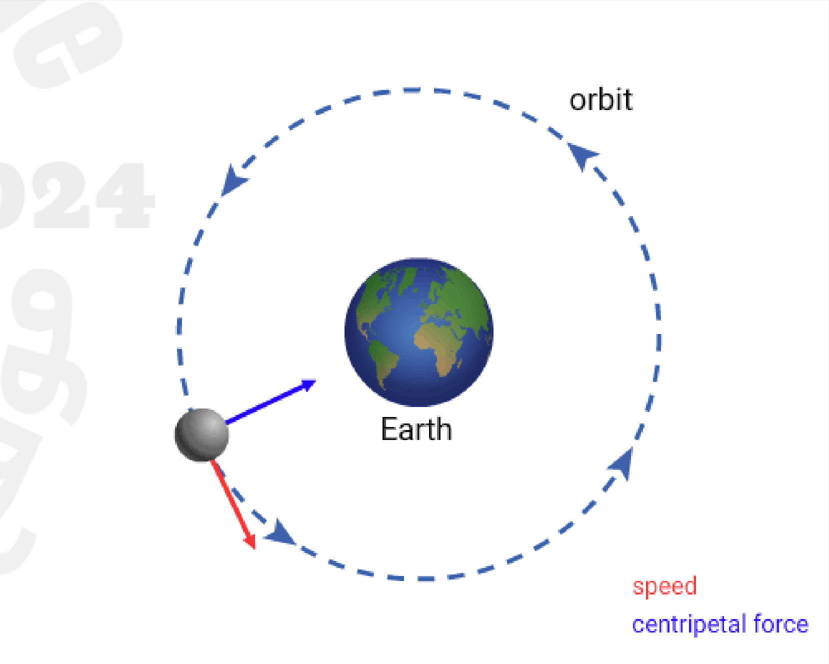
Unit 9: Circular motion

Section 9.3

Angular velocity, Angular frequency, and Period



Learning Objectives

Section 9.3

Angular velocity,

Angular frequency,

and Period

By the end of this section, you will be able to:

1) Define and calculate angular velocity of an object in circular motion.

2) Define and calculate angular frequency and period of an object in circular motion.

3) Describe the relationship between angular velocity and linear velocity.

Angular velocity

Angular frequency

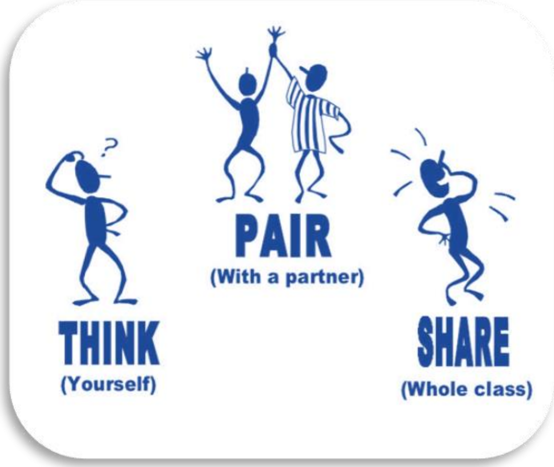
Period

linear velocity.

WARM UP

Discussion

How can you find the average linear velocity?



$$\bar{v} = \frac{\Delta x}{\Delta t}$$

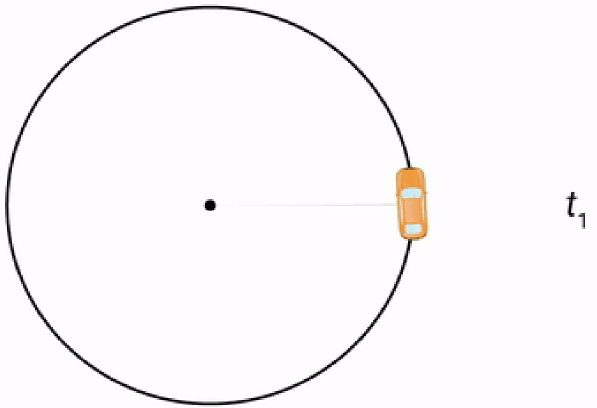
L.O: Define and calculate angular velocity of an object in circular motion.

