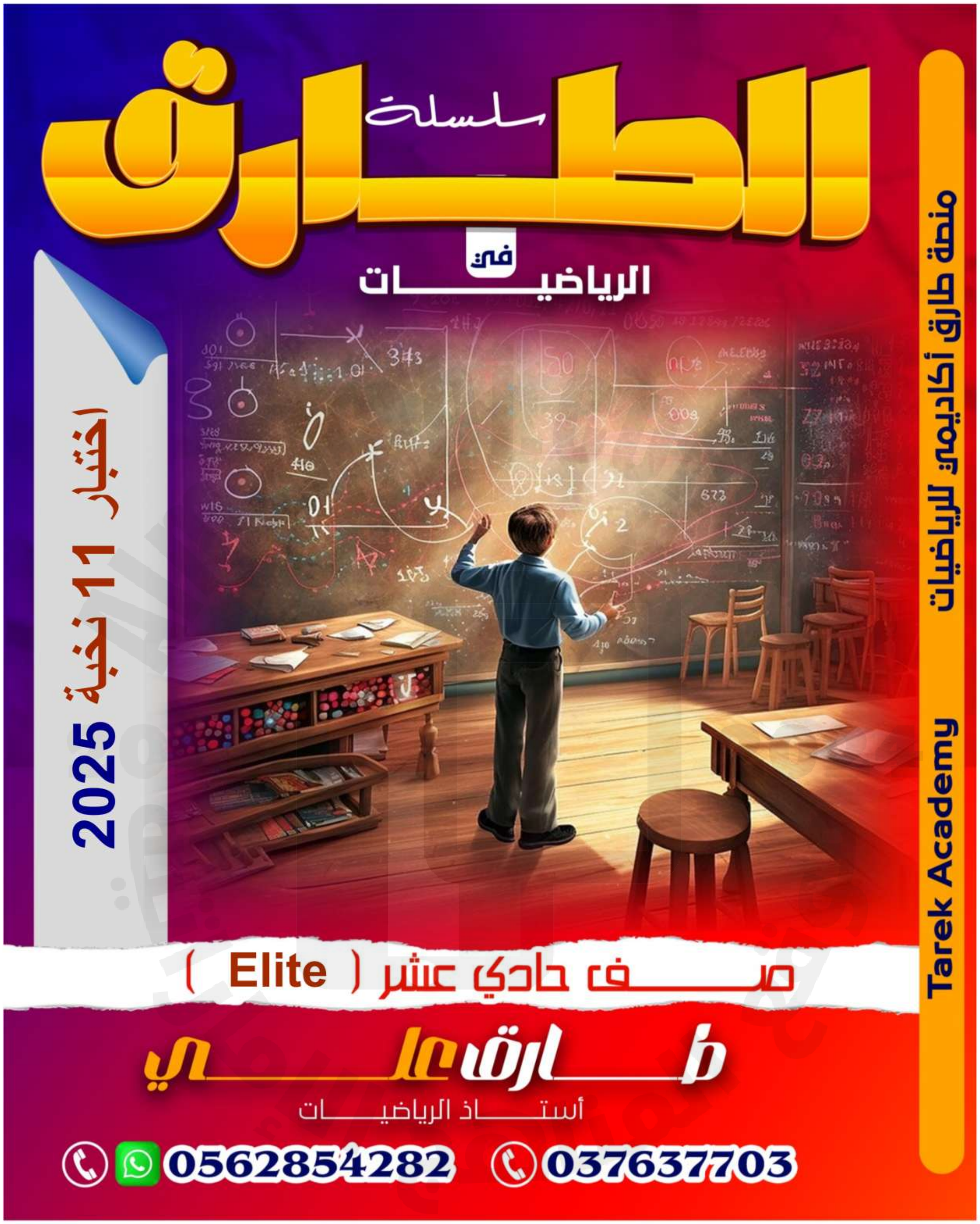
حل بالخطوات أسئلة امتحان نهائي سابق القسم الالكتروني المسار النخبة


