

سلسلة الطارق في الرياضيات



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف الحادي عشر المتقدم ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

تاريخ إضافة الملف على موقع المناهج: 00:28:10 2026-06-22

ملفات اكتب للمعلم اكتب للطالب | اختبارات الكترونية | اختبارات | حلول | عروض بوربوينت | أوراق عمل
منهج انجليزي | ملخصات وتقارير | مذكرات وبنوك | الامتحان النهائي | للمدرس

المزيد من مادة
رياضيات:

إعداد: طارق علي

التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم



صفحة المناهج
الإماراتية على
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

المواد على تلغرام

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الثالث

حلول ملزمة نهاية الفصل الثالث ريفيل وفق الهيكل

1

End of Term 3 Revision

2

End of Term 3 Revision Maths

3

كراسة تدريبية للاختبار النهائي 2026م

4

منهج ريفيل حلول مراجعة امتحانية تدريبات وأسئلة

5

الطارق

الرياضيات

منصة طارق أكاديمي للرياضيات

Tarek Academy

مرفق حادي عشر (Elite)

الطارق

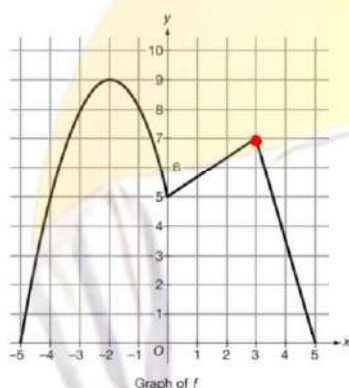
أستاذ الرياضيات

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1. The continuous function f is defined on the closed interval $[-5, 5]$. The graph of f consists of a parabola and two-line segments, as shown in the figure above. Let g be a function such that $g'(x) = f(x)$. Find all values of x in the open interval $(-5, 5)$ at which the graph of g has a point of inflection. Explain your reasoning.

x	$0 < x < 3$	$x = 3$	$3 < x < 9$	$x = 9$	$9 < x < 11$	$x = 11$	$11 < x < 16$
$f'(x)$	Positive	Und.	Negative	-3	Negative	0	Positive
$f''(x)$	Positive	Und.	Negative	0	Positive	0	Positive

The function f is continuous on the interval $(0, 16)$, and f is twice differentiable except at $x = 3$, where the derivatives are undefined. Information about the first and second derivatives of f for values of x in the interval $(0, 16)$ is given in the table above. At what values of x in the interval $(0, 16)$ does the graph of f have a point of inflection?



- A. $x = 9$ only
 B. $x = 3$ and $x = 9$
 C. $x = 3$ and $x = 11$
 D. $x = 9$ and $x = 11$

2. Let f be the function defined by $f(x) = \frac{1}{3}x^3 - 3x^2 - 16x$. On which of the following intervals is the graph of f both decreasing and concave down?

A. $(-\infty, 3)$

B. $(-2, 3)$ only

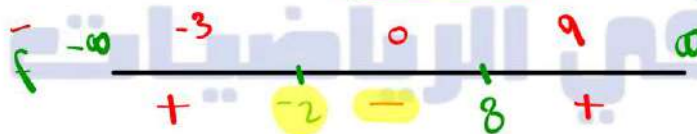
C. $(3, 8)$

D. $(8, \infty)$

$$f' = x^2 - 6x - 16 = 0 \quad x = 8$$

$$f' = 2x - 6 = 0 \quad x = -2$$

$$|x = 3|$$





4. Let f be the function defined by $f(x) = 13x^3 - 4x^2 - 9x + 5$. On which of the following intervals is the graph of f both decreasing and concave down?

A. $(-\infty, 4)$

B. $(-1, 4)$

C. $(4, 9)$

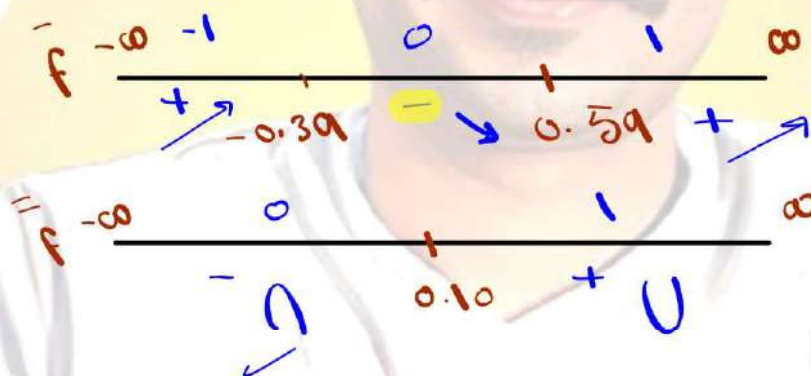
D. $(9, \infty)$

$$f' = 39x^2 - 8x - 9 = 0$$

$$x = 0.59 \quad | \quad x = -0.39$$

$$f'' = 78x - 8 = 0$$

$$x = 0.10$$



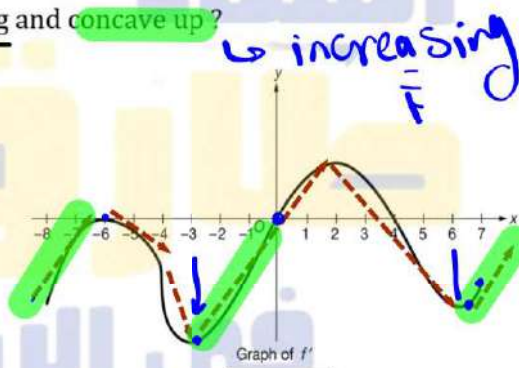
6. The graph of f' , the derivative of the continuous function f , is shown above on the interval $-8 < x < 7$. The graph of f' has horizontal tangent lines at $x = -6$, $x = -3$, $x = 2$, and $x = 6.3$, and a vertical tangent line at $x = -4$. On which of the following intervals is the graph of f both decreasing and concave up?

A. $(-8, 0)$ and $(4, 7)$

B. $(-6, -3)$ and $(4, 6.3)$ only

C. $(-4, -3)$ and $(4, 6.3)$ only

D. $(-8, -6)$, $(-3, 0)$, and $(6.3, 7)$ only



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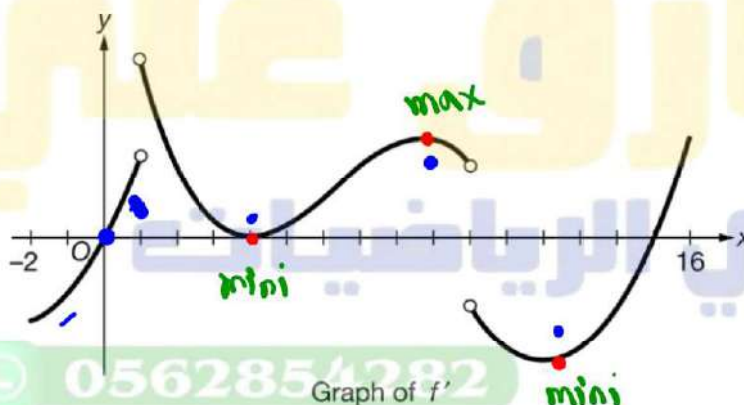
x	$0 < x < 2$	$x = 2$	$2 < x < 4$	$x = 4$	$4 < x < 6$	$x = 6$	$6 < x < 8$	$x = 8$	$8 < x < 9$
$f'(x)$	Unknown	0	Negative	0	Negative	DNE	Positive	0	Unknown
$f''(x)$	Negative	-1	Negative	0	Positive	DNE	Negative	0	Unknown

7. The function f is continuous on the interval $(0,9)$ and is twice differentiable except at $x = 6$, where the derivatives do not exist (DNE). Information about the first and second derivatives of f for some values of x in the interval $(0,9)$ is given in the table above. Which of the following statements could be false?

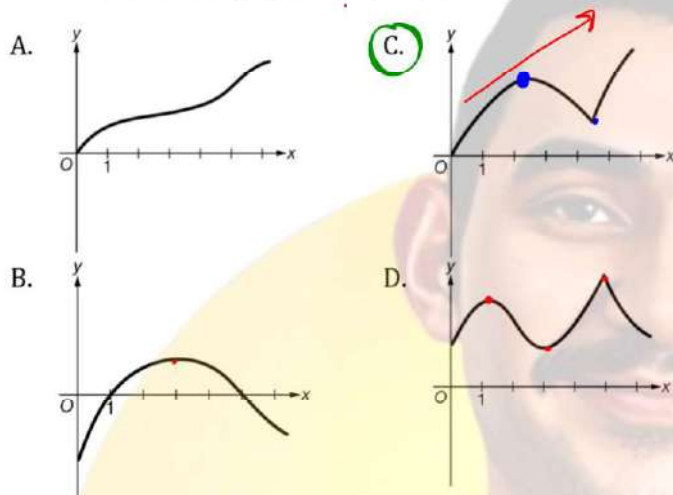
- ☒ A. The function f has a relative maximum at $x = 2$. ✓
☒ B. The graph of f has a point of inflection at $x = 4$. ✓
☒ C. The function f has a relative minimum at $x = 6$. ✓
☒ D. The graph of f has a point of inflection at $x = 8$. False
- inflection Point
 $x = 4$
 $x = 6$

8. The graph of f' , the derivative of the continuous function f , is shown above on the interval $-2 < x < 16$. Which of the following statements is true about f on the interval $-2 < x < 16$?

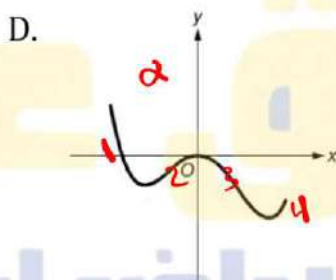
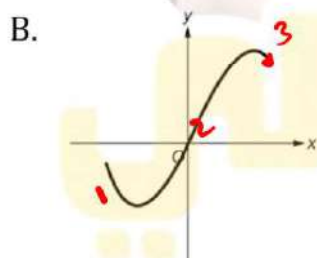
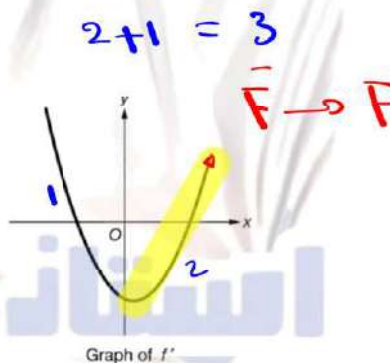
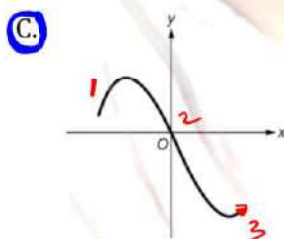
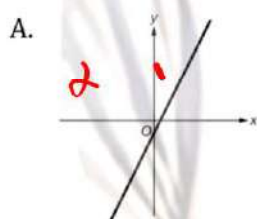
- ☒ A. f has two relative extrema, and the graph of f has three points of inflection.
☐ B. f has three relative extrema, and the graph of f has three points of inflection.
☒ C. f has three relative extrema, and the graph of f has four points of inflection.
☐ D. f has five relative extrema, and the graph of f has four points of inflection.



9. The function f is differentiable and increasing on the interval $0 \leq x \leq 6$, and the graph of f has exactly two points of inflection on this interval. Which of the following could be the graph of f' , the derivative of f ?



