

End of Term 3 Revision Maths



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف الحادي عشر المتقدم ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

تاريخ إضافة الملف على موقع المناهج: 2026-06-15 23:24:15

ملفات اكتب للمعلم اكتب للطالب | اختبارات الكترونية | اختبارات | حلول | عروض بوربوينت | أوراق عمل
منهج انجليزي | ملخصات وتقارير | مذكرات وبنوك | الامتحان النهائي | للمدرس

المزيد من مادة
رياضيات:

إعداد: Gomaa Ahmed

التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم



صفحة المناهج
الإماراتية على
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

المواد على تلغرام

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الثالث

كراسة تدريبية للاختبار النهائي 2026م

1

منهج ريفيل حلول مراجعة امتحانية تدريبات وأسئلة

2

منهج ريفيل مراجعة امتحانية تدريبات وأسئلة غير محلول

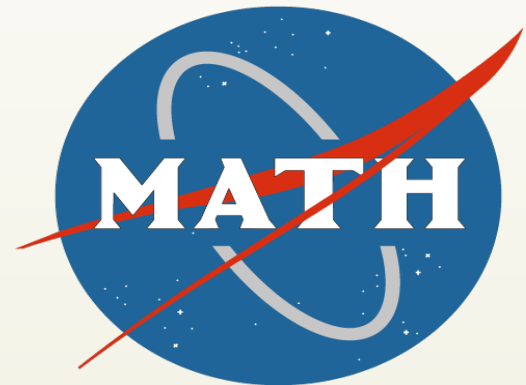
3

ملزمة هيكل واختياري وأسئلة

4

هيكل اختبار نهاية الفصل 2026م

5



BE
READY
END OF TERM 3
REVISION
MATHS
GRADE 11 ADV

COMPILED BY
MR. AHMED GOMAA

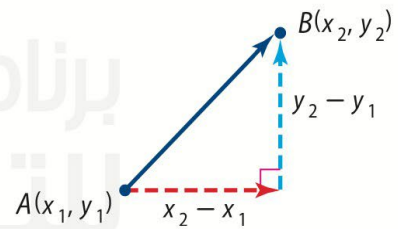
مدير المجمع
أحمد الجبر

وحدة الشؤون الأكاديمية
جلال عبدالرحمن صالح
حمدة العامري

KeyConcept Component Form of a Vector

The component form of a vector $\overrightarrow{AB}$ with initial point $A(x_1, y_1)$ and terminal point $B(x_2, y_2)$ is given by

$$\langle x_2 - x_1, y_2 - y_1 \rangle.$$



Example 1 Express a Vector in Component Form

Find the component form of $\overrightarrow{AB}$ with initial point $A(-4, 2)$ and terminal point $B(3, -5)$.

$$\begin{aligned} \overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle && \text{Component form} \\ &= \langle 3 - (-4), -5 - 2 \rangle && (x_1, y_1) = (-4, 2) \text{ and } (x_2, y_2) = (3, -5) \\ &= \langle 7, -7 \rangle && \text{Subtract.} \end{aligned}$$

GuidedPractice

Find the component form of $\overrightarrow{AB}$ with the given initial and terminal points.

1A. $A(-2, -7), B(6, 1)$

1B. $A(0, 8), B(-9, -3)$

extra Q's to practice....

Find the component form and magnitude of $\overrightarrow{AB}$ with the given initial and terminal points. (Examples 1 and 2)

1. $A(-3, 1), B(4, 5)$

2. $A(2, -7), B(-6, 9)$

3. $A(10, -2), B(3, -5)$

4. $A(-2, 7), B(-9, -1)$

5. $A(-5, -4), B(8, -2)$

6. $A(-2, 6), B(1, 10)$

7. $A(2.5, -3), B(-4, 1.5)$

8. $A(-4.3, 1.8), B(9.4, -6.2)$

9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$

10. $A\left(\frac{3}{5}, -\frac{2}{5}\right), B(-1, 7)$

2 Write a vector as a linear combination of unit vectors.

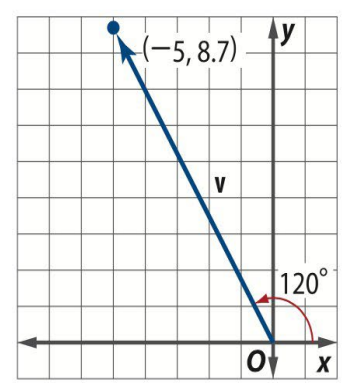
Example 6 & (6A,6B)

Example 6 Find Component Form

Find the component form of the vector $\mathbf{v}$ with magnitude 10 and direction angle 120° .

$$\begin{aligned}\mathbf{v} &= \langle |\mathbf{v}| \cos \theta, |\mathbf{v}| \sin \theta \rangle && \text{Component form of } \mathbf{v} \text{ in terms of } |\mathbf{v}| \text{ and } \theta \\ &= \langle 10 \cos 120^\circ, 10 \sin 120^\circ \rangle && |\mathbf{v}| = 10 \text{ and } \theta = 120^\circ \\ &= \left\langle 10 \left(-\frac{1}{2} \right), 10 \left(\frac{\sqrt{3}}{2} \right) \right\rangle && \cos 120^\circ = -\frac{1}{2} \text{ and } \sin 120^\circ = \frac{\sqrt{3}}{2} \\ &= \langle -5, 5\sqrt{3} \rangle && \text{Simplify.}\end{aligned}$$

CHECK Graph $\mathbf{v} = \langle -5, 5\sqrt{3} \rangle \approx \langle -5, 8.7 \rangle$. The measure of the angle $\mathbf{v}$ makes with the positive x -axis is about 120° as shown, and $|\mathbf{v}| = \sqrt{(-5)^2 + (5\sqrt{3})^2}$ or 10. ✓



Guided Practice

Find the component form of $\mathbf{v}$ with the given magnitude and direction angle.

6A. $|\mathbf{v}| = 8, \theta = 45^\circ$

6B. $|\mathbf{v}| = 24, \theta = 210^\circ$

extra Q's to practice....

Find the component form of $\mathbf{v}$ with the given magnitude and direction angle. (Example 6)

38. $|\mathbf{v}| = 12, \theta = 60^\circ$

39. $|\mathbf{v}| = 4, \theta = 135^\circ$

40. $|\mathbf{v}| = 6, \theta = 240^\circ$

41. $|\mathbf{v}| = 16, \theta = 330^\circ$

42. $|\mathbf{v}| = 28, \theta = 273^\circ$

43. $|\mathbf{v}| = 15, \theta = 125^\circ$

3 Find the dot product of two vectors and use the dot product to find the angle between them.

Example 3 & (3A,3B)

Example 3 Find the Angle Between Two Vectors

Find the angle θ between vectors $\mathbf{u}$ and $\mathbf{v}$ to the nearest tenth of a degree.

a. $\mathbf{u} = \langle 6, 2 \rangle$ and $\mathbf{v} = \langle -4, 3 \rangle$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|}$$

Angle between two vectors

$$\cos \theta = \frac{\langle 6, 2 \rangle \cdot \langle -4, 3 \rangle}{|\langle 6, 2 \rangle| |\langle -4, 3 \rangle|}$$

$$\mathbf{u} = \langle 6, 2 \rangle \text{ and } \mathbf{v} = \langle -4, 3 \rangle$$

$$\cos \theta = \frac{-24 + 6}{\sqrt{40} \sqrt{25}}$$

Evaluate.

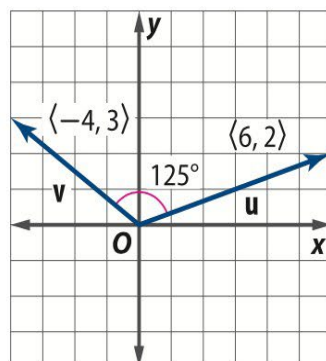
$$\cos \theta = \frac{-9}{5\sqrt{10}}$$

Simplify.

$$\theta = \cos^{-1} \frac{-9}{5\sqrt{10}} \text{ or about } 124.7^\circ$$

Solve for θ .

The measure of the angle between $\mathbf{u}$ and $\mathbf{v}$ is about 124.7° .



b. $\mathbf{u} = \langle 3, 1 \rangle$ and $\mathbf{v} = \langle 3, -3 \rangle$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|}$$

Angle between two vectors

$$\cos \theta = \frac{\langle 3, 1 \rangle \cdot \langle 3, -3 \rangle}{|\langle 3, 1 \rangle| |\langle 3, -3 \rangle|}$$

$$\mathbf{u} = \langle 3, 1 \rangle \text{ and } \mathbf{v} = \langle 3, -3 \rangle$$

$$\cos \theta = \frac{9 + (-3)}{\sqrt{10} \sqrt{18}}$$

Evaluate.

