused.)

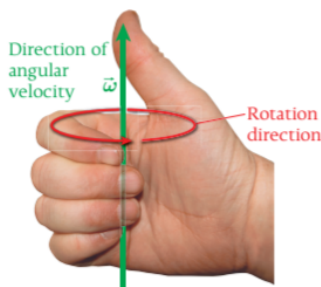


FIGURE 9.8 The right-hand rule for determining the direction of the angular velocity vector.



FIGURE 9.9 The tachometer of a car measures the frequency (in cycles per minute) of revolutions of the engine.

5	Relate the magnitudes of linear (tangential) and angular velocities for circular motion as $v = r\omega$, and explain that this relation does not hold for tangential and angular velocity vectors which point in different directions	M.C.Q(9.13) Additional Exe.Q. (9.62/a)	278 282
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9.13 A bicycle's wheels have a radius of 33.0 cm. The bicycle is traveling at a speed of 6.5 m/s. What is the angular speed of the front tire?

- a) 0.197 rad/s
- b) 1.24 rad/s
- c) 5.08 rad/s
- d) 19.7 rad/s
- e) 215 rad/s

9.62 Consider a 53.0 cm-long lawn mower blade rotating about its center at 3400. rpm.

a) Calculate the linear speed of the tip of the blade.

6	Sketch the path taken in circular motion (uniform and non-uniform) and explain the velocity and acceleration vectors (magnitudes and directions) during the motion.	Student Book	261~262
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9.4 Angular and Centripetal Acceleration

The rate of change of an object's angular velocity is its **angular acceleration**, denoted by the Greek letter α . The definition of the magnitude of the angular acceleration is analogous to that for the linear acceleration. Its time average is defined as

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}.$$

The instantaneous magnitude of the angular acceleration is obtained in the limit as the time interval approaches zero:

$$\alpha = \lim_{\Delta t \rightarrow 0} \bar{\alpha} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}. \quad (9.14)$$

Just as we related the linear velocity to the angular velocity, we can also relate the tangential acceleration to the angular acceleration. We start with the definition of the linear acceleration vector as the time derivative of the linear velocity vector. Then we substitute the expression for the linear velocity in circular motion from equation 9.12:

$$\vec{a}(t) = \frac{d}{dt} \vec{v}(t) = \frac{d}{dt} (v\hat{t}) = \left(\frac{dv}{dt}\right)\hat{t} + v\left(\frac{d\hat{t}}{dt}\right). \quad (9.15)$$

In the last step here, we used the product rule of differentiation. Thus, the acceleration in circular motion has two components. The first part arises from the change in the magnitude of the velocity; this is the **tangential acceleration**. The second part is due to the fact that the velocity vector always points in the tangential direction and thus has to change its direction continuously as the tip of the radial position vector moves around the circle; this is the **radial acceleration**.

Let's look at the two components individually. First, we can calculate the time derivative of the linear speed, v , using the relationship between linear speed and angular speed in equation 9.13 and again invoking the product rule:

$$\frac{dv}{dt} = \frac{d}{dt} (r\omega) = \left(\frac{dr}{dt}\right)\omega + r\frac{d\omega}{dt}.$$

Because r is constant for circular motion, $dr/dt = 0$, and the first term in the sum on the right-hand side is zero. From equation 9.14, $d\omega/dt = \alpha$, and so the second term in the sum is equal to $r\alpha$. Thus, the change in speed is related to the angular acceleration by

$$\frac{dv}{dt} = r\alpha. \quad (9.16)$$

However, the acceleration vector of equation 9.15 also has a second component, which is proportional to the time derivative of the tangential unit vector. For this quantity, we find

$$\begin{aligned} \frac{d}{dt} \hat{t} &= \frac{d}{dt} (-\sin\theta, \cos\theta) = \left(\frac{d}{dt}(-\sin\theta), \frac{d}{dt}(\cos\theta)\right) \\ &= \left(-\cos\theta \frac{d\theta}{dt}, -\sin\theta \frac{d\theta}{dt}\right) = -\frac{d\theta}{dt} (\cos\theta, \sin\theta) \\ &= -\omega\hat{r}. \end{aligned}$$

Therefore, we find that the time derivative of the tangential unit vector points in the direction opposite to that of the radial unit vector. With this result, we can finally write for the linear acceleration vector of equation 9.15:

$$\vec{a}(t) = r\alpha\hat{t} - v\omega\hat{r}. \quad (9.17)$$

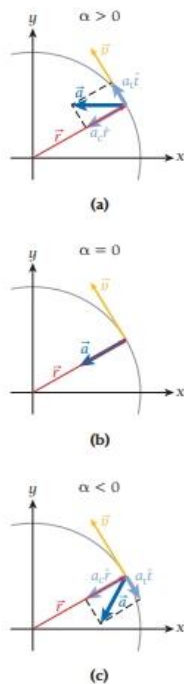


FIGURE 9.12 Relationships among linear acceleration, centripetal acceleration, and angular acceleration for (a) increasing speed; (b) constant speed; and (c) decreasing speed.

Again, for circular motion, the acceleration vector has two physical components (Figure 9.12): The first results from the change in speed and points in the tangential direction, and the second comes from the continuous change of direction of the velocity vector and points in the negative radial direction, toward the center of the circle. This second component is present even if the circular motion proceeds at constant speed. If the angular velocity is constant, the tangential angular acceleration is zero, but the velocity vector still changes direction continuously as the object moves in its circular path. The acceleration that changes the direction of the velocity vector without changing its magnitude is often called **centripetal acceleration** (*centripetal* means "center-seeking"), and it is directed in the inward radial direction. Thus, we can write equation 9.17 for the acceleration of an object in circular motion as the sum of the tangential acceleration and the centripetal acceleration:

$$\vec{a} = a_t \hat{t} - a_c \hat{r}. \quad (9.18)$$

The magnitude of the centripetal acceleration is

$$a_c = v\omega = \frac{v^2}{r} = \omega^2 r. \quad (9.19)$$

The first expression for the centripetal acceleration in equation 9.19 can be simply read off from equation 9.17 as the coefficient of the unit vector pointing in the negative radial direction. The second and third expressions for the centripetal acceleration then follow from the relationship between linear and angular speeds and the radius (equation 9.13).