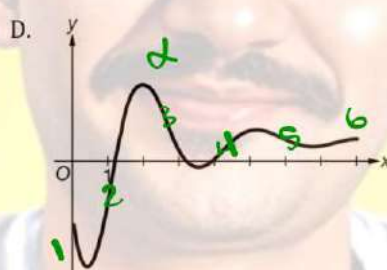
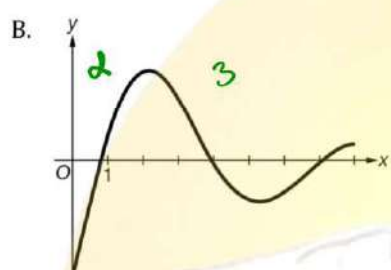
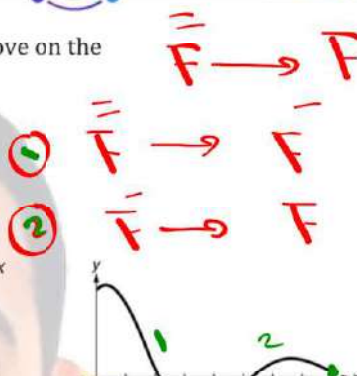
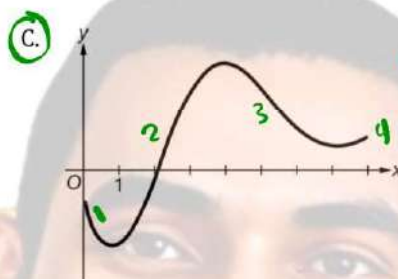
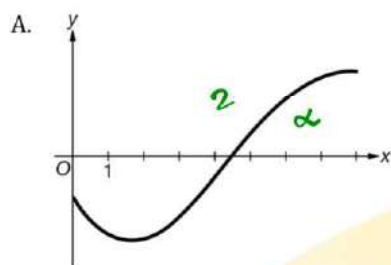
10. The graph of f' , the derivative of the function f , is shown above. Which of the following could be the graph of f ?



To Find F
① inverse directions
but after also
leave right side
② increasing lines
lines + 1

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11. The graph of f'' , the second derivative of the function f , is shown above on the interval $0 \leq x \leq 8$. Which of the following could be the graph of f ?

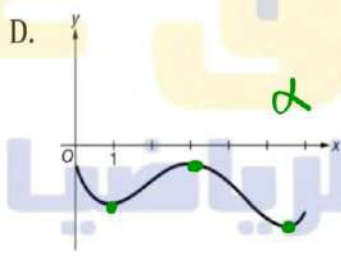
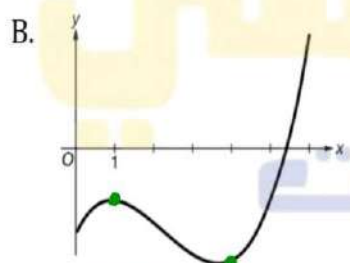
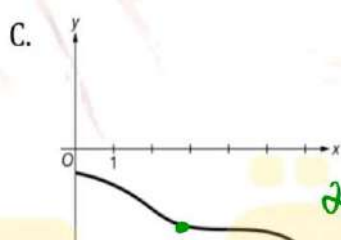
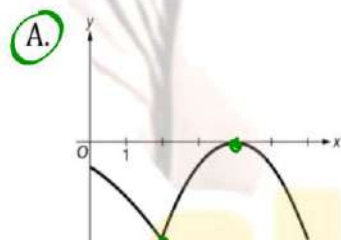
Graph of f''

$$2+1=3$$

$$3+1=4$$

under x-axis always

12. The function f is differentiable and decreasing on the interval $0 \leq x \leq 6$, and the graph of f has exactly two points of inflection on this interval. Which of the following could be the graph of f' , the derivative of f ?



$$F \rightarrow F$$

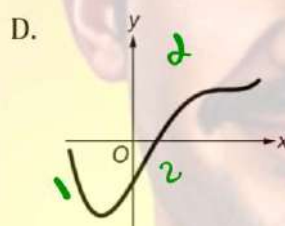
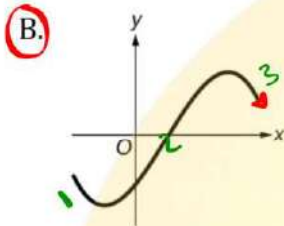
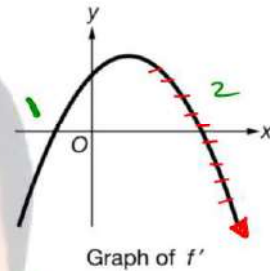
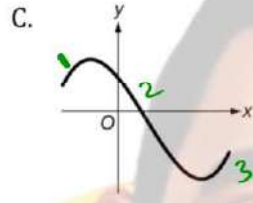
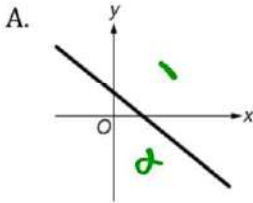
* inverse directions only

* lines - 1

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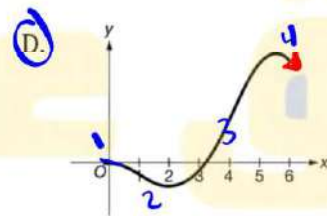
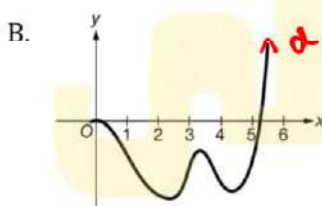
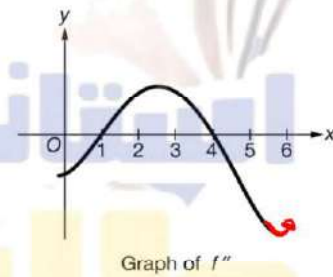
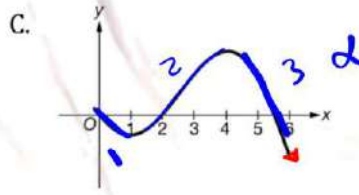
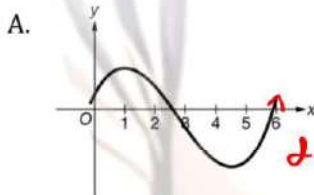


13. The graph of f' , the derivative of the function f , is shown above. Which of the following could be the graph of f ?



① $\bar{F} \rightarrow \bar{F}$
 ② lines + 1
 $2+1 = |3|$

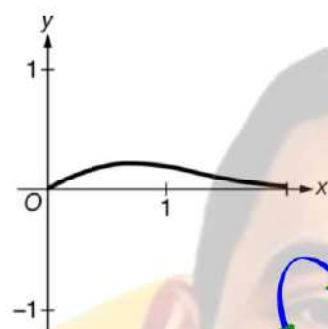
14. The graph of f'' , the second derivative of the function f , is shown above on the interval $0 \leq x \leq 6$. Which of the following could be the graph of f ?



① $\bar{F} \rightarrow \bar{F}$
 ② $\bar{F} \rightarrow \bar{F}$
 مذكر الإجابة ١٨٦٦٦

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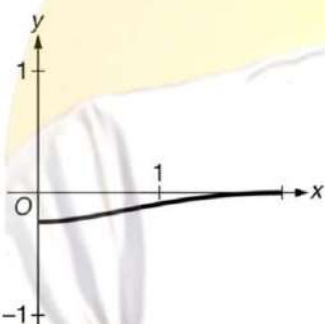


Graph of f

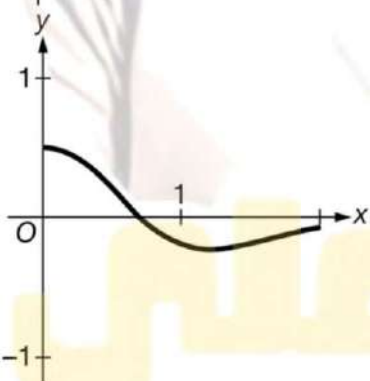
$f \rightarrow \bar{f}$
lines - 1
inverse directions
From Left $\rightarrow$ Right

15. The graph of $y = f(x)$ is shown above. Which of the following could be the graph of $y = f''(x)$?

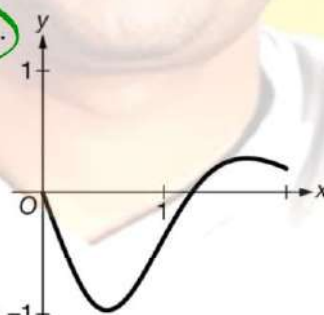
A.



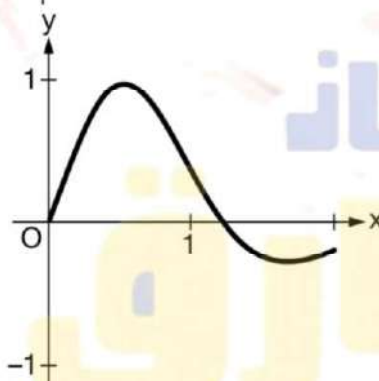
B.



C.

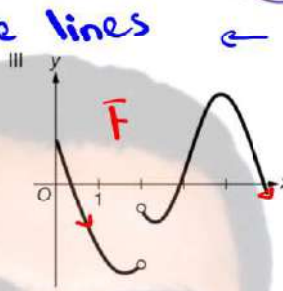
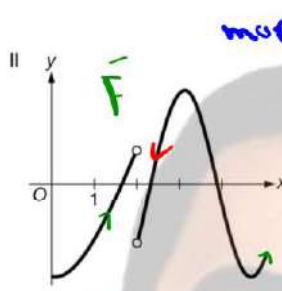
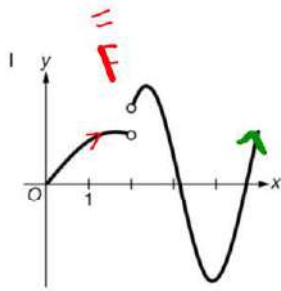


D.



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more lines

* f - original

* f' lines - 1
inverse

* f'' - inverse
of f'

16. Three graphs labeled I, II, and III are shown above. One is the graph of f , one is the graph of f' , and one is the graph of f'' . Which of the following correctly identifies each of the three graphs?

A. $f(x)$ $f'(x)$ $f''(x)$
I II III

C. $f(x)$ $f'(x)$ $f''(x)$
III I II

B. $f(x)$ $f'(x)$ $f''(x)$
II III I

D. $f(x)$ $f'(x)$ $f''(x)$
III II I

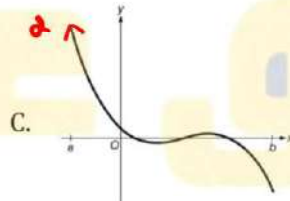
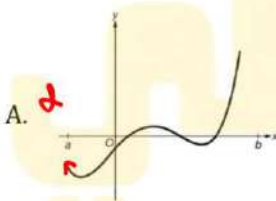
$f \rightarrow f'$

more right side
Same.



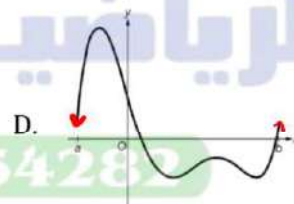
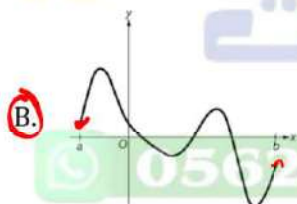
$f \rightarrow f'$

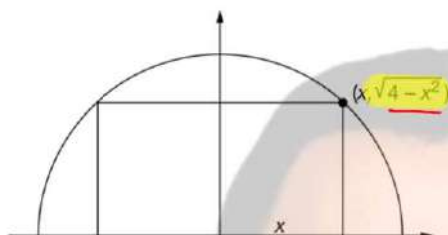
17. The figure above shows the graph of f on the interval $[a, b]$. Which of the following could be the graph of f' , the derivative of f , on the interval $[a, b]$?



* lines - 1 $\Rightarrow 4.1 = 3$

* inverse directions





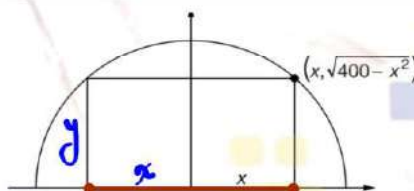
18. An electrical power station is located on the edge of a lake, as shown in the figure above. An electrical line needs to be connected from the station to an island in the lake that is located 4 miles due south and 1-mile due east of the station. It costs \$5,000 per mile to install an electrical line on land and \$10,000 per mile to install an electrical line underwater. If C represents a cost function, which of the following methods best explains how to determine the minimum cost, in dollars, for connecting the electrical line from the station to the island?