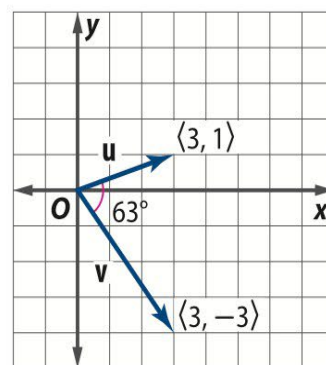
$$\cos \theta = \frac{1}{\sqrt{5}}$$

Simplify.

$$\theta = \cos^{-1} \frac{1}{\sqrt{5}} \text{ or about } 63.4^\circ$$

Solve for θ .

The measure of the angle between $\mathbf{u}$ and $\mathbf{v}$ is about 63.4° .



Guided Practice

3A. $\mathbf{u} = \langle -5, -2 \rangle$ and $\mathbf{v} = \langle 4, 4 \rangle$

3B. $\mathbf{u} = \langle 9, 5 \rangle$ and $\mathbf{v} = \langle -6, 7 \rangle$

extra Q's to practice....

Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$ to the nearest tenth of a degree. (Example 3)

16. $\mathbf{u} = \langle 0, -5 \rangle, \mathbf{v} = \langle 1, -4 \rangle$

17. $\mathbf{u} = \langle 7, 10 \rangle, \mathbf{v} = \langle 4, -4 \rangle$

18. $\mathbf{u} = \langle -2, 4 \rangle, \mathbf{v} = \langle 2, -10 \rangle$

19. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j}, \mathbf{v} = -4\mathbf{i} - 2\mathbf{j}$

20. $\mathbf{u} = \langle -9, 0 \rangle, \mathbf{v} = \langle -1, -1 \rangle$

21. $\mathbf{u} = -\mathbf{i} - 3\mathbf{j}, \mathbf{v} = -7\mathbf{i} - 3\mathbf{j}$

22. $\mathbf{u} = \langle 6, 0 \rangle, \mathbf{v} = \langle -10, 8 \rangle$

23. $\mathbf{u} = -10\mathbf{i} + \mathbf{j}, \mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$

24. **CAMPING** Omar and Ali set off from their campsite to search for firewood. The path that Omar takes can be represented by $\mathbf{u} = \langle 3, -5 \rangle$. The path that Ali takes can be represented by $\mathbf{v} = \langle -7, 6 \rangle$. Find the angle between the pair of vectors. (Example 3)

Example 1 Find the Dot Product to Determine Orthogonal Vectors in Space

Find the dot product of u and v . Then determine if u and v are orthogonal.

a. $u = \langle -7, 3, -3 \rangle, v = \langle 5, 17, 5 \rangle$

b. $u = \langle 3, -3, 3 \rangle, v = \langle 4, 7, 3 \rangle$

$$\begin{aligned} u \cdot v &= -7(5) + 3(17) + (-3)(5) \\ &= -35 + 51 + (-15) \text{ or } 1 \end{aligned}$$

$$\begin{aligned} u \cdot v &= 3(4) + (-3)(7) + 3(3) \\ &= 12 + (-21) + 9 \text{ or } 0 \end{aligned}$$

Since $u \cdot v \neq 0$, u and v are not orthogonal.

Since $u \cdot v = 0$, u and v are orthogonal.

Guided Practice

1A. $u = \langle 3, -5, 4 \rangle, v = \langle 5, 7, 5 \rangle$

1B. $u = \langle 4, -2, -3 \rangle, v = \langle 1, 3, -2 \rangle$

extra Q's to practice....

Find the dot product of u and v . Then determine if u and v are orthogonal. (Example 1)

1. $u = \langle 3, -9, 6 \rangle, v = \langle -8, 2, 7 \rangle$

2. $u = \langle 5, 0, -4 \rangle, v = \langle 6, -1, 4 \rangle$

3. $u = \langle 2, -8, -7 \rangle, v = \langle 5, 9, -7 \rangle$

4. $u = \langle -7, -3, 1 \rangle, v = \langle -4, 5, -13 \rangle$

5. $u = \langle 11, 4, -2 \rangle, v = \langle -1, 3, 8 \rangle$

6. $u = 6i - 2j - 5k, v = 3i - 2j + 6k$

7. $u = 3i - 10j + k, v = 7i + 2j - k$

8. $u = 9i - 9j + 6k, v = 6i + 4j - 3k$

Example 5 Area of a Parallelogram in Space

Find the area of the parallelogram with adjacent sides $\mathbf{u} = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$.

Step 1 Find $\mathbf{u} \times \mathbf{v}$.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & -3 \\ 1 & -5 & 3 \end{vmatrix}$$

$$\mathbf{u} = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} \text{ and } \mathbf{v} = \mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$$

$$= \begin{vmatrix} 4 & -3 \\ -5 & 3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & -3 \\ 1 & 3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & 4 \\ 1 & -5 \end{vmatrix} \mathbf{k}$$

Determinant of a 3×3 matrix

$$= -3\mathbf{i} - 9\mathbf{j} - 14\mathbf{k}$$

Determinants of 2×2 matrices

Step 2 Find the magnitude of $\mathbf{u} \times \mathbf{v}$.

$$|\mathbf{u} \times \mathbf{v}| = \sqrt{(-3)^2 + (-9)^2 + (-14)^2}$$

Magnitude of a vector in space

$$= \sqrt{286} \text{ or about } 16.9$$

Simplify.

The area of the parallelogram shown in Figure 7.5.4 is about 16.9 square units.

Guided Practice

5. Find the area of the parallelogram with adjacent sides $\mathbf{u} = -6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

extra Q's to practice....

Find the area of the parallelogram with adjacent sides $\mathbf{u}$ and $\mathbf{v}$. (Example 5)

24. $\mathbf{u} = \langle 2, -5, 3 \rangle, \mathbf{v} = \langle 4, 6, -1 \rangle$

25. $\mathbf{u} = \langle -9, 1, 2 \rangle, \mathbf{v} = \langle 6, -5, 3 \rangle$

26. $\mathbf{u} = \langle 4, 3, -1 \rangle, \mathbf{v} = \langle 7, 2, -2 \rangle$

27. $\mathbf{u} = 6\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}, \mathbf{v} = 5\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$

28. $\mathbf{u} = \mathbf{i} + 4\mathbf{j} - 8\mathbf{k}, \mathbf{v} = -2\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

29. $\mathbf{u} = -3\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}, \mathbf{v} = 4\mathbf{i} - \mathbf{j} + 6\mathbf{k}$

Example 3 Multiple Representations of Polar Coordinates

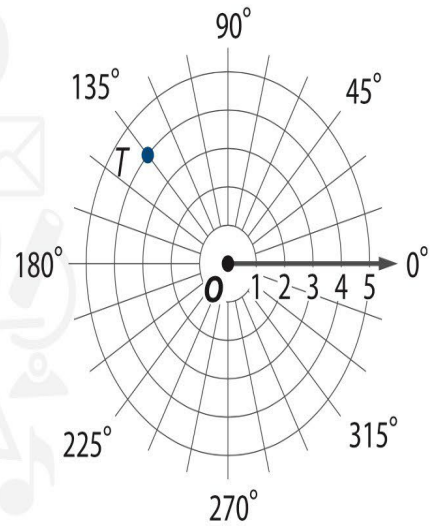
Find four different pairs of polar coordinates that name point T if $-360^\circ \leq \theta \leq 360^\circ$.

One pair of polar coordinates that name point T is $(4, 135^\circ)$.
 The other three representations are as follows.

$$\begin{aligned}(4, 135^\circ) &= (4, 135^\circ - 360^\circ) && \text{Subtract } 360^\circ \text{ from } \theta. \\ &= (4, -225^\circ)\end{aligned}$$

$$\begin{aligned}(4, 135^\circ) &= (-4, 135^\circ + 180^\circ) && \text{Replace } r \text{ with } -r \text{ and} \\ &= (-4, 315^\circ) && \text{add } 180^\circ \text{ to } \theta.\end{aligned}$$

$$\begin{aligned}(4, 135^\circ) &= (-4, 135^\circ - 180^\circ) && \text{Replace } r \text{ with } -r \text{ and} \\ &= (-4, -45^\circ) && \text{subtract } 180^\circ \text{ from } \theta.\end{aligned}$$



Guided Practice

Find three additional pairs of polar coordinates that name the given point if $-360^\circ \leq \theta \leq 360^\circ$ or $-2\pi \leq \theta \leq \pi$.

