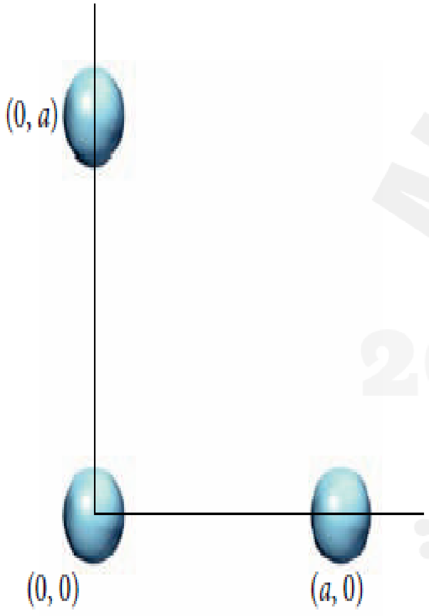
حل مراجعة أسئلة وفق الهيكل الوزاري منهج انسباير

11 ADV T3 coverage
Omar . B. Younes
PART 2 – Structured Questions

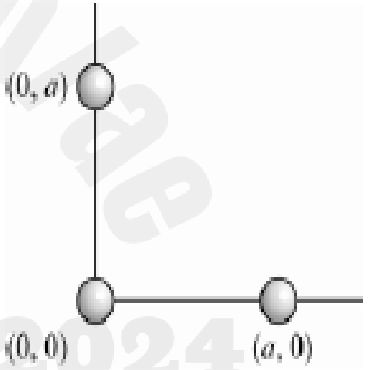
16	1st Part	↪ Determine the location of the center of mass of two or several particles or extended objects with uniform mass distribution (the object can be divided into simple geometric figures, each of which can be replaced by a particle at its center) by applying suitable mathematical equations.	Exercises Q. (8.30,8.58)	249,252
	2nd Part	↪ Convert angle measurements between degrees and radians. ↪ Relate the arc length (s), to the radius (r) of the circular path and the angle (θ), measured in radians, by ($S = r\theta$), and solve problems.	Student Book Exercises Q. (9.31)	256-257 280

8.54 Three identical balls of mass m are placed in the configuration shown in the figure. Find the location of the center of mass.



8.58. **THINK:** The system has three identical balls of mass m. The x and y coordinates of the balls are $\vec{r}_1 = (0\hat{x}, 0\hat{y})$, $\vec{r}_2 = (a\hat{x}, 0\hat{y})$ and $\vec{r}_3 = (0\hat{x}, a\hat{y})$. Determine the location of the system's center of mass, R.

SKETCH:



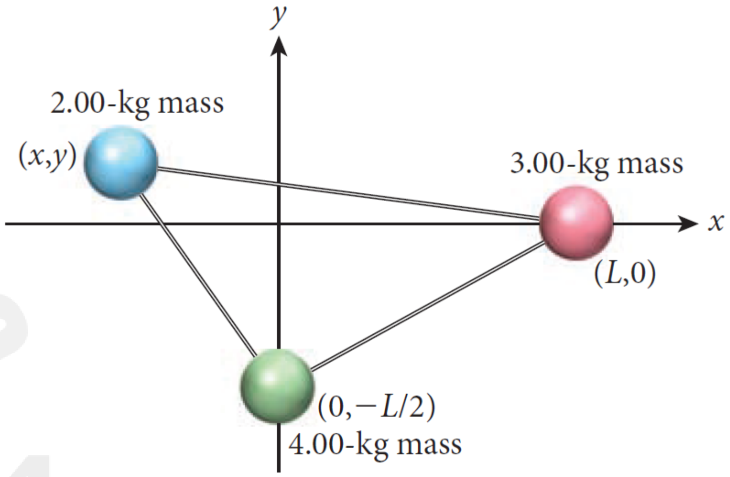
RESEARCH: The center of mass is a vector quantity, so the x and y components must be considered separately. The x- and y-components of the center of mass are given by

$$X_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \text{ and } Y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i.$$

For this system, the equations can be rewritten as

$$X_{com} = \frac{m(0) + ma\hat{x} + m(0)}{3m} = \frac{a}{3}\hat{x} \text{ and } Y_{com} = \frac{m(0) + m(0) + ma\hat{y}}{3m} = \frac{a}{3}\hat{y}.$$

Q8.30 The coordinates of the center of mass for the extended object shown in the figure are $(L/4, -L/5)$. What are the coordinates of the 2.00 kg mass?



$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$\frac{L}{4} = \frac{-2x + 3L + (4)(0)}{2+3+4}$$

$$\frac{L}{4} = \frac{-2x + 3L}{9}$$

$$\frac{9L}{4} = -2x + 3L$$

$$-2x = \frac{9L}{4} - 3L$$

$$-2x = \frac{9L}{4} - \frac{12L}{4} = -\frac{3L}{4}$$

$$x = \frac{3L}{8}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$-\frac{L}{5} = \frac{2y + (3)(0) + (4)\left(-\frac{L}{2}\right)}{2+3+4}$$

$$-\frac{L}{5} = \frac{2y - 2L}{9}$$

$$-\frac{9L}{5} = 2y - 2L$$

$$2y = -\frac{9L}{5} + 2L$$

$$2y = -\frac{9L}{5} + \frac{10L}{5} = \frac{L}{5}$$

$$y = \frac{L}{10}$$

Ex: The Cartesian coordinates of point P are (3,4) in units of cm. What are the polar coordinates of the point?

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{3^2 + 4^2} = \sqrt{25} = 5\ cm$$

$$\theta = \tan^{-1}\frac{y}{x}$$

$$\theta = \tan^{-1}\frac{4}{3} = 53.1°$$

Ex: A circle of radius r = 8 cm ,a point on the circle with x-component = 4 cm:

A- What is the angle θ:

$$x = r\cos\theta$$

$$\cos\theta = \frac{x}{r}$$

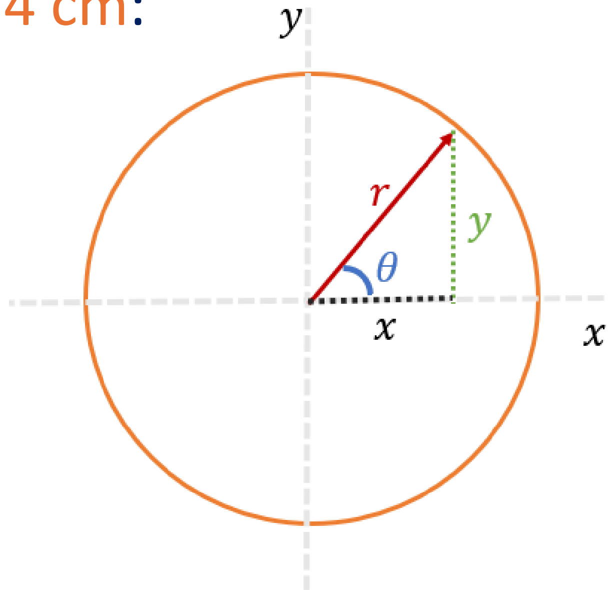
$$\theta = \cos^{-1}\frac{x}{r}$$

$$\theta = \cos^{-1}\frac{4}{8} = 60°$$

B- What is the y-component of that point:

$$y = r\sin\theta$$

$$y = 8\sin 60 = 6.93\ cm$$

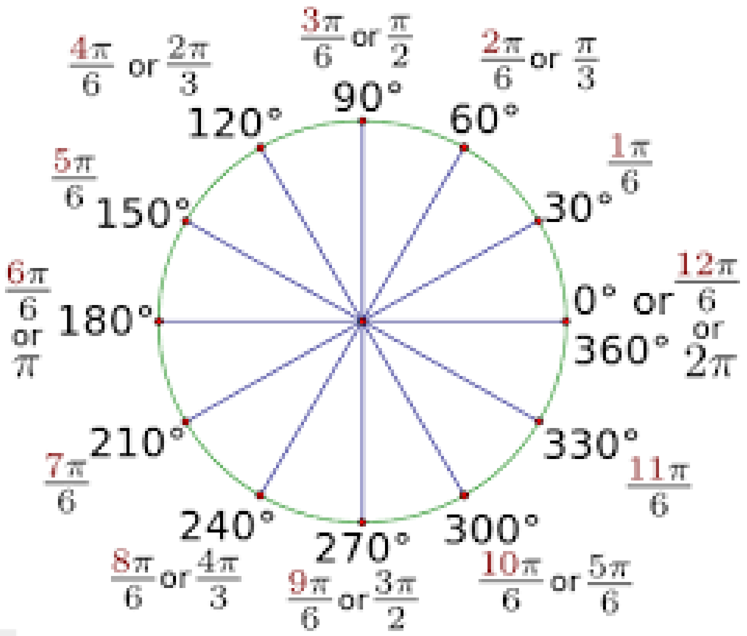


Angular Coordinates

Calculate relationships among the radius of a circle, the speed of an object (or period of revolution), and the magnitude of centripetal acceleration for an object moving in uniform circular motion.

$$\theta_{(radians)} = \theta_{(degree)} \times \frac{\pi}{180}$$

$$\theta_{(degree)} = \theta_{(radians)} \times \frac{180}{\pi}$$



1- convert the degree of the angle to radian measure:

A − 60°

$$rad = 60 \times \frac{\pi}{180} = \frac{\pi}{3} rad$$

B − 240°

$$rad = 240 \times \frac{\pi}{180} = \frac{4\pi}{3} rad$$

2- convert the radian measure to degree:

A − $\frac{2\pi}{3}$

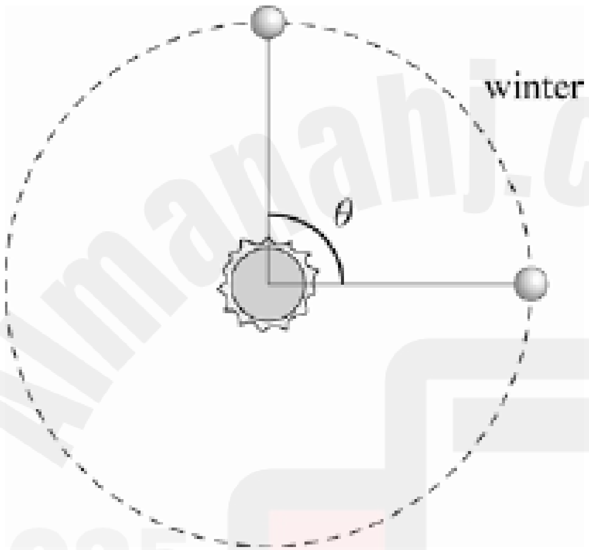
$$deg = \frac{2\pi}{3} \times \frac{180}{\pi} = 120°$$

B − $\frac{11\pi}{6}$

$$deg = \frac{11\pi}{6} \times \frac{180}{\pi} = 330°$$

9.31. THINK: Determine the change in the angular position in radians. Winter lasts roughly a fourth of a year. There are 2π radians in a circle. Consider the orbit of Earth to be circular.

SKETCH:



RESEARCH: The angular velocity of the earth is $\omega = 2\pi / \text{yr}$. The angular position is given by $\theta = \theta_0 + \omega_0 t$.

SIMPLIFY: $\Delta\theta = \theta - \theta_0 = \omega_0 t$

CALCULATE: $\Delta\theta = \dfrac{2\pi \text{ rad}}{\text{yr}}\left(\dfrac{1}{4}\text{ yr}\right) = \dfrac{\pi \text{ rad}}{2} = \dfrac{3.14 \text{ rad}}{2} = 1.57 \text{ rad}$

Angular Displacement

Calculate the translational kinematic quantities from an object's rotational kinematic quantities for objects that are rolling without slipping.