For the magnitude of the acceleration in circular motion, we thus have, from equations 9.17 and 9.19,

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}. \quad (9.20)$$

EXAMPLE 9.4 Ultracentrifuge

One of the most important pieces of equipment in biomedical labs is the ultracentrifuge (Figure 9.13). It is used for separation of substances (such as colloids or proteins) consisting of particles of different masses through the process of sedimentation from rotation.

7 Relate the magnitude of the net acceleration in circular motion to the tangential acceleration and centripetal acceleration as:

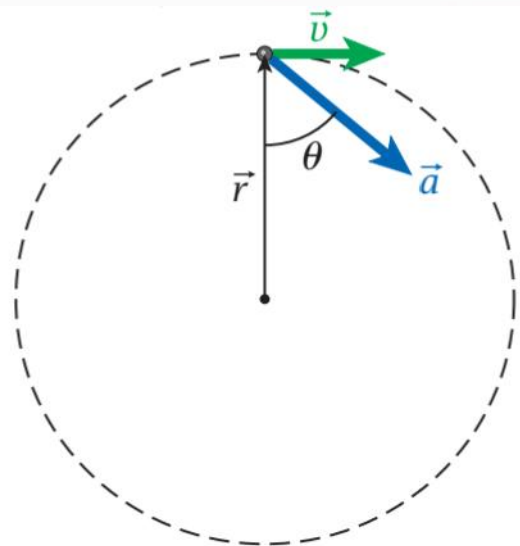
$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$$

Exercises
Q. (9.46)

281

•9.46 A particle is moving clockwise in a circle of radius 1.00 m. At a certain instant, the magnitude of its acceleration is $a = |\vec{a}| = 25.0 \text{ m/s}^2$, and the acceleration vector has an angle of $\theta = 50.0^\circ$ with the position vector, as shown in the figure. At this instant, find the speed, $v = |\vec{v}|$, of this particle.

•9.47 In a tape recorder, the magnetic tape moves at a constant linear speed of



- Describe centripetal force as the net inward force (towards the center of the circular path) needed to provide the centripetal acceleration necessary for circular motion.

- Solve problems related to acceleration in circular motion.

Student Book

M.C.Q(9.7)

Q.(9.90 / 9.9)

264

278

283

Concept Check 9.3

You are sitting on a carousel, which is in motion. Where should you sit so that the largest possible centripetal force is acting on you?

- close to the outer edge
- close to the center
- in the middle
- The force is the same everywhere.

9.5 Centripetal Force

The centripetal force, \vec{F}_c , is not another fundamental force of nature but is simply the net inward force needed to provide the centripetal acceleration necessary for circular motion. It has to point inward, toward the circle's center. Its magnitude is the product of the mass of the object and the centripetal acceleration required to force it onto a circular path:

$$F_c = ma_c = mv\omega = m\frac{v^2}{r} = m\omega^2 r. \quad (9.21)$$

To arrive at equation 9.21, we simply wrote the centripetal acceleration in terms of the linear velocity v , the angular velocity ω , and the radius r , as in equation 9.19, and multiplied it by the mass of the object forced onto a circular path by the centripetal force.

Figure 9.14 shows a top view of a spinning table with three identical (except for color) markers on it. The black marker is located close to the center, the red marker close to the outer edge, and the blue marker in the middle between them. If we spin the table slowly as in part (a), all three markers are in circular motion. In this case, the static friction force between the table and the markers provides the centripetal force required to keep the markers in circular motion. In parts (b), (c), and (d), the table is spinning progressively faster. A higher angular velocity means a larger centripetal force, according to equation 9.21. The markers slide when the friction force is not large enough to provide the centripetal force needed. As you can see, the outermost marker slides off first, and the innermost marker last. This clearly indicates that, for a given angular velocity, the centripetal force increases with the distance from the center. Equation 9.21 in the form $F_c = m\omega^2 r$ can explain this observed behavior. All points on the surface of the spinning table have the same angular velocity, ω , because all of them take the same time to complete one revolution. Thus, for the three markers, the centripetal force is proportional to the distance from the center, explaining why the red marker slides off first and the black marker slides off last.

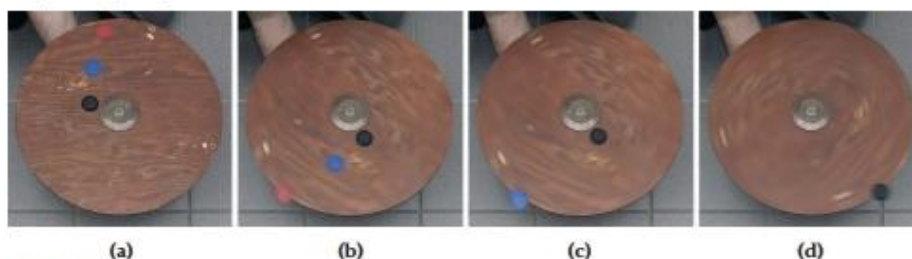


FIGURE 9.14 Markers on a spinning table. Shown from left to right are the initial positions of the markers and the moments when the three markers slide off during the process of circular motion.

9.7 A ball attached to the end of a string is swung around in a circular path of radius r . If the radius is doubled and the linear speed is kept constant, the centripetal acceleration

- remains the same.
- increases by a factor of 2.
- decreases by a factor of 2.
- increases by a factor of 4.
- decreases by a factor of 4.

9.9 You put three identical coins on a turntable at different distances from the center and then turn the motor on. As the turntable speeds up, the outermost coin slides off first, followed by the one at the middle distance, and, finally, when the turntable is going the fastest, the innermost one. Why is this?

- For greater distances from the center, the centripetal acceleration is higher, and so the force of friction becomes unable to hold the coin in place.
- The weight of the coin causes the turntable to flex downward, so the coin nearest the edge falls off first.
- Because of the way the turntable is made, the coefficient of static friction decreases with distance from the center.
- For smaller distances from the center, the centripetal acceleration is higher.

9.90 A flywheel of radius 27.01 cm rotates with a frequency of 4949 rpm. What is the centripetal acceleration at a point on the edge of the flywheel?

9	→ Identify that the centripetal force can be provided by different forces (frictional force, tension, gravitational force, Coulomb force, or the normal force.....).	Student Book	264
	→ Solve problems related to centripetal force	Solved Problem 9.1	266
		Additional Exercises 9.76	283

We have mentioned page 264 above.



FIGURE 9.17 Modern roller coaster with a vertical loop.