3A. $(5, 240^\circ)$

3B. $(2, \frac{\pi}{6})$

Find three different pairs of polar coordinates that name the given point if $-360^\circ \leq \theta \leq 360^\circ$ or $-2\pi \leq \theta \leq 2\pi$. (Example 3)

14. $(1, 150^\circ)$

15. $(-2, 300^\circ)$

16. $(4, -\frac{7\pi}{6})$

17. $(-3, \frac{2\pi}{3})$

18. $(5, \frac{11\pi}{6})$

19. $(-5, -\frac{4\pi}{3})$

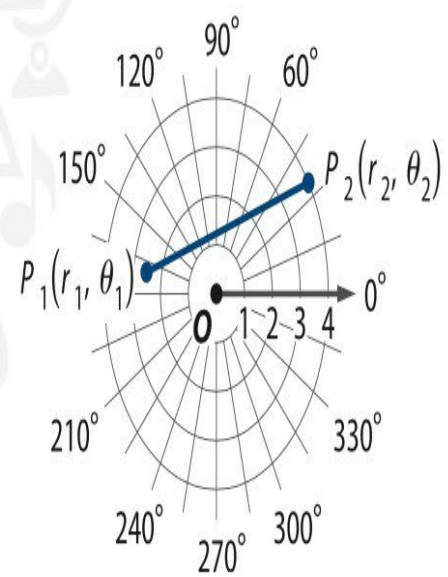
20. $(2, -30^\circ)$

21. $(-1, -240^\circ)$

KeyConcept Polar Distance Formula

If $P_1(r_1, \theta_1)$ and $P_2(r_2, \theta_2)$ are two points in the polar plane, then the distance P_1P_2 is given by

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_2 - \theta_1)}.$$



extra Q's to practice....

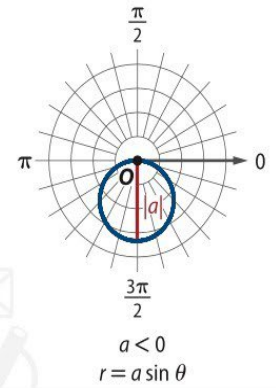
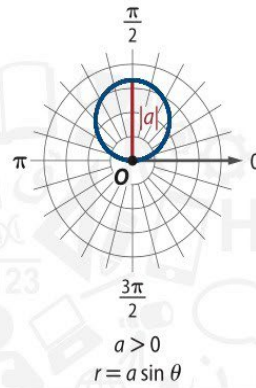
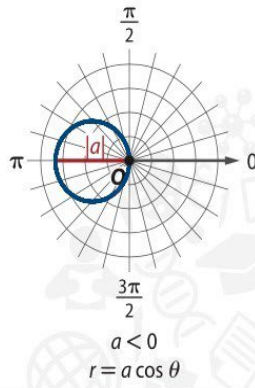
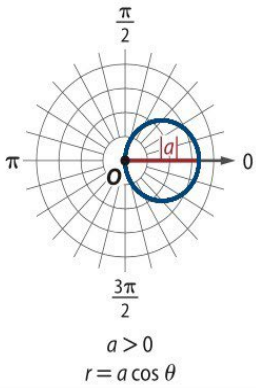
Find the distance between each pair of points. (Example 5)

- | | |
|--|---|
| 30. $(2, 30^\circ), (5, 120^\circ)$ | 31. $(3, \frac{\pi}{2}), (8, \frac{4\pi}{3})$ |
| 32. $(6, 45^\circ), (-3, 300^\circ)$ | 33. $(7, -\frac{\pi}{3}), (1, \frac{2\pi}{3})$ |
| 34. $(-5, \frac{7\pi}{6}), (4, \frac{\pi}{6})$ | 35. $(4, -315^\circ), (1, 60^\circ)$ |
| 36. $(-2, -30^\circ), (8, 210^\circ)$ | 37. $(-3, \frac{11\pi}{6}), (-2, \frac{5\pi}{6})$ |
| 38. $(1, -\frac{\pi}{4}), (-5, \frac{7\pi}{6})$ | 39. $(7, -90^\circ), (-4, -330^\circ)$ |
| 40. $(8, -\frac{2\pi}{3}), (4, -\frac{3\pi}{4})$ | 41. $(-5, 135^\circ), (-1, 240^\circ)$ |

ConceptSummary Special Types of Polar Graphs

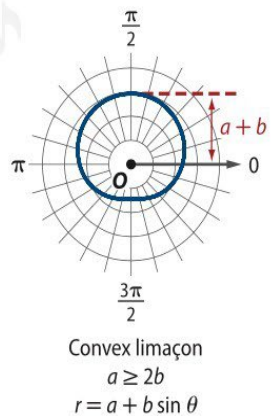
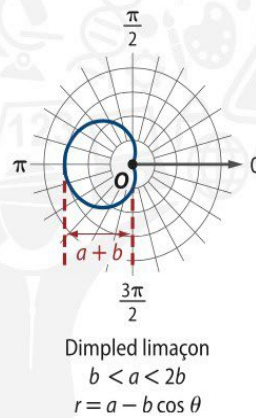
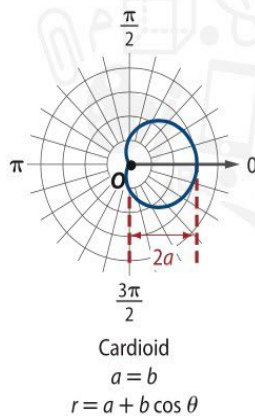
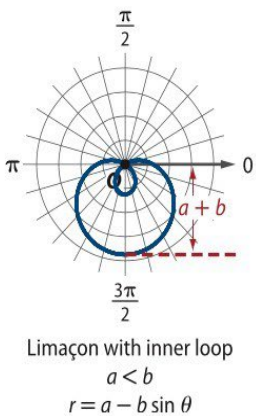
Circles

$$r = a \cos \theta \text{ or } r = a \sin \theta$$



Limaçons

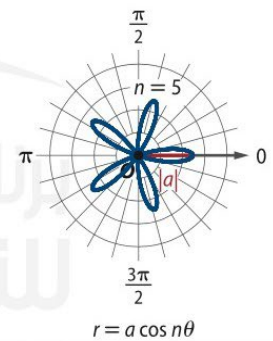
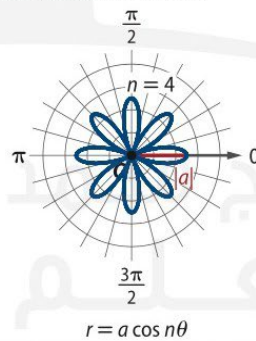
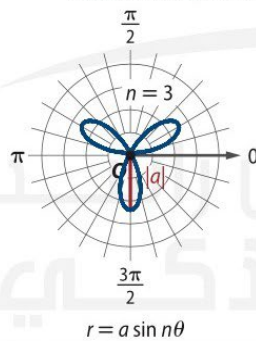
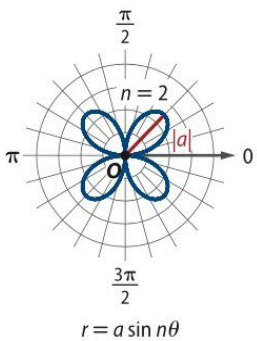
$$r = a \pm b \cos \theta \text{ or } r = a \pm b \sin \theta, \text{ where } a \text{ and } b \text{ are both positive}$$



Roses

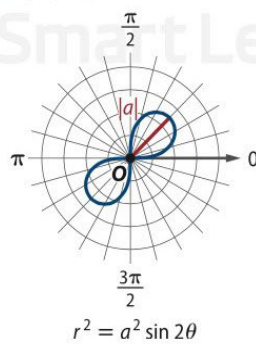
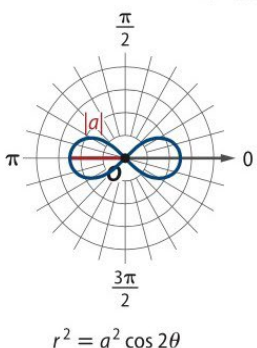
$$r = a \cos n\theta \text{ or } r = a \sin n\theta, \text{ where } n \geq 2 \text{ is an integer}$$

The rose has n petals if n is odd and $2n$ petals if n is even.