Ex: A person on a Ferris wheel with diameter of 12m has an angular displacement of π rad. How far has the person traveled around the circular path of the Ferris wheel?

$$S = r\,(\Delta\theta)$$

$$S = 6\,(\pi) = 18.84\, m$$

Ex: A boy swims for 20 m around the edge of a circular pool of 5 m radius . what is the boy angular displacement?

$$S = r\,(\Delta\theta)$$

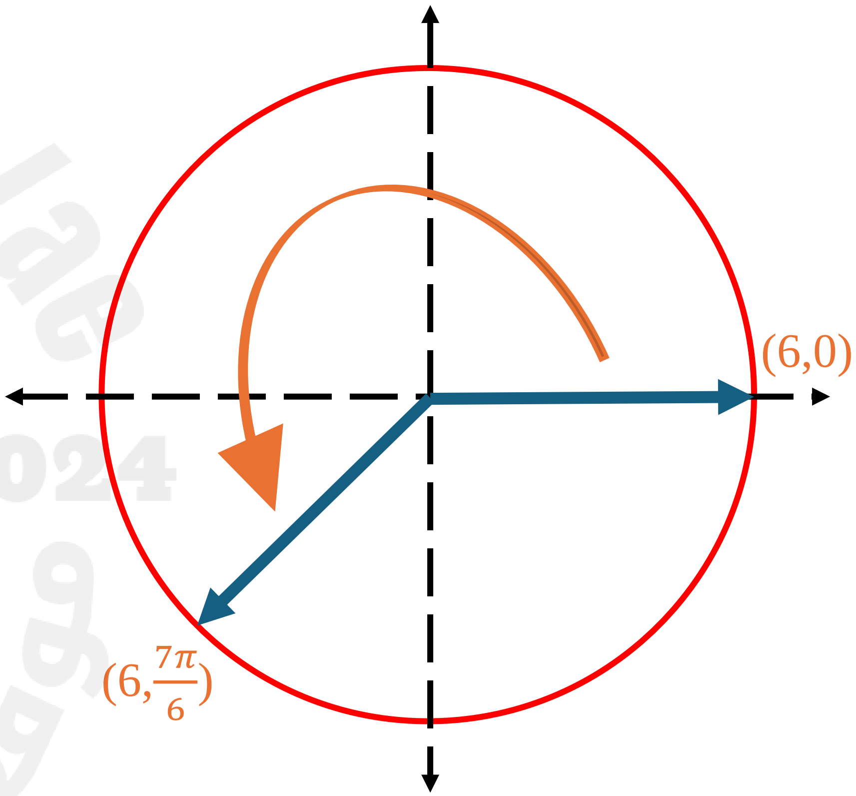
$$20 = 5\,(\Delta\theta)$$

$$\Delta\theta = \frac{20}{5} = 4\, rad$$

Ex: A car on a circular track move from point $(6,0)$ to $(6, \frac{7\pi}{6})$ in polar coordinates. The units for the radius are in meters. How far did the car moved?

$$S = r\,(\Delta\theta)$$

$$S = 6\left(0 - \frac{7\pi}{6}\right) = 22\ m$$



17	1st Part	↪ Relate the magnitudes of linear (tangential) and angular velocities for circular motion as $v = r\omega$, and explain that this relation does not hold for tangential and angular velocity vectors which point in different directions. ↪ Solve problems related to angular velocity, angular frequency and period.	Q.9.44(c,d)	281
	2nd Part	↪ Apply the kinematic relationships for circular motion with constant angular acceleration to calculate angular position, angular displacement, angular velocity, angular acceleration, or time.	Q.9.44(a) Q.9.45(d) Q.9.64	281 282

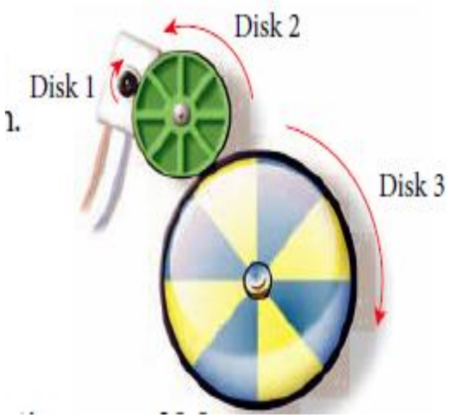
In a department store toy display, a small disk (disk 1) of radius 0.100 m is driven by a motor and turns a larger disk (disk 2) of radius 0.500 m. Disk 2, in turn, drives disk 3, whose radius is 1.00 m. The three disks are in contact, and there is no slipping. Disk 3 is observed to sweep through one complete revolution every 30.0 s.

a) What is the angular speed of disk 3?

b) What is the ratio of the tangential velocities of the rims
of the three disks?

c) What is the angular speed of disks 1 and 2?

d) If the motor malfunctions, resulting in an angular Acceleration of 0.100 rad/s2 for disk 1, what are disks 2 and
3's angular accelerations?



SIMPLIFY:

(a) $\omega_3 = 2\pi / T$

(b) $v = \omega_3 r_3$

(c) $\omega_1 = v / r_1$, and $\omega_2 = v / r_2$.

(d) $\alpha_2 = a / r_2$ and $\alpha_3 = a / r_3$, where $a = \alpha_1 r_1$.

CALCULATE:

(a) $\omega_3 = \dfrac{(2\pi \text{ rad/rev})}{30.0 \text{ s}} = 0.209 \text{ rad/s}$

(b) $v = (0.209 \text{ rad/s})(1.00 \text{ m}) = 0.209 \text{ m/s}$ for all three disks.

(c) $\omega_1 = \dfrac{0.209 \text{ m/s}}{0.100 \text{ m}} = 2.09 \text{ rad/s}$ and $\omega_2 = \dfrac{0.209 \text{ m/s}}{0.500 \text{ m}} = 0.419 \text{ rad/s}$.

(d) $a = (0.100 \text{ rad/s}^2)(0.100 \text{ m}) = 1.00 \cdot 10^{-2} \text{ m/s}^2$.

Therefore, $\alpha_2 = \dfrac{1.00 \cdot 10^{-2} \text{ m/s}^2}{0.500 \text{ m}} = 2.00 \cdot 10^{-2} \text{rad/s}^2$ and $\alpha_3 = \dfrac{1.00 \cdot 10^{-2} \text{ m/s}^2}{1.00 \text{ m}} = 1.00 \cdot 10^{-2} \text{ rad/s}^2$.

Q9.44 A discus thrower (with arm length of 1.2 m) starts from rest and begins to rotate counterclockwise with an angular acceleration of $2.5\ rad/s^2$.

a) How long does it take the discus thrower's speed to get to 4.70 rad/s?

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{4.7 - 0}{2.5} = 1.88\ s$$

b) What is the linear speed of the discus at 4.70 rad/s?

$$v = r\omega = (1.2)(4.7) = 5.64\ m/s$$

c) What is the linear acceleration of the discus thrower at this point?

$$a_t = r\alpha = (1.2)(2.5) = 3\ m/s^2$$

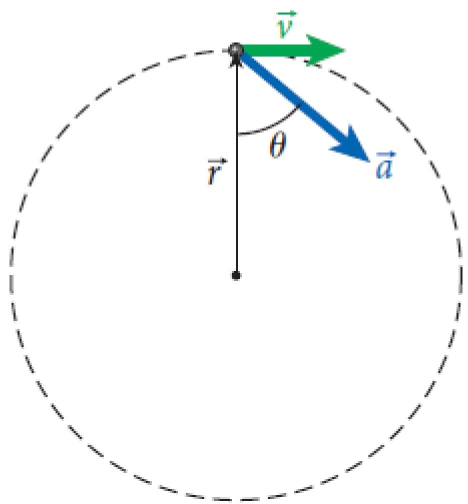
d) What is the magnitude of the centripetal acceleration of the discus thrown?

$$a_c = \frac{v^2}{r} = \frac{(5.64)^2}{1.2} = 26.5\ m/s^2$$

e) What is the magnitude of the discus's total acceleration?

$$a_{tot} = \sqrt{a_t^2 + a_c^2} = \sqrt{(3)^2 + (26.5)^2} = 26.66\ m/s^2$$

Q9.46 A particle is moving clockwise in a circle of radius 1.00 m. At a certain instant, the magnitude of its acceleration is $a = |a| = 25m/s^2$, and the acceleration vector has an angle of θ= 50.0° with the position vector, as shown in the figure. At this instant, find the speed, $v = |v|$, of this particle.



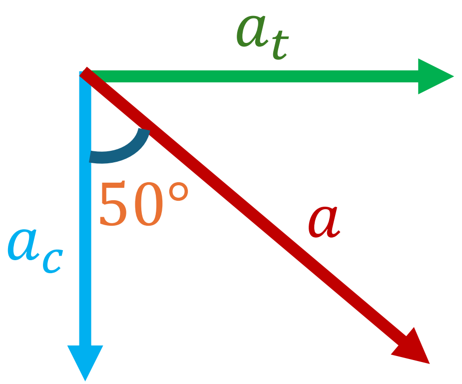
$$a_c = \frac{v^2}{r}$$

$$a_c = a\cos\theta$$

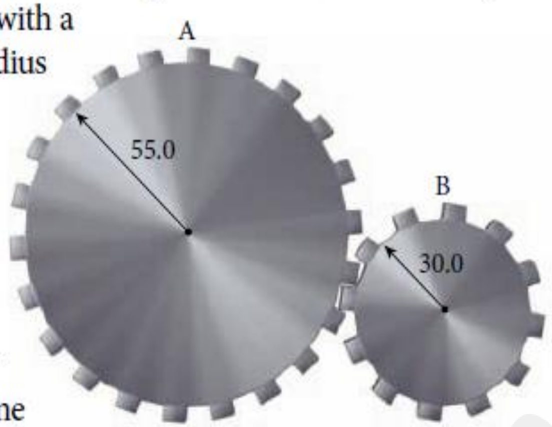
$$a_c = 25\cos 50 = 16\ m/s^2$$

$$v^2 = ra_c$$

$$v = \sqrt{ra_c} = \sqrt{(1)(16)} = 4m/s$$



9.60 Gear A, with a mass of 1.00 kg and a radius of 55.0 cm, is in contact with gear B, with a mass of 0.500 kg and a radius of 30.0 cm. The gears do not slip with respect to each other as they rotate. Gear A rotates at 120. rpm and slows to 60.0 rpm in 3.00 s. How many rotations does gear B undergo during this time interval?



Solution Summary Table

Step	Calculation	Result
1. Convert ω_A (rpm → rad/s)	$\omega_{A,i} = 120 \times \frac{2\pi}{60}$ $\omega_{A,f} = 60.0 \times \frac{2\pi}{60}$	4π rad/s 2π rad/s
2. Angular acceleration (α_A)	$\alpha_A = \frac{2\pi - 4\pi}{3.00}$	$-\frac{2\pi}{3}$ rad/s^2
3. Relate α_A and α_B	$\alpha_B = \alpha_A \times \frac{r_A}{r_B} = -\frac{2\pi}{3} \times \frac{0.550}{0.300}$	$-\frac{11\pi}{9}$ rad/s^2
4. Initial ω_B	$\omega_{B,i} = \omega_{A,i} \times \frac{r_A}{r_B} = 4\pi \times \frac{0.550}{0.300}$	$\frac{22\pi}{3}$ rad/s
5. Final ω_B	$\omega_{B,f} = \frac{22\pi}{3} + \left(-\frac{11\pi}{9}\right) \times 3.00$	$\frac{11\pi}{3}$ rad/s
6. Average ω_B	$\omega_{B,\text{avg}} = \frac{\frac{22\pi}{3} + \frac{11\pi}{3}}{2}$	$\frac{11\pi}{2}$ rad/s
7. Angular displacement ($\Delta\theta_B$)	$\Delta\theta_B = \frac{11\pi}{2} \times 3.00$	$\frac{33\pi}{2}$ rad
8. Convert to rotations	$\text{Rotations} = \frac{33\pi/2}{2\pi}$	**8.25 rotations**

18	1^{st} Part	↪ Relate the magnitude of the centripetal force to the centripetal acceleration by applying Newton's Second Law in the radial direction as: $F_c = ma_c = mv\omega = mr\omega^2 = m\frac{v^2}{r}$ and Slove problems related to centripetal force.	Solved Problem 9.1 Q.9.76	266 283
	2^{nd} Part	↪ Solve problems related to rotation with constant angular acceleration. $\theta = \theta_{\circ} + \omega_{\circ} + \frac{1}{2}\alpha t^2$ ‘‘ $\theta = \theta_{\circ} + \bar{\omega} t$ ‘‘ $\omega = \omega_{\circ} + \alpha t$ ‘‘ $\bar{\omega} = \frac{1}{2}(\omega + \omega_{\circ})$ $\omega^2 = \omega_{\circ}^2 + 2\alpha(\theta - \theta_{\circ})$	Q.9.35 Q.9.63/9.67	280 282

9.31 A vinyl record plays at 33.3 rpm. Assume it takes 5.00 s for it to reach this full speed, starting from rest.

a) What is its angular acceleration during the 5.00 s?

b) How many revolutions does the record make before reaching its final angular speed?

9.35. THINK: Determine the average angular acceleration of the record and its angular position after reaching full speed. The initial and final angular speeds are 0 rpm to 33.3 rpm. The time of acceleration is 5.00 s.

SKETCH:

RESEARCH: The equation for angular acceleration is $\alpha = (\omega_f - \omega_i)/\Delta t$. The angular position of an object under constant angular acceleration is given by $\theta = \frac{1}{2}\alpha t^2$.

SIMPLIFY: There is no need to simplify the equation.

CALCULATE: $\alpha = \dfrac{33.3 \text{ rpm} - 0 \text{ rpm}}{5.00 \text{ s}(60 \text{ s/min})} = 0.111 \text{ rev/s}^2 \cdot \dfrac{2\pi \text{ rad}}{1 \text{ rev}} = 0.6974 \text{ rad/s}^2$

$$\theta = \frac{1}{2}(0.111 \text{ rev/s})(5.00 \text{ s})^2 = 1.3875 \text{ rev} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 8.718 \text{ rad}$$

Use Newton's 2nd law to calculate unknown values of centripetal force, friction force, coefficient of friction, normal force, angle of banking, mass, speed, etc for a vehicle moving on a banked circular track.