سلسلة
الطارق
في الرياضيات
اختبار 11 نخبة 2025
منصة طارق أكاديمي للرياضيات
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Test 2024

Question 1:

$$\lim_{x \to -4} \frac{x+4}{x^3 - 16x} \text{ is}$$

( ) Calc -4.01 = = $\simeq$

- a) 0
- ~~b)~~ $\frac{1}{32}$
- c) 1
- d) nonexistent

Question 2:

The function f is defined as:

$$f(x) = \begin{cases} \frac{\sin(3x)}{6x} & \text{for } x \neq 0 \\ c & \text{for } x = 0 \end{cases}$$

For what value of c is f continuous at $x = 0$?

- a) 0
- ~~b)~~ $\frac{1}{2}$
- c) 1
- d) 2

$\lim_{x \to 0} \frac{\sin 3x}{6x} = c$

$\frac{3}{6} = c$

$c = \frac{1}{2}$

Question 3:

Let f be the function defined by $f(x) = \frac{3x^3+2x^2}{x^2-x}$. Which of the following statements is true?

- a) f has a discontinuity due to a vertical asymptote at $x = 0$ and at $x = 1$.
- b) f has a removable discontinuity at $x = 0$ and a jump discontinuity at $x = 1$.
- c) f has a removable discontinuity at $x = 0$ and a discontinuity due to a vertical asymptote at $x = 1$. ✓
- d) f is continuous at $x = 0$, and f has a discontinuity due to a vertical asymptote at $x = 1$.

$$\frac{\cancel{x}^{2}(3x+2)}{\cancel{x}(x-1)} \rightarrow \text{V.A} \quad \text{deno} = 0$$

$$\frac{x(3x+2)}{x-1}$$

$x-1=0 \rightarrow \boxed{x=1}$

hole $\leftarrow$ x 2-zip

removable

$\boxed{x=0}$

Question 4:

The table below shows selected values of a continuous function f:

x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4

For $0 \le x \le 13$, what is the fewest possible number of times $f(x) = 4$?

- a) one
- b) two
- c) three
- d) four

$\boxed{3 \rightarrow 4.5}$

$\boxed{4.5 \rightarrow 3}$

$\boxed{2.5 \rightarrow 4.4}$

Question 5:

Let g and h be the functions defined by $g(x) = -2x^2 + 4x + 1$ and $h(x) = \frac{1}{2}x^2 - x + \frac{11}{2}$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x\to1} f(x)$?

- a) 3
- b) 4
- c) 5
- d) The limit cannot be determined from the information given.

$$\lim_{x\to1} -2x^2+4x+1 \leq \lim_{x\to1} f(x) \leq \lim_{x\to1} \frac{1}{2}x^2 - x + \frac{11}{2}$$

$$-2(1)^2+4(1)+1 \leq \lim_{x\to1} f(x) \leq \frac{1}{2}(1)^2 - 1 + \frac{11}{2}$$

$$3 \leq \lim_{x\to1} f(x) \leq 5$$

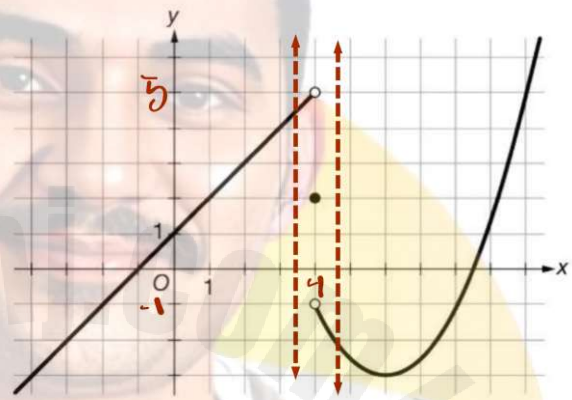
$$3 \neq 5$$

$$\boxed{\text{DNE}}$$

Question 6:

The graph of the function h is shown above. What is $\lim_{x\to 4} h(x)$?