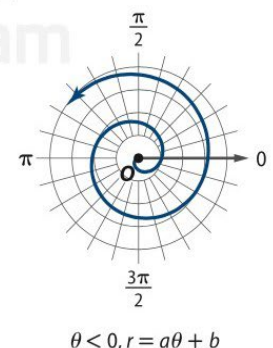
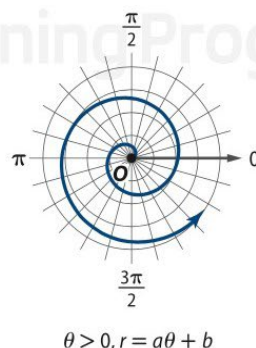
Lemniscates (LEM-nis-keyts)

$$r^2 = a^2 \cos 2\theta \text{ or } r^2 = a^2 \sin 2\theta$$



Spirals of Archimedes (ahr-kuh-MEE-deez)

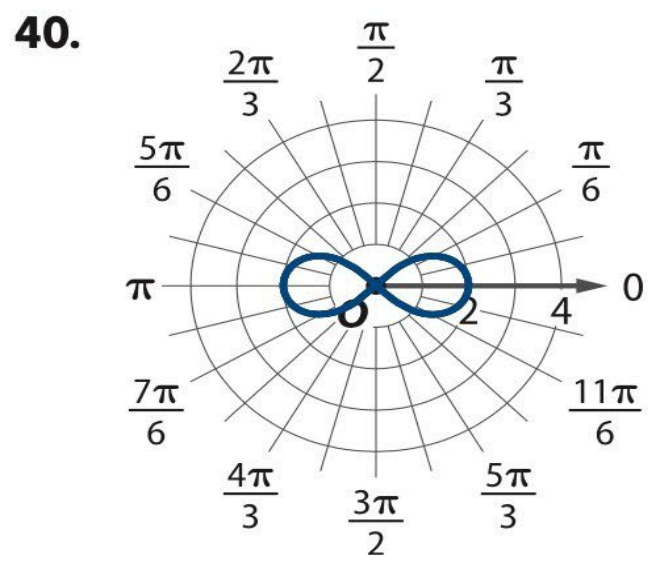
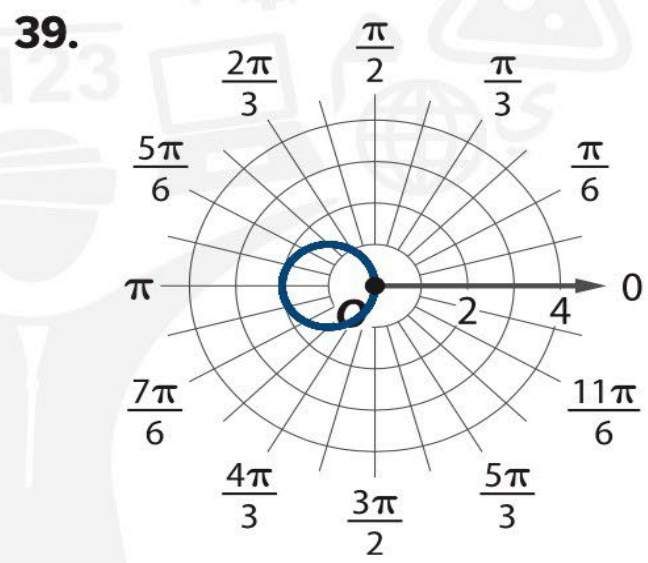
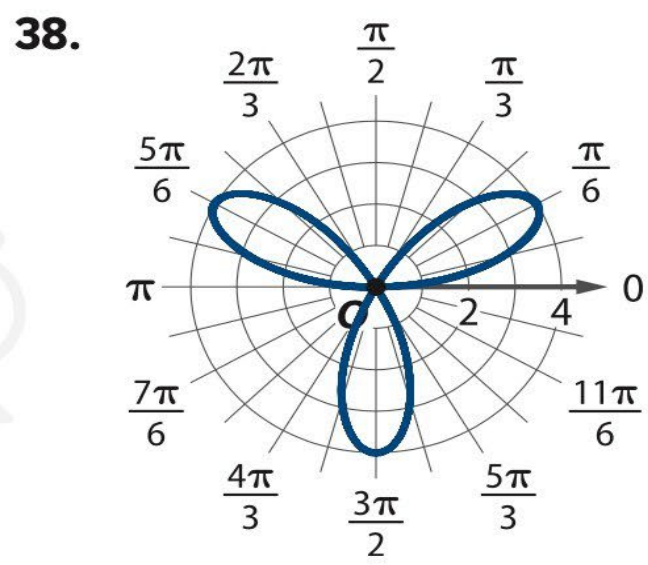
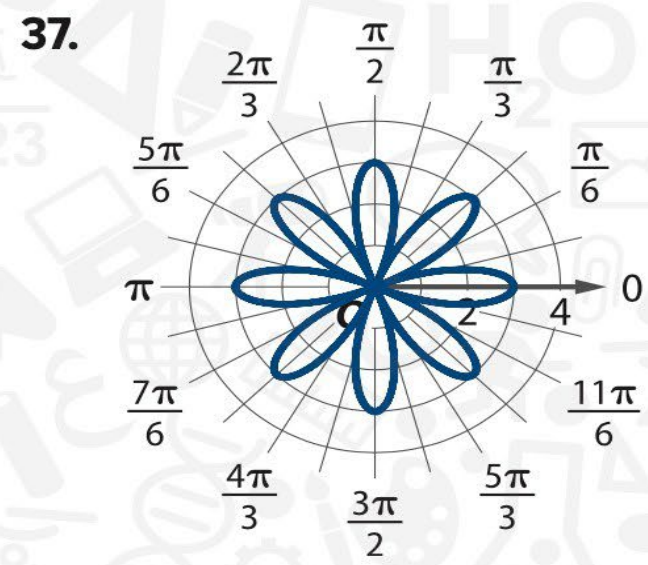
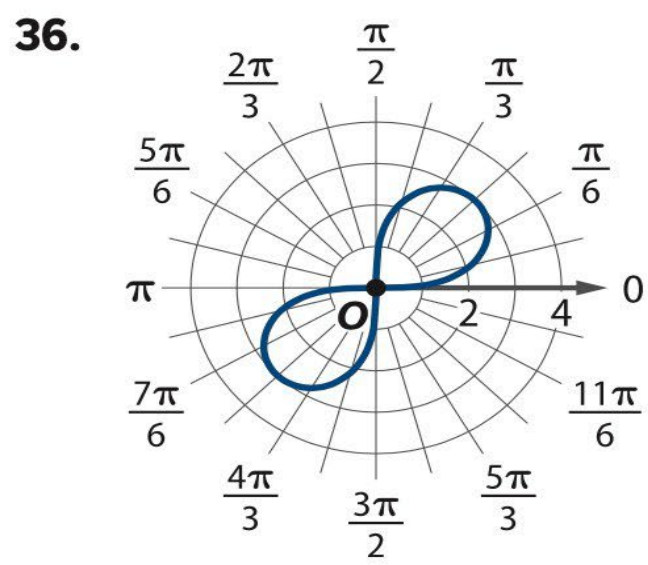
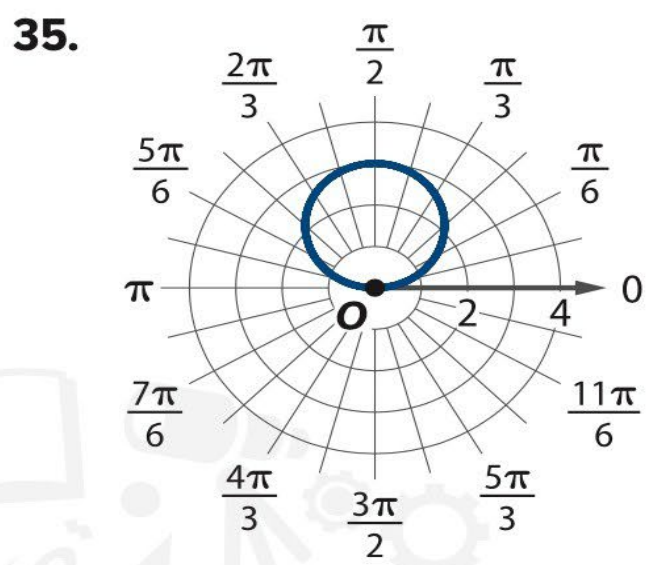
$$r = a\theta + b$$



8 Identify and graph classical curves.

Concept Summary • Special Types of Polar Graphs
 (35-40)

Write an equation for each graph.



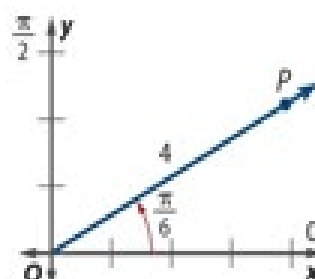
Example 1 Polar Coordinates to Rectangular Coordinates

Find the rectangular coordinates for each point with the given polar coordinates.

a. $P\left(4, \frac{\pi}{6}\right)$

For $P\left(4, \frac{\pi}{6}\right)$, $r = 4$ and $\theta = \frac{\pi}{6}$.