Per
every
each
5000x

- A. Let $C(x) = 10,000\sqrt{(4-x)^2 + 1} + 5,000x$. Solve $C'(x) = 0$ and find the values of x where $C'(x)$ changes sign from positive to negative. Evaluate C for those values of x to determine the minimum cost.
- B. Let $C(x) = 10,000\sqrt{(4-x)^2 + 1} + 5,000x$. Solve $C'(x) = 0$ and find the values of x where $C'(x)$ changes sign from negative to positive. Evaluate C for those values of x to determine the minimum cost.
- C. Let $C(x) = 10,000\sqrt{x^2 + 1} + 5,000x$. Solve $C'(x) = 0$ and find the values of x where $C'(x)$ changes sign from positive to negative. Evaluate C for those values of x to determine the minimum cost.
- D. Let $C(x) = 10,000\sqrt{x^2 + 1} + 5,000x$. Solve $C'(x) = 0$ and find the values of x where $C'(x)$ changes sign from negative to positive. Evaluate C for those values of x to determine the minimum cost.

$$A = 2 \cdot \sqrt{2} \cdot \sqrt{4 - (2)^2}$$

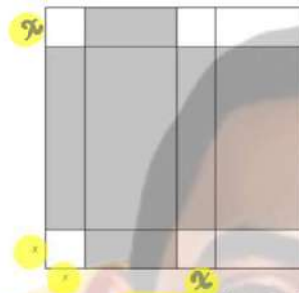
$$= 2\sqrt{2} \cdot \sqrt{2}$$



19. The figure above shows a rectangle inscribed in a semicircle with a radius of 2. The area of such a rectangle is given by $A(x) = 2x\sqrt{4-x^2}$, where the width of the rectangle is $2x$. It can be shown that $A'(x) = -\frac{2x^2}{\sqrt{4-x^2}} + 2\sqrt{4-x^2}$ and A has critical values of $-2, -\sqrt{2}, \sqrt{2}$ and 2 . It can also be shown that $A'(x)$ changes from positive to negative at $x = \sqrt{2}$. Which of the following statements is true?

- A. The inscribed rectangle with maximum area has dimensions $\sqrt{2}$ by $\sqrt{2}$.
- B. The inscribed rectangle with minimum area has dimensions $\sqrt{2}$ by $\sqrt{2}$.
- C. The inscribed rectangle with maximum area has dimensions $2\sqrt{2}$ by $\sqrt{2}$.
- D. The inscribed rectangle with minimum area has dimensions $2\sqrt{2}$ by $\sqrt{2}$.

maximum



$$h = x$$

$$l = 20 - 2x$$

$$w = \frac{l}{2} = \frac{20}{2} - \frac{2x}{2} = 10 - x$$

20. The figure above represents a square sheet of cardboard with side length 20 inches. The sheet is cut, and pieces are discarded. When the cardboard is folded, it becomes a rectangular box with a lid. The pattern for the rectangular box with a lid is shaded in the figure. Four squares with side length x and two rectangular regions are discarded from the cardboard. Which of the following statements is true? (The volume V of a rectangular box is given by $V = lwh$.)

- A. When $x = 10$ inches, the box has a minimum possible volume.
- B. When $x = 10$ inches, the box has a maximum possible volume.
- C. When $x = \frac{10}{3}$ inches, the box has a minimum possible volume.
- ☒ D. When $x = \frac{10}{3}$ inches, the box has a maximum possible volume.

$$V = l \times w \times h = (20 - 2x)(10 - x)(x)$$

$$V = (20 - 2x)(10x - x^2)$$

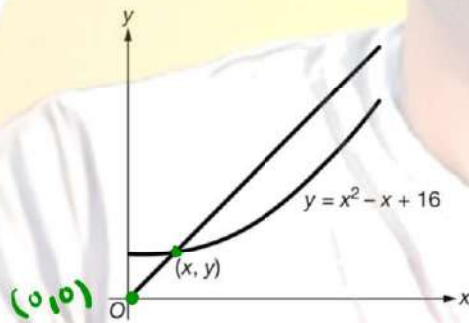
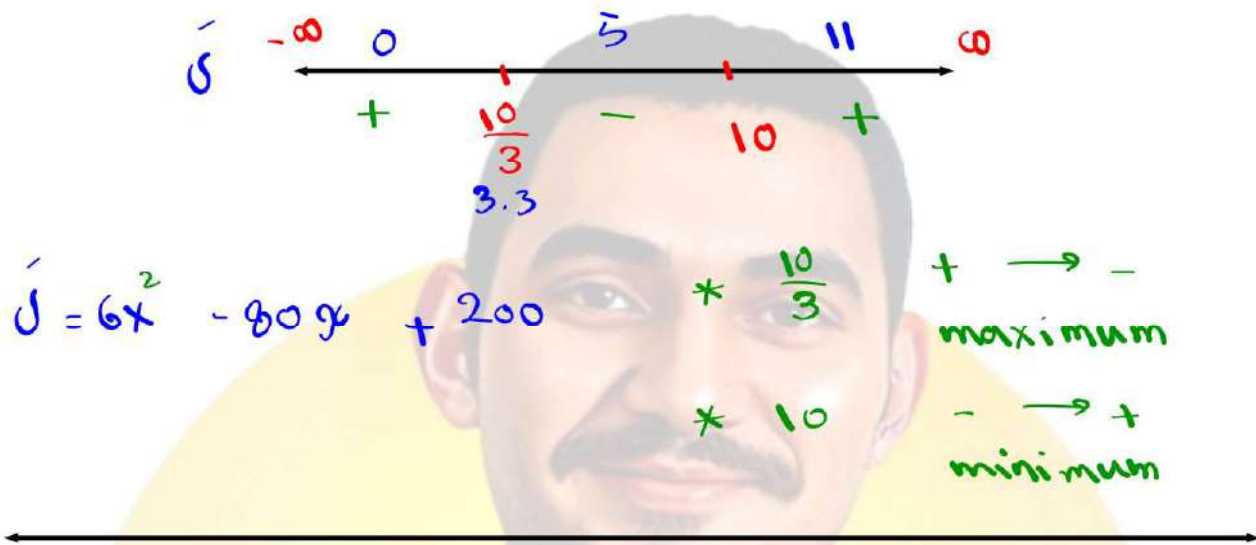
$$V = 200x - 20x^2 - 20x^2 + 2x^3$$

$$V = 2x^3 - 40x^2 + 200x$$

$$\rightarrow V' = 6x^2 - 80x + 200 = 0$$

$$x = 10, x = \frac{10}{3}$$

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21. Consider all lines in the xy -plane that pass through both the origin and a point (x, y) on the graph of $y = x^2 - x + 16$ for $1 \leq x \leq 8$. The figure above shows one such line and the graph of $y = x^2 - x + 16$. Which of the following statements is true?

- A. The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 1$.
- ☒ B. The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 4$.
- C. The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 7$.
- D. The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 8$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - 0}{x - 0} = \frac{y}{x}$$

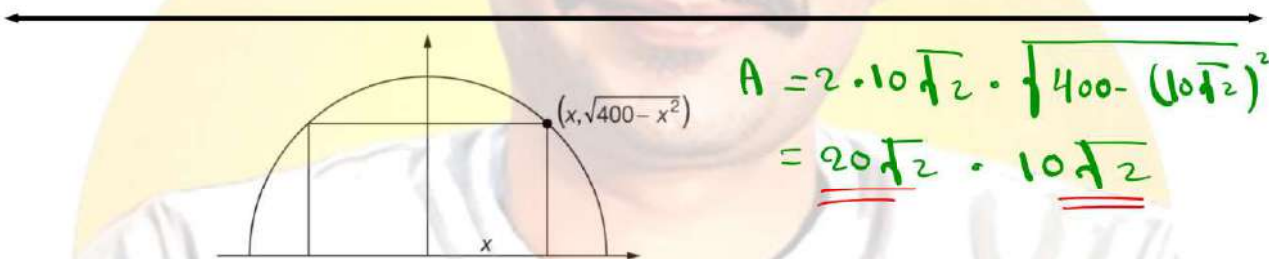
$$m = \frac{x^2 - x + 16}{x} = x - 1 + \frac{16}{x} \quad \#$$

$$m' = 1 - \cancel{0} - 16x^{-2} = 1 - \frac{16}{x^2} = 0 \quad \frac{16}{x} = 16x^{-1}$$

$$1 = \frac{16}{x^2} \rightarrow x^2 = 16 \rightarrow |x = \pm 4|$$

$$1 \leq x \leq 8$$

$$x = -4 \text{ مرفوض}, \quad |x = 4| \checkmark$$



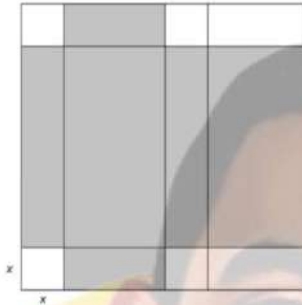
22. The figure above shows a rectangle inscribed in a semicircle with a radius of 20. The area of such a rectangle is given by $A(x) = 2x\sqrt{400 - x^2}$, where the width of the rectangle is $2x$. It can be shown that $A'(x) = -\frac{2x^2}{\sqrt{400 - x^2}} + 2\sqrt{400 - x^2}$ and A has critical values of -20 , $-10\sqrt{2}$, $10\sqrt{2}$, and 20 . It can also be shown that $A'(x)$ changes from positive to negative at $x = 10\sqrt{2}$. Which of the following statements is true?

- $\rightarrow + \rightarrow - \text{ max}$
- A. The inscribed rectangle with maximum area has dimensions $10\sqrt{2}$ by $10\sqrt{2}$.
 - B. The inscribed rectangle with minimum area has dimensions $10\sqrt{2}$ by $10\sqrt{2}$.
 - ☒ C. The inscribed rectangle with maximum area has dimensions $20\sqrt{2}$ by $10\sqrt{2}$.
 - D. The inscribed rectangle with minimum area has dimensions $20\sqrt{2}$ by $10\sqrt{2}$.

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$$l = 40 - 2x$$

$$w = \frac{40 - 2x}{2} = 20 - x$$

$$h = x$$

23. The figure above represents a square sheet of cardboard with side length 40 inches. The sheet is cut, and pieces are discarded. When the cardboard is folded, it becomes a rectangular box with a lid. The pattern for the rectangular box with a lid is shaded in the figure. Four squares with side length x and two rectangular regions are discarded from the cardboard. Which of the following statements is true? (The volume V of a rectangular box is given by $V = lwh$.)

- A. When $x = 20$ inches, the box has a minimum possible volume. → خطأ
- B. When $x = 20$ inches, the box has a maximum possible volume.
- C. When $x = \frac{20}{3}$ inches, the box has a minimum possible volume.
- ☒ D. When $x = \frac{20}{3}$ inches, the box has a maximum possible volume.

$$V = l + w + h = (40 - 2x)(20 - x)(x)$$

$$V = (40 - 2x)(20x - x^2)$$

$$= 800x - 40x^2 - 40x^2 + 2x^3$$

$$V = 2x^3 - 80x^2 + 800x$$

$$V' = 6x^2 - 160x + 800 = 0$$

$$x = 20$$

$$x = \frac{20}{3}$$

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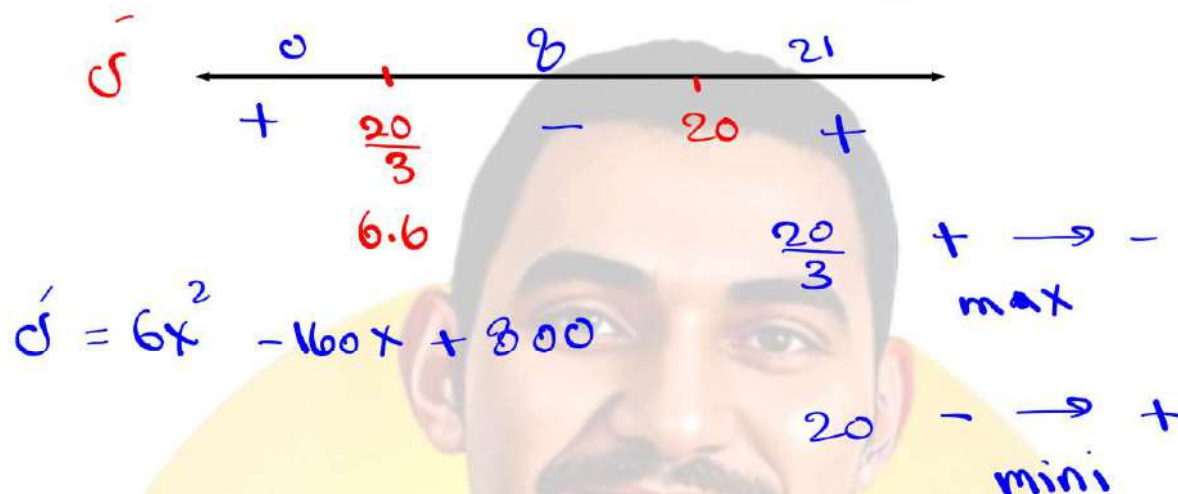
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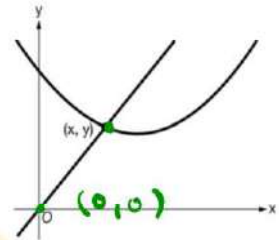
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24. Consider all lines in the xy -plane that pass through both the origin and a point (x, y) on the graph of $y = x^2 - 4x + 9$ for $1 \leq x \leq 4$. The figure above shows one such line and the graph of $y = x^2 - 4x + 9$. Which of the following statements is true?