SOLVED PROBLEM 9.1

Analysis of a Roller Coaster

Perhaps the biggest thrill to be had at an amusement park is on a roller coaster with a vertical loop in it (Figure 9.17), where passengers feel almost weightless at the top of the loop.

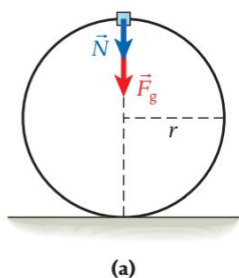
PROBLEM

Suppose the vertical loop has a radius of 5.00 m. What does the linear speed of the roller coaster have to be at the top of the loop for the passengers to feel weightless? (Assume that friction between roller coaster and rails can be neglected.)

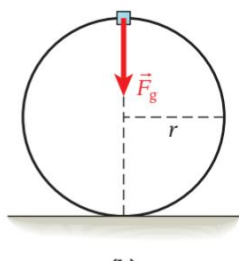
SOLUTION

THINK A person feels weightless when there is no supporting force, from a seat or a restraint, acting to counter his or her weight. For a person to feel weightless at the top of the loop, no normal force can be acting on him or her at this point.





(a)



SKETCH The free-body diagrams in Figure 9.18 may help to conceptualize the situation. The force of gravity and the normal force acting on a passenger in the roller coaster at the top of the loop are shown in Figure 9.18a. The sum of these two forces is the net force, which has to equal the centripetal force in circular motion. If the net force (centripetal force here) is equal to the gravitational force, then the normal force is zero, and the passenger feels weightless. This situation is illustrated in Figure 9.18b.

RESEARCH We have just stated that the net force is equal to the centripetal force and that the net force is the sum of the normal force and the force of gravity:

$$\vec{F}_c = \vec{F}_{\text{net}} = \vec{F}_g + \vec{N}.$$

For the feeling of weightlessness at the top of the loop, we need $\vec{N} = 0$, and thus

$$\vec{F}_c = \vec{F}_g \Rightarrow F_c = F_g. \quad (i)$$

As always, we have $F_g = mg$. For the magnitude of the centripetal force, we use equation 9.21:

$$F_c = ma_c = m \frac{v^2}{r}.$$

SIMPLIFY After substituting the expressions for the centripetal and the gravitational forces into equation (i), we solve for the linear speed at the top of the loop:

$$F_c = F_g \Rightarrow m \frac{v_{\text{top}}^2}{r} = mg \Rightarrow v_{\text{top}} = \sqrt{rg}.$$

CALCULATE Using $g = 9.81 \text{ m/s}^2$ and the value of 5.00 m given for the radius, we obtain

$$v_{\text{top}} = \sqrt{(5.00 \text{ m})(9.81 \text{ m/s}^2)} = 7.00357 \text{ m/s}.$$

10

Define angular acceleration as the rate of change of an object's angular velocity
Solve problems related to rotation with constant angular acceleration.

M.C.Q(9.8)
Q.(9.60,9.61)

278
282

9.8 The angular speed of the hour hand of a clock (in radians per second) is

- a) $\frac{\pi}{21,600}$ c) $\frac{\pi}{3600}$ e) $\frac{\pi}{60}$
b) $\frac{\pi}{7200}$ d) $\frac{\pi}{1800}$

Additional Exercises

9.60 A particular Ferris wheel takes riders in a vertical circle of radius 9.00 m once every 12.0 s.

- Calculate the speed of the riders, assuming it to be constant.
- Draw a free-body diagram for a rider at a time when she is at the bottom of the circle. Calculate the normal force exerted by the seat on the rider at that point in the ride.
- Perform the same analysis as in part (b) for a point at the top of the ride.

9.61 A boy is on a Ferris wheel, which takes him in a vertical circle of radius 9.00 m once every 12.0 s.

- What is the angular speed of the Ferris wheel?
- Suppose the wheel comes to a stop at a uniform rate during one quarter of a revolution. What is the angular acceleration of the wheel during this time?
- Calculate the tangential acceleration of the boy during the time interval described in part (b).

SOLVED PROBLEM 9.4 NASCAR Racing

As a NASCAR racer moves through a banked curve, the banking helps the driver achieve higher speeds. Let's see how. Figure 9.25 shows a race car on a banked curve.

PROBLEM

If the coefficient of static friction between the track surface and the car's tires is $\mu_s = 0.620$ and the radius of the turn is $R = 110$ m, what is the maximum speed with which a driver can take a curve banked at $\theta = 21.1^\circ$? (This is a fairly typical banking angle for NASCAR tracks. Indianapolis has only 9° banking, but there are some tracks with banking angles over 30° , including Daytona (31°), Talladega (33°), and Bristol (36°).)

SOLUTION

THINK The three forces acting on the race car are gravity, \vec{F}_g , the normal force, \vec{N} , and friction, \vec{f} . The curve is banked at an angle θ , which is also the angle between the normal to the track surface and the gravity force vector, as shown in Figure 9.26a. To draw the vector for the force of friction, we have assumed that the car has entered the curve at high speed, so the direction of the force of friction is down the incline. In contrast to the situation of static equilibrium, these three forces do not add up to zero, but instead add up to a net force, \vec{F}_{net} , as shown in Figure 9.26b. This net force has to provide the centripetal force, \vec{F}_c , which forces the car to move in a circle. Thus, the net force has to act in the horizontal direction because this is the direction of the center of the circle in which the car is moving.



FIGURE 9.25 Race car on a banked curve.

SKETCH The free-body diagram for the race car on the banked curve, showing the x - and y -components of the forces, is presented in Figure 9.26c. The orientation for the coordinate system was selected to give a horizontal x -axis and a vertical y -axis.

RESEARCH Like problems involving linear motion, we can solve problems involving circular motion by starting with the familiar Newton's Second Law: $\sum \vec{F} = m\vec{a}$. And just as in the linear case, we can generally work the problems in Cartesian components. From the free-body diagram in Figure 9.26c, we can see that the x -components of the forces acting on the race car are

$$N \sin \theta + f \cos \theta = F_{\text{net}}. \quad (\text{i})$$

Similarly, the forces acting in the y -direction are

$$N \cos \theta - F_g - f \sin \theta = 0. \quad (\text{ii})$$

As usual, the maximum friction force is given by the product of the coefficient of friction and the normal force: $f = \mu_s N$. The gravitational force is the product of mass and gravitational acceleration: $F_g = mg$.