Angular velocity of an object

Angular velocity measures how fast the angle θ changes in time.

- We have seen that the change of an object's linear coordinates in time is its velocity.
- Similarly, the change of an object's angular coordinate in time is its **angular velocity.**
- The average magnitude of the angular velocity is defined as:

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

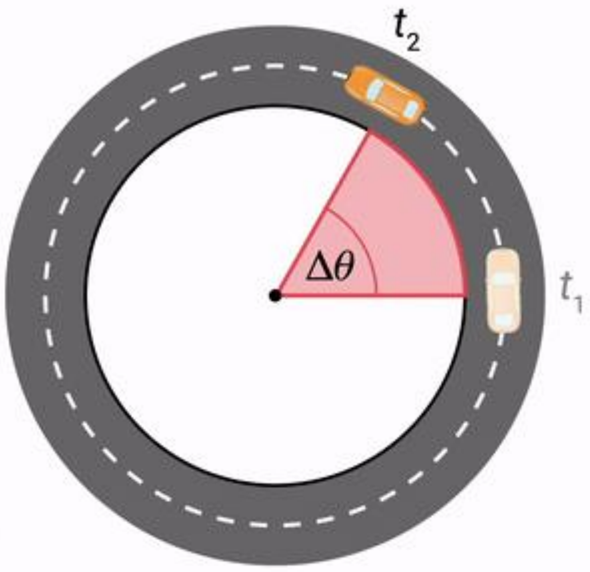


L.O: Define and calculate angular velocity of an object in circular motion.

By taking the limit of average angular velocity as the time interval approaches zero, we find the instantaneous value of the magnitude of the angular velocity:

$$\omega = \lim_{\Delta t \to 0} \bar{\omega} = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \equiv \frac{d\theta}{dt}$$

$$\omega = \frac{d\theta}{dt}$$



$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

L.O: Define and calculate angular velocity of an object in circular motion.

The most common unit of angular velocity is radians per second (rad/s).

Degrees per second is not generally used.

L.O: Define and calculate angular velocity of an object in circular motion.

Example:

A computer fan completes one rotation every 55 ms.
What is the magnitude of the fan's angular velocity?

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$\bar{\omega} = \frac{2\pi}{55 \times 10^{-3}}$$

$$\bar{\omega} = 114.2 \text{ rad/s}$$

L.O: Define and calculate angular velocity of an object in circular motion.

- **Angular velocity is a vector quantity.**

- **Its direction is that of an axis through the center of the circular path and perpendicular to the plane of the circle. (This axis is a rotation axis).**

- **This definition allows for two possibilities for the direction in which the vector $\overrightarrow{\omega}$ can point:**

  parallel or antiparallel to the axis of rotation.

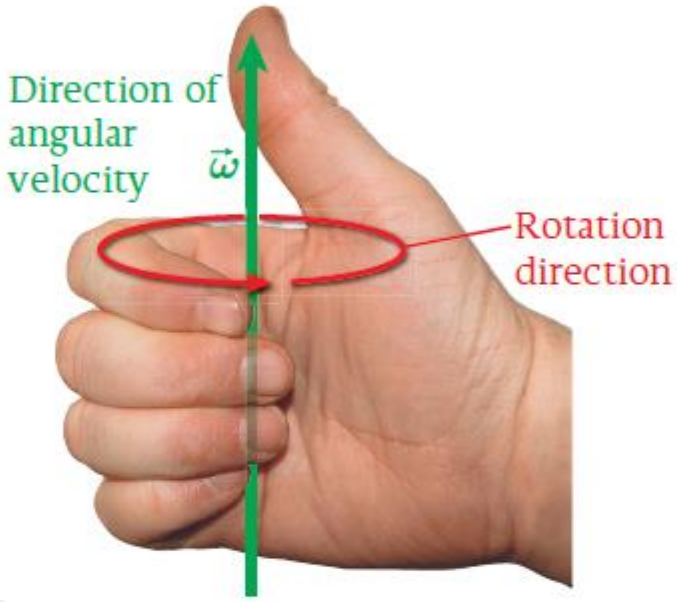


FIGURE 9.8 The right-hand rule for determining the direction of the angular velocity vector.