SOLVED PROBLEM 4.1 Analysis of a Roller Coaster:

Suppose the vertical loop has a radius of 5.00 m. What does the linear speed of the roller coaster have to be at the top of the loop for the passengers to feel weightless? (Assume that friction between roller coaster and rails can be neglected.)

A person feels weightless when there is no supporting force, from a seat or a restraint, acting to counter his or her weight. For a person to feel weightless at the top of the loop, no normal force can be acting on him or her at this point.

$$F_c = F_{net}$$

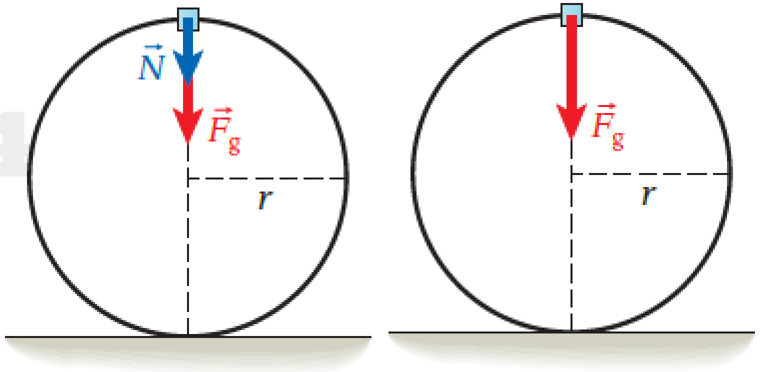
$$F_{net} = mg - N$$ Weightless N=0

$$F_c = m\frac{v^2}{r}$$

$$m\frac{v^2}{r} = mg$$

$$v = \sqrt{gr}$$

$$v = \sqrt{(5)(9.812)} = 7.00357\ m/s$$



(a) (b)

Use Newton's 2nd law to calculate unknown values of centripetal force, friction force, coefficient of friction, normal force, angle of banking, mass, speed, etc for a vehicle moving on a banked circular track.

Q9.54 A race car is making a U-turn at constant speed. The coefficient of friction between the tires and the track is $\mu_s = 1.20$. If the radius of the curve is 10.0 m, what is the maximum speed at which the car can turn without sliding? Assume that the car is undergoing uniform circular motion

F_c is the force of static friction

$$F_c = m\frac{v^2}{r}$$

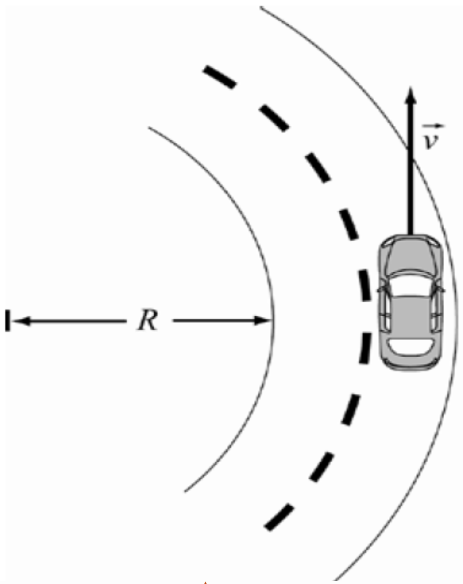
$$\mu_s N = m\frac{v^2}{r}$$

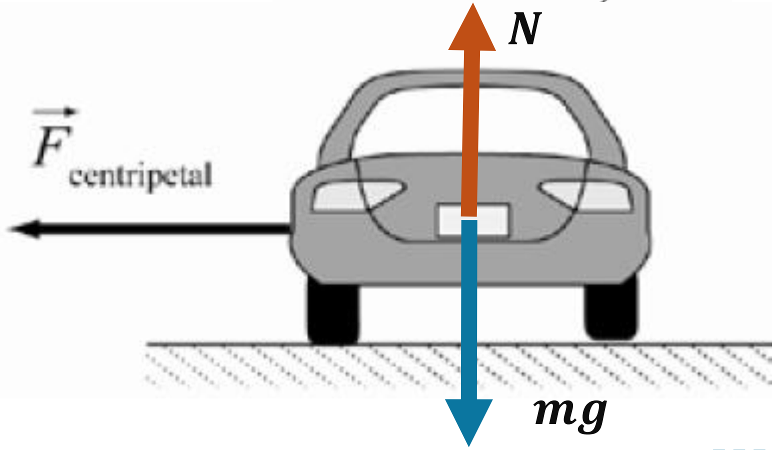
$$\mu_s mg = m\frac{v^2}{r}$$

$$v^2 = \mu_s gr$$

$$v = \sqrt{\mu_s gr}$$

$$v = \sqrt{(1.2)(9.8)(10)} = 10.84\ m/s$$





Q9.55 A car speeds over the top of a hill. If the radius of curvature of the hill at the top is 9.00 m, how fast can the car be traveling and maintain constant contact with the ground?

As the car travels over the top of the hill it undergoes circular motion in the vertical plane. The only force that can provide the centripetal force for this motion is gravity. Clearly, for small speeds the car remains in contact with the road due to gravity. But the car will lose contact if the centripetal acceleration exceeds gravity.

$$F_c = m\frac{v^2}{r}$$

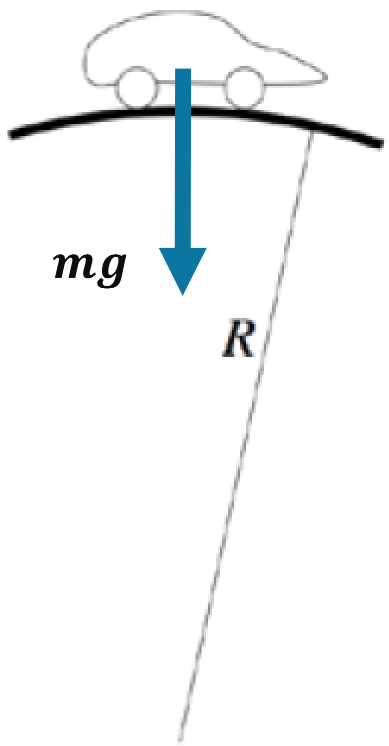
$$mg = m\frac{v^2}{r}$$

$$g = \frac{v^2}{r}$$

$$v^2 = gr$$

$$v = \sqrt{gr}$$

$$v = \sqrt{(9.8)(9)} = 9.40\ m/s$$



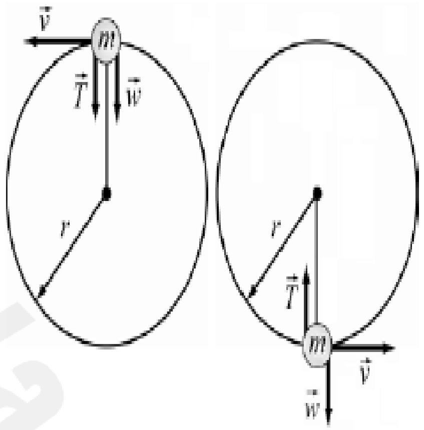
•9.72 A ball having a mass of 1.00 kg is attached to a string 1.00 m long and is whirled in a vertical circle at a constant speed of 10.0 m/s.

a) Determine the tension in the string when the ball is at the top of the circle.

b) Determine the tension in the string when the ball is at the bottom of the circle.

c) Consider the ball at some point other than the top or bottom. What can you say about the tension in the string at this point?

SKETCH:



RESEARCH: $F_{\text{net}} = \dfrac{mv^2}{r}$

(a) $F_{\text{net}} = T + w$

(b) $F_{\text{net}} = T - w$

SIMPLIFY:

(a) $T = F_{\text{net}} - w = \dfrac{mv^2}{r} - mg$

(b) $T = F_{\text{net}} + w = \dfrac{mv^2}{r} + mg$

CALCULATE: $\dfrac{mv^2}{r} = \dfrac{(1.00\text{ kg})(10.0\text{ m/s})^2}{1.00\text{ m}} = 100.\text{ N},\ \ mg = (1.00\text{ kg})(9.81\text{ m/s}^2) = 9.81\text{ N}$

9.63 A vinyl record that is initially turning at 33 1 rpm slows uniformly to a stop in a time of 15 s. How many rotations are made by the record while stopping?

9.67. THINK: The acceleration is uniform during the given time interval. The average angular speed during this time interval can be determined and from this, the angular displacement can be determined.

SKETCH:

RESEARCH: $\omega_{avg} = \dfrac{\omega_f + \omega_i}{2}$, $\Delta\theta = \omega_{avg}\Delta t$

SIMPLIFY: Simplification is not necessary.

CALCULATE: $\omega_i = 33.33 \text{ rpm} = 33.33 \text{ rpm}\left(\dfrac{2\pi}{60 \text{ s}}\right) = 3.491 \text{ rad/s}$, $\omega_f = 0$

$$\Delta\theta = \left(\frac{3.491}{2} \text{ rad s}^{-1}\right)(15.0 \text{ s}) = 26.18 \text{ rad}$$

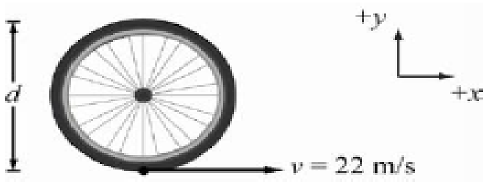
$$\text{number of rotations} = \frac{\Delta\theta}{2\pi} = 4.167$$

9.59 A car accelerates uniformly from rest and reaches a speed of 22.0 m/s in 9.00 s. The diameter of a tire on this car is 58.0 cm.

a) Find the number of revolutions the tire makes during the car's motion, assuming that no slipping occurs.

b) What is the final angular speed of a tire in revolutions per second?

SKETCH:



RESEARCH: The circumference of a circle is given by $C = 2\pi r = \pi d$. The displacement at constant acceleration is $\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$, where $v = \omega r$.

SIMPLIFY:

(a) $v_i = 0 \Rightarrow \Delta x = \frac{1}{2} a \Delta t^2,\ a = \frac{\Delta v}{\Delta t} \Rightarrow \Delta x = \frac{1}{2}\frac{\Delta v}{\Delta t}\Delta t^2 = \frac{1}{2}\Delta v \Delta t$

Let N = number of revolutions and the displacement is given by $\Delta x = \left(\frac{\text{displacement}}{\text{revolution}}\right) N$. The displacement per revolution is simply the circumference, C, so

$$\Delta x = CN \Rightarrow N = \frac{\Delta x}{C} = \frac{1}{\pi d}\left(\frac{1}{2}\Delta v \Delta t\right) = \frac{\Delta v \Delta t}{2\pi d}.$$

(b) $\omega = \frac{v}{r} = \frac{v}{d/2} = \frac{2v}{d}$

CALCULATE:

(a) $N = \frac{(22.0 \text{ m/s})(9.00 \text{ s})}{2\pi(0.58 \text{ m})} = 54.33$ revolutions

(b) $\omega = \frac{2(22.0 \text{ m/s})}{0.58 \text{ m}} = 75.86 \text{ rad/s} = \frac{75.86}{2\pi} \text{ rev/s} = 12.07 \text{ rev/s}$

19	↪ Calculate the moment of inertia of a point particle or a group of several point particles rotating about an axis of rotation. $I = mr^2$,, $I = \sum_{i=1}^{n} m_i r_i^2$ ↪ Calculate the rotational kinetic energy of a point particle, or several point particles, rotating about a fixed axis of rotation by applying the expression for the rotational kinetic energy in terms of the rational inertia and angular speed. $K_{Rot} = \frac{1}{2}\sum_{i=1}^{n} m_i r_i^2 \omega_i^2 = \frac{1}{2} I\omega^2$	Exercises Q. (10.38, 10.39)	318

10.38 A uniform solid cylinder of mass *M* = 5.00 kg is rolling without slipping along a horizontal surface. The velocity of its center of mass is 30.0 m/s. Calculate its energy.

About its central axis (longitudinal axis)

Formula:

$$I = \frac{1}{2}MR^2$$

$$K_{\text{total}} = \frac{1}{2}Mv^2(1+c) = \frac{1}{2} \times 5.00 \times (30.0)^2 \times \left(1 + \frac{1}{2}\right)$$

$$K_{\text{total}} = \frac{1}{2} \times 5.00 \times 900 \times \frac{3}{2}$$

$$K_{\text{total}} = 2250 \times \frac{3}{2} = 3375\,\text{J}$$

$$\boxed{3375\,\text{J}}$$

10.39. Determine the moment of inertia for three children weighing 60.0 kg, 45.0 kg and 80.0 kg sitting at different points on the edge of a rotating merry-go-round, which has a radius of 12.0 m.