The key to solving this problem is to realize that the net force has to be the force that causes the race car to move through the curve, that is, that provides the centripetal force. Therefore, using the expression for the centripetal force from equation 9.21, we have

$$F_{\text{net}} = F_c = m \frac{v^2}{R},$$

where R is the radius of the curve.

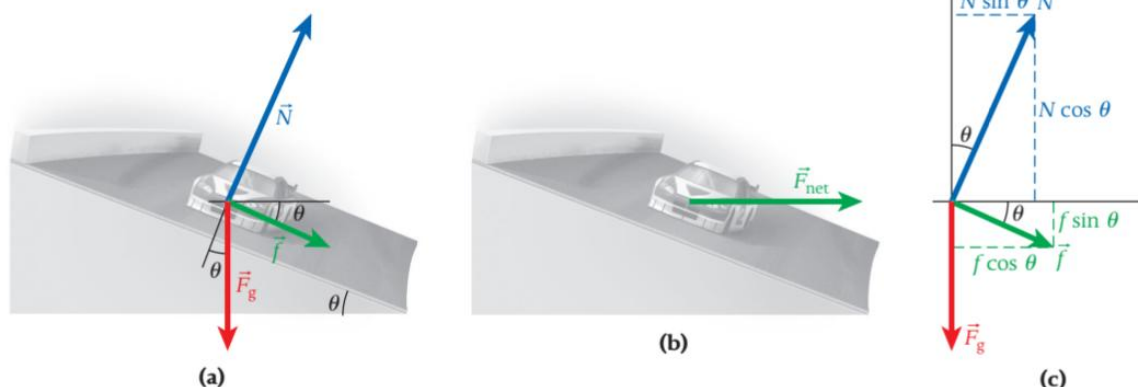


FIGURE 9.26 (a) Forces on a race car going around a banked curve on a racetrack. (b) The net force, the sum of the three forces in part (a). (c) A free-body diagram showing the x - and y -components of the forces acting on the car.

- Continued

SIMPLIFY We insert the expressions for the maximum friction force, the gravitational force, and the net force into equations (i) and (ii) for the x - and y -components of the forces:

$$N \sin \theta + \mu_s N \cos \theta = m \frac{v^2}{R} \Rightarrow N(\sin \theta + \mu_s \cos \theta) = m \frac{v^2}{R}$$

$$N \cos \theta - mg - \mu_s N \sin \theta = 0 \Rightarrow N(\cos \theta - \mu_s \sin \theta) = mg.$$

This is a system of two equations for two unknown quantities: the magnitude of the normal force, N , and the speed of the car, v . It is easy to eliminate N by dividing the first equation above by the second:

$$\frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} = \frac{v^2}{gR}.$$

We solve for v :

$$v = \sqrt{\frac{Rg(\sin \theta + \mu_s \cos \theta)}{\cos \theta - \mu_s \sin \theta}}. \quad (\text{iii})$$

Note that the mass of the car, m , canceled out. Thus, what matters in this situation is the coefficient of friction between the tires and the track surface, the radius of the turn, and the angle of banking.

CALCULATE Putting in the numbers, we obtain

$$v = \sqrt{\frac{(110. \text{ m})(9.81 \text{ m/s}^2)[\sin 21.1^\circ + 0.620(\cos 21.1^\circ)]}{\cos 21.1^\circ - 0.620(\sin 21.1^\circ)}} = 37.7726 \text{ m/s}.$$

ROUND Expressing our result to three significant figures gives us

$$v = 37.8 \text{ m/s}.$$

●●9.59 A speedway turn, with radius of curvature R , is banked at an angle θ above the horizontal.

- a) What is the optimal speed at which to take the turn if the track's surface is iced over (that is, if there is very little friction between the tires and the track)?
- b) If the track surface is ice-free and there is a coefficient of friction μ_s between the tires and the track, what are the maximum and minimum speeds at which this turn can be taken?
- c) Evaluate the results of parts (a) and (b) for $R = 400. \text{ m}$, $\theta = 45.0^\circ$, and $\mu_s = 0.700$.



الفصل العاشر

12

Describe that the moment of inertia plays the same role for rotational (or circular) motion as the mass does for linear motion.

Student Book
Solved Problem 10.1

285~286
294

10.1 Kinetic Energy of Rotation

In Chapter 8, we saw that we can describe the motion of an extended object in terms of the path that its center of mass follows and the rotation of the object around its center of mass. However, even though we covered circular motion for point particles in Chapter 9, we have not yet considered the rotation of extended objects. Analyzing this motion is the purpose of this chapter.

Point Particle in Circular Motion

Chapter 9 introduced the kinematical quantities of circular motion. Angular speed, ω , and angular acceleration, α , were defined in terms of the time derivatives of the angular displacement, θ :

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

We saw that the angular quantities are related to the linear quantities as follows:

$$s = r\theta, \quad v = r\omega,$$

$$a_t = r\alpha, \quad a_c = \omega^2 r, \quad a = \sqrt{a_c^2 + a_t^2},$$

where s is the arc length, v is the linear speed of the center of mass, a_t is the tangential acceleration, a_c is the centripetal acceleration, and a is the linear acceleration.

The most straightforward way to introduce the physical quantities for the description of rotation is through the kinetic energy of rotation of an extended object. In Chapter 5 on work and energy, the kinetic energy of a moving object was defined as

$$K = \frac{1}{2}mv^2. \quad (10.1)$$

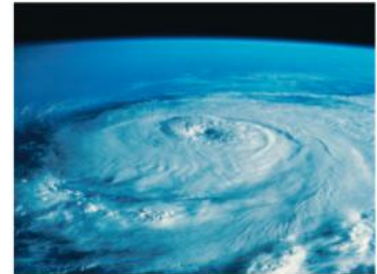
If the motion of this object is circular, we can use the relationship between linear and angular velocity to obtain

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}mr^2\omega^2, \quad (10.2)$$

which is the **kinetic energy of rotation** for a point particle's motion on the circumference of a circle of radius r about a fixed axis, as illustrated in Figure 10.3.

Several Point Particles in Circular Motion

Just as we proceeded in Chapter 8 in finding the location of the center of mass of a system of particles, we start with a collection of individual rotating objects and then



(a)



(b)

FIGURE 10.2 (a) Rotating air mass forming a hurricane. (b) Rotation on an enormous scale—spiral galaxy M74.