L.O: Define and calculate angular velocity of an object in circular motion.

We use the right-hand rule to determine the direction of the angular velocity.

When the fingers point in the direction of rotation along the circle's circumference, the thumb points in the direction of $\overrightarrow{\omega}$, as shown in Figure 9.8.

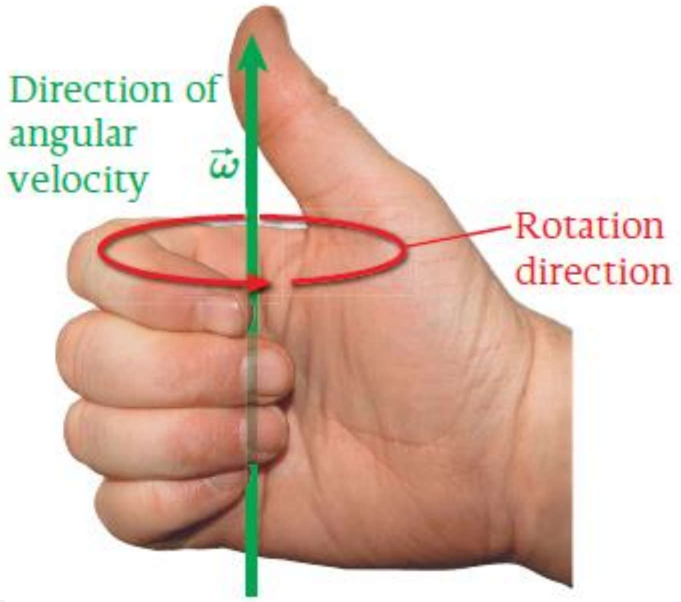


FIGURE 9.8 The right-hand rule for determining the direction of the angular velocity vector.

L.O: Define and calculate angular velocity of an object in circular motion.

Check your understanding:

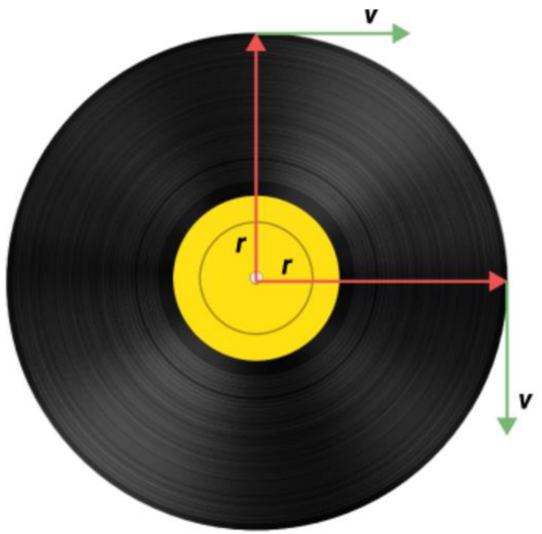
Khalifa is listening to music on a vinyl record. If the record is rotating clockwise, what is the direction of the angular velocity?

❑ The angular velocity is clockwise.

❑ The angular velocity is counterclockwise.

❑ The angular velocity is oriented into the screen.

❑ The angular velocity is out of the screen.



Angular velocity

Angular frequency

Period

linear velocity.

L.O: Define and calculate angular frequency and period of an object in circular motion.