Since all children are on the edge ($r = 12.0\,\mathrm{m}$):

$$I = m_1 r^2 + m_2 r^2 + m_3 r^2 = (m_1 + m_2 + m_3) r^2$$

Given:

- $m_1 = 60.0\,\mathrm{kg}$, $m_2 = 45.0\,\mathrm{kg}$, $m_3 = 80.0\,\mathrm{kg}$
- $r = 12.0\,\mathrm{m}$

Calculation:

1. **Total mass:**

$$M = m_1 + m_2 + m_3 = 60.0 + 45.0 + 80.0 = 185.0\,\mathrm{kg}$$

2. **Moment of inertia:**

$$I = Mr^2 = 185.0 \times (12.0)^2 = 185.0 \times 144 = 26,640$$

MCQ

1	↳ Locate the center of mass of an extended, symmetric object of uniform mass distribution by using the symmetry. ↳ Recall that center of gravity is equivalent to center of mass in situations where the gravitational force is constant everywhere throughout the object.	Student Book	226

8.1 Center of Mass and Center of Gravity

So far, we have represented the location of an object by coordinates of a single point. However, a statement such as "a car is located at $x = 3.2$ m" surely does not mean that the entire car is located at that point. So, what does it mean to give the coordinate of one particular point to represent an extended object? Answers to this question depend on the particular application. In auto racing, for example, a car's location is represented by the coordinate of the frontmost part of the car. When this point crosses the finish line, the race is decided. On the other hand, in soccer, a goal is counted only if the entire ball has crossed the goal line; in this case, it makes sense to represent the soccer ball's location by the coordinates of the rearmost part of the ball. However, these examples are exceptions. In almost all situations, there is a natural choice of a point to represent the location of an extended object. This point is called the *center of mass*.

Definition
The **center of mass** is the point at which we can imagine all the mass of an object to be concentrated.

Concept	Definition/Description
Center of Mass	The point where all the mass of an object can be imagined to be concentrated.
Center of Gravity	The point where the force of gravity on the entire object can be considered to act.
Representation	A single point used to describe the location of an extended object.
Mass Density	If an object's mass density is constant, the center of mass coincides with the geometric center.
Exceptions	Some applications use non-center points (e.g., auto racing uses the frontmost part, soccer uses the rearmost).

Questions:

1.Under what condition can the center of mass and center of gravity be used interchangeably?

1. ***Answer:*** **They can be used interchangeably when the gravitational force is constant throughout the object. (This is typically true for everyday-sized objects but not for very large objects where gravity varies.)**

2.Why is the center of mass often located at the geometric center of an object?

1. ***Answer:*** **This occurs when the object has a constant mass density, meaning the mass is evenly distributed, making the center of mass coincide with the geometric center.**

2	Describe that the center of mass of two-point masses (or two objects each of which can be replaced by a particle having its mass and located at its center) always lies on the connecting line between the two masses.	Student Book Figure 8.2 Solved problem 8.1	227

Combined Center of Mass for Two Objects

If we have two identical objects of equal mass and want to find the center of mass for the combination of the two, it is reasonable to assume from considerations of symmetry that the combined center of mass of this system lies exactly midway between the individual centers of mass of the two objects. If one of the two objects is more massive, then it is equally reasonable to assume that the center of mass for the combination is closer to that of the more massive one. Thus, we have a general formula for calculating the location of the center of mass, $\vec{R}$, for two masses m_1 and m_2 located at positions $\vec{r}_1$ and $\vec{r}_2$ in an arbitrary coordinate system (Figure 8.2):

$$\vec{R} = \frac{\vec{r}_1 m_1 + \vec{r}_2 m_2}{m_1 + m_2}. \tag{8.1}$$

This equation says that the center-of-mass position vector is an average of the position vectors of the individual objects, weighted by their mass. Such a definition is consistent with the empirical evidence we have just cited. For now, we will use this equation as an operating definition and gradually work out its consequences. Later in this chapter and in the following chapters, we will see additional reasons why this definition makes sense.

Note that we can immediately write vector equation 8.1 in Cartesian coordinates as follows:

$$X = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}, \quad Y = \frac{y_1 m_1 + y_2 m_2}{m_1 + m_2}, \quad Z = \frac{z_1 m_1 + z_2 m_2}{m_1 + m_2}. \tag{8.2}$$

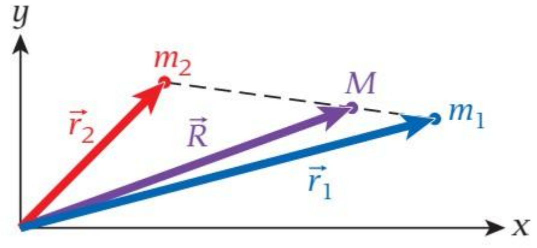


FIGURE 8.2 Location of the center of mass for a system of two masses m_1 and m_2, where $M = m_1 + m_2$.

2	Describe that the center of mass of two-point masses (or two objects each of which can be replaced by a particle having its mass and located at its center) always lies on the connecting line between the two masses.	Student Book Figure 8.2 Solved problem 8.1	227

SOLVED PROBLEM 8.1 Center of Mass of Earth and Moon

The Earth has a mass of 5.97×10^{24} kg, and the Moon has a mass of 7.36×10^{22} kg. The Moon orbits the Earth at a distance of 384,000 km; that is, the center of the Moon is a distance of 384,000 km from the center of Earth, as shown in Figure 8.3a.

PROBLEM

How far from the center of the Earth is the center of mass of the Earth-Moon system?

SKETCH A sketch showing Earth and Moon to scale is presented in Figure 8.3b.

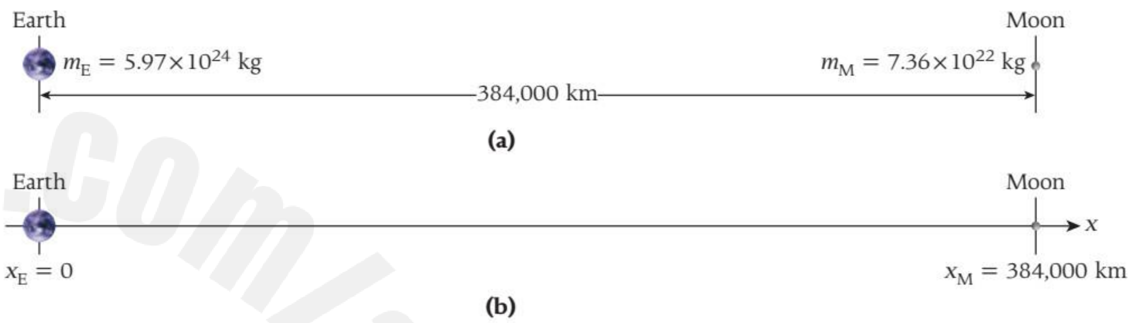


FIGURE 8.3 (a) The Moon orbits the Earth at a distance of 384,000 km (drawing to scale). (b) A sketch showing the Earth at $x_E = 0$ and the Moon at $x_M = 384,000$ km.

– *Continued*

3	↳ Express the Cartesian coordinates (x, y) in terms of the polar coordinates (r, θ) and vice versa. ↳ Convert polar coordinates to Cartesian coordinates and vice versa.	Student Book Example 9.1	255~256 256

During an object's **circular motion,** its x- and y-coordinates change continuously, but the distance from the object to the center of the circular path stays the same. We can take advantage of this fact by using **polar coordinates** to study circular motion. Shown in Figure 9.3 is the position vector, $\vec{r}$, of an object in circular motion. This vector changes as a function of time, but its tip always moves on the circumference of a circle. We can specify $\vec{r}$ by giving its x- and y-components. However, we can specify the same vector by giving two other numbers: the angle of $\vec{r}$ relative to the x-axis, θ, and the length of $\vec{r}$, $r = |\vec{r}|$ (Figure 9.3).

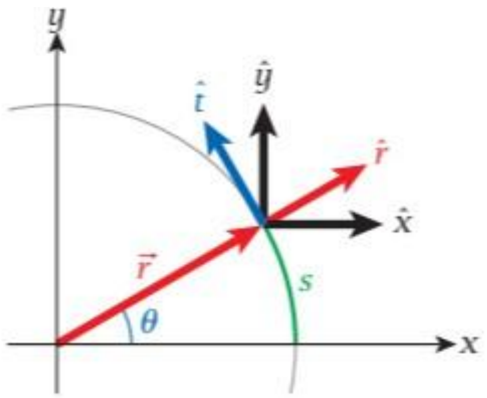


FIGURE 9.3 Polar coordinate system for circular motion.

Trigonometry provides the relationship between the Cartesian coordinates x and y and the polar coordinates θ and r:

$$r = \sqrt{x^2 + y^2} \tag{9.1}$$

$$\theta = \tan^{-1}(y/x). \tag{9.2}$$

The inverse transformation from polar to Cartesian coordinates is given by

$$x = r\cos\theta \tag{9.3}$$

$$y = r\sin\theta. \tag{9.4}$$

3	↳ Express the Cartesian coordinates (x, y) in terms of the polar coordinates (r, θ) and vice versa. ↳ Convert polar coordinates to Cartesian coordinates and vice versa.	Student Book Example 9.1	255~256 256

EXAMPLE 9.1 Locating a Point with Cartesian and Polar Coordinates

A point has a location given in Cartesian coordinates as (4,3), as shown in Figure 9.5.

PROBLEM

How do we represent the position of this point in polar coordinates?

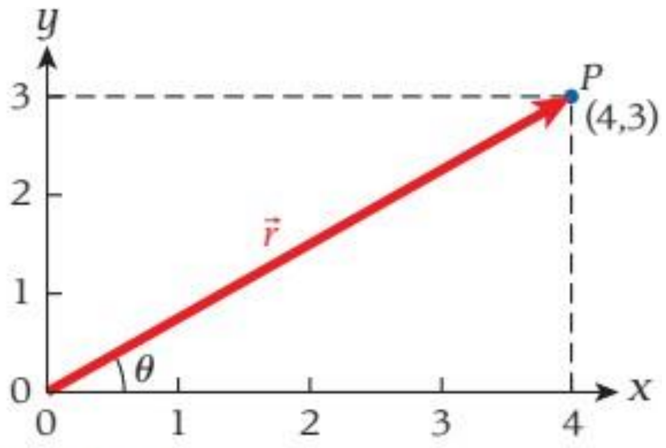


FIGURE 9.5 A point located at (4,3) in a Cartesian coordinate system.

Ex: The Cartesian coordinates of point P are (3,4) in units of cm. What are the polar coordinates of the point?

$$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\frac{y}{x}$$

$$r = \sqrt{3^2 + 4^2} = \sqrt{25} = 5\ cm \qquad \theta = \tan^{-1}\frac{4}{3} = 53.1°$$

4	Recall that the common unit for angular velocity is radian per second (rad/s).	Student Book	256,258

9.3 Angular Velocity, Angular Frequency, and Period

We have seen that the change of an object's linear coordinates in time is its velocity. Similarly, the change of an object's angular coordinate in time is its **angular velocity.** The average magnitude of the angular velocity is defined as

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}.$$

This definition uses the notation $\theta_1 \equiv \theta(t_1)$ and $\theta_2 \equiv \theta(t_2)$. The horizontal bar above the symbol ω for angular velocity again indicates a time average. By taking the limit of this expression as the time interval approaches zero, we find the instantaneous value of the magnitude of the angular velocity:

$$\omega = \lim_{\Delta t \to 0} \bar{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \equiv \frac{d\theta}{dt}. \qquad (9.8)$$

The most common unit of angular velocity is radians per second (rad/s); degrees per second is not generally used.

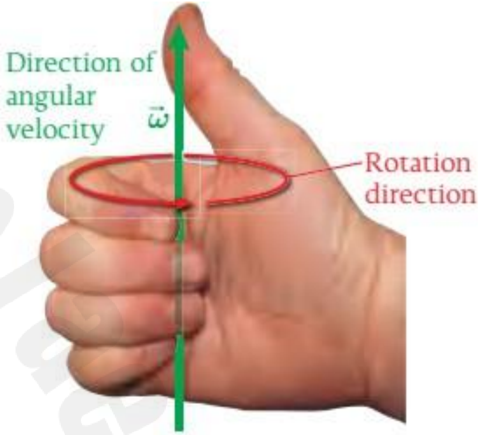


FIGURE 9.8 The right-hand rule for determining the direction of the angular velocity vector.

5	Relate the magnitudes of linear (tangential) and angular velocities for circular motion as $v = r\omega$, and explain that this relation does not hold for tangential and angular velocity vectors which point in different directions	M.C.Q(9.13) Additional Exe.Q. (9.62/a)	278 282

9.13 A bicycle's wheels have a radius of 33.0 cm. The bicycle is traveling at a speed of 6.5 m/s. What is the angular speed of the front tire?

a) 0.197 rad/s
b) 1.24 rad/s
c) 5.08 rad/s
d) 19.7 rad/s
e) 215 rad/s

where:

- $v = 6.5\,\mathrm{m/s}$ (linear speed),
- $r = 33.0\,\mathrm{cm} = 0.33\,\mathrm{m}$ (radius of the wheel).