- a) -1
- b) 2
- c) 5
- ~~d)~~ nonexistent



Graph of h

$* \lim_{x\to 4^+} h(x) = -1$

$* \lim_{x\to 4^-} h(x) = 5$

$-1 \neq 5$

DNE

Question 7:

The function g is continuous at all x except $x = 4$. If $\lim_{x\to4} g(x) = \infty$, which of the following statements about g must be true?

- a) $g(4) = \infty$
- b) The line $x = 4$ is a horizontal asymptote to the graph of g.
- c) The line $x = 4$ is a vertical asymptote to the graph of g.
- d) The line $y = 4$ is a vertical asymptote to the graph of g.

$* \lim_{x\to\pm\infty} \to$ H.A

$y =$

$* \lim_{x\to \text{عدد}} = \infty$

V.A

$x =$ عدد

Question 8:

Selected values of a function g are shown in the table below:

x	-3	-1	1	3
$g(x)$	12	0	1	-4

What is the average rate of change of g over the interval $[-3, 3]$?

- a) $\frac{3-(-3)}{(-4)-12}$
- b) $\frac{(-4)-12}{3-(-3)}$
- c) $\frac{12+(-4)}{2}$
- d) $\frac{12+0+1+(-4)}{4}$

$$\frac{g(3)-g(-3)}{3--3}$$

$$\frac{-4-12}{3--3}$$

Question 9:

Let f be the function defined by:

$$f(x) = \begin{cases} x^2 - 20 & \text{for } x < 5 \\ -x^2 + 20 & \text{for } x \geq 5 \end{cases}$$

Which of the following statements is true?

- a) f is not differentiable at $x = 5$ because f is not continuous at $x = 5$.
- b) f is not differentiable at $x = 5$ because the graph of f has a sharp corner at $x = 5$.
- c) f is not differentiable at $x = 5$ because the graph of f has a vertical tangent at $x = 5$.
- d) f is not differentiable at $x = 5$ because f is not defined at $x = 5$.

$* \lim_{x \to 5^-} x^2 - 20 = 25 - 20 = 5$

$* \lim_{x \to 5^+} -x^2 + 20 = -5^2 + 20 = -5$

$\lim_{x \to 5} f(x)$ DNE

not Continuous.

Question 10:

Let f be the function given by $f(x) = 5x^3 - 3x - 7$. What is the value of $f'(-2)$?

- a) -114
- b) 17
- c) 50
- d) 57

* df! Shift → $\frac{d}{dx}(\quad)_{x=-2}$

$\boxed{\text{or}}$

$f'(-2) = 15x^2 - 3 = 15(-2)^2 - 3$
$= 57$

Question 11:

What is the value of $\lim_{h \to 0} \frac{(8+h)^{\frac{1}{3}} - 2}{h}$?

- a) 0
- b) $\frac{1}{12}$
- c) $\frac{1}{4}$
- d) 1

$(\quad)$ Calc → 0.01 = = ≃

0.0833

$= \frac{1}{12}$

Question 12:

If $f(x) = \ln x . \cos x$, then

product rule

$f'(x) =$

- a) $\frac{-\sin x}{x}$
- b) $\frac{\sin x}{x}$
- c) $\frac{\cos x}{x} - \ln x \sin x$
- d) $\frac{\cos x}{x} + \ln x \sin x$

$\ln x \cdot \cos x$

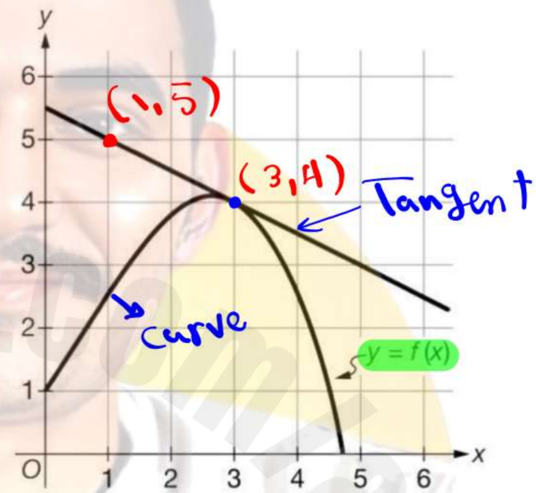
$+$

$\frac{1}{x}$ $-\sin x$

$$f'(x) = \frac{\cos x}{x} - \ln x \sin x$$

Question 13:

The graph of the differentiable function f is shown above, along with the line tangent to the graph of f at $x = 3$. What is the value of $f'(3)$? = slope



- a) $-\frac{1}{2}$
- b) -2
- c) 4
- d) $\frac{11}{2}$

$$\frac{y_2 - y_1}{x_2 - x_1}$$

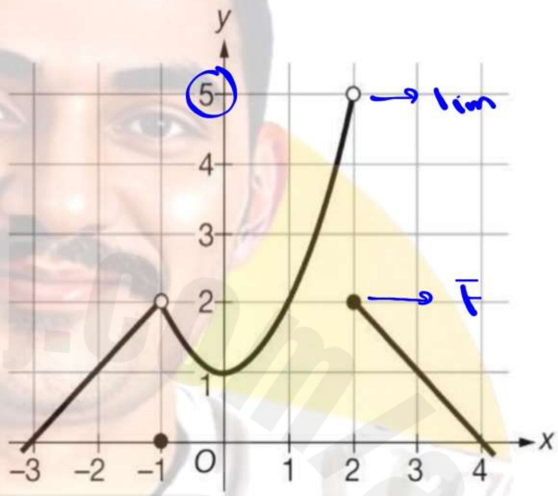
$$\frac{5 - 4}{1 - 3}$$

$$= \boxed{\frac{1}{-2}}$$

Question 14:

The graph of the function f is shown above. What is $\lim_{x\to-1} f(f(x))$?

- a) 1
- b) 2
- c) 5
- d) nonexistent



Graph of f

F → closed

→ open

$$\lim_{x\to -1} f(f(-1)) = \lim_{x\to -1} f(2)$$

$$= 5$$

Question 15:

Let f be the function defined by $f(x) = \frac{1-5x-2x^2}{3x^2+7}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f?

H. A

* 3 Types

- a) $y = -\frac{2}{3}$
- b) $y = \frac{1}{3}$
- c) $y = \frac{2}{3}$
- d) There is no horizontal asymptote to the graph of f.

① Power num = Power deno

y = divide leading = $\frac{-2}{3}$

② Power num less always

y = 0

③ power num more

No H.A

or $\pm\infty$

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

The graphs of the functions f and g are shown above on the interval $0 \leq x \leq 5$.

(a) Write a difference quotient that best approximates the instantaneous rate of change of g at $x = 2.5$.

(b) Let h be the function defined by $h(x) = g(f(x))$. Find the value of $\lim_{x \to 3} h(x)$ or explain why the limit does not exist. Use correct limit notation in your answer.

(c) Let k be the function defined by $k(x) = (4 - f(x))g(x)$. Consider $x = 2$ and $x = 4$. Determine whether k is continuous at each of these values. Justify your answers using correct limit notation.

(d) Values of $f(x)$ at selected values of x are given in the table below. What type of discontinuity does f have at $x = 1$? Based on the values in the table, what is a reasonable estimate for $\lim_{x \to 1} f(x)$? Give a reason for your answer.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	2.634	2.770	2.784	Undefined	2.787	2.801	2.946

(a) $\dfrac{g(3)-g(2)}{3-2} = \dfrac{1-4}{3-2} = \boxed{-3}$ $(3,1)$ $(2,4)$

(b) $\lim\limits_{x\to 3^-} h(x) = \lim\limits_{x\to 3^-} g(f(x))$

$\lim\limits_{x\to 2^+} g(x) = \boxed{4}$

$f(3)=2$

Right

$\lim\limits_{x\to 3^+} h(x) = \lim\limits_{x\to 3^+} g(f(x))$

$= \lim\limits_{x\to 2^+} g(x) = \boxed{4}$

$\lim\limits_{x\to 3} g(f(x)) = \boxed{4}$

(c) $* \lim\limits_{x\to 2^-} (4-f(x))\,g(x) = \left(4-\lim\limits_{x\to 2^-} f(x)\right)\cdot \lim\limits_{x\to 2^-} g(x)$