Angular velocity measures how fast the angle θ changes in time.
Another quantity also specifies how fast this angle changes in time—the frequency f.

the frequency, f, measures cycles per unit time, instead of radians per unit time as the angular velocity does.
The frequency is related to the magnitude of the angular velocity, ω, by:

$$f = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f$$

ω Must be in *rad/s* to calculate frequency.

L.O: Define and calculate angular frequency and period of an object in circular motion.

For example, the rpm number on the tachometer in your car indicates how many times per minute the engine cycles and thus specifies the frequency of engine revolution.

FIGURE 9.9 The tachometer of a car measures the frequency (in cycles per minute) of revolutions of the engine.

L.O: Define and calculate angular frequency and period of an object in circular motion.

What is the unit used to measure angular frequency?

- Frequency is measured in hertz (Hz).
- 1 Hz = 1 s^{-1}
- Because the unit inverse second is used so widely, it was given its own name, the hertz (Hz), for the physicist Hertz.

L.O: Define and calculate angular frequency and period of an object in circular motion.

- The period of rotation (T) is the inverse of the frequency:

$$T = \frac{1}{f}$$

- Period measures the time interval between two successive instances where the angle has the same value (the time taken to pass once around the circle).
- The unit of the period is the same as that of time, which is the second (s).

WS # 6: (Exercise 1)

Complete the table with the appropriate information.

Angular velocity	measures how fast the angle θ changes in time.
Frequency	measures cycles per unit time.
Frequency	measured in hertz (Hz).
Angular velocity	measured in rad/s.
Period	the inverse of the frequency.
Period	measures the time interval between two successive instances where the angle has the same value.
Period	the time taken to pass once around the circle.
second	The unit of the period .

WS # 6: (Exercise 2)

Write the formulas needed to calculate each of the following :

Average angular velocity	Instantaneous angular velocity	Frequency	Period
$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$	$\omega = \frac{d\theta}{dt}$	$f = \frac{\omega}{2\pi}$	$T = \frac{1}{f}$

anglar disp.

time derivative of angular disp.

should be in rad/s

$T = \frac{2\pi}{\omega}$

$1\ rev. = 2\pi\ rad$

WS # 6: (Exercise 3)

If a wheel turning at a constant rate completes 100 revolutions in 10 s, what is its angular velocity?

Δt

$\Delta\theta = 100 \times 2\pi$

$\omega = ?$

A. $0.63\ rad/s$

B. $10\ rad/s$

C. $31\ rad/s$

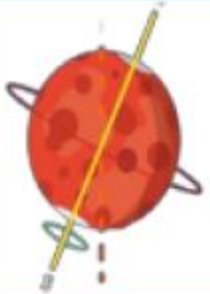
D. $63\ rad/s$

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$\bar{\omega} = \frac{100 \times 2\pi}{10}$$

$$\bar{\omega} = 62.8\ rad/s$$

WS # 6: (Exercise 4)



يدور المريخ حول محوره الذي يمتد من القطب إلى القطب بسرعة زاوية $7.1\times10^{-5}\ rad/s$
ما الزمن الذي يحتاجه كوكب المريخ لإكمال دورة واحدة؟

The planet Mars rotates on its pole-to-pole axis, with angular velocity $7.1\times10^{-5}\ rad/s$.
What is the period of rotation Mars needed to complete one rotation?

ω

$T = ?$

$\Delta\theta = 2\pi$ rad

24.0 h　　36.8 h　　12.0 h　　24.6 h

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$7.1\times10^{-5} = \frac{2\pi}{\Delta t}$$

$$\Delta t = 88495.6\ s$$

convert to h

$$\Delta t = \frac{88495.6}{3600} = 24.6\ \text{h}$$

OR

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{7.1\times10^{-5}}$$

WS # 6: (Exercise 5)

FINAL EXAM

Hz

$\omega = ?$

A disc rotates at a frequency of (1.3 s^{-1}), what is the angular velocity of the disc?

f

$$f = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f$$

$$\omega = 2\pi \times 1.3$$

$$\omega = 8.2\ rad/s$$

WS # 6: (Exercise 6)

If a wheel is turning at 3.0 rad/s, what is the time it takes to complete one revolution?