- A. The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 1$.
- B. The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 2$.
- ☒ C. The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 3$.
- D. The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 4$.



$$* m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - 0}{x - 0} = \frac{y}{x} = \frac{x^2 - 4x + 9}{x}$$

$$m = x - 4 + \frac{9}{x} = x - 4 + 9x^{-1}$$

$$m' = 1 - 0 - 9x^{-2} = 1 - 9x^{-2} = 0$$

$$1 = \frac{9}{x^2} \rightarrow x^2 = 9$$

$$x = \pm 3$$

$$* x = 3 \in [1, 4]$$

$$x = -3 \notin [1, 4]$$

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25. The acceleration, in meters per second per second, of a race car is modeled by $A(t) = t^3 - \frac{15}{2}t^2 + 12t + 10$, where t is measured in seconds. What is the car's maximum acceleration on the time interval $0 \leq t \leq 6$?

- A. The maximum acceleration of the race car is 2 meters per second per second and occurs at $t = 4$ seconds.
 B. The maximum acceleration of the race car is 6 meters per second per second and occurs at $t = 28$ seconds.
 C. The maximum acceleration of the race car is 15.5 meters per second per second and occurs at $t = 1$ second.
 D. The maximum acceleration of the race car is 28 meters per second per second and occurs at $t = 6$ seconds.

$$\bar{A}(t) = 3t^2 - 15t + 12 = 0$$

$$t = 1$$

$$t = 4$$

$$0 \leq t \leq 6$$

$$* A(0) = 10$$

$$* A(1) = 15.5$$

$$* A(6) = 28 \quad \checkmark \quad \text{Big max}$$

$$* A(4) = 2$$

26. The total cost, in dollars, to order x units of a certain product is modeled by $C(x) = 5x^2 + 320$. According to the model, for what size order is the cost per unit a minimum?

- A. An order of 1 unit has a minimum cost per unit.
 B. An order of 8 units has a minimum cost per unit.
 C. An order of 80 units has a minimum cost per unit.
 D. An order of 320 units has a minimum cost per unit.

$$\textcircled{1} \bar{C}(x) = \frac{C(x)}{x}$$

$$\bar{C}(x) = \frac{5x^2 + 320}{x}$$

$$= 5x + \frac{320}{x}$$

$$\textcircled{2} \bar{C}'(x) = 5 - \frac{320}{x^2} = 0$$

$$5 - \frac{320}{x^2} = 0 \rightarrow 5x^2 = 320$$

$$x^2 = 64 \rightarrow |x = \pm 8| \checkmark$$

$$* \overline{C}''(x) = 0 + 640x^{-3} = \frac{640}{x^3} > 0$$

$$* x = 8 \rightarrow \frac{640}{8^3} = 1.25 > 0$$

27. An electrical power station is located on the edge of a lake, as shown in the figure above. An electrical line needs to be connected from the station to an island in the lake that is located 4 miles due south and 1-mile due east of the station. It costs \$5,000 per mile to install an electrical line on land and \$10,000 per mile to install an electrical line underwater. If C represents a cost function, which of the following methods best explains how to determine the minimum cost, in dollars, for connecting the electrical line from the station to the island?

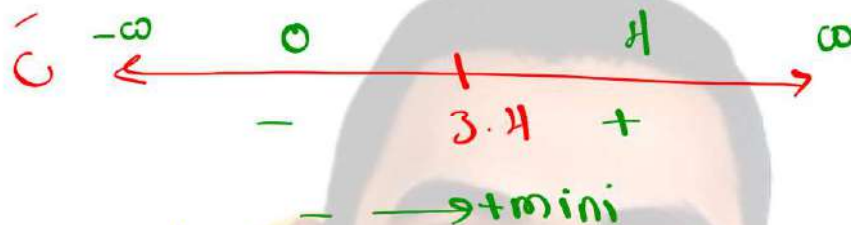
- A. Let $C(x) = 10,000\sqrt{(4-x)^2 + 1} + 5,000x$. Solve $C'(x) = 0$ and find the values of x where $C'(x)$ changes sign from positive to negative. Evaluate C for those values of x to determine the minimum cost. $+\rightarrow -$ max
- ☒ B. Let $C(x) = 10,000\sqrt{(4-x)^2 + 1} + 5,000x$. Solve $C'(x) = 0$ and find the values of x where $C'(x)$ changes sign from negative to positive. Evaluate C for those values of x to determine the minimum cost. $-\rightarrow +$ mini
- C. Let $C(x) = 10,000\sqrt{x^2 + 1} + 5,000x$. Solve $C'(x) = 0$ and find the values of x where $C'(x)$ changes sign from positive to negative. Evaluate C for those values of x to determine the minimum cost.
- D. Let $C(x) = 10,000\sqrt{x^2 + 1} + 5,000x$. Solve $C'(x) = 0$ and find the values of x where $C'(x)$ changes sign from negative to positive. Evaluate C for those values of x to determine the minimum cost.

$$C = 10,000 \sqrt{(4-x)^2 + 1} + 5,000x$$

$$\overline{C} = 10,000 \cdot \frac{1}{2} ((4-x)^2 + 1)^{-\frac{1}{2}} \cdot 2(4-x) \cdot (-1) + 5,000 = 0$$



$$x = 3.4$$



28. If $xy^2 + \frac{x^2}{y} = 5$, then $\frac{dy}{dx} =$

A. $\frac{-2xy}{x(2y^2 - x)}$

B. $\frac{-y(y^3 + 2x)}{x(2y^3 - x)}$

C. $\frac{-y^4}{x(2y^3 - x)}$

D. $\frac{5y^2 - y^4 - 2xy}{x(2y^3 + x)}$

$$1 \cdot y^2 + x \cdot 2y y' + 2x y^{-1} + x^2 (-y^{-2} y') = 0$$

$$y' (2xy - \frac{x^2}{y^2}) = -y^2 - \frac{2x}{y}$$

$$y' = \frac{-y^3 - 2x}{2xy^3 - x^2}$$

$$y' = \frac{(-y^3 - 2x)y}{2xy^3 - x^2}$$

$$= \frac{-y(y^3 + 2x)}{x(2y^3 - x)}$$

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29. Consider the curve in the xy - plane defined by $x^2 - \frac{y^2}{5} = 1$. It is known that $\frac{dy}{dx} = \frac{5x}{y}$ and $\frac{d^2y}{dx^2} = -\frac{25}{y^3}$. Which of the following statements is true about the curve in Quadrant IV?

- A. The curve is concave up because $\frac{dy}{dx} > 0$.
- B. The curve is concave down because $\frac{dy}{dx} < 0$.
- C. The curve is concave up because $\frac{d^2y}{dx^2} > 0$.
- D. The curve is concave down because $\frac{d^2y}{dx^2} < 0$.

$$\bar{y} = -\frac{25}{(-1)^3} = \frac{-25}{-1} = + > 0$$

30. If $x^2y + \frac{x}{y} = -2$, then $\frac{dy}{dx} =$

A. $\frac{y}{x + x^2y^2}$

B. $\frac{y - 2xy^3}{x + x^2y^2} =$

C. $\frac{y + 2xy^3}{x - x^2y^2}$

D. $\frac{y - 2xy^3}{x^2y^2 - x} =$

$$\bar{y} = \frac{y(-2xy^3 + 1)}{x^2y^2 + x} = \frac{-2xy^3 + y}{x^2y^2 + x}$$



31. Consider the curve in the xy - plane defined by $\frac{x^2}{16} - \frac{y^2}{9} = 1$. It is known that $\frac{dy}{dx} = \frac{9x}{16y}$ and $\frac{d^2y}{dx^2} = -\frac{81}{16y^3}$. Which of the following statements is true about the curve in Quadrant IV?

- A. The curve is concave up because $\frac{dy}{dx} > 0$.
- B. The curve is concave down because $\frac{dy}{dx} < 0$.
- ☒ C. The curve is concave up because $\frac{d^2y}{dx^2} > 0$.
- D. The curve is concave down because $\frac{d^2y}{dx^2} < 0$.

$$y' > 0 \quad (+)$$

$$y = \frac{-81}{16(-1)^3} = \frac{-81}{-16} = + > 0$$

(+x, -y)

32. Let C be the curve defined by $xy = -4$. Which of the following statements is true of curve C at the point $(-2, 2)$?

- A. It has a relative minimum because $y' = 0$ and $y'' > 0$.
- B. It has a relative maximum because $y' = 0$ and $y'' < 0$.
- ☒ C. It is increasing and concave up because $y' > 0$ and $y'' > 0$.
- D. It is decreasing and concave down because $y' > 0$ and $y'' < 0$.

$$1 \cdot y + x \cdot y' = 0$$

$$y' = \frac{-y}{x} \quad \#$$

$$* y'(-2, 2) = \frac{-2}{-2} = 1 \rightarrow + > 0$$

$$* y + x y' = 0 \quad y' = 1$$

$$y' + 1 \cdot y' + x y'' = 0$$

$$y'' = \frac{-2y'}{x} = \frac{-2(1)}{-2} = 1 \rightarrow + > 0$$



33. Which of the following describes the x - coordinates of the points on the curve $\cos x = e^y$ in the xy - plane where the curve has a horizontal tangent line.

- A. The values $n\pi$ for all integers n •
- B. The values $\frac{\pi}{2} + n\pi$ for all integers n
- C. The values $(2n + 1)\pi$ for all integers n
- D. The values $2n\pi$ for all integers n •

$$\cos x = e^y$$

→ num = 0
in y

$$-\sin x = e^y$$

$$y = \frac{-\sin x}{e^y} \rightarrow \#$$

H. tangent $\rightarrow -\sin x = 0$

$$x = \sin^{-1}(0) \rightarrow x = 0$$

$$x = n\pi \rightarrow \cos(n\pi) = \boxed{-1}$$

$$\text{or } \boxed{2n\pi}$$

نصف

$$* \cos x = e^y$$

$$n = 0, 1, 2, 3, \dots$$

$$n = 1 \rightarrow \cos 2(1)(\pi) = \boxed{1} \checkmark$$

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34. Consider the curve in the xy - plane defined by $y\sqrt{y^3+1} = x$ for $y > -1$. For what value of y , $y > -1$, does the derivative of the y with respect to x not exist?

A. $y = \left(-\frac{2}{5}\right)^{\frac{1}{3}}$ ✓

B. $y = \left(-\frac{1}{4}\right)^{\frac{1}{3}}$

C. $y = 0$

D. The derivative of y with respect to x exists for all $y > -1$.

$$x = y \cdot (y^3 + 1)^{\frac{1}{2}}$$

$$\dot{x} = 1 \cdot (y^3 + 1)^{\frac{1}{2}} + y \cdot \frac{1}{2} (y^3 + 1)^{-\frac{1}{2}} \cdot 3y^2$$

$$= 0$$

$$y = -0.7368$$

Shift $\rightarrow$ solve $\rightarrow$ =

35. At how many points on the curve $x^{\frac{2}{5}} + y^{\frac{2}{5}} = 1$ in the xy - plane does the curve have a tangent line that is horizontal?