$=(4-4)(-1)=0$

$* \lim\limits_{x\to 2^+} (4-f(x))\,g(x) = \left(4-\lim\limits_{x\to 2^+} f(x)\right)\cdot \lim\limits_{x\to 2^+} g(x)$

$=(4-4)(4)=0$

$\lim\limits_{x\to 2} (4-f(x))(g(x)) = \boxed{0}$

* $k(2) = (4 - f(2))\, g(2) = (4-4)(4) = 0$

$k(x)$ is Continuous at

$\boxed{x = 2}$

$$\lim_{x\to 4^-} k(x) = \lim_{x\to 4^-} (4 - f(x))g(x) = \left(4 - \lim_{x\to 4^-} f(x)\right) \lim_{x\to 4^-} g(x) = (4-3)(1) = 1$$

$$\lim_{x\to 4+} k(x) = \lim_{x\to 4^+} (4 - f(x))g(x) = \left(4 - \lim_{x\to 4^+} f(x)\right) \lim_{x\to 4^+} g(x) = (4-3)(4) = 4$$

Because $\lim_{x\to 4^-} k(x) \neq \lim_{x\to 4^+} k(x)$,

$\lim_{x\to 4} k(x)$ does not exist and therefore k is not continuous at $x = 4$.

(d) Values of $f(x)$ at selected values of x are given in the table below. What type of discontinuity does f have at $x = 1$? Based on the values in the table, what is a reasonable estimate for $\lim_{x\to 1} f(x)$? Give a reason for your answer.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	2.634	2.770	2.784	Undefined	2.787	2.801	2.946

removable.

* estimate $\lim_{x\to 1} f(x) = \dfrac{f(0.999) + f(1.001)}{2}$ average

$= \dfrac{2.784 + 2.787}{2} = 2.7855$

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Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$$f(x) = \begin{cases} \frac{x^2+kx-2}{3x^2+4x+3} & \text{for } x < -2 \quad L \\ x^3+2 & \text{for } -2 \le x < 0 \quad R \\ 0 & \text{for } x = 0 \\ \frac{2e^x}{2-e^x} & \text{for } x > 0 \end{cases}$$

Let f be the function defined above, where k is a constant.

(a) For what value of k, if any, is f continuous at $x = -2$? Justify your answer.

(b) What type of discontinuity does f have at $x = 0$? Give a reason for your answer.

(c) Find all horizontal asymptotes to the graph of f. Show the work that leads to your answer.

(a) $\lim\limits_{x \to -2^-} \frac{(-2)^2 - 2k - 2}{3(-2)^2 + 4(-2) + 3} = \frac{2-2k}{7}$

$\lim\limits_{x \to -2^+} (-2)^3 + 2 = -6$

$\rightarrow \frac{2-2k}{7} = -6 \rightarrow 2 - 2k = -42$

$2 - -42 = 2k$

$42 = 2k \rightarrow k = \frac{42}{2} = \boxed{21}$

(b)

$$f(x)=\begin{cases}\frac{x^2+kx-2}{3x^2+4x+3} & \text{for } x<-2 \quad L\\ x^3+2 & \text{for } -2\le x<0 \quad R\\ 0 & \text{for } x=0\\ \frac{2e^x}{2-e^x} & \text{for } x>0\end{cases}$$

$* \lim_{x\to 0^-} 0^3+2=\boxed{2}$ $\quad * \lim_{x\to 0^+} \frac{2e^0}{2-e^0}=\frac{2}{2-1}=\boxed{2}$

$\lim_{x\to 0} f(x)=2$

$f(0)=0$

$\lim_{x\to 0} \neq f(0)$ removable.