A. 0.67 s

B. 1.0 s

C. 1.3 s

D. 2.1 s

$\Delta\theta = 2\pi$ rad

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$3 = \frac{2\pi}{\Delta t}$$

$\Delta t = 2.1\,s$

WS # 6: (Exercise 7)

. One revolution per minute is ______.

A. 0.0524 rad/s

B. 0.105 rad/s

C. 0.950 rad/s

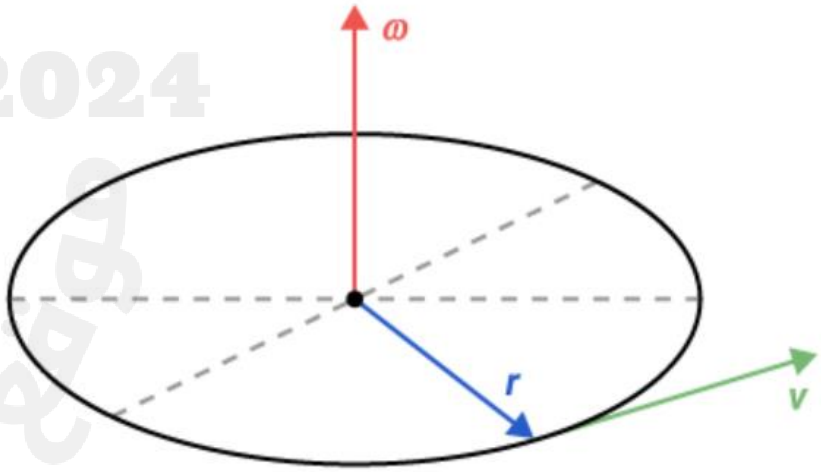
D. 1.57 rad/s

$$\frac{1\,rev.}{min.} \times \frac{2\pi}{60\,s}$$

$$= 0.105\ rad/s$$

WS # 6: (Exercise 8)

Angular velocity points up If the object moves counterclockwise.
If the object moves clockwise, the angular velocity points …. down …….



WS # 6: (Exercise 9)

A disk with a diameter of (0.5 m) started rotating from rest, and the angular displacement of the disk changed with time according to the function ($\theta = 6t^2 + 12$). What is the angular velocity of the disk after (4.0 s) of starting the rotation?

- ❑ 60 rad/s
- ❑ 48 rad/s
- ❑ 96 rad/s
- ❑ 108 rad/s

$$\boldsymbol{\omega = \frac{d\theta}{dt}}$$

$$\boldsymbol{\omega = 12t}$$

$$\boldsymbol{\omega(4) = 12 \times 4}$$

$$\boldsymbol{\omega(4) = 48 rad/s}$$

Learning Objectives

Section 9.3

Angular velocity,

Angular frequency,

and Period

1) Describe the relationship between angular velocity and linear velocity.

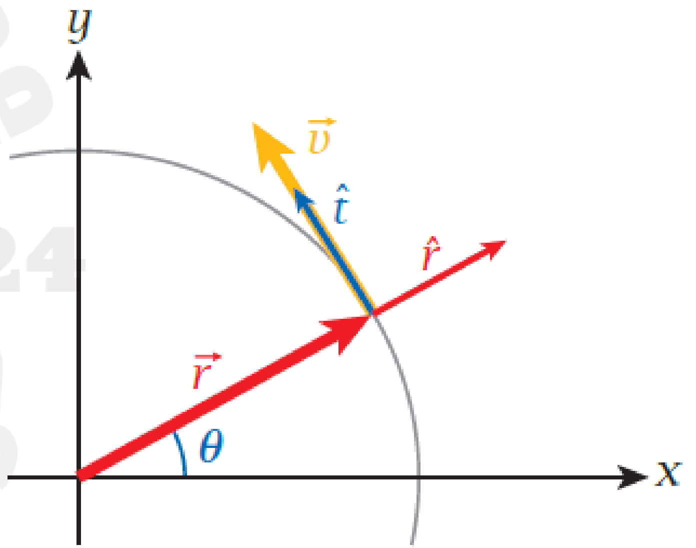
2) Solve problems to find the linear and angular velocity.