$x = r \cos \theta$	Conversion formula	$y = r \sin \theta$
$= 4 \cos \frac{\pi}{6}$	$r = 4$ and $\theta = \frac{\pi}{6}$	$= 4 \sin \frac{\pi}{6}$
$= 4\left(\frac{\sqrt{3}}{2}\right)$	Simplify.	$= 4\left(\frac{1}{2}\right)$
$= 2\sqrt{3}$		$= 2$

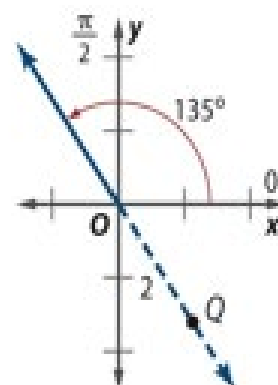


The rectangular coordinates of P are $(2\sqrt{3}, 2)$ or approximately $(3.46, 2)$ as shown.

b. $Q(-2, 135^\circ)$

For $Q(-2, 135^\circ)$, $r = -2$ and $\theta = 135^\circ$.

$x = r \cos \theta$	Conversion formula	$y = r \sin \theta$
$= -2 \cos 135^\circ$	$r = -2$ and $\theta = 135^\circ$	$= -2 \sin 135^\circ$
$= -2\left(-\frac{\sqrt{2}}{2}\right)$	Simplify.	$= -2\left(\frac{\sqrt{2}}{2}\right)$
$= \sqrt{2}$		$= -\sqrt{2}$

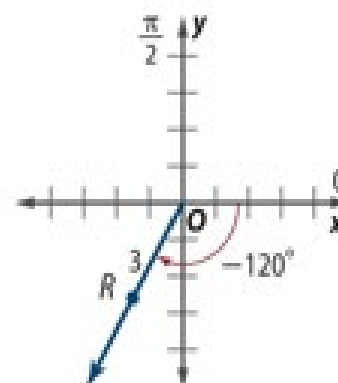


The rectangular coordinates of Q are $(\sqrt{2}, -\sqrt{2})$ or approximately $(1.41, -1.41)$ as shown.

c. $V(3, -120^\circ)$

For $V(3, -120^\circ)$, $r = 3$ and $\theta = -120^\circ$.

$x = r \cos \theta$	Conversion formula	$y = r \sin \theta$
$= 3 \cos -120^\circ$	$r = 3$ and $\theta = -120^\circ$	$= 3 \sin -120^\circ$
$= 3\left(-\frac{1}{2}\right)$	Simplify.	$= 3\left(-\frac{\sqrt{3}}{2}\right)$
$= -\frac{3}{2}$		$= -\frac{3\sqrt{3}}{2}$



The rectangular coordinates of V are $\left(-\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$ or approximately $(-1.5, -2.6)$ as shown.

Guided Practice

1A. $R(-6, -120^\circ)$

1B. $S\left(5, \frac{\pi}{3}\right)$

1C. $T(-3, 45^\circ)$

extra Q's to practice....

Find the rectangular coordinates for each point with the given polar coordinates. Round to the nearest hundredth, if necessary. (Example 1)

1. $\left(2, \frac{\pi}{4}\right)$

2. $\left(\frac{1}{4}, \frac{\pi}{2}\right)$

3. $(5, 240^\circ)$

4. $(2.5, 250^\circ)$

5. $\left(-2, \frac{4\pi}{3}\right)$

6. $(-13, -70^\circ)$

7. $\left(3, \frac{\pi}{2}\right)$

8. $\left(\frac{1}{2}, \frac{3\pi}{4}\right)$

9. $(-2, 270^\circ)$

10. $(4, 210^\circ)$

11. $\left(-1, -\frac{\pi}{6}\right)$

12. $\left(5, \frac{\pi}{3}\right)$

Example 4 Rectangular Equations to Polar Equations

Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.

a. $(x - 4)^2 + y^2 = 16$

The graph of $(x - 4)^2 + y^2 = 16$ is a circle with radius 4 centered at (4, 0). To find the polar form of this equation, replace x with $r \cos \theta$ and y with $r \sin \theta$. Then simplify.

$$(x - 4)^2 + y^2 = 16$$

Original equation

$$(r \cos \theta - 4)^2 + (r \sin \theta)^2 = 16$$

$x = r \cos \theta$ and $y = r \sin \theta$

$$r^2 \cos^2 \theta - 8r \cos \theta + 16 + r^2 \sin^2 \theta = 16$$

Multiply.

$$r^2 \cos^2 \theta - 8r \cos \theta + r^2 \sin^2 \theta = 0$$

Subtract 16 from each side.

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 8r \cos \theta$$

Isolate the squared terms.

$$r^2(\cos^2 \theta + \sin^2 \theta) = 8r \cos \theta$$

Factor.

$$r^2(1) = 8r \cos \theta$$

Pythagorean Identity

$$r = 8 \cos \theta$$

Divide each side by r .

The graph of this polar equation (Figure 8.3.3) is a circle with radius 4 centered at (4, 0).

b. $y = x^2$

The graph of $y = x^2$ is a parabola with vertex at the origin that opens up.

$$y = x^2$$

Original equation

$$r \sin \theta = (r \cos \theta)^2$$

$x = r \cos \theta$ and $y = r \sin \theta$

$$r \sin \theta = r^2 \cos^2 \theta$$

Multiply.

$$\frac{\sin \theta}{\cos^2 \theta} = r$$

Divide each side by $r \cos^2 \theta$.

$$\frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} = r$$

Rewrite.

$$\tan \theta \sec \theta = r$$

Quotient and Reciprocal Identities

10 Convert between polar and rectangular equations.

Example 4 & (4A,4B)

The graph of the polar equation $r = \tan \theta \sec \theta$ (Figure 8.3.4) is a parabola with vertex at the pole that opens up.

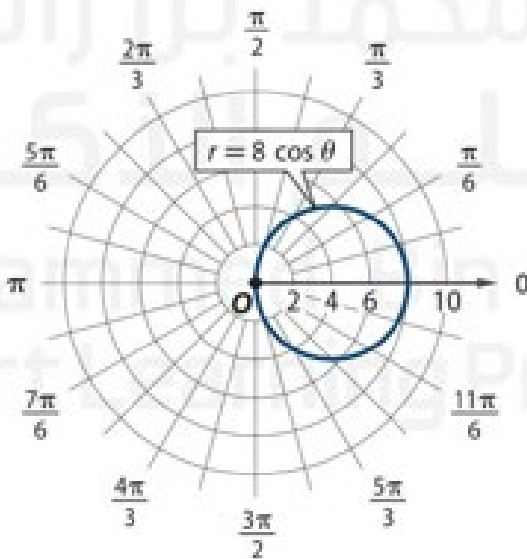


Figure 8.3.3

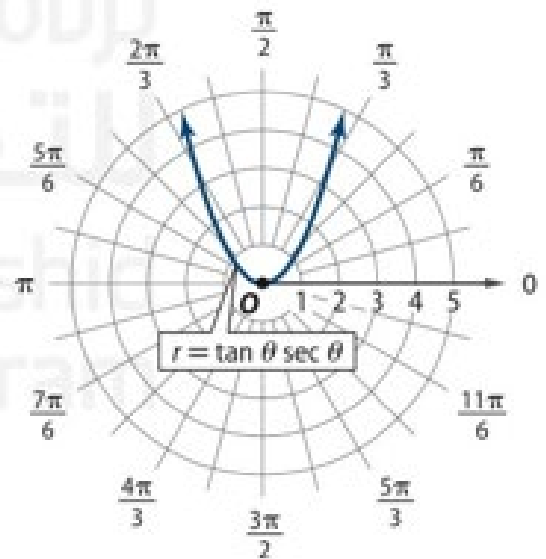


Figure 8.3.4

Guided Practice

4A. $x^2 + (y - 3)^2 = 9$

4B. $x^2 - y^2 = 1$

extra Q's to practice....

Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation. (Example 4)

26. $x = -2$

27. $(x + 5)^2 + y^2 = 25$

28. $y = -3$

29. $x = y^2$

30. $(x - 2)^2 + y^2 = 4$

31. $(x - 1)^2 - y^2 = 1$

32. $x^2 + (y + 3)^2 = 9$

33. $y = \sqrt{3}x$

34. $x^2 + (y + 1)^2 = 1$

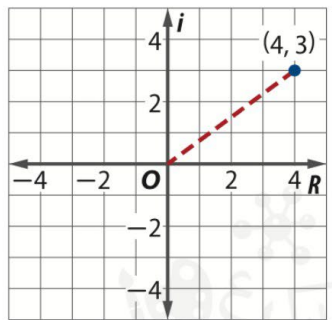
35. $x^2 + (y - 8)^2 = 64$

Example 1 Graphs and Absolute Values of Complex Numbers

Graph each number in the complex plane, and find its absolute value.

a. $z = 4 + 3i$

$(a, b) = (4, 3)$



$$|z| = \sqrt{a^2 + b^2} \quad \text{Absolute value formula}$$

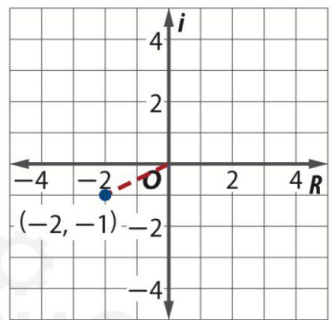
$$= \sqrt{4^2 + 3^2} \quad a = 4 \text{ and } b = 3$$

$$= \sqrt{25} \text{ or } 5 \quad \text{Simplify.}$$

The absolute value of $4 + 3i = 5$.

b. $z = -2 - i$

$(a, b) = (-2, -1)$



$$|z| = \sqrt{a^2 + b^2} \quad \text{Absolute value formula}$$

$$= \sqrt{(-2)^2 + (-1)^2} \quad a = -2 \text{ and } b = -1$$

$$= \sqrt{5} \text{ or } 2.24 \quad \text{Simplify.}$$

The absolute value of $-2 - i \approx 2.24$.

Guided Practice

1A. $5 + 2i$

1B. $-3 + 4i$

extra Q's to practice....

Graph each number in the complex plane, and find its absolute value. (Example 1)

1. $z = 4 + 4i$

2. $z = -3 + i$

3. $z = -4 - 6i$

4. $z = 2 - 5i$

5. $z = 3 + 4i$

6. $z = -7 + 5i$

7. $z = -3 - 7i$

8. $z = 8 - 2i$

9. **VECTORS** The force on an object is given by $z = 10 + 15i$, where the components are measured in newtons (N). (Example 1)

- a. Represent z as a vector in the complex plane.
- b. Find the magnitude and direction angle of the vector.

Example 4 Product of Complex Numbers in Polar Form

Find $2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right) \cdot 4\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$ in polar form. Then express the product in rectangular form.

$$2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right) \cdot 4\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right) \quad \text{Original expression}$$

$$= 2(4)\left[\cos \left(\frac{5\pi}{3} + \frac{\pi}{6}\right) + i \sin \left(\frac{5\pi}{3} + \frac{\pi}{6}\right)\right] \quad \text{Product Formula}$$

$$= 8\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right) \quad \text{Simplify.}$$

Now find the rectangular form of the product.

$$8\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right) \quad \text{Polar form}$$

$$= 8\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) \quad \text{Evaluate.}$$

$$= 4\sqrt{3} - 4i \quad \text{Distributive Property}$$

The polar form of the product is $8\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right)$. The rectangular form of the product is $4\sqrt{3} - 4i$.

Guided Practice

Find each product in polar form. Then express the product in rectangular form.

4A. $3\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) \cdot 5\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

4B. $-6\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) \cdot 2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

extra Q's to practice....

Find each product or quotient, and express it in rectangular form. (Examples 4 and 5)

26. $6\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) \cdot 4\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

27. $5(\cos 135^\circ + i \sin 135^\circ) \cdot 2(\cos 45^\circ + i \sin 45^\circ)$

28. $3\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) \div \frac{1}{2}(\cos \pi + i \sin \pi)$

29. $2(\cos 90^\circ + i \sin 90^\circ) \cdot 2(\cos 270^\circ + i \sin 270^\circ)$

30. $3\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right) \div 4\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

31. $4\left(\cos \frac{9\pi}{4} + i \sin \frac{9\pi}{4}\right) \div 2\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$

32. $\frac{1}{2}(\cos 60^\circ + i \sin 60^\circ) \cdot 6(\cos 150^\circ + i \sin 150^\circ)$

33. $6\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) \div 2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

34. $5(\cos 180^\circ + i \sin 180^\circ) \cdot 2(\cos 135^\circ + i \sin 135^\circ)$

35. $\frac{1}{2}\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) \div 3\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

Example 1 Identify Arithmetic Sequences

Determine whether each sequence is arithmetic.

a. $5, -6, -17, -28, \dots$



The common difference is -11 .
 The sequence is arithmetic.

b. $-4, 12, 28, 42, \dots$



There is no common difference.
 This is not an arithmetic sequence.

Guided Practice

1A. $7, 12, 16, 20, \dots$

1B. $-6, 3, 12, 21, \dots$

extra Q's to practice....

Determine whether each sequence is arithmetic. Write *yes* or *no*.

1. $8, -2, -12, -22,$

2. $-19, -12, -5, 2, 9$

3. $1, 2, 4, 8, 16$

4. $0.6, 0.9, 1.2, 1.8, \dots$

21. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$

22. $-9, -3, 0, 3, 9$

23. $14, -5, -19, \dots$

24. $\frac{2}{9}, \frac{5}{9}, \frac{8}{9}, \frac{11}{9}, \dots$

14 Relate geometric sequences to exponential functions.

(14-17)

Find the next three terms of each geometric sequence. Then graph the sequence.

14. $8, 12, 18, 27, \dots$

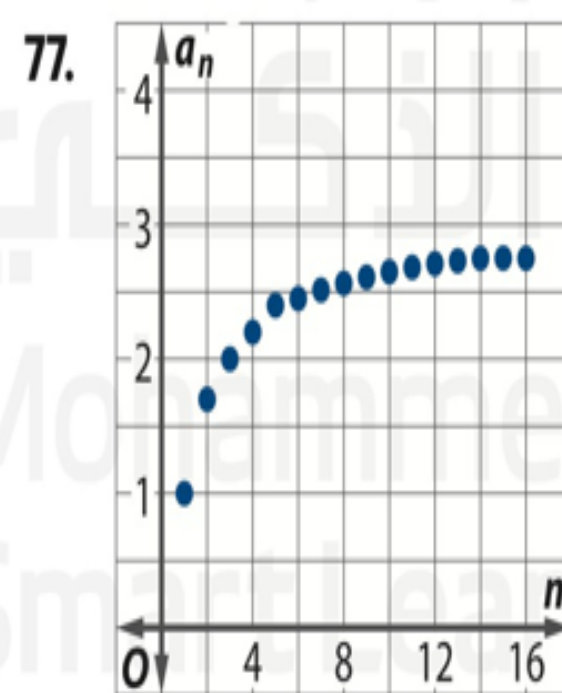
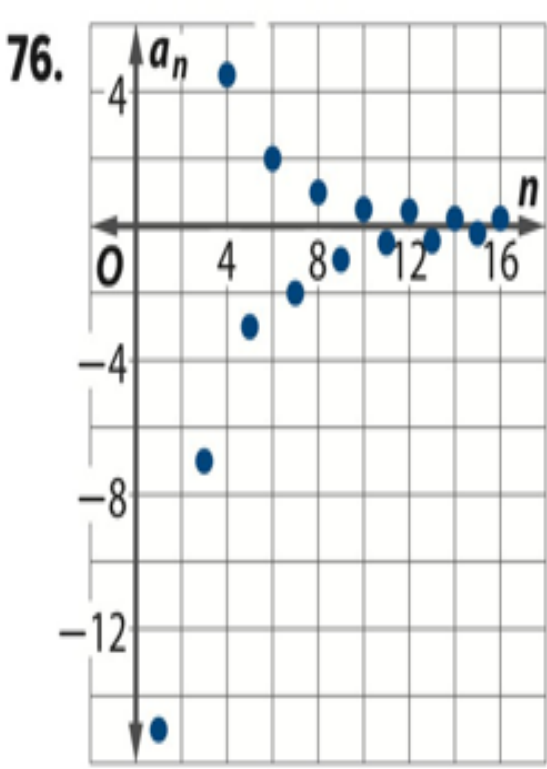
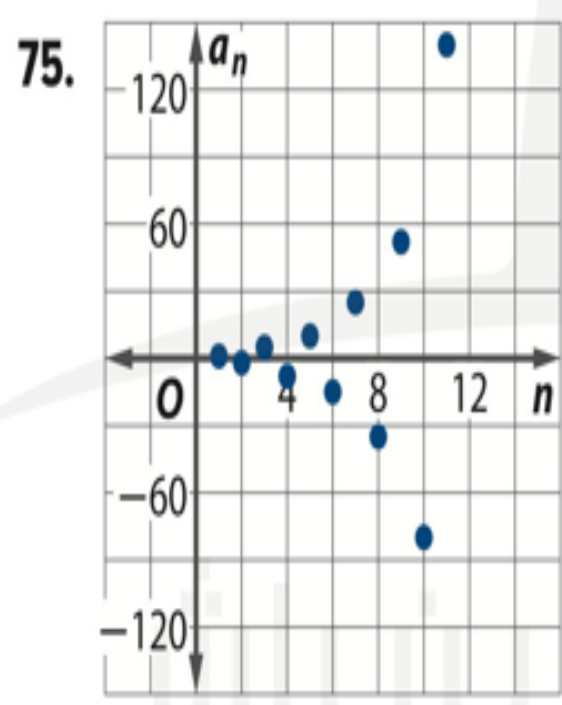
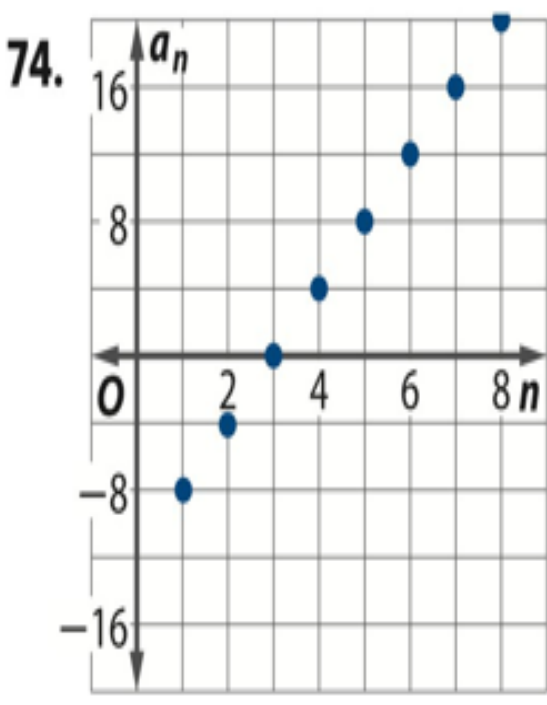
15. $8, 16, 32, 64, \dots$