(c)

$$f(x)=\begin{cases}\frac{x^2+kx-2}{3x^2+4x+3} & \text{for } x<-2 \quad L\\ x^3+2 & \text{for } -2\le x<0 \quad R\\ 0 & \text{for } x=0\\ \frac{2e^x}{2-e^x} & \text{for } x>0\end{cases}$$

H.A fraction

$x>0$

$* \lim_{x\to\infty} \frac{2e^x}{2-e^x}=\lim_{x\to\infty}\frac{2\cancel{e^x}}{-\cancel{e^x}}=\boxed{-2}$

$* \lim_{x\to-\infty} \frac{x^2+kx-2}{3x^2+4x+3}=\lim_{x\to-\infty}\frac{x^2}{3x^2}=\boxed{\frac{1}{3}}$

H.A are

$y=-2$, $y=\frac{1}{3}$

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Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$$R(t) = \begin{cases} 41\sqrt{\frac{t}{20}} & \text{for } 0 \le t < 20 \\ \frac{100t^2+1000}{t^2+600} & \text{for } t \ge 20 \end{cases}$$

* The total accumulated revenue a company has received up to time t is modeled by the function R defined above, where $R(t)$ is measured in millions of United States dollars and t is the time in years since 1980.

(a) Find $\lim_{t\to\infty} R(t)$. Explain the meaning of $\lim_{t\to\infty} R(t)$ in the context of the problem.

more

(b) Is the function R continuous at $t = 20$? Justify your answer.

(c) The company's total accumulated expenses up to time t is modeled by the function E defined by $E(t) = 3\log_2(12t+4)$, where $E(t)$ is measured in millions of United States dollars and t is the time in years since 1980. According to these models, is there a time t, for $0 \le t \le 5$, at which the total accumulated profit, $R(t) - E(t)$, is equal to 0 ? Justify your answer.

(a) $\lim_{t\to\infty} \frac{100t^2+1000}{t^2+600} = \lim_{t\to\infty} \frac{100t^2}{t^2} = \boxed{100}$

As time increasing revenue approaches 100 million

(b) $\lim_{t\to 20^+} \frac{100t^2+1000}{t^2+600} = \frac{100(20)^2+1000}{20^2+600} = 41$

$\lim_{t\to 20^-} 41\sqrt{\frac{t}{20}} = 41\sqrt{\frac{20}{20}} = 41$

$$R(20) = \frac{100(20)^2 + 1000}{(20)^2 + 600} = 41$$

$$\lim_{t \to 20} R(t) = R(20) \rightarrow \text{continuous at } t = 20$$

(c) $R(t) - E(t) \quad 0 \le t \le 5$

$$*\, R(0) - E(0) = 41\sqrt{\frac{0}{20}} - 3\log_2(12(0)+4)$$
$$= \boxed{-6} < 0$$

$$*\, R(5) - E(5) = 41\sqrt{\frac{5}{20}} - 3\log_2(12(5)+4)$$
$$= \boxed{2.5} > 0$$

$$R(0) - E(0) < 0 < R(5) - E(5)$$

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$$f(t) = \begin{cases} 48t + t^2 - \frac{t^3}{12} & \text{for } 0 \le t < 6 \\ g(t) & \text{for } 6 \le t \le 12 \end{cases}$$

t (hours)	6	8	10	12
$g(t)$ (cubic meters)	$\boxed{306}$	376	428	$\boxed{474}$

At an excavation site, the amount of dirt that has been removed, in cubic meters, is modeled by the function f defined above, where g is a differentiable function and t is measured in hours. Values of $g(t)$ at selected values of t are given in the table above.

(a) According to the model f, what is the average rate of change of the amount of dirt removed over the time interval $6 \le t \le 12$ hours?

(b) Use the data in the table to approximate $f'(9)$, the instantaneous rate of change in the amount of dirt removed, in cubic meters per hour, at time $t = 9$ hours. Show the computations that lead to your answer.

(c) Is f continuous for $0 \le t \le 12$? Justify your answer.

(d) Find $f'(2)$, the instantaneous rate of change in the amount of dirt removed, in cubic meters per hour, at time $t = 2$ hours.