Rearranging the formula to solve for ω:

$$\omega = \frac{v}{r} = \frac{6.5\,\mathrm{m/s}}{0.33\,\mathrm{m}} \approx 19.7\,\mathrm{rad/s}$$

9.62 Consider a 53.0 cm-long lawn mower blade rotating about its center at 3400 rpm.

a) Calculate the linear speed of the tip of the blade.

b) If safety regulations require that the blade be stoppable within 3.00 s, what minimum angular acceleration will accomplish this? Assume that the angular acceleration is constant.

Part (a): Linear Speed of the Blade Tip

1. **Convert rotational speed to radians per second ($\mathrm{rad/s}$):**

$$\omega_0 = 3400\,\mathrm{rpm} \times \frac{2\pi\,\mathrm{rad}}{1\,\mathrm{rev}} \times \frac{1\,\mathrm{min}}{60\,\mathrm{s}} = 3400 \times \frac{2\pi}{60} \approx 356\,\mathrm{rad/s}$$

2. **Calculate linear speed (v) at the tip:**

$$v = r\omega = 0.265\,\mathrm{m} \times 356\,\mathrm{rad/s} \approx 94.3\,\mathrm{m/s}$$

1. **Use the kinematic equation for rotational motion:**

$$\omega = \omega_0 + \alpha t$$

Since the blade must stop ($\omega = 0$):

$$0 = 356\,\mathrm{rad/s} + \alpha(3.00\,\mathrm{s})$$

2. **Solve for angular acceleration (α):**

$$\alpha = \frac{-\omega_0}{t} = \frac{-356\,\mathrm{rad/s}}{3.00\,\mathrm{s}} \approx -119\,\mathrm{rad/s^2}$$

(The negative sign indicates deceleration.)

6	Sketch the path taken in circular motion (uniform and non-uniform) and explain the velocity and acceleration vectors (magnitudes and directions) during the motion.	Student Book	261~262

The acceleration in circular motion has two components.

$$\boldsymbol{a}(t) = \frac{d\boldsymbol{v}(t)}{dt} = \frac{d(v\hat{\boldsymbol{t}})}{dt}$$

$$\frac{dv}{dt} = \frac{d(r\omega)}{dt} = \omega\frac{dr}{dt} + r\frac{d\omega}{dt} = r\alpha$$

The first part arises from the change in the magnitude of the velocity; this is the **tangential acceleration.**

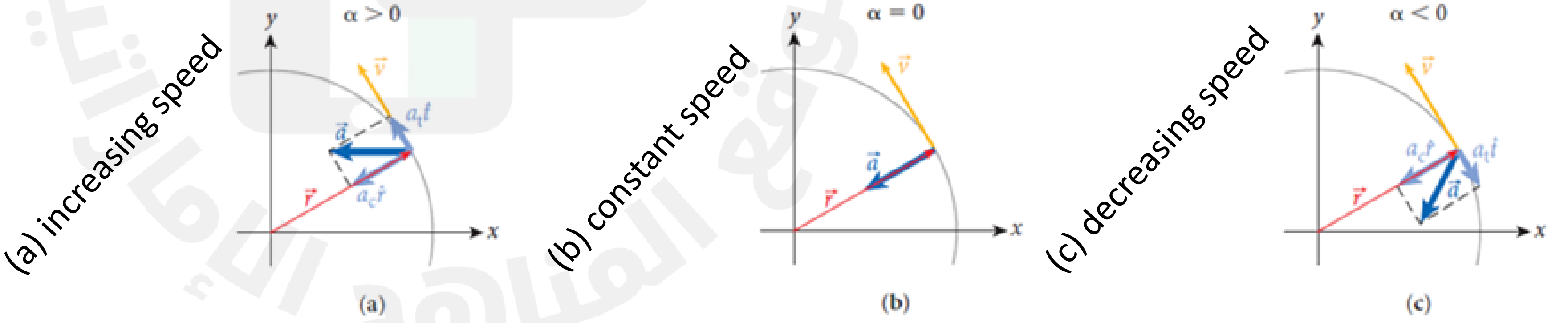
$$\boldsymbol{a}(t) = \frac{dv}{dt}\hat{\boldsymbol{t}} + v\frac{d\hat{\boldsymbol{t}}}{dt}$$

$$v\frac{d\hat{t}}{dt} = v\frac{d}{dt}(-\sin\theta, \cos\theta) = \left(-\cos\theta\frac{d\theta}{dt}, -\sin\theta\frac{d\theta}{dt}\right) = -\frac{d\theta}{dt}(\cos\theta, \sin\theta) = -\omega r$$

$$\boldsymbol{a}(t) = r\alpha\hat{\boldsymbol{t}} - \omega r\hat{\boldsymbol{r}}$$

The second part is due to the fact that the velocity vector always points in the tangential direction and thus has to change its direction continuously as the tip of the radial position vector moves around the circle; this is the **radial acceleration.**

$$\boldsymbol{a} = a_t\hat{\boldsymbol{t}} - a_c\hat{\boldsymbol{r}}$$



Non-uniform circular motion

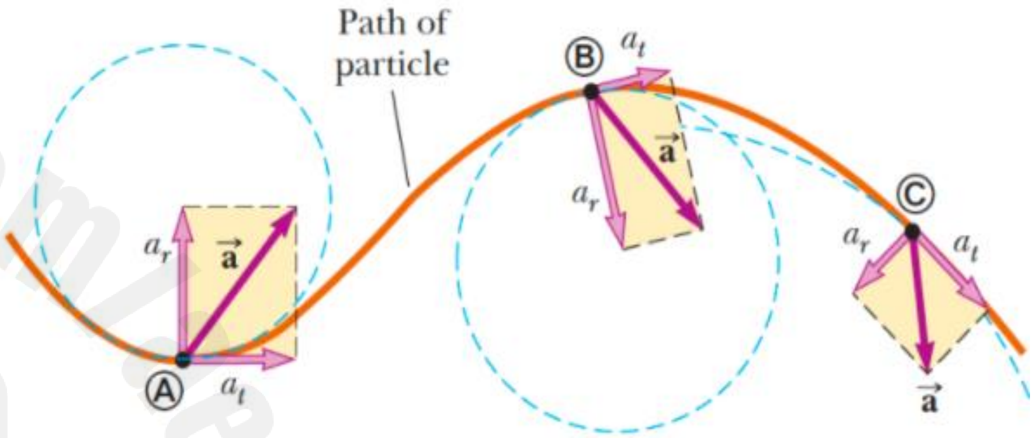
Describe the direction of the velocity and acceleration vector for an object moving in two dimensions, circular motion, or uniform circular motion.

Tangential acceleration is the instantaneous *linear* acceleration of a particle.

$$a_t = \frac{dv}{dt}$$

The magnitude of velocity is changed.

Net force and acceleration either in same direction or opposite direction of motion



Centripetal acceleration is the acceleration of a moving particle in uniform circular motion

$$a_c = \frac{v^2}{r} = v\omega = \omega^2 r$$

The direction of velocity is changed

Net force and acceleration are perpendicular to direction of motion.

$$\theta = \tan^{-1}\frac{a_y}{a_x}$$

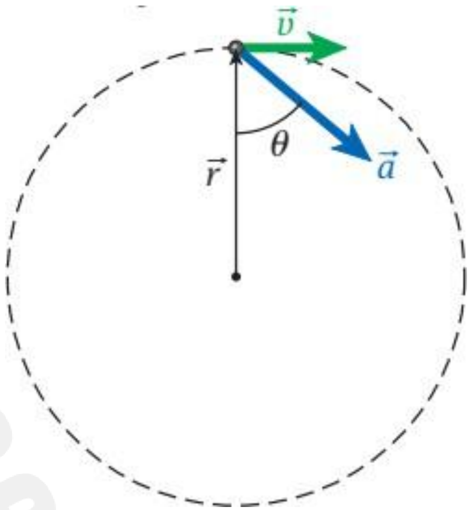
Total acceleration is combination between tangential and centripetal acceleration for a moving particle. in non-uniform circular motion (moving on a curve).

$$a_{tot} = \sqrt{a_t^2 + a_c^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{(\alpha)^2 + (\omega)^4}$$

Net force and acceleration make an angle with direction of motion.
Total acceleration's direction between tangential and centripetal acceleration.

7	Relate the magnitude of the net acceleration in circular motion to the tangential acceleration and centripetal acceleration as: $a = \sqrt{{a_c}^2 + {a_t}^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$	Exercises Q. (9.46)	281

•9.46 A particle is moving clockwise in a circle of radius 1.00 m. At a certain instant, the magnitude of its acceleration is $a = |\vec{a}| = 25.0\ \text{m/s}^2$, and the acceleration vector has an angle of $\theta = 50.0°$ with the position vector, as shown in the figure. At this instant, find the speed, $v = |\vec{v}|$, of this particle.



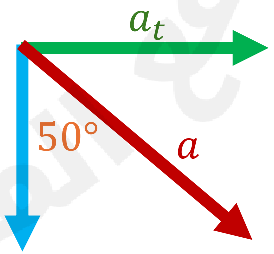
$$a_c = \frac{v^2}{r}$$

$$a_c = a\cos\theta$$

$$a_c = 25\cos 50 = 16\ m/s^2$$

$$v^2 = ra_c$$

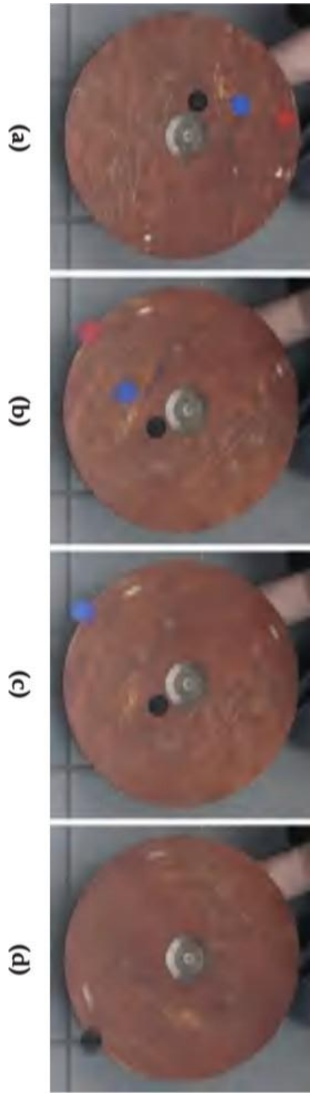
$$v = \sqrt{ra_c} = \sqrt{(1)(16)} = 4m/s$$



8	↳ Describe centripetal force as the net inward force (towards the center of the circular path) needed to provide the centripetal acceleration necessary for circular motion. ↳ Solve problems related to acceleration in circular motion.	Student Book M.C.Q(9.7) Q.(9.90 / 9.9)	264 278 283

Centripetal Force and Spinning Table Experiment

Key Point	Explanation
Formula	Centripetal force: $F_c = m\omega^2 r$ (depends on mass, angular velocity, radius).
Experiment Setup	Spinning table with three markers: black (center), blue (midway), red (edge).
Slow Spin (Part a)	Static friction provides enough F_c to keep all markers in circular motion.
Faster Spin (Parts b–d)	Higher ω increases F_c. Markers slide when friction is insufficient.
Observation	Red (outermost) slides first, black (innermost) last. Shows $F_c \propto r$.
Conclusion	Same ω for all points, but F_c increases with radius r.



The centripetal force, $\vec{F}_c$, is not another fundamental force of nature but is simply the net inward force needed to provide the centripetal acceleration necessary for circular motion. It has to point inward, toward the circle's center. Its magnitude is the product of the mass of the object and the centripetal acceleration required to force it onto a circular path:

$$F_c = ma_c = mv\omega = m\frac{v^2}{r} = m\omega^2 r. \quad (9.21)$$

8	↳ Describe centripetal force as the net inward force (towards the center of the circular path) needed to provide the centripetal acceleration necessary for circular motion. ↳ Solve problems related to acceleration in circular motion.	Student Book M.C.Q(9.7) Q.(9.90 / 9.9)	264 278 283

9.7 A ball attached to the end of a string is swung around in a circular path of radius r. If the radius is doubled and the linear speed is kept constant, the centripetal acceleration

a) remains the same.

b) increases by a factor of 2.

c) decreases by a factor of 2.

d) increases by a factor of 4.

e) decreases by a factor of 4.

c) decreases by a factor of 2.

New centripetal acceleration (a_{c2}) after doubling the radius:

$$a_{c2} = \frac{v^2}{2r}$$

Compare a_{c2} to a_{c1}:

$$a_{c2} = \frac{1}{2} \cdot \frac{v^2}{r} = \frac{1}{2} a_{c1}$$

8	↳ Describe centripetal force as the net inward force (towards the center of the circular path) needed to provide the centripetal acceleration necessary for circular motion. ↳ Solve problems related to acceleration in circular motion.	Student Book M.C.Q(9.7) Q.(9.90 / 9.9)	264 278 283

9.90 A flywheel of radius 27.01 cm rotates with a frequency of 4949 rpm. What is the centripetal acceleration at a point on the edge of the flywheel?