A. None

B. One

C. Two

D. Three

$$\frac{2}{5} x^{-\frac{3}{5}} + \frac{2}{5} y^{-\frac{3}{5}} \cdot \dot{y} = 0$$

$$\dot{y} = \frac{-\frac{2}{5} x^{-\frac{3}{5}}}{\frac{2}{5} y^{-\frac{3}{5}}} = -\frac{y^{\frac{3}{5}}}{x^{\frac{3}{5}}} = 0$$

$$\text{num} = 0 \rightarrow y = 0 \rightarrow |y = 0|$$

$$* x^{\frac{2}{5}} + 0 = 1 \rightarrow x^{\frac{2}{5}} = 1 \quad \frac{2}{5} \rightarrow \text{even}$$

$$|x = \pm 1| \text{ Two}$$

even $\rightarrow \pm 1$, odd $\rightarrow$ same sign

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37. At how many points on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 9$ in the xy - plane does the curve have a tangent line that is horizontal?

A. None

B. One

C. Two

$$\frac{2}{3} x^{-\frac{1}{3}} + \frac{2}{3} y^{-\frac{1}{3}} y' = 0$$

$$y' = \frac{-\frac{2}{3} x^{-\frac{1}{3}}}{\frac{2}{3} y^{-\frac{1}{3}}} = - \frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

$$= 0 \rightarrow \boxed{y = 0}$$

$\frac{2}{3} \rightarrow \text{even}$

$$* x^{\frac{2}{3}} + 0 = 9 \rightarrow x = 9 \quad (\pm)$$

38. A curve in the xy -plane is defined by the equation $x^3 + y^2 - 12x + 16y = 28$. Which of the following statements are true?

- I. At points where $x = -2$, the lines tangent to the curve are horizontal. ✓ num = 0
- II. At points where $y = -8$, the lines tangent to the curve are vertical. ✓ deno = 0
- III. The line tangent to the curve at the point $(-1, 1)$ has slope $\frac{1}{2}$. ✓

A. I and II only

B. II and III only

C. I and III only

D. I, II, and III

$$3x^2 + 2yy' - 12 + 16y' = 0$$

$$y'(2y + 16) = 12 - 3x^2$$

$$y' = \frac{12 - 3x^2}{2y + 16} \rightarrow = 0 \quad \text{H.T}$$

$$12 - 3x^2 = 0 \rightarrow 3x^2 = 12 \rightarrow x^2 = 4 \rightarrow x = \pm 2$$

$$V.T \rightarrow 2y + 16 = 0 \rightarrow 2y = -16 \rightarrow y = -8$$

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$$f(-1, 1) = \frac{12 - 3x^2}{2y + 16} = \frac{12 - 3(-1)^2}{2(1) + 16} = \frac{9}{18} = \left| \frac{1}{2} \right|$$

39. In the xy -plane, how many horizontal or vertical tangent lines does the curve $xy^2 = 2 + xy$ have?

- A. None
 B. One horizontal but no vertical
 C. One vertical but no horizontal
 D. No vertical but one horizontal

$$1 \cdot y^2 + x \cdot 2yy' = 0 + 1 \cdot y + x \cdot y'$$

$$y^2 + 2xyy' = y + xy'$$

$$y'(2+y-x) = y - y^2 \Rightarrow y' = \frac{y - y^2}{2xy - x}$$

$$H.T \rightarrow num = 0$$

$$y - y^2 = 0$$

$$\begin{cases} y=0 \\ y=1 \end{cases}$$

$$\Rightarrow xy^2 = 2 + xy$$

$$x(0) = 2 + x(0) \rightarrow 0 \neq 2 \text{ False}$$

$$\Rightarrow x(1)^2 = 2 + x(1) \rightarrow x = 2 + x$$

$$x - x = 2 \rightarrow 0 \neq 2 \text{ False}$$

No H.T

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40. Let C be the curve defined by $x^2 + y^2 = 36$. Consider all points (x, y) on curve C where $y > 0$. Which of the following statements provides a justification for the concavity of the curve?

- A. The curve is concave down because $y'' = -\frac{2y}{x^2} < 0$
- B. The curve is concave up because $y'' = -\frac{2y}{x^2} > 0$
- ☒ C. The curve is concave down because $y'' = -\frac{36}{y^3} < 0$
- D. The curve is concave up because $y'' = -\frac{36}{y^3} > 0$

$$2x + 2y y' = 0 \rightarrow y' = \frac{-2x}{2y} = \left| \frac{-x}{y} \right|$$

$$2 + 2y' \cdot y' + 2y \cdot y'' = 0$$

$$2 + 2y'^2 + 2y y'' = 0$$

$$2 + 2\left(-\frac{x}{y}\right)^2 + 2y y'' = 0$$

$$2 + \frac{2x^2}{y^2} + 2y y'' = 0$$

$$y'' = \frac{-2 - \frac{2x^2}{y^2}}{2y} = \frac{-2y^2 - 2x^2}{2y^3}$$

$$y'' = \frac{-2y^2 - 2x^2}{2y^3} = -\left(\frac{y^2 + x^2}{y^3}\right)$$

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$$y' = - \frac{x^2 + y^2}{y^3} = - \frac{36}{y^3}$$

41. The point $(-3, 4)$ is on the curve defined by $x^2 y^3 = 576$. Which of the following statements is true about the curve at the point $(-3, 4)$?

A. $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} < 0$

B. $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$

C. $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$

D. $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$

$$* 2xy^3 + x^2 \cdot 3y^2 y' = 0$$

$$2(-3)(4)^3 + (-3)^2 \cdot 3(4)^2 y' = 0$$

$$-384 + 432 y' = 0$$

$$y' = \frac{384}{432} = \frac{8}{9} (+) > 0$$

$$* y' = \frac{-2xy}{3x^2 y^2} = \frac{-2y \rightarrow -2y}{3x \rightarrow 3} \text{ quotient rule}$$

$$* y' = \frac{-2y \cdot 3x - (-2y \cdot 3)}{(3x)^2} \quad \begin{matrix} x = -3 \\ y = 4 \\ y' = \frac{8}{9} \end{matrix}$$

$$= \frac{-2(\frac{8}{9}) \cdot 3 \cdot (-3) + 2(4) \cdot 3}{(3(-3))^2} + > 0$$

$$y' > 0 \quad y' > 0$$

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42. In the xy - plane, the point $(0, -2)$ is on the curve C . If $\frac{dy}{dx} = \frac{4x}{9y}$ for the curve, which of the following statements is true?

- $\checkmark$ A. At the point $(0, -2)$, the curve C has a relative minimum because $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$.
- $\checkmark$ B. At the point $(0, -2)$, the curve C has a relative minimum because $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} < 0$.
- $\checkmark$ C. At the point $(0, -2)$, the curve C has a relative minimum because $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$.
- D. At the point $(0, -2)$, the curve C has a relative minimum because $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$.

$$* \checkmark \bar{y} = \frac{y(0)}{y(-2)} = 0$$

$$* \checkmark \bar{y} = \frac{4x}{9y} \rightarrow \bar{y} = \frac{4 \cdot 9y - 4x \cdot 9\bar{y}}{(9y)^2}$$

$$\bar{y}(0, -2) = \frac{36(-2) - 36(0)(0)}{(9(-2))^2} = \frac{-72}{324}$$

$$\bar{y} < 0$$

$$* \text{if } \bar{y} = 0, \bar{y} < 0 \rightarrow$$

$$\boxed{\bar{y} > 0}$$

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43. Let C be the curve defined by $x^2y = 4$. Which of the following statements is true of curve C at the point (2, 1)?

$$* 2xy + x^2 \dot{y} = 0 \rightarrow \dot{y} = \frac{-2xy}{x^2} = \boxed{\frac{-2y}{x}}$$

$$\dot{y} < 0$$

$$* \dot{y} = \frac{-2y}{x} \rightarrow -2\dot{y} = \frac{-2\dot{y}x - 2y \cdot 1}{x^2}$$

$$\dot{y} = \frac{-2x\dot{y} + 2y}{x^2} = \frac{-2x \cdot \frac{-2y}{x} + 2y}{x^2}$$

$$\dot{y} = \frac{4y + 2y}{x^2} = \frac{6y}{x^2} > 0$$

$$\rightarrow \dot{y}(2,1) = \frac{-2(1)}{2} = -1 < 0$$

$$\rightarrow \dot{y}(2,1) = \frac{6(1)}{(2)^2} = \frac{3}{2} > 0$$

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44. Given the curve $y2^y = x$ in the xy - plane, for what value of y , if any. Does the derivative of y with respect to x not exist?

$$\begin{aligned} \frac{d}{dx}(y \cdot 2^y) &= \frac{d}{dx}x \\ y \cdot 2^y \cdot \ln 2 \cdot \frac{dy}{dx} + y \cdot 2^y &= 1 \\ \frac{dy}{dx} (y \cdot 2^y \ln 2 + y \cdot 2^y) &= 1 \rightarrow \frac{dy}{dx} = \frac{1}{y \cdot 2^y \ln 2 + y \cdot 2^y} \\ \text{deno} = 0 &\quad y \cdot 2^y (1 + \ln 2) = 0 \\ y &\neq 0, \quad 1 + y \ln 2 = 0 \\ y &= \frac{-1}{\ln 2} \end{aligned}$$

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45. Which of the following equations represents the parabola graphed below?

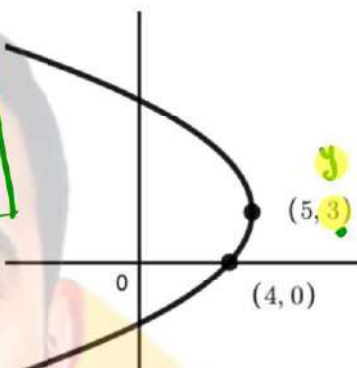
A. $(y-3)^2 = -9(x-5)$

B. $(y-3)^2 = 6(x-5)$

C. $(x-3)^2 = -9(y-5)$

D. $(y-5)^2 = -9(x-3)$

open left
 $x = -y^2$



right $x = y^2$ * left $x = -y^2$

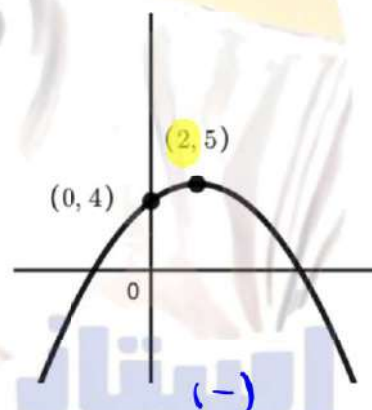
46. Which of the following equations represents the parabola graphed below?