(a) $\dfrac{f(12) - f(6)}{12 - 6} = \dfrac{474 - 306}{12 - 6} = 28$

$8 \leftarrow 9 \rightarrow 10$

(b) $f'(9) = \text{Slope} = \dfrac{g(10) - g(8)}{10 - 8} = \dfrac{428 - 376}{2}$
$= \boxed{26}$

(c)

$$f(t) = \begin{cases} 48t + t^2 - \frac{t^3}{12} & \text{for } 0 \le t < 6 \\ g(t) & \text{for } 6 \le t \le 12 \end{cases}$$

t (hours)	6	8	10	12
$g(t)$ (cubic meters)	306	376	428	474

$\lim_{t \to 6^-} 48(6) + 6^2 - \frac{6^3}{12} = 306$

$\lim_{t \to 6^+} g(6) = 306$

$f(6) = g(6) = 306$

$\lim_{t \to 6} f(t) = f(6) = 306$

Continuous at $t = 6$

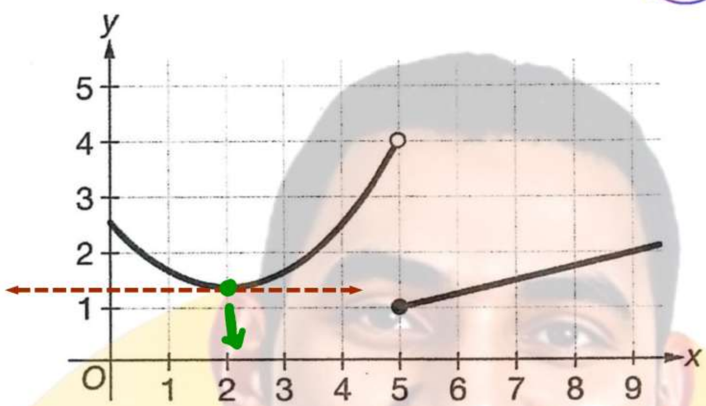
Then Continuous $0 \le t \le 12$

(d) $f'(2) = 48 + 2t - \frac{\cancel{3}t^2}{\cancel{12}} = 48 + 2t - \frac{t^2}{4}$

$= 48 + 2(2) - \frac{2^2}{4}$

$= \boxed{51}$

H.T



Graph of f

8 9 10

x	3	3.5	4	$\boxed{4.5}$	5	5.5	6
$f(x)$	1.653	2.02	2.533	3.193	1	1.125	1.25

The graph of the function f and a table of selected values of $f(x)$ are shown above. The graph of f has a horizontal tangent line at $x = 2$, is concave up for $0 < x < 5$, and is linear for $x \geq 5$.

A. Approximate the value of $f'(4.5)$ using data from the table. Show the computations that lead to your answer.

$$f'(4.5) = \frac{f(4.5) - f(4)}{4.5 - 4} = \frac{3.193 - 2.533}{4.5 - 4}$$

$$= 1.320$$

نعم

B. Is there a value of x, for $\boxed{0 < x < 5,}$ such that $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = 0$?

Give a reason for your answer.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x) = 0$$

at $x = 2$ as has Horizental Tangent.