1. **Convert the frequency to angular velocity (▮):**

   Given the frequency $f = 4949\,\mathrm{rpm}$, first convert it to revolutions per second (Hz):

   $$f = \frac{4949}{60} \approx 82.4833\,\mathrm{Hz}$$

   The angular velocity ω in radians per second is:

   $$\omega = 2\pi f = 2\pi \times 82.4833 \approx 518.25\,\mathrm{rad/s}$$

2. **Convert the radius to meters:**

   Given the radius $r = 27.01\,\mathrm{cm}$, convert it to meters:

   $$r = 0.2701\,\mathrm{m}$$

3. **Calculate the centripetal acceleration (a_c):**

   The formula for centripetal acceleration is:

   $$a_c = \omega^2 r$$

   Substituting the values:

   $$a_c = (518.25)^2 \times 0.2701 \approx 72491.06\,\mathrm{m/s^2}$$

8	↳ Describe centripetal force as the net inward force (towards the center of the circular path) needed to provide the centripetal acceleration necessary for circular motion. ↳ Solve problems related to acceleration in circular motion.	Student Book M.C.Q(9.7) Q.(9.90 / 9.9)	264 278 283

9.9 You put three identical coins on a turntable at different distances from the center and then turn the motor on. As the turntable speeds up, the outermost coin slides off first, followed by the one at the middle distance, and, finally, when the turntable is going the fastest, the innermost one. Why is this?

a) For greater distances from the center, the centripetal acceleration is higher, and so the force of friction becomes unable to hold the coin in place.

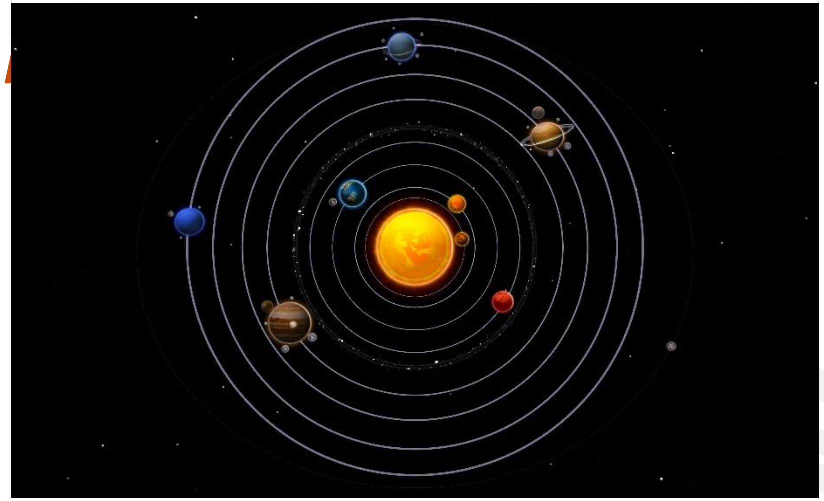
b) The weight of the coin causes the turntable to flex downward, so the coin nearest the edge falls off first.

c) Because of the way the turntable is made, the coefficient of static friction decreases with distance from the center.

d) For smaller distances from the center, the centripetal acceleration is higher.

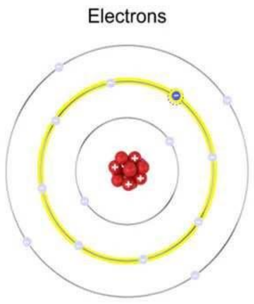
9	↳ Identify that the centripetal force can be provided by different forces (frictional force, tension, gravitational force, Coulomb force, or the normal force……). ↳ Solve problems related to centripetal force	Student Book Solved Problem 9.1 Additional Exercises 9.76	264 266 283

Examples of objects move in a circular



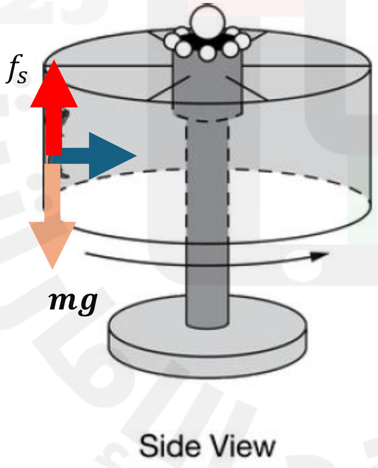
For Earth circling the Sun, the force is the Sun's gravitational force on Earth.

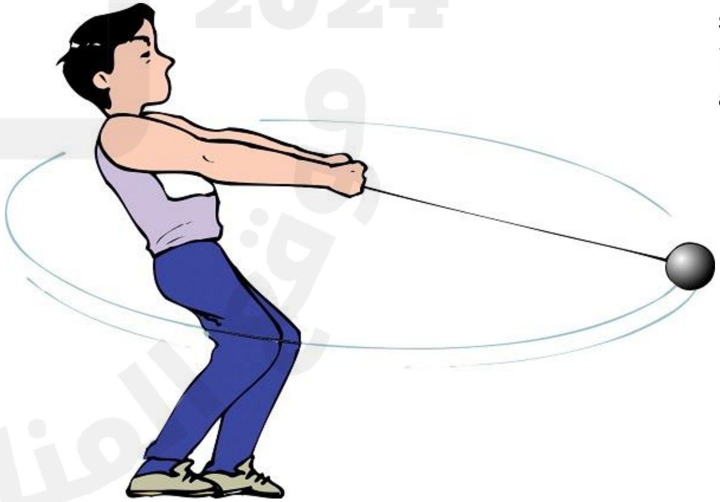
when a car is moving in a circle around the round about the force is the friction between the wheels of the car and the ground.



when electrons move around the nucleus the force is the electric force of attraction between the positive charge of the nucleus and the negative charge of the electrons.

In a Carnival Ride normal force provides CM





When a hammer thrower swings the hammer, the force is the tension in the chain attached to the massive ball.

9	↳ Identify that the centripetal force can be provided by different forces (frictional force, tension, gravitational force, Coulomb force, or the normal force......). ↳ Solve problems related to centripetal force	Student Book Solved Problem 9.1 Additional Exercises 9.76	264 266 283

SOLVED PROBLEM 9.1 Analysis of a Roller Coaster

Perhaps the biggest thrill to be had at an amusement park is on a roller coaster with a vertical loop in it (Figure 9.17), where passengers feel almost weightless at the top of the loop.

PROBLEM

Suppose the vertical loop has a radius of 5.00 m. What does the linear speed of the roller coaster have to be at the top of the loop for the passengers to feel weightless? (Assume that friction between roller coaster and rails can be neglected.)

FIGURE 9.17 Modern roller coaster with a vertical loop.

$$F_c = F_{net}$$

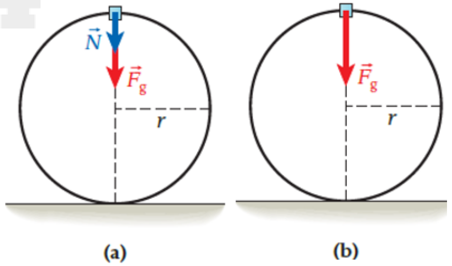
$$F_{net} = mg - N$$ Weightless N=0

$$F_c = m\frac{v^2}{r}$$

$$m\frac{v^2}{r} = mg$$

$$v = \sqrt{gr}$$

$$v = \sqrt{(5)(9.812)} = 7.00357\ m/s$$



9	↳ Identify that the centripetal force can be provided by different forces (frictional force, tension, gravitational force, Coulomb force, or the normal force......). ↳ Solve problems related to centripetal force	Student Book Solved Problem 9.1 Additional Exercises 9.76	264 266 283

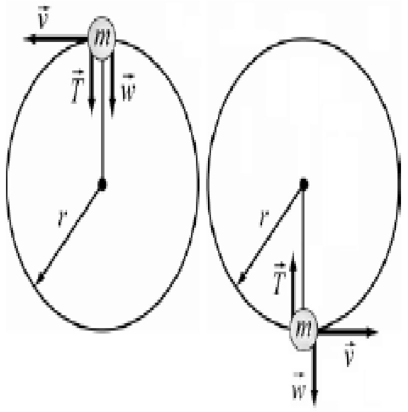
•9.76 A ball that has a mass of 1.00 kg is attached to a string 1.00 m long and is whirled in a vertical circle at a constant speed of 10.0 m/s.

a) Determine the tension in the string when the ball is at the top of the circle.

b) Determine the tension in the string when the ball is at the bottom of the circle.

c) Consider the ball at some point other than the top or bottom. What can you say about the tension in the string at this point?

SKETCH:



RESEARCH: $F_{net} = \dfrac{mv^2}{r}$

(a) $F_{net} = T + w$

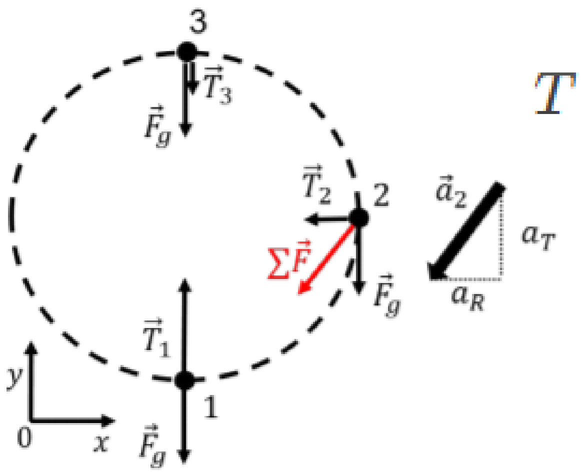
(b) $F_{net} = T - w$

SIMPLIFY:

(a) $T = F_{net} - w = \dfrac{mv^2}{r} - mg$

(b) $T = F_{net} + w = \dfrac{mv^2}{r} + mg$

CALCULATE: $\dfrac{mv^2}{r} = \dfrac{(1.00\text{ kg})(10.0\text{ m/s})^2}{1.00\text{ m}} = 100.\text{ N},\ mg = (1.00\text{ kg})(9.81\text{ m/s}^2) = 9.81\text{ N}$



$$T = \frac{mv^2}{r} + mg\cos\theta$$

10	↳ Define angular acceleration as the rate of change of an object's angular velocity ↳ Solve problems related to rotation with constant angular acceleration.	M.C.Q(9.8) Q.(9.60,9.61)	278 282

9.8 The angular speed of the hour hand of a clock (in radians per second) is

a) $\frac{\pi}{21,600}$ c) $\frac{\pi}{3600}$ e) $\frac{\pi}{60}$

b) $\frac{\pi}{7200}$ d) $\frac{\pi}{1800}$

- **12 hours** = 12 × 60 × 60 = **43,200 seconds**.

Step 3: Calculate Angular Speed (▮)

Angular speed is given by:

$$\omega = \frac{\text{Total angle rotated}}{\text{Time taken}} = \frac{2\pi \text{ radians}}{43,200 \text{ seconds}}$$

$$\omega = \frac{\pi}{21,600} \text{ radians/second}$$

Step 4: Match with Given Options

The correct answer is:

$$\boxed{a} \quad \left(\frac{\pi}{21,600} \text{ rad/s}\right)$$

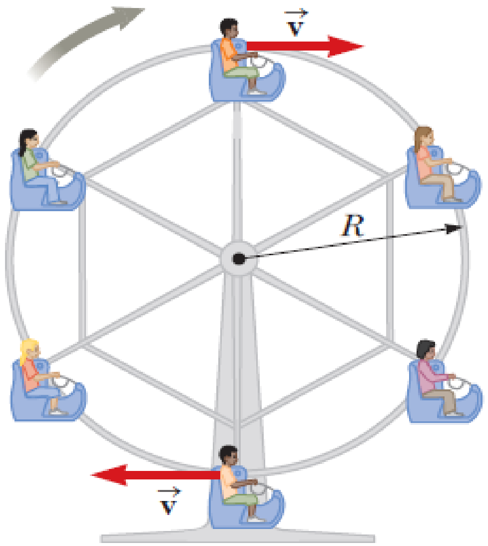
10	↳ Define angular acceleration as the rate of change of an object's angular velocity ↳ Solve problems related to rotation with constant angular acceleration.	M.C.Q(9.8) Q.(9.60,9.61)	278 282

9.61 A boy is on a Ferris wheel, which takes him in a vertical circle of radius 9.00 m once every 12.0 s.

a) What is the angular speed of the Ferris wheel?

b) Suppose the wheel comes to a stop at a uniform rate during one quarter of a revolution. What is the angular acceleration of the wheel during this time?

c) Calculate the tangential acceleration of the boy during the time interval described in part (b).