L.O: Describe the relationship between angular velocity and linear velocity.

Angular velocity and linear velocity

we have the relationship between angular and linear velocities for circular motion:

$$\boxed{v = r\omega\,\hat{t}}$$



(Again, $\hat{t}$ is the symbol for the tangential unit vector and has no connection with the time, t!)

L.O: Describe the relationship between angular velocity and linear velocity.

The relationship between angular and linear velocity of circular motion:

$$v = r\omega\,\hat{t}$$

note:

The symbol $\hat{t}$ represents the tangential unit vector and has no relation to time t.

The linear velocity vector is always perpendicular to the position vector.

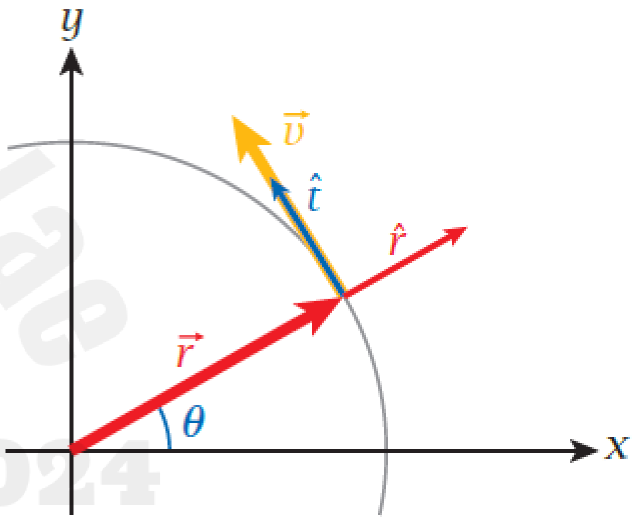


FIGURE 9.10 Linear velocity and coordinate vectors.

L.O: Describe the relationship between angular velocity and linear velocity.

$$v = r\omega$$

This relationship applies only to linear and angular velocity magnitudes.

But their vectors point in different directions.

Remember that this relationship holds only for the *magnitudes* of the linear and angular velocities. Their vectors point in different directions and, for uniform circular motion, are perpendicular to each other, with $\vec{\omega}$ pointing in the direction of the rotation axis and $\vec{v}$ tangential to the circle.

L.O: Describe the relationship between angular velocity and linear velocity.

WS # 7: (Exercise 1)

A bicycle's wheels have a radius R.
The bicycle is traveling with speed v.
Which one of the following expressions describes the angular speed of the front tire?

a) $\omega = \frac{1}{2}Rv^2$

b) $\omega = \frac{1}{2}vR^2$

c) $\omega = R/v$

d) $\omega = Rv$

e) $\omega = v/R$

Answer:

e

L.O: Solve problems to find the linear and angular velocity.

WS # 7: (Exercise 2)

A bicycle's wheels have a radius of 33.0 cm. The bicycle is traveling at a speed of 6.5 m/s. What is the angular speed of the front tire?

A. $0.197\ rad/s$

B. $1.24\ rad/s$

C. $5.08\ rad/s$

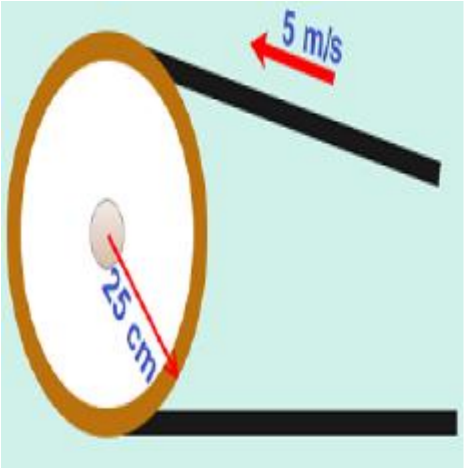
√ D. $19.7\ rad/s$

$v = r\omega$

L.O: Solve problems to find the linear and angular velocity.