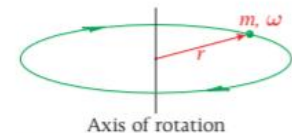


FIGURE 10.3 A point particle moving in a circle about the axis of rotation.

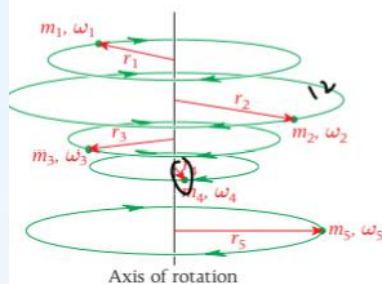
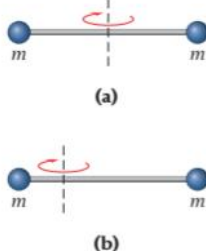


FIGURE 10.4 Five point particles moving in circles about a common axis of rotation.

Concept Check 10.1

Consider two equal masses, m , connected by a thin, massless rod. As shown in the figures, the two masses spin in a horizontal plane around a vertical axis represented by the dashed line. Which system has the highest moment of inertia?



approach the continuous limit. The kinetic energy of a collection of rotating objects is given by

$$K = \sum_{i=1}^n K_i = \frac{1}{2} \sum_{i=1}^n m_i v_i^2 = \frac{1}{2} \sum_{i=1}^n m_i r_i^2 \omega_i^2.$$

This result is simply a consequence of using equation 10.2 for several point particles and writing the total kinetic energy as the sum of the individual kinetic energies. Here ω_i is the angular velocity of particle i and r_i is the perpendicular distance from i to a fixed axis. This fixed axis is the **axis of rotation** for these particles. An example of a system of five rotating point particles is shown in Figure 10.4.

Now we assume that all of the point particles whose kinetic energies we have summed keep their distances with respect to one another and with respect to the axis of rotation fixed. Then, all of the point particles in the system will undergo circular motion around the common axis of rotation with the same angular velocity. With this constraint, the sum of the particles' kinetic energies becomes

$$K = \frac{1}{2} \sum_{i=1}^n m_i r_i^2 \omega^2 = \frac{1}{2} \left(\sum_{i=1}^n m_i r_i^2 \right) \omega^2 = \frac{1}{2} I \omega^2. \quad (10.3)$$

The quantity I introduced in equation 10.3 is called the **moment of inertia**, also known as the *rotational inertia*. It depends only on the masses of the individual particles and their distances to the axis of rotation:

$$I = \sum_{i=1}^n m_i r_i^2. \quad (10.4)$$

In Chapter 9, we saw that all quantities associated with circular motion have equivalents in linear motion. The angular velocity ω and the linear velocity v form such a pair. Comparing the expressions for the kinetic energy of rotation (equation 10.3) and the kinetic energy of linear motion (equation 10.1), we see that the moment of inertia I plays the same role for circular motion as the mass m does for linear motion.

SOLVED PROBLEM 10.1 Sphere Rolling Down an Inclined Plane

PROBLEM

A solid sphere with a mass of 5.15 kg and a radius of 0.340 m starts from rest at a height of 2.10 m above the base of an inclined plane and rolls down without sliding under the influence of gravity. What is the linear speed of the center of mass of the sphere just as it leaves the incline and rolls onto a horizontal surface?

SOLUTION

THINK At the top of the incline, the sphere is at rest. At that point, the sphere has gravitational potential energy and no kinetic energy. As the sphere starts to roll, it loses potential energy and gains kinetic energy of linear motion and kinetic energy of rotation. At the bottom of the inclined plane, all of the original potential energy is in the form of kinetic energy. The kinetic energy of linear motion is linked to the kinetic energy of rotation through the radius of the sphere.

SKETCH A sketch of the problem situation is shown in Figure 10.12 with the zero of the y -coordinate at the bottom of the incline.

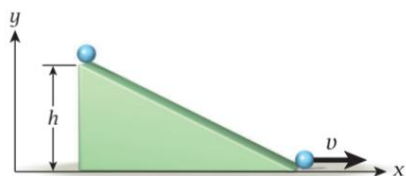


FIGURE 10.12 Sphere rolling down an inclined plane.

RESEARCH At the top of the incline, the sphere is at rest and has zero kinetic energy. At the top, its energy is therefore its potential energy, mgh :

$$E_{\text{top}} = K_{\text{top}} + U_{\text{top}} = 0 + mgh = mgh,$$

where m is the mass of the sphere, h is the height of the sphere above the horizontal surface, and g is the acceleration due to gravity. At the bottom of the incline, just as the sphere starts rolling onto the horizontal surface, the potential energy is zero. According to equation 10.14, the sphere has a total kinetic energy (sum of translational and rotational kinetic energy) of $(1 + c)\frac{1}{2}mv^2$. Thus, the total energy at the bottom of the incline is

$$E_{\text{bottom}} = K_{\text{bottom}} + U_{\text{bottom}} = (1 + c)\frac{1}{2}mv^2 + 0 = (1 + c)\frac{1}{2}mv^2.$$

Because the moment of inertia of a sphere is $I = \frac{2}{5}mR^2$ (see equation 10.10), the constant c has the value $\frac{2}{5}$ in this case.

SIMPLIFY Because of the conservation of energy, the energy at the top of the incline is equal to that at the bottom:

$$mgh = (1 + c)\frac{1}{2}mv^2.$$

Solving for the linear velocity gives us

$$v = \sqrt{\frac{2gh}{1 + c}}.$$

For a sphere, $c = \frac{2}{5}$, as noted above, and so the speed of the rolling object is in this case

$$v = \sqrt{\frac{2gh}{1 + \frac{2}{5}}} = \sqrt{\frac{10}{7}gh}.$$

CALCULATE Putting in the numerical values gives us

$$v = \sqrt{\frac{10}{7}(9.81 \text{ m/s}^2)(2.10 \text{ m})} = 5.42494 \text{ m/s}.$$

ROUND Expressing our result with three significant figures leads to

$$v = 5.42 \text{ m/s}.$$

10.4 Torque

So far, in discussing forces, we have seen that a force can cause linear motion of an object, which can be described in terms of the motion of the center of mass of the object. However, we have not addressed one general question: Where are the force vectors acting on an extended object placed in a free-body diagram? A force can be exerted on an extended object at a point away from its center of mass, which can cause the object to rotate as well as move linearly.