Given:

- Radius of Ferris wheel, $r = 9.00\,\text{m}$
- Period of rotation, $T = 12.0\,\text{s}$
- Stopping condition: Uniformly stops in $\frac{1}{4}$ revolution.

Part (a): Angular Speed of the Ferris Wheel

1. **Calculate angular speed (ω):**

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{12.0\,\text{s}} \approx 0.524\,\text{rad/s}$$

Answer for (a):

The angular speed of the Ferris wheel is $\boxed{0.524\,\text{rad/s}}$.

Alternatively, using $\omega = \omega_0 + \alpha t$ with $\omega = 0$:

$$\alpha = \frac{-\omega_0}{t} = \frac{-0.524}{3.00} \approx -0.175\,\text{rad/s}^2$$

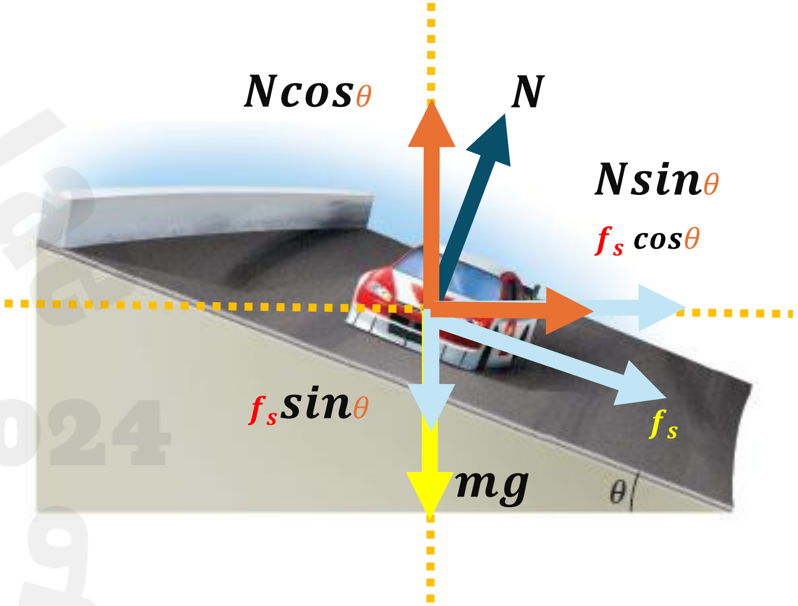
1. **Relate angular acceleration (α) to tangential acceleration (a_t):**

$$a_t = r\alpha = 9.00\,\text{m} \times (-0.175\,\text{rad/s}^2) \approx -1.58\,\text{m/s}^2$$

(Negative sign indicates deceleration.)

SOLVED PROBLEM 4.4 NASCAR Racing

As a NASCAR racer moves through a banked curve, the banking helps the driver achieve higher speeds. Let's see how. Figure 4.25 shows a race car on a banked curve. If the coefficient of static friction between the track surface and the car's tires is $\mu_s = 0.62$ and the radius of the turn is $R = 110m$. what is the maximum speed with which a driver can take a curve banked at $\theta = 21.1°$?



$$F_{net-y} = 0$$

$$N\cos\theta - f_s\sin\theta - mg = 0$$

$$N\cos\theta - \mu_s N\sin\theta - mg = 0$$

$$N(\cos\theta - \mu_s\sin\theta) = mg \text{.....(1)}$$

$$F_{net-x} = F_c$$

$$\mu_s N\cos\theta + N\sin\theta = \frac{mv^2}{R}$$

$$N(\mu_s\cos\theta + \sin\theta) = \frac{mv^2}{R} \text{......(2)}$$

$$\frac{N(\mu_s\cos\theta + \sin\theta)}{N(\cos\theta - \mu_s\sin\theta)} = \frac{\frac{mv^2}{R}}{mg}$$

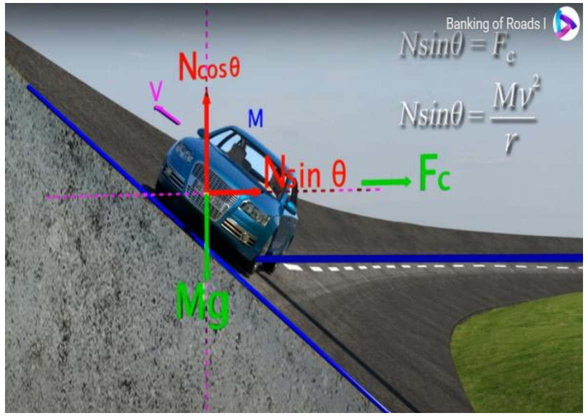
$$\frac{(\mu_s\cos\theta + \sin\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{v^2}{gR}$$

$$v = \sqrt{gR\frac{(\mu_s\cos\theta + \sin\theta)}{(\cos\theta - \mu_s\sin\theta)}}$$

$$v = \sqrt{(9.8)(110)\frac{(0.62)(\cos 21.1) + (\sin 21.1)}{(\cos 21.1) - (0.62)(\sin 21.1)}} = 37.8\ m/s$$

Centripetal Force

Use Newton's 2nd law to calculate unknown values of centripetal force, friction force, coefficient of friction, normal force, angle of banking, mass, speed, etc for a vehicle moving on a banked circular track.



A highway engineer needs to design a banked curve so that cars traveling at 90 km/h (25 m/s) can safely take the turn without relying on friction. If the curve must have a radius of 150 m, what should the banking angle (θ) be?

Given:

- **Speed, v=25 m/s**
- **Radius, r=150 m**
- **Frictionless surface**
- **g=9.81 m/s2**

Objective:

Find the required banking angle θ.

$$F_{net-y} = 0$$

$$N\cos\theta - mg = 0$$

$$N\cos\theta = mg \dots (1)$$

$$F_{net-x} = F_c = \frac{mv^2}{R}$$

$$N\sin\theta = \frac{mv^2}{R} \dots (2)$$

$$\frac{N\sin\theta}{N\cos\theta} = \frac{\frac{mv^2}{R}}{mg}$$

$$\tan\theta = \frac{v^2}{gR}$$

$$v = \sqrt{gR\tan\theta}$$

$$25 = \sqrt{(9.8)(150)\tan(x)}$$

θ = 23

11	Solve problems related to circular motion.	Solved Problem 9.4 Q. (9.59)	275 282

••9.59 A speedway turn, with radius of curvature R, is banked at an angle θ above the horizontal.

a) What is the optimal speed at which to take the turn if the track's surface is iced over (that is, if there is very little friction between the tires and the track)?

b) If the track surface is ice-free and there is a coefficient of friction μ_s between the tires and the track, what are the maximum and minimum speeds at which this turn can be taken?

c) Evaluate the results of parts (a) and (b) for $R = 400$ m, $\theta = 45.0°$, and $\mu_s = 0.700$.

$$v = \sqrt{gR\,tan\theta}$$ .No friction

$$v = \sqrt{gR\frac{(\mu_s cos\theta + sin\theta)}{(cos\theta - \mu_s sin\theta)}}$$. With friction

12	Describe that the moment of inertia plays the same role for rotational (or circular) motion as the mass does for linear motion.	Student Book Solved Problem10.1	285~286 294

Several Point Particles in Circular Motion

Just as we proceeded in Chapter 8 in finding the location of the center of mass of a system of particles, we start with a collection of individual rotating objects and then approach the continuous limit. The kinetic energy of a collection of rotating objects is given by

$$K = \sum_{i=1}^{n} K_i = \tfrac{1}{2}\sum_{i=1}^{n} m_i v_i^2 = \tfrac{1}{2}\sum_{i=1}^{n} m_i r_i^2 \omega_i^2.$$

This result is simply a consequence of using equation 10.2 for several point particles and writing the total kinetic energy as the sum of the individual kinetic energies. Here ω_i is the angular velocity of particle i and r_i is the perpendicular distance from i to a fixed axis. This fixed axis is the **axis of rotation** for these particles. An example of a system of five rotating point particles is shown in Figure 10.4.

Now we assume that all of the point particles whose kinetic energies we have summed keep their distances with respect to one another and with respect to the axis of rotation fixed. Then, all of the point particles in the system will undergo circular motion around the common axis of rotation with the same angular velocity. With this constraint, the sum of the particles' kinetic energies becomes

$$K = \tfrac{1}{2}\sum_{i=1}^{n} m_i r_i^2 \omega^2 = \tfrac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega^2 = \tfrac{1}{2} I \omega^2. \qquad (10.3)$$

The quantity I introduced in equation 10.3 is called the **moment of inertia**, also known as the *rotational inertia*. It depends only on the masses of the individual particles and their distances to the axis of rotation:

$$I = \sum_{i=1}^{n} m_i r_i^2. \qquad (10.4)$$

In Chapter 9, we saw that all quantities associated with circular motion have equivalents in linear motion. The angular velocity ω and the linear velocity v form such a pair. Comparing the expressions for the kinetic energy of rotation (equation 10.3) and the kinetic energy of linear motion (equation 10.1), we see that the moment of inertia I plays the same role for circular motion as the mass m does for linear motion.

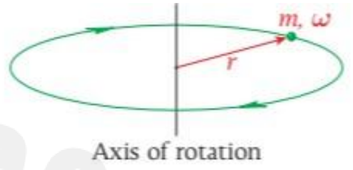


FIGURE 10.3 A point particle moving in a circle about the axis of rotation.

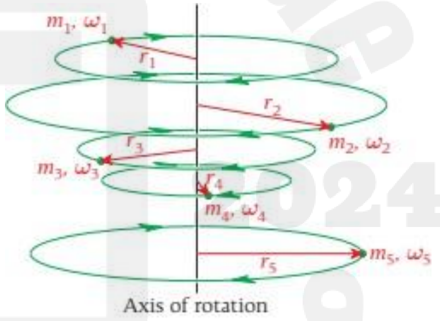
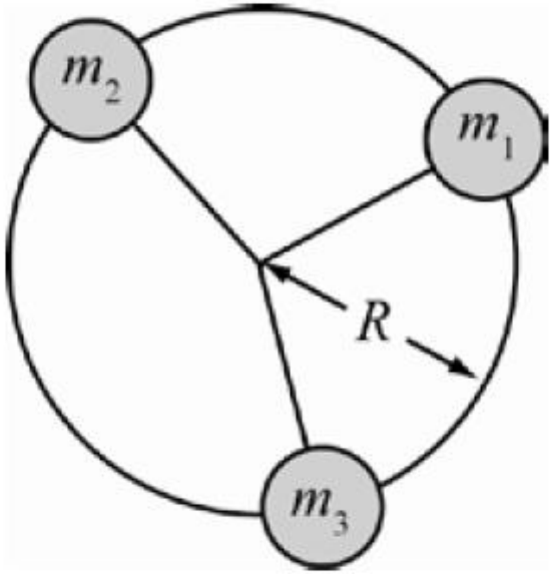


FIGURE 10.4 Five point particles moving in circles about a common axis of rotation.

Q10.39 Determine the moment of inertia for three children weighing 60.0 kg, 45.0 kg, and 80.0 kg sitting at different points on the edge of a rotating merry-go-round, which has a radius of 2.0 m.

$$I = \sum mr^2 \quad = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= (60)(2)^2 + (45)(2)^2 + (80)(2)^2 = 740\ Kg.m^2$$



Rotational Kinetic Energy of Rigid Bodies

Calculate the rotational kinetic energy of a point particle, or several point particles, rotating about a fixed axis

Ex: A cylinder rotates with constant angular acceleration about a fixed axis. The cylinder's moment of inertia about the axis is $4\ kg.m^2$. At time $t = 0$ the cylinder is at rest. At time $t = 2$ seconds its angular velocity is 1 radian per second.

A- What is the angular acceleration of the cylinder between $t = 0$ and $t = 2$ seconds?

A) $0.5\ rad/s^2$ B) $1\ rad/s^2$ C) $2\ rad/s^2$ D) $4\ rad/s^2$ E) $5\ rad/s^2$

$$\alpha = \frac{\Delta\omega}{\Delta t} \qquad = \frac{1-0}{2} = 0.5\ rad/s^2$$

B- What is the kinetic energy of the cylinder at time $t = 2$ seconds?

A) $1\ J$ B) $2\ J$ C) $3\ J$ D) $4\ J$

E) *connot be determind without Knowing the radius*

$$K.E = \frac{1}{2} I\omega^2 \qquad = \frac{1}{2}(4)(1)^2 = 2\ J$$

12	Describe that the moment of inertia plays the same role for rotational (or circular) motion as the mass does for linear motion.	Student Book Solved Problem10.1	285~286 294

SOLVED PROBLEM 10.1 Sphere Rolling Down an Inclined Plane

PROBLEM

A solid sphere with a mass of 5.15 kg and a radius of 0.340 m starts from rest at a height of 2.10 m above the base of an inclined plane and rolls down without sliding under the influence of gravity. What is the linear speed of the center of mass of the sphere just as it leaves the incline and rolls onto a horizontal surface?