WS # 7: (Exercise 3)

FINAL EXAM



يمر حزام على دولاب نصف قطره 25 cm. إذا كانت سرعة نقطة على الحزام 5 m/s .
ما السرعة الزاوية للحزام؟

A belt passes over a wheel of radius 25 cm. if a point on the belt has a speed of 5 m/s, what is the angular velocity of the belt?

$20\ rad/s$ Comes out/للخارج	$20\ rad/s$ Comes in/للداخل	$2.5\ rad/s$ Comes out/للخارج

$$v = r\omega$$

$$5 = 0.25 \times \omega$$

$$\omega = 20\ rad/s$$ *Comes* out

L.O: Solve problems to find the linear and angular velocity.

WS # 7: (Exercise 4)

FINAL EXAM

السرعة الزاوية كمية متجهة، أي مما يلي صحيح بالنسبة لاتجاهها؟

Angular velocity is a vector quantity, which of the following is correct about its direction?

Parallel to the radius of rotation.	موازية لنصف قطر مسار الدوران.
Along the axis of rotation.	على طول محور الدوران.
Perpendicular to the axis of rotation.	عمودياً على محور الدوران.
Tangential to the circular path.	مماسيا للمسار الدائري.

L.O: Solve problems to find the linear and angular velocity.

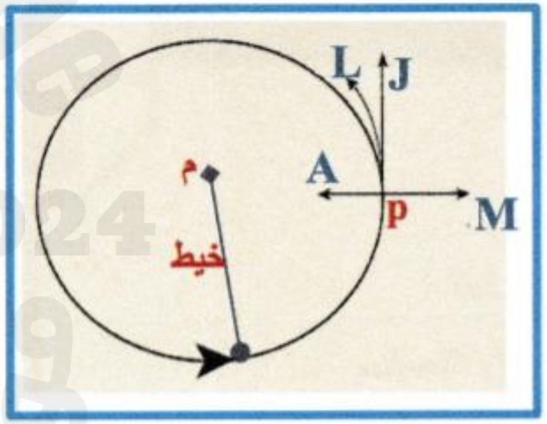
Write the relationship between angular velocity and linear velocity.

$$v = r\omega$$

L.O: Solve problems to find the linear and angular velocity.

WS # 7: (Exercise 5)

An object tied to a string moves in uniform circular motion in a smooth horizontal plane as shown in the adjacent figure. If the string is cut at point P , in what direction will the object move at the moment the string is cut?



- ☐ A
- ☑ J
- ☐ L
- ☐ M

L.O: Solve problems to find the linear and angular velocity.

WS # 7: (Exercise 6)

FINAL EXAM

If the linear displacement of a body moving in circular motion as a function of time is given by the relation ($s = 5t^2 + 7$)

Which of the following represents angular velocity?

ω

- ❑ $10t$
- ❑ $\frac{10\,r}{t}$
- ❑ $\frac{10\,t}{r}$
- ❑ $\frac{5t^2+7}{r}$

$v = r\omega$

$\omega = \frac{v}{r}$

$v = \frac{ds}{dt}$

$v = 10t$

$\omega = \frac{10t}{r}$

Example 3 page 260: WS # 8

The Earth orbits around the Sun and also rotates on its pole-to-pole axis.

Find each of the following:

If you know that the distance between the Earth and the Sun is 1.49×10^{11} m .