A. $(x+2)^2 = 4(y+5)$

B. $(y-5)^2 = 4(x-2)$

C. $(x-2)^2 = -4(y-5)$

D. $(x-5)^2 = -4(y-2)$



up $y = x^2$
down $y = -x^2$

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47. Write the equation of the parabola whose graph is shown below.

$$y = a(x - h)^2 + k$$

vertex = $(h, k) = (2, 3)$

$$y = a(x - 2)^2 + 3$$

→ $(4, -1)$

$$-1 = a(4 - 2)^2 + 3$$

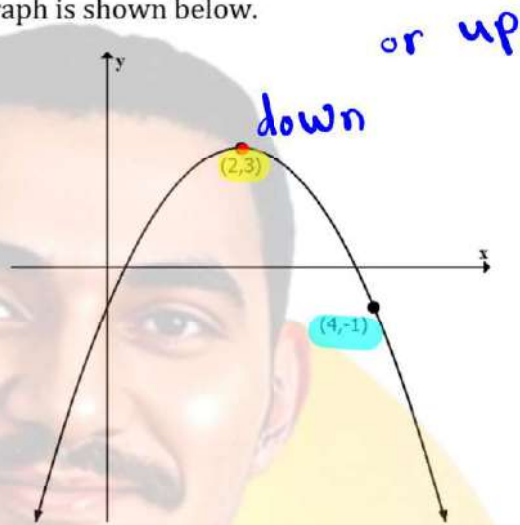
$$-1 - 3 = 4a$$

$$-4 = 4a \rightarrow \boxed{a = -1}$$

$$\boxed{y = -(x - 2)^2 + 3}$$

or

$$(y - 3) = -(x - 2)^2$$



48. Find the foci of the ellipse: $\frac{(x-3)^2}{16} + \frac{(y-2)^2}{12} = 1$

A. $(1, 2)$ and $(5, 2)$

B. $(1, -3)$ and $(1, 6)$

C. $(5, -4)$ and $(5, 2)$

D. $(5, -3)$ and $(5, 1)$

$$(h \pm c, k)$$

$$c = \sqrt{a^2 - b^2} = \sqrt{16 - 12} = 2$$

$$(3 \pm 2, 2) \Rightarrow (5, 2) \text{ and } (1, 2)$$

$$h = 3$$

$$k = 2$$

$$a^2 = 16$$

$$b^2 = 12$$

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49. Find the coordinates of the center and vertices for the ellipse with the equation

$$\frac{(x+4)^2}{49} + \frac{(y-3)^2}{25} = 1.$$

* Center = (h, k) = (-4, 3) *change*

* vertices = (h ± a, k) *Big*
 $a = \sqrt{49} = 7$

$(-4 \pm 7, 3) = (3, 3)$
 $(-11, 3)$

50. Find the coordinates of the center, vertices, and foci for the ellipse with the equation

$$\frac{(y+4)^2}{49} + \frac{(x-3)^2}{25} = 1.$$

* Center = (h, k) = (3, -4)

* vertices = (h, k ± a) *a = \sqrt{49} = 7*

$= (3, -4 \pm 7) = \begin{bmatrix} (3, 3) \\ (3, -11) \end{bmatrix}$

* Foci :- (h, k ± c) *c = \sqrt{a^2 - b^2}*
 $= \sqrt{49 - 25} = 2\sqrt{6}$
 $(3, -4 \pm 2\sqrt{6})$

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51. Find the coordinates of the centre, the vertices, the co-vertices and the lengths of the major and minor axes for the ellipse with equation

$$\frac{(x-4)^2}{25} + \frac{(y+2)^2}{9} = 1$$

* Center = $(h, k) = (4, -2)$

* vertices = $(h \pm a, k)$ $a = \sqrt{25} = 5$

$(4 \pm 5, -2) = \begin{bmatrix} (9, -2) \\ (-1, -2) \end{bmatrix}$

* co-vertices: $(h, k \pm b)$ $b = \sqrt{9} = 3$

$(4, -2 \pm 3) = \begin{bmatrix} (4, 1) \\ (4, -5) \end{bmatrix}$

* length of major = $2a$
 $= 2(5) = 10$

* length of minor = $2b$
 $= 2(3) = 6$

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52. Find the coordinates of the foci for the ellipse of equation $4x^2 + 9y^2 = 36$.

divided all by 36 to reach 1

$$\frac{4x^2}{36} + \frac{9y^2}{36} = \frac{36}{36}$$

$$\left(\frac{x^2}{9}\right) + \frac{y^2}{4} = 1 \quad \text{center} = (0,0)$$

$$a^2 = 9 \rightarrow a = 3, \quad b^2 = 4, \quad b = 2$$

$$c = \sqrt{a^2 - b^2} = \sqrt{9 - 4} = \sqrt{5}$$

* Foci $(h \pm c, k)$

$$(\cancel{0} \pm \sqrt{5}, 0)$$

$$= (\pm \sqrt{5}, 0)$$

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53. Write the equation of the hyperbola $2x^2 - 3y^2 - 12x - 24y + 60 = 0$ in standard form

* x in same side , y

$$2x^2 - 12x - 3y^2 - 24y = -60$$

* GCF

$$2(x^2 - 6x) - 3(y^2 + 8y) = -60$$

$$* \frac{b}{2} = \frac{-6}{2} = -3 \quad \left| \quad * \frac{b}{2} = \frac{8}{2} = 4 \right.$$

$$2[(x-3)^2 - (-3)^2] - 3[(y+4)^2 - (4)^2] = -60$$

$$2(x-3)^2 - 18 - 3(y+4)^2 + 48 = -60$$

$$2(x-3)^2 - 3(y+4)^2 = -60 + 18 - 48$$

$$\frac{2(x-3)^2}{-90} - \frac{3(y+4)^2}{-90} = \frac{-90}{-90} \quad \text{equals to } 1$$

$$\frac{(x-3)^2}{-45} + \frac{(y+4)^2}{30} = 1$$

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$$\frac{(y+4)^2}{30} - \frac{(x-3)^2}{45} = 1$$

54. Write the equation of the hyperbola in standard form

$$25x^2 - 4y^2 - 100x + 48y - 144 = 0$$

$$* 25x^2 - 100x - 4y^2 + 48y = 144$$

$$* 25(x^2 - 4x) - 4(y^2 - 12y) = 144$$

$$* \frac{b}{2} = \frac{-4}{2} = -2$$

$$* \frac{b}{2} = \frac{-12}{2} = -6$$

$$25[(x-2)^2 - (-2)^2] - 4[(y-6)^2 - (-6)^2] = 144$$

$$25(x-2)^2 - 100 - 4(y-6)^2 + 144 = 144$$

$$25(x-2)^2 - 4(y-6)^2 = 144 - 144 + 100$$

$$\frac{25(x-2)^2}{100} - \frac{4(y-6)^2}{100} = \frac{100}{100}$$

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$$\frac{(x-2)^2}{4} - \frac{(y-6)^2}{25} = 1$$

55. Which of the following is an equation of one asymptote of the hyperbola $\frac{(y+3)^2}{16} - \frac{x^2}{49} = 1$?

A. $y = \frac{7}{4}x + 3$

B. $y = \frac{7}{4}x - 3$

C. $y = \frac{4}{7}x - 3$

D. $y = \frac{4}{7}x + 4$

$(h, k) = (0, -3)$

(vertical) start with y (-)

$$y = \pm \frac{a}{b}(x-h) + k$$

$a = \sqrt{16} = 4$, $b = \sqrt{49} = 7$

$$y = \pm \frac{4}{7}(x - \cancel{0}) - 3$$

$$y = \pm \frac{4}{7}x - 3$$

56. The slopes of the asymptotes of the hyperbola $\frac{x^2}{4} - \frac{y^2}{3} = 1$ are _____.

A. ± 1

B. $\pm \frac{3}{2}$

C. $\pm \frac{\sqrt{3}}{2}$

D. $\pm \frac{2}{\sqrt{3}}$

(Horizontal) start with x (-)

$$y = \pm \frac{b}{a}(x-h) + k$$

$(h, k) = (0, 0)$

$a = \sqrt{4} = 2$

$b = \sqrt{3}$

$$y = \pm \frac{\sqrt{3}}{2}(x - \cancel{0}) + \cancel{0}$$

$$= \pm \frac{\sqrt{3}}{2}x$$

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57. The vertices of the given hyperbola $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{16} = 1$ are _____.

A. $(-2, 2)$ and $(5, 1)$

B. $(-1, 2)$ and $(4, 2)$

C. $(-2, 2)$ and $(4, 2)$

D. $(-3, 2)$ and $(5, 2)$

Horizontal

* Center $(h, k) = (1, 2)$

* $a = \sqrt{9} = 3$

* $b = \sqrt{16} = 4$

vertices $= (h \pm a, k) = (1 \pm 3, 2)$
 $= \begin{bmatrix} (4, 2) \\ (-2, 2) \end{bmatrix}$

58. Find the equations of asymptotes to the hyperbola $\frac{(x+4)^2}{9} - \frac{(y-2)^2}{16} = 1$.

Horizontal

$a = \sqrt{9} = 3$

$b = \sqrt{16} = 4$

$(h, k) = (-4, 2)$

$y = \pm \frac{b}{a} (x - h) + k$

$y = \pm \frac{4}{3} (x + 4) + 2$

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59. Find the coordinates of the centre, the vertices and the co-vertices. Also, find the lengths of the transverse and conjugate axes and the equations of the asymptotes for the hyperbola with the equation

$$\frac{(x-4)^2}{9} - \frac{(y+2)^2}{4} = 1$$

* Horizontal

* $(h, k) = \text{Center} = (4, -2)$

$a = \sqrt{9} = 3$, $b = \sqrt{4} = 2$

* Vertices = $(h \pm a, k) = (4 \pm 3, -2)$
 $\rightarrow (7, -2)$ and $\rightarrow (1, -2)$

* Co-vertices :- $(h, k \pm b) = (4, -2 \pm 2)$
 $\rightarrow (4, 0)$ and $(4, -4)$

* Asymptotes : $y = \pm \frac{b}{a}(x-h) + k$

$y = \pm \frac{2}{3}(x-4) - 2$

* length of Transverse (major) = $2a$
 $= 2(3) = 6$

* length of Conjugate (minor) = $2b$
 $= 2(2) = 4$



60. Find the foci of the hyperbola $\frac{(y+2)^2}{64} - \frac{(x-3)^2}{16} = 1$.

Vertical

hyper

$$= \sqrt{a^2 + b^2}$$

$$= \sqrt{a^2 - b^2}$$

$$a = \sqrt{64} = 8$$

$$b = \sqrt{16} = 4$$

$$c = \sqrt{a^2 + b^2} = \sqrt{64 + 16} = 4\sqrt{5}$$

$$\text{Foci } (h, k \pm c) = (3, -2 \pm 4\sqrt{5})$$

61. There are **two** major roads from downtown Dubai to Dubai Marina and **five** roads from Dubai Marina to Dubai Mall. How many different routes are there from downtown Dubai to Dubai Mall by way of Dubai Marina?

- A. 7
- ☒ B. 10
- C. 14
- D. 20

times

$$2 \times 5 = 10$$

62. A license plate consists of 4 digits (0 through 9). Find the number of different plates that could be formed if there are no restrictions on the digits that can be used.

- ☒ A. 10000
- B. 1000
- C. 5040
- D. 3024

$$0 \rightarrow 9 = 10$$

$$10 \times 10 \times 10 \times 10 = 10000$$

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63. A multiple-choice test consists of 5 questions with each question having 3 possible answers. How many ways are there to mark the answers?

- A. 125
B. 3^4
C. 243
D. 120

$$3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$$

64. Abby is registering at a web site and must select a six – character password. The password can contain either letters or digits. How many passwords are possible if:

a. Characters can be repeated.

$$6 \times 6 = 36$$

$$36^6 = 2146782336$$

b. No characters can be repeated.

$$36 \times 35 \times 34 \times 33 \times 32 \times 31 = 1402410240$$

65. In how many ways can a 5-person committee be formed out of a class of 18 students?

- A. 90
B. $15!$
C. 8568
D. 1028160

No order

$$18C5$$

66. In how many ways can a president and vice president of a class of 12 students be selected?

- A. 24
B. 66
C. 132

order

$$12P2$$

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67. How many ways can 1st, 2nd, 3rd, and 4th place prize be awarded among 14 contestants?