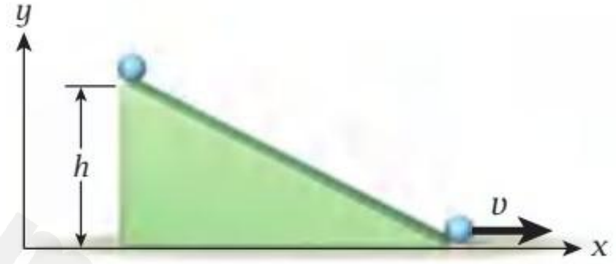


FIGURE 10.12 Sphere rolling down an inclined plane.

$$M.E_A = M.E_B$$

$$(K_{total}+U)_A = (K_{total}+U)_B$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{v^2}{R^2}$$

$$v^2 = \frac{2gh}{1+c} \implies v = \sqrt{\frac{2gh}{1+c}}$$

. **Solid Sphere** $(c = \frac{2}{5})$:

$$v = \sqrt{\frac{2gh}{1+\frac{2}{5}}} = \sqrt{\frac{10gh}{7}} \approx 1.195\sqrt{gh}$$

13	Identify that torque is a vector quantity, measured in the SI units of Nm.	Student Book	297~298

and downright impossible in Figure 10.16a. This example shows that the magnitude of the force is not the only relevant quantity. The perpendicular distance from the line of action of the force to the axis of rotation, called the **moment arm,** is also important. In addition, the angle at which the force is applied, relative to the moment arm, matters as well. In parts (b) and (c) of Figure 10.16, this angle is 90°. (An angle of 270° would be just as effective, but then the force would act in the opposite direction.) An angle of 180° or 0° (Figure 10.16a) will not turn the bolt.

These considerations are quantified by the concept of torque, τ. **Torque** (also called *moment*) is the vector product of the force $\vec{F}$ and the position vector $\vec{r}$:

$$\vec{\tau} = \vec{r} \times \vec{F}. \tag{10.16}$$

The position vector $\vec{r}$ is measured with the origin at the axis of rotation. The symbol × denotes the **vector product,** or *cross product*. (We introduced vector products in Chapter 1, which you may want to consult for a review.)

The SI unit of torque is N m, not to be confused with the unit of energy, which is the joule (J = N m)

$$[\tau] = [F] \cdot [r] = \text{N m}.$$

In English units, torque is often expressed in foot-pounds (ft-lb).

The magnitude of the torque is the product of the magnitude of the force and the distance to the axis of rotation (the magnitude of the position vector, or the moment arm) times the sine of the angle between the force vector and the position vector (see Figure 10.17):

$$\tau = rF\sin\theta. \tag{10.17}$$

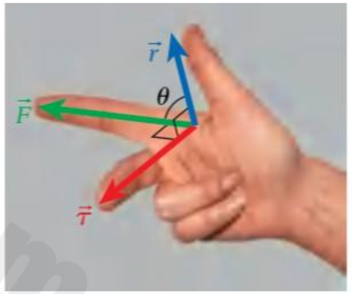


FIGURE 10.17 Right-hand rule for the direction of the torque for a given force and position vector.

14	Describe that a torque ($\vec{\tau}$) on a body involves a force ($\vec{F}$) and a position vector ($\vec{r}$), which extends from a rotation axis to the point where the force is applied.	Student Book Concept Check 10.4	297~298 298

and downright impossible in Figure 10.16a. This example shows that the magnitude of the force is not the only relevant quantity. The perpendicular distance from the line of action of the force to the axis of rotation, called the **moment arm,** is also important. In addition, the angle at which the force is applied, relative to the moment arm, matters as well. In parts (b) and (c) of Figure 10.16, this angle is 90°. (An angle of 270° would be just as effective, but then the force would act in the opposite direction.) An angle of 180° or 0° (Figure 10.16a) will not turn the bolt.

These considerations are quantified by the concept of torque, τ. **Torque** (also called *moment*) is the vector product of the force $\vec{F}$ and the position vector $\vec{r}$:

$$\vec{\tau} = \vec{r} \times \vec{F}. \tag{10.16}$$

The position vector $\vec{r}$ is measured with the origin at the axis of rotation. The symbol × denotes the **vector product,** or *cross product*. (We introduced vector products in Chapter 1, which you may want to consult for a review.)

The SI unit of torque is N m, not to be confused with the unit of energy, which is the joule (J = N m)

$$[\tau] = [F] \cdot [r] = \text{N m}.$$

In English units, torque is often expressed in foot-pounds (ft-lb).

The magnitude of the torque is the product of the magnitude of the force and the distance to the axis of rotation (the magnitude of the position vector, or the moment arm) times the sine of the angle between the force vector and the position vector (see Figure 10.17):

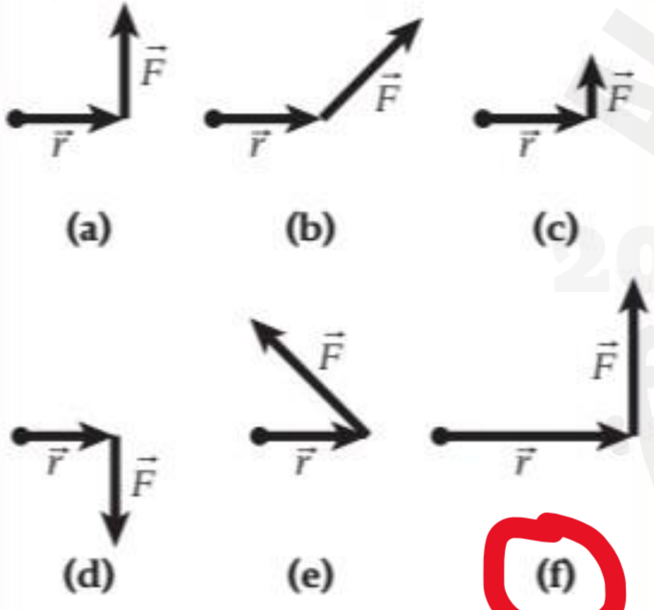
$$\tau = rF\sin\theta. \tag{10.17}$$

(a) (b) (c) (d)

14	Describe that a torque ($\vec{\tau}$) on a body involves a force ($\vec{F}$) and a position vector ($\vec{r}$), which extends from a rotation axis to the point where the force is applied.	Student Book Concept Check 10.4	297~298 298

Concept Check 10.4

Choose the combination of position vector, $\vec{r}$, and force vector, $\vec{F}$, that produces the torque of highest magnitude around the point indicated by the black dot.



Key Observations for Maximum Torque:

1. **Perpendicularity**: Torque is maximized when $\vec{F}$ is perpendicular to $\vec{r}$ ($\theta = 90^\circ$, so $\sin\theta = 1$).
2. **Large Magnitudes**: Both $|\vec{r}|$ and $|\vec{F}|$ should be as large as possible.
3. **Lever Arm**: A longer $\vec{r}$ (greater distance from pivot) increases torque.

15	Calculate the torque due to a force on a particle by taking the cross product of the particle's position vector and the force vector. $\vec{\tau} = \vec{r} \times \vec{F}$,, $\tau = rFsin(\theta)$	Student Book Q.(10.47 / 10.48) Q.(10.49/a)	297~298 318 319

These considerations are quantified by the concept of torque, τ. **Torque** (also called *moment*) is the vector product of the force $\vec{F}$ and the position vector $\vec{r}$

$$\vec{\tau} = \vec{r} \times \vec{F}. \tag{10.16}$$

The position vector $\vec{r}$ is measured with the origin at the axis of rotation. The symbol × denotes the **vector product,** or *cross product*. (We introduced vector products in Chapter 1, which you may want to consult for a review.)

The SI unit of torque is N m, not to be confused with the unit of energy, which is the joule (J = N m)

$$[\tau] = [F] \cdot [r] = \text{N m}.$$

In English units, torque is often expressed in foot-pounds (ft-lb).

The magnitude of the torque is the product of the magnitude of the force and the distance to the axis of rotation (the magnitude of the position vector, or the moment arm) times the sine of the angle between the force vector and the position vector (see Figure 10.17):

$$\tau = rF\sin\theta. \tag{10.17}$$

15	Calculate the torque due to a force on a particle by taking the cross product of the particle's position vector and the force vector. $\vec{\tau} = \vec{r} \times \vec{F}$,, $\tau = rF\sin(\theta)$	Student Book Q.(10.47 / 10.48) Q.(10.49/a)	297~298 318 319

•10.47 A disk with a mass of 30.0 kg and a radius of 40.0 cm is mounted on a frictionless horizontal axle. A string is wound many times around the disk and then attached to a 70.0 kg block, as shown in the figure. Find the acceleration of the block, assuming that the string does not slip.

$$a = \frac{m_1 - m_2}{m_1 + m_2 + \frac{1}{2}m_p} g.$$

$$a = \frac{(70.0)(9.81)}{70.0 + \frac{1}{2}(30.0)} = \frac{686.7}{70.0 + 15.0} = \frac{686.7}{85.0} \approx 8.08\,\mathrm{m/s^2}$$

•10.48 A force, $\vec{F} = (2\hat{x} + 3\hat{y})$ N, is applied to an object at a point whose position vector with respect to the pivot point is $\vec{r} = (4\hat{x} + 4\hat{y} + 4\hat{z})$ m. Calculate the torque created by the force about that pivot point.

1. **Express $\vec{r}$ and $\vec{F}$ in component form:**

$$\vec{r} = 4\hat{x} + 4\hat{y} + 4\hat{z}\,\mathrm{m}, \quad \vec{F} = 2\hat{x} + 3\hat{y}\,\mathrm{N}$$

2. **Compute the cross product $\vec{r} \times \vec{F}$:**

$$\vec{\tau} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 4 & 4 & 4 \\ 2 & 3 & 0 \end{vmatrix}$$

   - **$\hat{x}$-component:**
   - **$\hat{y}$-component:**
   - **$\hat{z}$-component:**

3. **Combine the components to write the torque vector:**

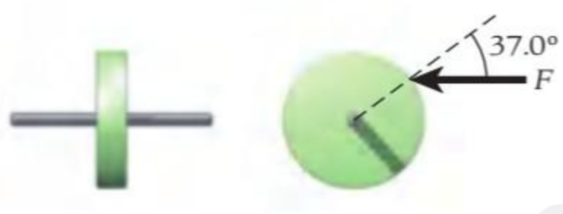
$$\vec{\tau} = -12\hat{x} + 8\hat{y} + 4\hat{z}$$

4. **Magnitude of the Torque (Optional):**

$$|\vec{\tau}| = \sqrt{(-12)^2 + 8^2 + 4^2} = \sqrt{144 + 64 + 16} = \sqrt{224} \approx 14.97$$

15	Calculate the torque due to a force on a particle by taking the cross product of the particle's position vector and the force vector. $\vec{\tau} = \vec{r} \times \vec{F}$,, $\tau = rF\sin(\theta)$	Student Book Q.(10.47 / 10.48) Q.(10.49/a)	297~298 318 319

••10.49 A disk with a mass of 14.0 kg, a diameter of 30.0 cm, and a thickness of 8.00 cm is mounted on a rough horizontal axle as shown on the left in the figure. (There is a friction force between the axle and the disk.) The disk is initially at rest. A constant force, $F = 70.0$ N, is applied to the edge of the disk at an angle of 37.0°, as shown on the right in the figure. After 2.00 s, the force is reduced to $F = 24.0$ N, and the disk spins with a constant angular velocity.



a) What is the magnitude of the torque due to friction between the disk and the axle?

To calculate the **torque due to the applied force** during the initial 2.00 s (when the force is $F_1 = 70.0\,\mathrm{N}$), we use the torque formula:

$$\tau_{\text{applied}} = R \cdot F_1 \cdot \sin(\theta)$$

Given:

- $R = 0.150\,\mathrm{m}$
- $F_1 = 70.0\,\mathrm{N}$
- $\theta = 37.0^\circ \Rightarrow \sin(37.0^\circ) \approx 0.6018$

Calculation:

$$\tau_{\text{applied}} = 0.150 \cdot 70.0 \cdot 0.6018$$

$$\tau_{\text{applied}} \approx 6.31\,\mathrm{N} \cdot \mathrm{m}$$

Torque from the applied force:

$$\tau_{\text{applied}} = R \cdot F \cdot \sin(\theta)$$

At steady-state (constant angular velocity), net torque is zero, so:

$$\tau_{\text{applied}} = \tau_{\text{friction}} \quad \text{(since they balance)}$$

Use the **reduced force** $F_2 = 24.0\,\mathrm{N}$:

$$\tau_{\text{friction}} = R \cdot F_2 \cdot \sin(37.0^\circ)$$

$$\tau_{\text{friction}} = 0.150 \cdot 24.0 \cdot \sin(37.0^\circ)$$

$$\tau_{\text{friction}} = 0.150 \cdot 24.0 \cdot 0.6018 \approx 2.17\,\mathrm{N} \cdot \mathrm{m}$$