	Earth's rotation around its axis	Earth's rotation around the sun
1) Period in s ?	$T_E = 24\ h$ $T_E = 24 \times 3600$ $T_E = 8.64 \times 10^4 s$	$T_s = 365\ day$ $T_s = 365 \times 24 \times 3600$ $T_E = 3.15 \times 10^7 s$
2) Frequency?	$f = \frac{1}{T} = \frac{1}{8.64 \times 10^4} = 1.16 \times 10^{-5} Hz$	$f = \frac{1}{T} = \frac{1}{3.15 \times 10^7} = 3.17 \times 10^{-8} Hz$
3) Angular velocity?	$\omega = 2\pi\ f$ $\omega = 2\pi\ \times 1.16 \times 10^{-5}$ $\omega = 7.3 \times 10^{-5} \mathrm{rad/s}$	$\omega = 2\pi\ f$ $\omega = 2\pi\ \times 3.17 \times 10^{-8}$ $\omega = 1.99 \times 10^{-7}\ \mathrm{rad/s}$
4) Linear velocity? $v = r\omega$ $v = 6380 \times 10^3 \times \cos\vartheta \times 7.3 \times 10^{-5}$ $v = 466 \cos\vartheta$	If you know that At the Equator, the radius of rotation is r= R_{earth} = 6380 km. Away from the Equator, the radius of rotation as a function of the latitude angle is r= $R_{earth} \cos\vartheta$ ϑUsed to express latitude angles.	$v = r\omega$ $v = 1.49 \times 10\ 11 \times 1.99 \times 10^{-7}$ $v = 2.97 \times 10^4 m/s$

FIGURE 9.11 Earth's rotation axis is indicated by the vertical line. Points at different latitudes on the Earth's surface move at different speeds.

4) Linear velocity? (Earth's rotation around its axis)

If you know that At the Equator, the radius of rotation is r= R_{earth} = 6380 km.

Away from the Equator, the radius of rotation as a function of the latitude angle is r= $R_{earth} \cos\vartheta$

(ϑ Used to express latitude angles).

$$v = r\omega$$
$$v = 6380 \times 10^3 \times \cos\vartheta \times 7.3 \times 10^{-5}$$
$$v = 466 \cos\vartheta$$

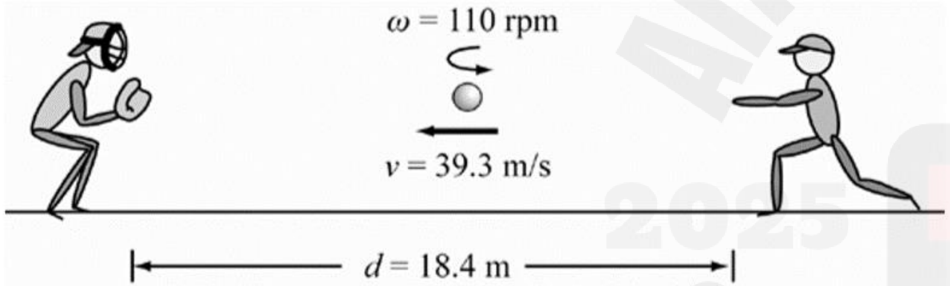
Question 34 / page 280:

9.34 A baseball is thrown at approximately 141.6 kph and with a spin rate of 110. rpm. If the distance between the pitcher's point of release and the catcher's glove is 18.5 m, how many full turns does the ball make between release and catch? Neglect any effect of gravity or air resistance on the ball's flight.

v

How many revolutions?

4



$$v = \frac{141.6 \times 10^3}{3600} = 39.3\ m/s$$

$$w = 110 \times \frac{2\pi}{60} = 11.5\ rad/s$$

$$w = \frac{\Delta\theta}{\Delta t} \Rightarrow \Delta\theta = w\,\Delta t$$

$$\Delta\theta = 11.5 \times 0.468 = 5.38\ rad$$

$$v = \frac{d}{\Delta t}$$

$$39.3 = \frac{18.4}{\Delta t} \Rightarrow \Delta t = 0.468\,s$$

Answer:

0.85 rev

convert to revolutions

$$\frac{5.38}{2\pi} = 0.86\ rev.$$

End of the section