Moment Arm

Consider the hand attempting to use a wrench to loosen a bolt in Figure 10.16. It is clear that it will be easiest to turn the bolt in Figure 10.16c, quite a bit harder in Figure 10.16b,

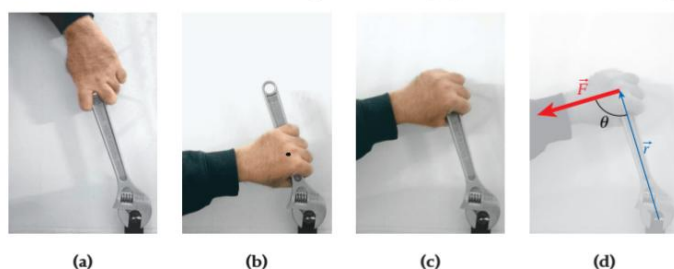


FIGURE 10.16 (a)–(c) Three ways to use a wrench to loosen a bolt. (d) The force \vec{F} and moment arm \vec{r} , with the angle θ between them.

and downright impossible in Figure 10.16a. This example shows that the magnitude of the force is not the only relevant quantity. The perpendicular distance from the line of action of the force to the axis of rotation, called the **moment arm**, is also important. In addition, the angle at which the force is applied, relative to the moment arm, matters as well. In parts (b) and (c) of Figure 10.16, this angle is 90° . (An angle of 270° would be just as effective, but then the force would act in the opposite direction.) An angle of 180° or 0° (Figure 10.16a) will not turn the bolt.

These considerations are quantified by the concept of torque, τ . **Torque** (also called *moment*) is the vector product of the force \vec{F} and the position vector \vec{r}

$$\vec{\tau} = \vec{r} \times \vec{F}. \quad (10.16)$$

The position vector \vec{r} is measured with the origin at the axis of rotation. The symbol \times denotes the **vector product**, or *cross product*. (We introduced vector products in Chapter 1, which you may want to consult for a review.)

The SI unit of torque is N m, not to be confused with the unit of energy, which is the joule ($\text{J} = \text{N m}$)

$$[\tau] = [F] \cdot [r] = \text{N m}.$$

In English units, torque is often expressed in foot-pounds (ft-lb).

The magnitude of the torque is the product of the magnitude of the force and the distance to the axis of rotation (the magnitude of the position vector, or the moment arm) times the sine of the angle between the force vector and the position vector (see Figure 10.17):

$$\tau = rF \sin \theta. \quad (10.17)$$

Angular quantities can also be vectors, called *axial vectors*. (An axial vector is any vector that points along the rotation axis.) Torque is an example of an axial vector, and its magnitude is given by equation 10.17. The direction of the torque is given by a right-hand rule (Figure 10.17). Note that right-hand rules apply to all vector products! The torque points in a direction perpendicular to the plane spanned by the force and position vectors. Thus, if the position vector points along the thumb and the force vector points along the index finger, then the direction of the axial torque vector is the direction of the middle finger, as shown in Figure 10.17. Note that the torque vector is perpendicular to both the force vector and the position vector.

With the mathematical definition of the torque, its magnitude, and its relation to the force vector, the position vector, and their relative angle, we can understand why the approach shown in part (c) of Figure 10.16 yields maximum torque for a given magnitude of the force, whereas that in part (a) yields zero torque. We see that the magnitude of the torque is the decisive factor in determining how easy or hard it is to loosen (or tighten) a bolt.

Torques around any fixed axis of rotation can be clockwise or counterclockwise. As indicated by the force vector in Figure 10.16d, the torque generated by the hand pulling on the wrench would be counterclockwise. The **net torque** is defined as the difference between the sum of all clockwise torques and the sum of all counterclockwise torques:

$$\tau_{\text{net}} = \sum_i \tau_{\text{counterclockwise},i} - \sum_j \tau_{\text{clockwise},j}$$

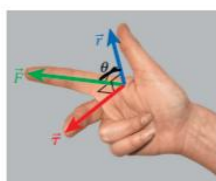
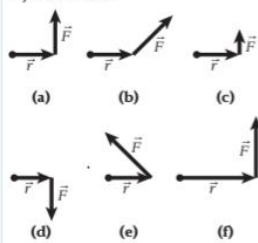


FIGURE 10.17 Right-hand rule for the direction of the torque for a given force and position vector.

Concept Check 10.4

Choose the combination of position vector, \vec{r} , and force vector, \vec{F} , that produces the torque of highest magnitude around the point indicated by the black dot.

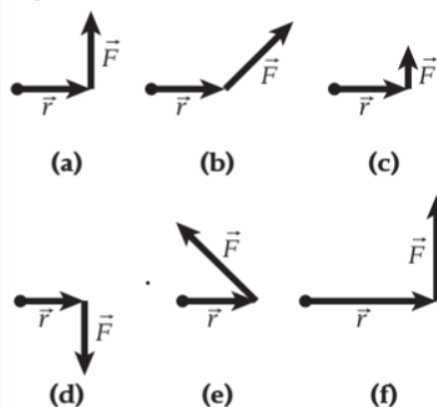


14	Describe that a torque ($\vec{\tau}$) on a body involves a force (\vec{F}) and a position vector (\vec{r}), which extends from a rotation axis to the point where the force is applied.	Student Book Concept Check 10.4	297~298 298
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Pages 297 and 298 are in the previous page.

Concept Check 10.4

Choose the combination of position vector, \vec{r} , and force vector, \vec{F} , that produces the torque of highest magnitude around the point indicated by the black dot.



15	Calculate the torque due to a force on a particle by taking the cross product of the particle's position vector and the force vector. $\vec{\tau} = \vec{r} \times \vec{F} \quad , , \quad \tau = rF\sin(\theta)$	Student Book Q.(10.47 / 10.48) Q.(10.49/a)	297~298 318 319
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Pages 297 and 298 are in the previous page.

so both the linear and rotational motion stop when the object reaches its maximum height.