- A. 56
B. 336
C. 1001
D. 24024

order

 ${}^{14}P_4$

68. Find the sixth term of the expansion $(\sqrt{a} + \sqrt{b})^{12}$ Explain your reasoning (Synthesis)

$$r = p - 1 = 6 - 1 = 5, \quad n = 12$$

$${}^{12}C_5 (\sqrt{a})^{12-5} (\sqrt{b})^5 = 792 (\sqrt{a})^7 (\sqrt{b})^5$$

$$792 \cdot a^{\frac{6}{2}} \cdot \sqrt{a} \cdot b^{\frac{5}{2}} \sqrt{b}$$

$$792 a^3 \sqrt{a} b^2 \sqrt{b} = 792 a^3 b^2 \sqrt{ab}$$

69. Expand the following binomial: $(3x + 2y)^5$

$$(3x)^5 + {}^5C_1 (3x)^4 (2y) + {}^5C_2 (3x)^3 (2y)^2 + {}^5C_3 (3x)^2 (2y)^3 + {}^5C_4 (3x) (2y)^4 + (2y)^5 =$$

$$= 243x^5 + 810x^4y + 1080x^3y^2 + 720x^2y^3 + 210xy^4 + 32y^5$$

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70. Expand the following binomial: $(3a - \frac{1}{3})^5 = (3a + (-\frac{1}{3}))^5$

$$\begin{aligned}
 & (3a)^5 + {}^5C_1 (3a)^4 (-\frac{1}{3}) + {}^5C_2 (3a)^3 (-\frac{1}{3})^2 \\
 & + {}^5C_3 (3a)^2 (-\frac{1}{3})^3 + {}^5C_4 (3a) (-\frac{1}{3})^4 + (-\frac{1}{3})^5 = \\
 & = 243a^5 - 135a^4 + 30a^3 - \frac{10}{3}a^2 + \frac{5}{27}a - \frac{1}{243}
 \end{aligned}$$

71. Write the equation of the n^{th} term of the arithmetic sequence $-8, -2, 4, 10, \dots$

- A. $a_n = -14 + 8n$
 B. $a_n = 14 - 6n$
 C. $a_n = -8 + 6n$
 ✓ D. $a_n = -14 + 6n$

$$a_n = a_1 + (n-1)d$$

$$a_1 = -8$$

$$d = -2 - (-8) = 6$$

$$a_n = -8 + (n-1)6 = -8 + 6n - 6$$

$$\boxed{a_n = -14 + 6n}$$

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72. Given the arithmetic sequence $-4, 2, 8, \dots$. Write an equation for the n^{th} term, and then find the 90^{th} term.

$$a_1 = -4, \quad d = 2 - (-4) = 6$$

$$a_n = -4 + (n-1)6 = -4 + 6n - 6$$

$$\boxed{a_n = -10 + 6n}$$

$$* a_{90} = -10 + 6(90) = 530$$

73. Write the equation of the n^{th} term of the arithmetic sequence $a_8 = -8, d = -2$

$$a_n = a_1 + (n-1)d$$

$$-8 = a_1 + (8-1)(-2)$$

$$-8 = a_1 - 14$$

$$\rightarrow \begin{array}{l} a_1 = -8 + 14 \\ \hline a_1 = 6 \end{array}$$

$$\rightarrow a_n = 6 + (n-1)(-2)$$

$$a_n = 6 - 2n + 2$$

$$\boxed{a_n = 8 - 2n}$$

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74. Find the sum of the arithmetic series $a_n = 154, d = 8, n = 19$

- A. $S_n = 82$
 B. $S_n = 1558$
 C. $S_n = 19$
 D. $S_n = 1446$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$* a_n = a_1 + (n-1)d$$

$$154 = a_1 + (19-1)8$$

$$\boxed{a_1 = 10}$$

$$* S_n = \frac{19}{2} (10 + 154) = 1558$$

75. Evaluate the sum of the arithmetic series below.

$$8 + 12 + 16 + \dots + 80.$$

a.

an

$$d = 12 - 8 = 4$$

$$a_n = a_1 + (n-1)d$$

$$80 = 8 + (n-1)(4)$$

$$\boxed{n = 19}$$

$$* S_n = \frac{19}{2} (8 + 80)$$

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76. Evaluate the arithmetic series $\sum_{i=1}^5 (5i - 6)$

* $n - i + 1 \rightarrow i \neq 1$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

* $i = 1 \rightarrow n = 5$
Same upper value.

$$a_1 = 5(1) - 6 = -1$$

$$a_n = a_5 = 5(5) - 6 = 19$$

$$S_n = \frac{5}{2} (-1 + 19) = 45$$

77. Find the sum of the arithmetic series $a_1 = -16, d = 6, n = 24$

$$\begin{aligned} * a_n &= a_1 + (n-1)d \\ &= -16 + (24-1)6 = 122 \end{aligned}$$

$$\begin{aligned} * S_n &= \frac{n}{2} (a_1 + a_n) = \frac{24}{2} (-16 + 122) \\ &= 1272 \end{aligned}$$

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78. Find the sum of the arithmetic sequence: -7, -1, 5, 11, ..., 41.

$$a_1 = -7, \quad d = -1 - (-7) = 6, \quad a_n = 41$$

$$* a_n = a_1 + (n-1)d$$

$$41 = -7 + (n-1)6 \rightarrow \boxed{n = 9}$$

$$* S_n = \frac{9}{2} (-7 + 41) = 153$$

79. Find the sum of the arithmetic sequence: 2, 5, 8, 11, ..., 29.

$$a_1 = 2, \quad d = 5 - 2 = 3, \quad a_n = 29$$

$$* a_n = a_1 + (n-1)d$$

$$29 = 2 + (n-1)3 \rightarrow \boxed{n = 10}$$

$$* S_n = \frac{10}{2} (2 + 29) = 155$$

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80. Find the sum of the geometric series $a_1 = 16, r = \frac{1}{2}, n = 9$

$$S_n = \frac{a_1 (1 - r^n)}{1 - r} = \frac{16 (1 - (\frac{1}{2})^9)}{1 - \frac{1}{2}}$$

$$\approx 31.9375$$

81. Evaluate the geometric series $\sum_{m=1}^6 2^{m-1}$

$$S_n = \frac{a_1 (1 - r^n)}{1 - r}, \quad r = 2 \text{ under Power}$$

$$a_1 = 2^{1-1} = 1, \quad \boxed{n = \text{Same} = 6}$$

$$S_n = \frac{1 (1 - 2^6)}{1 - 2} = 63$$

82. Find the sum of the geometric series $\sum_{k=1}^7 4(-3)^{k-1}$

$$a_1 = 4(-3)^{1-1} = 4, \quad r = -3$$

$$n = \text{Same} = 7$$

$$S_n = \frac{4 (1 - (-3)^7)}{1 - (-3)} = 2188$$



83. Find the second term in a geometric series for which the common ratio $r = \frac{1}{4}$ and the sum of the first 5 terms is $\frac{1705}{16}$.

$$S_n = \frac{a_1(1-r^n)}{1-r} \Rightarrow \frac{1705}{16} = \frac{a_1(1-(\frac{1}{4})^5)}{1-\frac{1}{4}}$$

$$a_1 = 80$$

$$\rightarrow a_2 = a_1 \times r = 80 \times \frac{1}{4} = 20$$

84. Find the sum of the infinite geometric series, S_∞ , if $a_1 = 240$ and $r = \frac{3}{4}$

A. 320

B. 720

☒ C. 960

D. Does not exist.

$$S_n = \frac{a_1}{1-r} = \frac{240}{1-\frac{3}{4}} = 960$$

85. Evaluate $\sum_{k=1}^{\infty} (60) \left(\frac{2}{3}\right)^{k-1}$.

A. 40

B. 60

☒ C. 180

D. Does not exist.

replace $|a_1 = 60|$

$$\sum_{k=1}^{\infty} 60 \left(\frac{2}{3}\right)^{k-1} = 180$$

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$$\sum_{k=1}^{\infty} \left(60 \left(\frac{2}{3}\right)^{k-1}\right)$$

180

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86. What is the sum of an infinite geometric series with first term 10 and second term -2 ?

- A. $\frac{1}{3}$
 B. $\frac{25}{3}$
 C. $\frac{32}{3}$
 D. $\frac{25}{2}$

$$* r = \frac{a_2}{a_1} = \frac{-2}{10} = \boxed{-\frac{1}{5}}$$

$$* S_{\infty} = \frac{a_1}{1-r} = \frac{10}{1 - (-\frac{1}{5})} = \frac{25}{3}$$

in sum of geometric ∞ (٢٥)
 Sure First if $|r| < 1$ or not
 why as if $|r| < 1$ has sum
 but if $|r| \geq 1$ No sum

87. In a survey conducted among students, the distribution of favorite subjects is 30% Math, 25% Science, 20% English, and 25% Art. If 2 students are chosen at random, find the probability that neither has Science as their favorite subject.

- A. 0.25
 B. 0.0625
 C. 0.75
 D. 0.5625

$$P(\text{not S}) = 1 - 0.25 = 0.75$$

$$= 0.75 \times 0.75$$

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88. In a survey conducted among gym members, the distribution of favorite workouts is **40% Cardio, 30% Weightlifting, 20% Yoga, and 10% Swimming**. If **2 gym members are chosen at random**, find the probability that **neither** has Yoga as their favorite workout.

- A. 0.04
- ☒ B. 0.64
- C. 0.80
- D. 0.36

$$P(\text{not yoga}) = 1 - 0.20$$

$$= 0.80$$

$$\rightarrow 0.80 \times 0.80 = 0.64$$

89. In a survey conducted among travelers, the distribution of favorite destinations is **45% Beach, 20% Mountains, 25% Cities, and 10% Desert**. If **2 travelers are chosen at random**, find the probability that **neither** has Beach as their favorite destination.

- A. 0.2025
- ☒ B. 0.3025
- C. 0.45
- D. 0.55

$$1 - 0.45 = 0.55$$

$$0.55 \times 0.55$$

90. A basket contains 5 apples, 3 bananas and 4 pears. Yousef randomly chooses one piece of fruit, eats it, and chooses another. What is the probability that he chose a banana and then a pear?

- A. 1/22
- ☒ B. 1/11
- C. 4/11
- D. 5/11

$$\text{Total} = 5 + 3 + 4 = 12$$

$$P(b) \times P(p)$$

$$\frac{3}{12} \times \frac{4}{11} = \frac{1}{11}$$

* in the same basket *

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91. A bag contains 4 red marbles, 5 blue marbles, and 3 yellow marbles. A marble is selected, **replaced**, and a second marble is selected. Find the probability of selecting a red marble, then a blue marble.

- A. $\frac{3}{50}$
 B. $\frac{5}{36}$
 C. $\frac{6}{100}$
 D. $\frac{2}{15}$

$$\text{Total} = 4 + 5 + 3 = 12$$

$$\frac{4}{12} \times \frac{5}{12} = \frac{5}{36}$$

replaced → not subtract by 1

92. A bag contains 2 black marbles, 3 blue marbles, 2 yellow marbles, and 2 green marbles. A marble is selected, **replaced**, and a second marble is selected. Determine if the event is independent or dependent. Find the probability of selecting a black marble, then a blue marble.

$$\text{Total} = 2 + 3 + 2 + 2 = 9$$

$$P = \frac{2}{9} \times \frac{3}{9} = \frac{2}{27}$$

93. The table below summarizes the responses by gender. Answer (i-iv).

Primary interest	Men	Women	Total
The game	260	220	480
The commercials	90	140	230
Won't watch	130	160	290
Total	480	520	1000

- i. What percent of the men planned to skip the game?

- A. 27.08%
 B. 13.00%
 C. 43%

$$\frac{130}{480} \times 100\%$$



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ii. What percent of the people who planned to skip the game were men?

Answer: $\frac{130}{290} \times 100\% \approx 44.83\%$

Primary interest	Men	Women	Total
The game	260	220	480
The commercials	90	140	230
Won't watch	130	160	290
Total	480	520	1000

iii. What percent of the people planned to skip the game?

Answer: $\frac{290}{1000} \times 100\% \approx 29\%$

Primary interest	Men	Women	Total
The game	260	220	480
The commercials	90	140	230
Won't watch	130	160	290
Total	480	520	1000

iv. What percent of the people were men who planned to skip the game?

Answer: $\frac{130}{1000} \times 100\% \approx 13\%$

Primary interest	Men	Women	Total
The game	260	220	480
The commercials	90	140	230
Won't watch	130	160	290
Total	480	520	1000

94. The table below shows the results of a survey about whether students planned to attend the school talent show.

Primary interest	Boys	Girls	Total
Singing acts	180	220	400
Dance acts	120	160	280
Won't attend	100	120	220
Total	400	500	900

i. What percent of the boys planned to skip the talent show?

A. 22%

B. 25%

C. 45.45%

D. 11.11%

$\frac{100}{400} \times 100\% = 25\%$

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ii. What percent of the people who planned to skip the talent show were boys?

$$\frac{100}{220} \times 100\% \approx 45.45\%$$

Primary interest	Boys	Girls	Total
Singing acts	180	220	400
Dance acts	120	160	280
Won't attend	100	120	220
Total	400	500	900

iii. What percent of the people planned to skip the talent show?

$$\frac{220}{900} \times 100\% \approx 24.44\%$$

Primary interest	Boys	Girls	Total
Singing acts	180	220	400
Dance acts	120	160	280
Won't attend	100	120	220
Total	400	500	900

iv. What percent of the people were boys who planned to skip the talent show?

$$\frac{100}{900} \times 100\% \approx 11.11\%$$

Primary interest	Boys	Girls	Total
Singing acts	180	220	400
Dance acts	120	160	280
Won't attend	100	120	220
Total	400	500	900

Write a brief interpretation on each of the stemplots in questions (4-6).