16. $250, 50, 10, 2, \dots$

17. $9, -3, 1, -\frac{1}{3}, \dots$



Determine whether each sequence is *convergent* or *divergent*.



Key Concept Sigma Notation

For any sequence $a_1, a_2, a_3, a_4, \dots$, the sum of the first k terms is denoted

$$\sum_{n=1}^k a_n = a_1 + a_2 + a_3 + \dots + a_k,$$

where n is the index of summation, k is the upper bound of summation, and 1 is the lower bound of summation.

Reading Math

Sigma Notation $\sum_{n=1}^k a_n$ is read
 the summation from $n = 1$
 to k of a sub n .

Write each series in sigma notation. The lower bound is given.

57. $-2 - 1 + 0 + 1 + 2 + 3 + 4 + 5; n = 1$

58. $\frac{1}{20} + \frac{1}{25} + \frac{1}{30} + \frac{1}{35} + \frac{1}{40} + \frac{1}{45}; n = 4$

59. $8 + 27 + 64 + \dots + 1000; n = 2$

60. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{128}; n = 1$

61. $-8 + 16 - 32 + 64 - 128 + 256 - 512; n = 3$

62. $8\left(-\frac{1}{3}\right) + 8\left(\frac{1}{9}\right) + 8\left(-\frac{1}{27}\right) + \dots + 8\left(-\frac{1}{243}\right); n = 1$

Key Concept n th Term of an Arithmetic Sequence

The n th term a_n of an arithmetic sequence in which the first term is a_1 and the common difference is d is given by the following formula, where n is any natural number.

$$a_n = a_1 + (n - 1)d$$

Example 1 Find the n th Term

Find the 12th term of the arithmetic sequence 9, 16, 23, 30, ...

Step 1 Find the common difference.

$$16 - 9 = 7 \quad 23 - 16 = 7 \quad 30 - 23 = 7$$

$$\text{So, } d = 7.$$

Step 2 Find the 12th term.

$$a_n = a_1 + (n - 1)d \quad \text{\textit{n}th term of an arithmetic sequence}$$

$$\begin{aligned} a_{12} &= 9 + (12 - 1)(7) & a_1 = 9, d = 7, \text{ and } n = 12 \\ &= 9 + 77 \text{ or } 86 & \text{Simplify.} \end{aligned}$$

Guided Practice

Find the indicated term of each arithmetic sequence.

1A. $a_1 = -4, d = 6, n = 9$

1B. a_{20} for $a_1 = 15, d = -8$

extra Q's to practice....

Find the indicated term of each arithmetic sequence.

14. $a_1 = -18, d = 12, n = 16$

15. $a_1 = -12, n = 66, d = 4$

16. $a_1 = 9, n = 24, d = -6$

17. a_{15} for $-5, -12, -19, \dots$

18. a_{10} for $-1, 1, 3, \dots$

19. a_{24} for $8.25, 8.5, 8.75, \dots$

extra Q's to practice....

Standardized Test Example 6 Use Sigma Notation

Find $\sum_{k=4}^{18} (6k - 1)$.

- A 846 B 910 C 975 D 1008

Read the Test Item

You need to find the sum of the series. Find a_1 , a_n , and n .

Solve the Test Item

There are $18 - 4 + 1$ or 15 terms, so $n = 15$.

$a_1 = 6(4) - 1$ or 23 $a_n = 6(18) - 1$ or 107

Find the sum.

$S_n = n \left(\frac{a_1 + a_n}{2} \right)$ Sum formula

$S_{15} = 15 \left(\frac{23 + 107}{2} \right)$ $n = 15$, $a_1 = 23$, and $a_n = 107$

$S_{15} = 15(65)$ or 975 The correct answer is C.

Guided Practice

6. Find $\sum_{m=9}^{21} (5m + 6)$.

- F 972 G 1053 H 1281 J 1701

Find the sum of each arithmetic series.

57. $\sum_{k=1}^{16} (4k - 2)$

58. $\sum_{k=4}^{13} (4k + 1)$

59. $\sum_{k=5}^{16} (2k + 6)$

60. $\sum_{k=0}^{12} (-3k + 2)$

Example 3 Find Geometric Means

Find three geometric means between 2 and 1250.

Step 1 Since there are three terms between the first and last term, there are $3 + 2$ or 5 total terms, so $n = 5$.

Step 2 Find r .

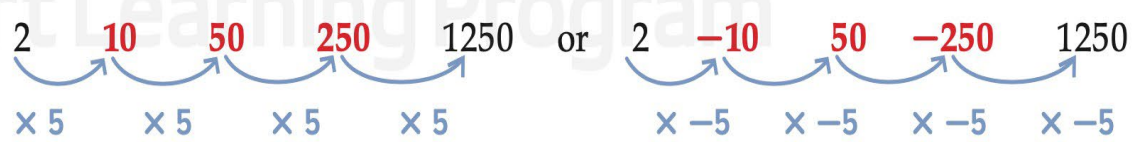
$$a_n = a_1 r^{n-1} \quad \text{nth term of a geometric sequence}$$

$$1250 = 2r^{5-1} \quad a_n = 1250, a_1 = 2, \text{ and } n = 5$$

$$625 = r^4 \quad \text{Divide each side by 2.}$$

$$\pm 5 = r \quad \text{Take the 4th root of each side.}$$

Step 3 Use r to find the four arithmetic means.



The geometric means are 10, 50, and 250 or -10, 50, and -250.

Guided Practice

3. Find four geometric means between 0.5 and 512.

extra Q's to practice....

Find the geometric means of each sequence.

- 35. 810, , , , 10
- 36. 640, , , , 2.5
- 37. $\frac{7}{2}$, , , , $\frac{56}{81}$
- 38. $\frac{729}{64}$, , , , $\frac{324}{9}$
- 39. Find two geometric means between 3 and 375.
- 40. Find two geometric means between 16 and -2.

Example 6 Find the First Term of a Series

Find a_1 in a geometric series for which $S_n = 13,116$, $n = 7$, and $r = 3$.