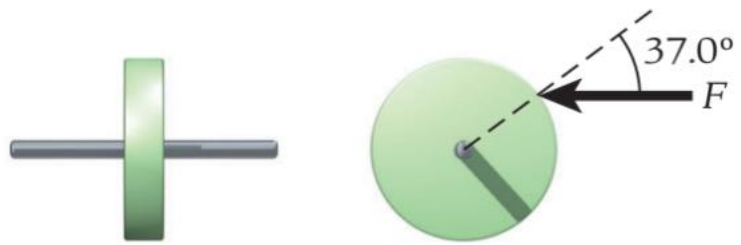
Section 10.4

•**10.47** A disk with a mass of 30.0 kg and a radius of 40.0 cm is mounted on a frictionless horizontal axle. A string is wound many times around the disk and then attached to a 70.0 kg block, as shown in the figure. Find the acceleration of the block, assuming that the string does not slip.



●●**10.48** A force, $\vec{F} = (2\hat{x} + 3\hat{y})$ N, is applied to an object at a point whose position vector with respect to the pivot point is $\vec{r} = (4\hat{x} + 4\hat{y} + 4\hat{z})$ m. Calculate the torque created by the force about that pivot point.

●●**10.49** A disk with a mass of 14.0 kg, a diameter of 30.0 cm, and a thickness of 8.00 cm is mounted on a rough horizontal axle as shown



on the left in the figure. (There is a friction force between the axle and the disk.) The disk is initially at rest. A constant force, $F = 70.0$ N, is applied to the edge of the disk at an angle of 37.0° , as shown on the right in the figure. After 2.00 s, the force is reduced to $F = 24.0$ N, and the disk spins with a constant angular velocity.

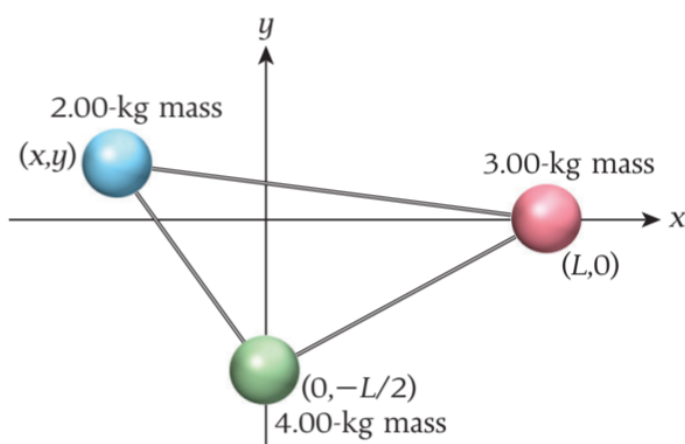
a) What is the magnitude of the torque due to friction between the disk and the axle?

القسم الكتابي (Written Part)

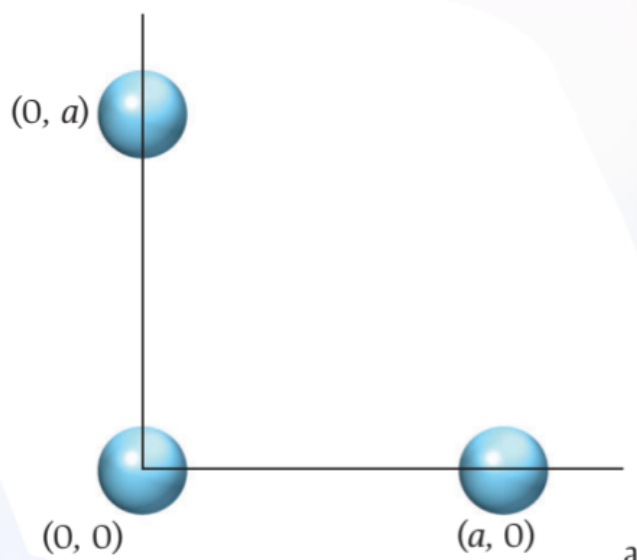
16	1 st Part	<ul style="list-style-type: none"> Determine the location of the center of mass of two or several particles or extended objects with uniform mass distribution (the object can be divided into simple geometric figures, each of which can be replaced by a particle at its center) by applying suitable mathematical equations. 	Exercises Q. (8.30, 8.58)	249, 252
	2 nd Part	<ul style="list-style-type: none"> Convert angle measurements between degrees and radians. Relate the arc length (s), to the radius (r) of the circular path and the angle (θ), measured in radians, by ($s=r\theta$), and solve problems. 	Student Book Exercises Q. (9.31)	256-257 280

Part 1:

●8.30 The coordinates of the center of mass for the extended object shown in the figure are $(L/4, -L/5)$. What are the coordinates of the 2.00-kg mass?



8.58 Three identical balls of mass m are placed in the configuration shown in the figure. Find the location of the center of mass.



Part 2:

We took page 257 in the MCQ part.

Arc Length

Figure 9.3 also shows (in green) the path on the circumference of the circle traveled by the tip of the vector \vec{r} in going from an angle of zero to θ . This path is called the *arc length*, s . It is related to the radius and angle via

$$s = r\theta. \quad (9.7)$$

For this relationship to work out numerically, the angle has to be measured in radians. The fact that the circumference of a circle is $2\pi r$ is a special case of equation 9.7 with $\theta = 2\pi$ rad, corresponding to one full turn around the circle. The arc length has the same unit as the radius.

For small angles, measuring a degree or less, the sine of the angle is approximately (to four significant figures) equal to the angle measured in radians. Because of this fact as well as the need to use equation 9.7 to solve problems, the preferred unit for angular coordinates is the radian. But the use of degrees is common, and this book uses both units.

Self-Test Opportunity 9.1

Use polar coordinates and calculus to show that the circumference of a circle with radius R is $2\pi R$.

Self-Test Opportunity 9.2

Use polar coordinates and calculus to show that the area of a circle with radius R is πR^2 .