C. For each of the following limits, find the value or explain why it does not exist.

i. $\lim\limits_{h\to 0^+} \dfrac{f(5+h)-f(5)}{h} =$

ii. $\lim\limits_{h\to 0^-} \dfrac{f(5+h)-f(5)}{h}$

iii. $\lim\limits_{h\to 0} \dfrac{f(5+h)-f(5)}{h}$

x	3	3.5	4	$\boxed{4.5}$	5	5.5	6
$f(x)$	1.653	2.02	2.533	3.193	1	1.125	1.25

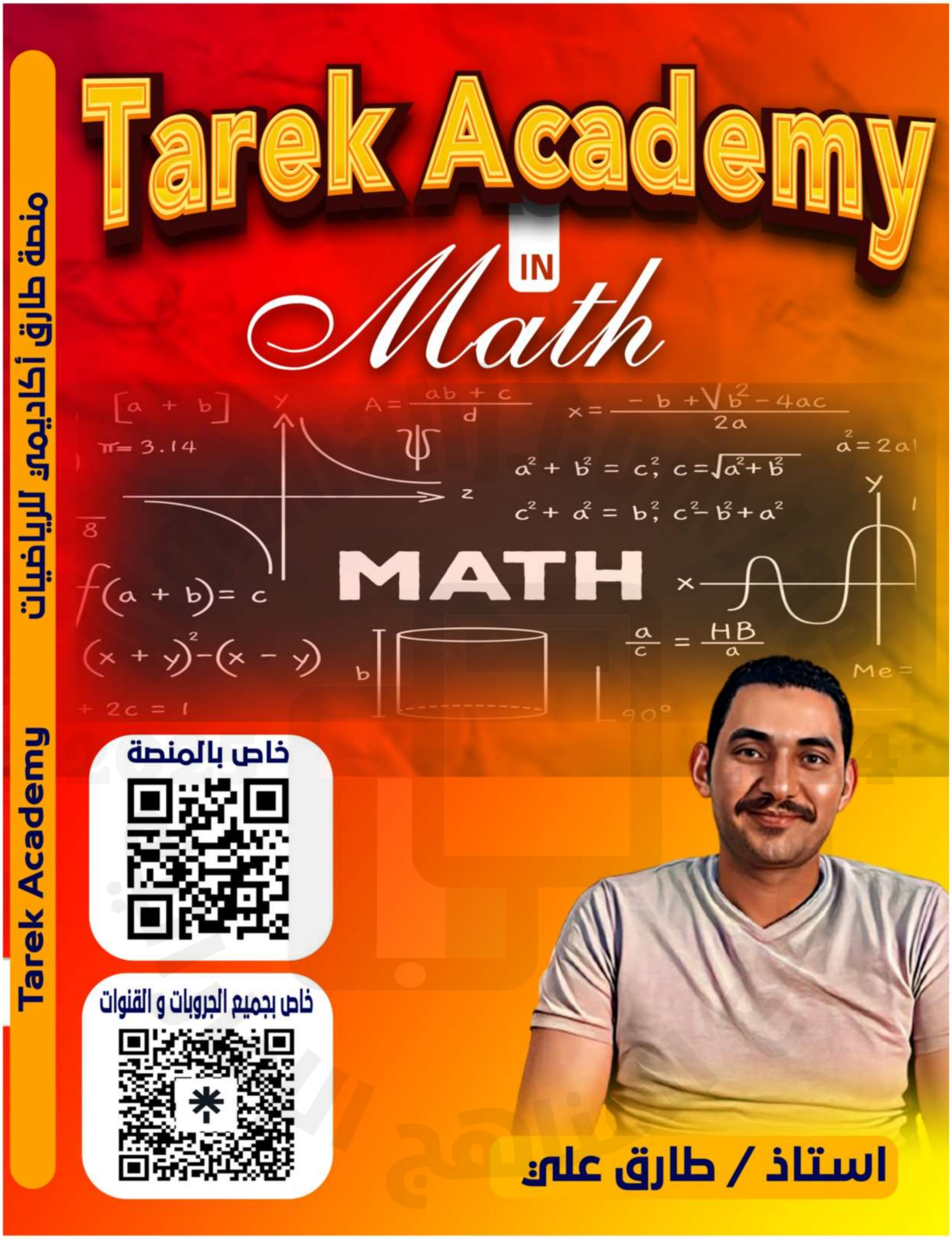
(i) $f'(5^+) = \dfrac{f(5.5)-f(5)}{5.5-5} = \dfrac{1.125-1}{5.5-5}$

$= \boxed{\dfrac{1}{4}}$

(ii) $f'(5^-) = \lim\limits_{h\to 0^-} \dfrac{f(5+h)-f(5)}{h}$ DNE

$1 \neq 3$

(iii) $\lim\limits_{h\to 0} \dfrac{f(5+h)-f(5)}{h}$ DNE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