95. A stemplot showing life expectations of males.

are clustered near 70 and lower 64

Males

6	4
6	5 8
7	0 1 2 2 2 2 3 3
7	6

96. A stemplot showing life expectations of females.

are clustered between 70 and 80

Females

6	9 9
7	4 4
7	8 8 8 9 9
8	0 0 1

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97. A back-to-back stemplot showing life expectations of males and females.

women are higher than the men

Males		Females	
3	3	4	6
2	2	8	5
2	2	6	6
2	2	7	9
1	0	7	4
		6	7
		8	8
			8
			9
			9
			0
			0
			1

98. The distribution below shows the scores on a quiz for 15 students

Scores	4	6	8	10	12	14	16
Frequencies	1	2	3	2	3	2	2

What is the median score?

- A. 8
B. 10
C. 12
D. 14

order → cut

4, 6, 6, 8, 8, 8, 10, 10, 12, 12, 12, 14, 14, 16, 16

99. The distribution below shows the number of books read by 17 students in one month.

Number of Books	1	2	3	4	5	6	8
Frequencies	2	3	2	4	3	2	1

What is the five-number summary?

- A. {1, 2, 4, 5, 8}
B. {1, 3, 4, 5, 8}
C. {1, 4, 4, 5, 8}
D. {1, 2.5, 4, 5.5, 8}

1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 4, 5, 5, 5, 8

* lower = min = 1

* max = upper = 8

* Median = 4

* Q_1 = left middle = $\frac{2+2}{2} = 2$

* Q_3 = R. middle = $\frac{5+5}{2} = 5$



100. The distribution below shows the scores on a driving test for 19 applicants.

Scores →	5	7	10	15	19	21	22	23	24	25
Frequencies	1	1	1	1	1	2	2	5	4	1

Find the five-number summary for this data. Identify any outliers.

5, 7, 10, 15, 19, 21, 22, 22, 23, 23, 23, 23, 23, 24, 24, 24, 24, 25

The median: 23

Q1: 19

, max = upper = 25

Q3 : 24

, mini = lower = 5

The five-number summary: {5, 19, 23, 24, 25}

IQR: $Q_3 - Q_1 = 24 - 19 = 5$

* Outliers (show your work):

$$① Q_1 - 1.5 \times IQR = 19 - 1.5 \times 5 = 11.5$$

$$② Q_3 + 1.5 \times IQR = 24 + 1.5 \times 5 = 31.5$$

11.5 → 31.5

less 11.5 ← outliers → more 31.5

are 5, 7, 10

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Mr. Tarek Ali

سلسلة الطارق في الرياضيات منصة طارق أكاديمي

11

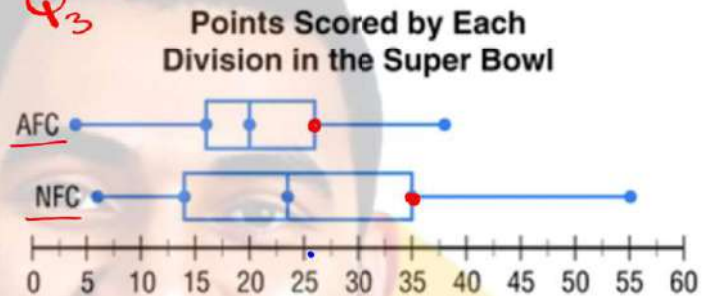
Elite



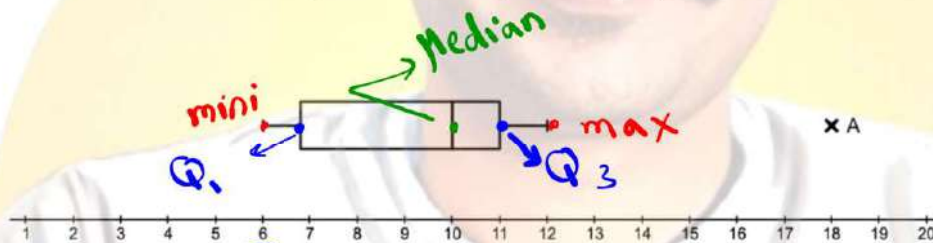
101. The box-and-whisker plots show the points scored in each Super Bowl by the AFC and NFC. About how many points higher is the upper quartile for the NFC than the AFC?

- A. 5 points
 B. 10 points
 C. 15 points
 D. 40 points

$$35 - 25 = 10$$



Refer to the below box-whisker plot to answer questions (i-ii).



102. Find the following:

- i. Min: 6
 ii. Q1: 7

iii. Median: 10

iv. Q3: 12

v. Range: $\text{max} - \text{mini} = 12 - 6 = 6$

103. Point A is much larger than any of the other points. Is point A an outlier according to the $1.5 \times IQR$ criterion? Explain your answer.

* Q_1 , Q_3

* $Q_1 - 1.5 \times IQR$

* $Q_3 + 1.5 \times IQR$

* $IQR = Q_3 - Q_1$, let = 4



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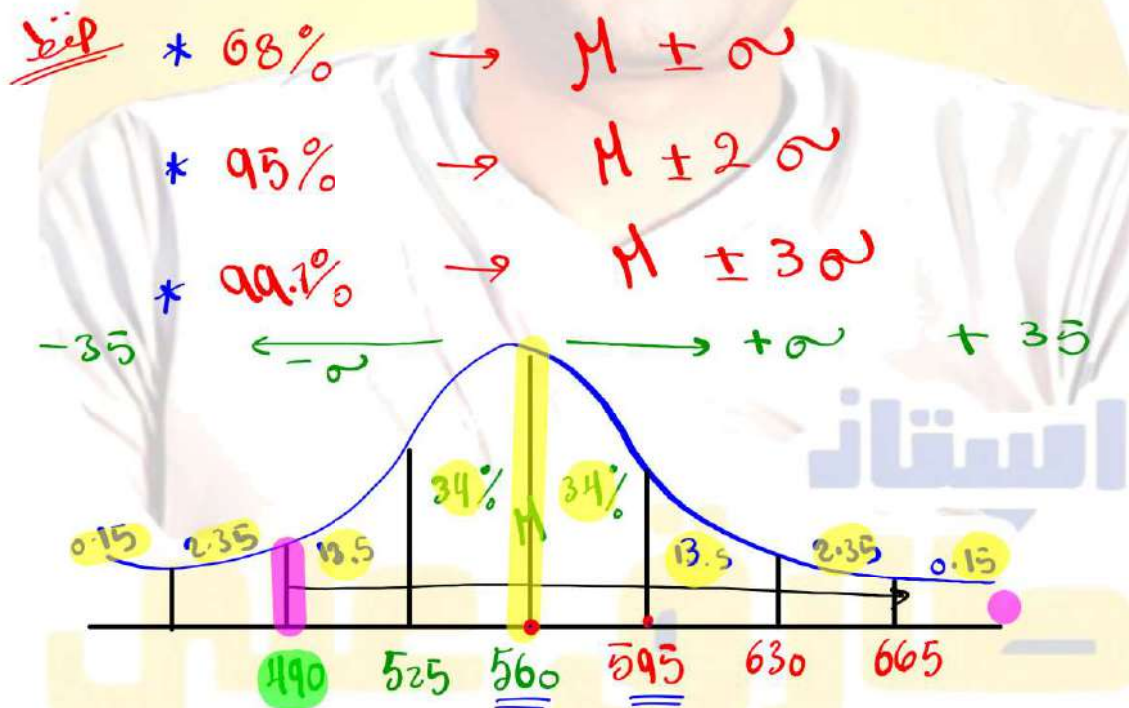
$$\text{if } Q_3 + 1.5 \times IQR = 17$$

$A=18$ then A outlier

why as $18 > 17$

104. The general population scores on a standardized exam approximate a normal distribution with a mean of 560, and an approximate standard deviation of 35. ~

1. Approximately, what percentage of the scores should be higher than 490?



$$13.5 + 34 + 34 + 13.5 + 2.35 + 0.15 = 97.5\%$$

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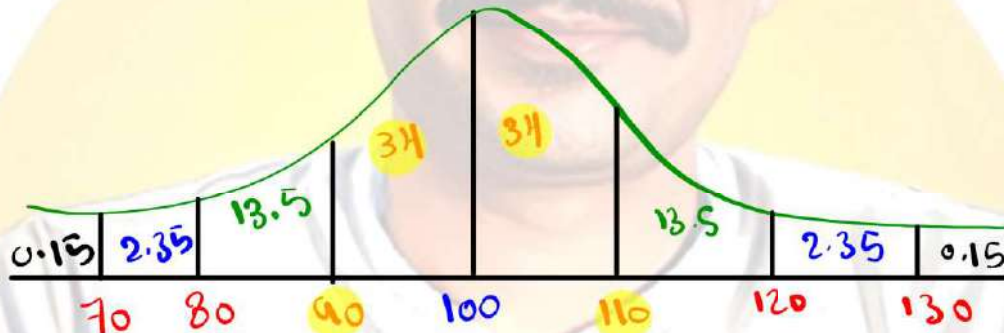


2. Approximately what score would one need to make on this exam to be marked among the top 16% of all who took the test?

$$\text{lie in } \mu + \sigma = 560 + 35 = 595$$

105. The heights of 50 trees are normally distributed with a mean of 100cm and a standard deviation of 10cm.

- i. How many trees are with height between 90cm and 110cm?



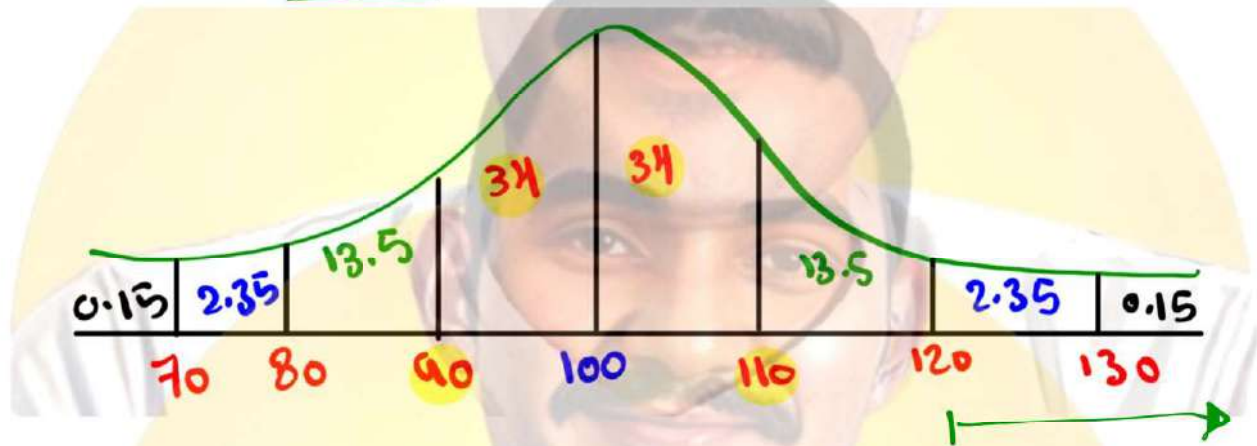
$$90 \rightarrow 110 = 34\% + 34\% = 68\%$$

$$\text{How many} = \frac{68}{100} \times 50 = 34 \text{ trees}$$

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- ii. What is the probability that a selected tree is taller than 120 cm?



$$\begin{aligned} \text{more } 120 &= 2.35\% + 0.15\% \\ &= 2.5\% \end{aligned}$$

Percent = probability

استاذ

طارق علي
في الرياضيات

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IN
Math

منصة طارق أكاديمي للرياضيات

$\pi = 3.14$

$A = \frac{ab + c}{d}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a^2 + b^2 = c^2, c = \sqrt{a^2 + b^2}$

$c^2 + a^2 = b^2, c^2 = b^2 + a^2$

$a^2 = 2a$

$f(a + b) = c$

$(x + y)^2 - (x - y)$

$+ 2c = 1$

MATH

$\frac{a}{c} = \frac{HB}{a}$

$Me =$

خاص بالمنصة

خاص بجميع الجروبات و القنوات

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