9.31 What is the angle in radians that the Earth sweeps out in its orbit during winter?

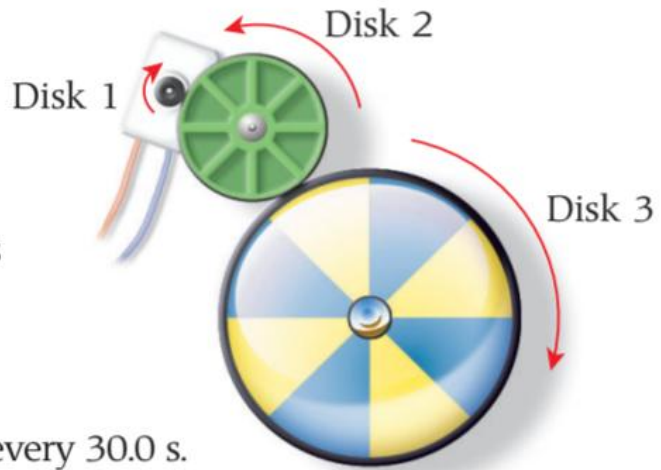
17	1 st Part	<ul style="list-style-type: none"> Relate the magnitudes of linear (tangential) and angular velocities for circular motion as $v = r\omega$, and explain that this relation does not hold for tangential and angular velocity vectors which point in different directions. Solve problems related to angular velocity, angular frequency and period. 	Q.9.44(c,d)	281
	2 nd Part	<ul style="list-style-type: none"> Apply the kinematic relationships for circular motion with constant angular acceleration to calculate angular position, angular displacement, angular velocity, angular acceleration, or time. 	Q.9.44(a) Q.9.45(d) Q.9.64	281 282

9.44 A discus thrower (with arm length of 1.20 m) starts from rest and begins to rotate counterclockwise with an angular acceleration of 2.50 rad/s^2 .

- How long does it take the discus thrower's speed to get to 4.70 rad/s ?
- How many revolutions does the thrower make to reach the speed of 4.70 rad/s ?
- What is the linear speed of the discus at 4.70 rad/s ?
- What is the linear acceleration of the discus thrower at this point?



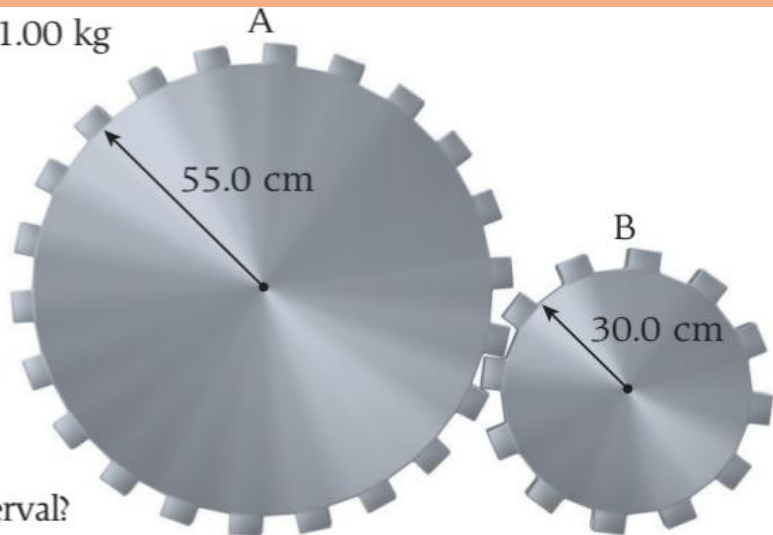
●**9.45** In a department store toy display, a small disk (disk 1) of radius 0.100 m is driven by a motor and turns a larger disk (disk 2) of radius 0.500 m. Disk 2, in turn, drives disk 3, whose radius is 1.00 m. The three disks are in contact, and there is no slipping. Disk 3 is observed to sweep through one complete revolution every 30.0 s.



- What is the angular speed of disk 3?
- What is the ratio of the tangential velocities of the rims of the three disks?
- What are the angular speeds of disks 1 and 2?
- If the motor malfunctions, resulting in an angular acceleration of 0.100 rad/s^2 for disk 1, what are disks 2 and 3's angular accelerations?

ملاحظة: فقط الطلب d مطلوب ولكنه مرتبط بما قبله.

9.64 Gear A, with a mass of 1.00 kg and a radius of 55.0 cm, is in contact with gear B, with a mass of 0.500 kg and a radius of 30.0 cm. The gears do not slip with respect to each other as they rotate. Gear A rotates at 120. rpm and slows to 60.0 rpm in 3.00 s. How many rotations does gear B undergo during this time interval?



18	1 st Part	<p>↪ Relate the magnitude of the centripetal force to the centripetal acceleration by applying Newton's Second Law in the radial direction as: $F_c = ma_c = mv\omega = mr\omega^2 = m\frac{v^2}{r}$ and Solve problems related to centripetal force.</p>	Solved Problem 9.1 Q.9.76	266 283
	2 nd Part	<p>↪ Solve problems related to rotation with constant angular acceleration.</p> $\theta = \theta_0 + \omega_0 t + \frac{1}{2}at^2 \quad \omega = \omega_0 + at \quad \bar{\omega} = \frac{1}{2}(\omega + \omega_0)$ $\omega^2 = \omega_0^2 + 2a(\theta - \theta_0)$	Q.9.35 Q.9.63/9.67	280 282

📌 ملاحظة: المسألة 9.1 Solved Problem 9.1 وضعنا أعلاه في الأسئلة الاختيارية في الهدف 12

9.63 A car accelerates uniformly from rest and reaches a speed of 22.0 m/s in 9.00 s. The diameter of a tire on this car is 58.0 cm.

a) Find the number of revolutions the tire makes during the car's motion, assuming that no slipping occurs.

● **9.76** A ball that has a mass of 1.00 kg is attached to a string 1.00 m long and is whirled in a vertical circle at a constant speed of 10.0 m/s.

a) Determine the tension in the string when the ball is at the top of the circle.

b) Determine the tension in the string when the ball is at the bottom of the circle.

c) Consider the ball at some point other than the top or bottom. What can you say about the tension in the string at this point?

19	<p>↪ Calculate the moment of inertia of a point particle or a group of several point particles rotating about an axis of rotation. $I = mr^2$, , $I = \sum_{i=1}^n m_i r_i^2$</p> <p>↪ Calculate the rotational kinetic energy of a point particle, or several point particles, rotating about a fixed axis of rotation by applying the expression for the rotational kinetic energy in terms of the rotational inertia and angular speed.</p> $K_{Rot} = \frac{1}{2} \sum_{i=1}^n m_i r_i^2 \omega_i^2 = \frac{1}{2} I \omega^2$	Exercises Q. (10.38, 10.39)	318
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10.38 A uniform solid cylinder of mass $M = 5.00$ kg is rolling without slipping along a horizontal surface. The velocity of its center of mass is 30.0 m/s. Calculate its energy.

10.39 Determine the moment of inertia for three teenagers weighing 60.0 kg, 45.0 kg, and 80.0 kg sitting at different points on the edge of a rotating stage, which has a radius of 12.0 m.