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \quad \text{Sum formula}$$

$$13,116 = \frac{a_1 - a_1(3^7)}{1 - 3} \quad S_n = 13,116, r = 3, \text{ and } n = 7$$

$$13,116 = \frac{a_1(1 - 3^7)}{1 - 3} \quad \text{Distributive Property}$$

$$13,116 = \frac{-2186a_1}{-2} \quad \text{Subtract.}$$

$$13,116 = 1093a_1 \quad \text{Simplify.}$$

$$12 = a_1 \quad \text{Divide each side by 1093.}$$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}, r \neq 1$$

$$S_n = \frac{a_1 - a_n r}{1 - r}, r \neq 1$$

Guided Practice

6. Find a_1 in a geometric series for which $S_n = -26,240$, $n = 8$, and $r = -3$.

extra Q's to practice....

Find a_1 for each geometric series described.

51. $S_n = -2912$, $r = 3$, $n = 6$

52. $S_n = -10,922$, $r = 4$, $n = 7$

53. $S_n = 1330$, $a_n = 486$, $r = \frac{3}{2}$

54. $S_n = 4118$, $a_n = 128$, $r = \frac{2}{3}$

55. $a_n = 1024$, $r = 8$, $n = 5$

56. $a_n = 1875$, $r = 5$, $n = 7$

Example 3 Infinite Series in Sigma Notation

Find $\sum_{k=1}^{\infty} 18\left(\frac{4}{5}\right)^{k-1}$.

$$S = \frac{a_1}{1-r}$$

Sum formula

$$= \frac{18}{1-\frac{4}{5}}$$

$$a_1 = 18 \text{ and } r = \frac{4}{5}$$

$$= \frac{18}{\frac{1}{5}} \text{ or } 90$$

Simplify.

Guided Practice

3. Find $\sum_{k=1}^{\infty} 12\left(\frac{3}{4}\right)^{k-1}$.

extra Q's to practice....

Find the sum of each infinite series, if it exists.

$$29. \sum_{k=1}^{\infty} \frac{4}{3} \cdot \left(\frac{5}{4}\right)^{k-1}$$

$$30. \sum_{k=1}^{\infty} \frac{1}{4} \cdot 3^{k-1}$$

$$31. \sum_{k=1}^{\infty} \frac{5}{3} \cdot \left(\frac{3}{7}\right)^{k-1}$$

$$32. \sum_{k=1}^{\infty} \frac{2}{3} \cdot \left(\frac{4}{3}\right)^{k-1}$$

$$33. \sum_{k=1}^{\infty} \frac{8}{3} \cdot \left(\frac{5}{6}\right)^{k-1}$$

$$34. \sum_{k=1}^{\infty} \frac{1}{8} \cdot \left(\frac{1}{12}\right)^{k-1}$$

Example 2 Write Recursive Formulas

Write a recursive formula for each sequence.

a. 2, 10, 18, 26, 34, ...

Step 1 Determine whether the sequence is arithmetic or geometric.
 The sequence is arithmetic because each term after the first can be found by adding a common difference.

Step 2 Find the common difference.

$$d = 10 - 2 \text{ or } 8$$

Step 3 Write the recursive formula.

$$a_n = a_{n-1} + d \quad \text{Recursive formula for arithmetic sequence}$$

$$a_n = a_{n-1} + 8 \quad d = 8$$

A recursive formula for the sequence is $a_n = a_{n-1} + 8, a_1 = 2$.

b. 16, 56, 196, 686, 2401, ...

Step 1 Determine whether the sequence is arithmetic or geometric.
 The sequence is geometric because each term after the first can be found after multiplying by a common ratio.

Step 2 Find the common ratio.

$$r = \frac{56}{16} \text{ or } 3.5$$

Step 3 Write the recursive formula.

$$a_n = r \cdot a_{n-1} \quad \text{Recursive formula for geometric sequence}$$

$$a_n = 3.5a_{n-1} \quad r = 3.5$$

A recursive formula for the sequence is $a_n = 3.5a_{n-1}, a_1 = 16$.

c. $a_4 = 108$ and $r = 3$

Step 1 Determine whether the sequence is arithmetic or geometric.
 Because r is given, the sequence is geometric.

Step 2 Write the recursive formula.

$$a_n = r \cdot a_{n-1} \quad \text{Recursive formula for geometric sequence}$$

$$a_n = 3a_{n-1} \quad r = 3$$

A recursive formula for the sequence is $a_n = 3a_{n-1}, a_1 = 4$.

Guided Practice

Write a recursive formula for each sequence.

- 2A. 8, 20, 50, 125, 312.5, ... 2B. 8, 17, 26, 35, 44, ... 2C. $a_3 = 16$ and $r = 4$

extra Q's to practice....

Write a recursive formula for each sequence.

- | | |
|-------------------------------|----------------------------|
| 24. 16, 10, 7, 5.5, 4.75, ... | 25. 32, 12, 7, 5.75, ... |
| 26. 4, 15, 224, 50, 175, ... | 27. 1, 2, 9, 730, ... |
| 28. 9, 33, 129, 513, ... | 29. 480, 128, 40, 18, ... |
| 30. 393, 132, 45, 16, ... | 31. 68, 104, 176, 320, ... |

Example 4 Iterate a Function

Find the first three iterates x_1 , x_2 , and x_3 of $f(x) = 5x + 4$ for an initial value of $x_0 = 2$.

$$x_1 = f(x_0) \quad \text{Iterate the function.}$$

$$= 5(2) + 4 \text{ or } 14 \quad x_0 = 2$$

$$x_2 = f(x_1) \quad \text{Iterate the function.}$$

$$= 5(14) + 4 \text{ or } 74 \quad x_1 = 14$$

$$x_3 = f(x_2) \quad \text{Iterate the function.}$$

$$= 5(74) + 4 \text{ or } 374 \quad x_2 = 74$$

The first three iterates are 14, 74, and 374.

Guided Practice

4. Find the first three iterates x_1 , x_2 , and x_3 of $f(x) = -3x + 8$ for an initial value of $x_0 = 6$.

extra Q's to practice....

Find the first three iterates of each function for the given initial value.

33. $f(x) = 12x + 8, x_0 = 4$

34. $f(x) = -9x + 1, x_0 = -6$

35. $f(x) = -6x + 3, x_0 = 8$

36. $f(x) = 8x + 3, x_0 = -4$

37. $f(x) = -3x^2 + 9, x_0 = 2$

38. $f(x) = 4x^2 + 5, x_0 = -2$

39. $f(x) = 2x^2 - 5x + 1, x_0 = 6$

40. $f(x) = -0.25x^2 + x + 6, x_0 = 8$

41. $f(x) = x^2 + 2x + 3, x_0 = \frac{1}{2}$

42. $f(x) = 2x^2 + x + 1, x_0 = -\frac{1}{2}$

24 Use the Binomial Theorem to expand powers of binomials.

Example 4 & (4)

Example 4 Determine a Single Term

Find the fifth term of $(y + z)^{11}$.

Step 1 Use the Binomial Theorem to write the expansion in sigma notation.

$$(y + z)^{11} = \sum_{k=0}^{11} \frac{11!}{k!(11-k)!} y^{11-k} z^k$$

Step 2 $\frac{11!}{k!(11-k)!} y^{11-k} z^k = \frac{11!}{4!(11-4)!} y^{11-4} z^4$
 $= 330y^7z^4$

For the fifth term, $k = 4$.

$$C(11, 4) = 330$$

Guided Practice

4. Find the sixth term of $(c + d)^{10}$.

extra Q's to practice....

Find the indicated term of each expression.

23. third term of $(x + 2z)^7$

24. fourth term of $(y - 3x)^6$

25. seventh term of $(2a - 2b)^8$

26. sixth term of $(4x + 5y)^6$

27. fifth term of $(x - 4)^9$

28. fourth term of $(c + 6)^8$

Example 3 Use a Counterexample to Disprove

Find a counterexample to disprove the statement that $2^n + 2n^2$ is divisible by 4 for any natural number n .

Test different values of n .

n	$2^n + 2n^2$	Divisible by 4?
1	$2^1 + 2(1)^2 = 2 + 2$ or 4	yes
2	$2^2 + 2(2)^2 = 4 + 8$ or 12	yes
3	$2^3 + 2(3)^2 = 8 + 18$ or 26	no

The value $n = 3$ is a counterexample for the statement.

Guided Practice

3. Find a counterexample to disprove $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(3n - 1)}{2}$.

extra Q's to practice....

Find a counterexample to disprove each statement.

6. $3^n + 1$ is divisible by 4.
7. $2^n + 3^n$ is divisible by 4.

21. $1 + 2 + 3 + \dots + n = n^2$
22. $1 + 8 + 27 + \dots + n^3 = (2n + 2)^2$

23. $n^2 - n + 15$ is prime.
24. $n^2 + n + 23$ is prime.

اللهم إني استودعك ما قرأت و ما حفظت و ما تعلمت فرده عند حاجتي إليه انك على كل شيء قدير