

سلسلة الطارق في الرياضيات



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف الحادي عشر المتقدم ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

تاريخ إضافة الملف على موقع المناهج: 00:24:03 2026-06-22

ملفات اكتب للمعلم اكتب للطالب | اختبارات الكترونية | اختبارات | حلول | عروض بوربوينت | أوراق عمل
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المزيد من مادة
رياضيات:

إعداد: طارق علي

التواصل الاجتماعي بحسب الصف الحادي عشر المتقدم



صفحة المناهج
الإماراتية على
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

المواد على تلغرام

المزيد من الملفات بحسب الصف الحادي عشر المتقدم والمادة رياضيات في الفصل الثالث

End of Term 3 Revision

1

End of Term 3 Revision Maths

2

كراسة تدريبية للاختبار النهائي 2026م

3

منهج ريفيل حلول مراجعة امتحانية تدريبات وأسئلة

4

منهج ريفيل مراجعة امتحانية تدريبات وأسئلة غير محلول

5

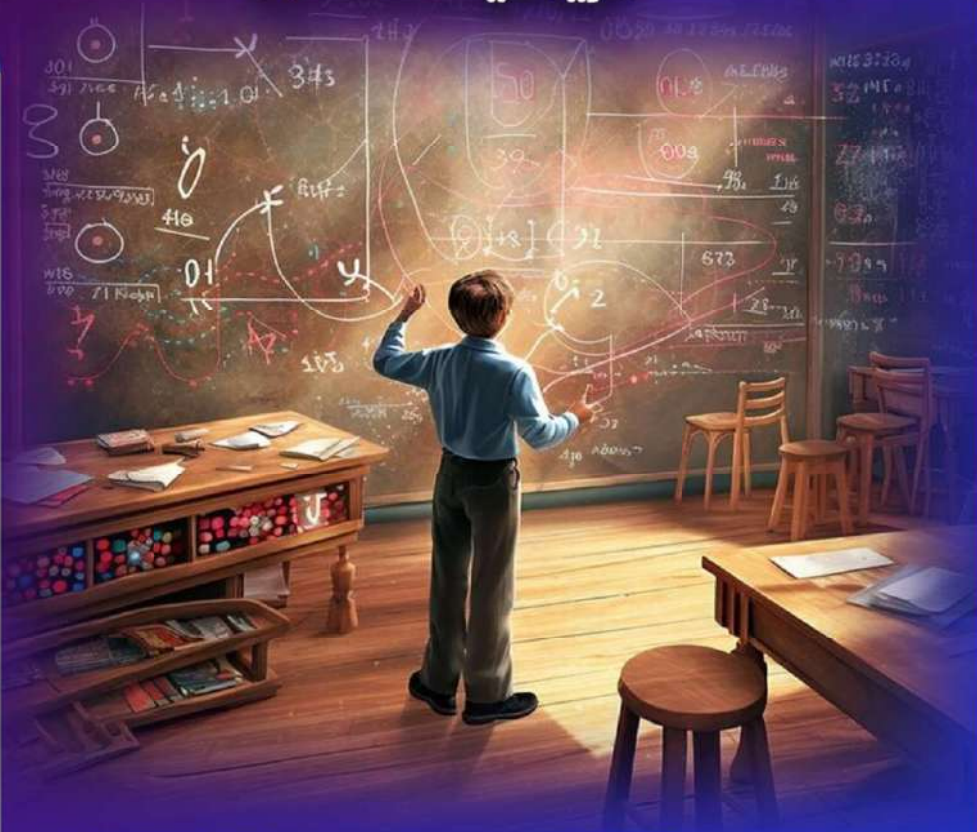
الطارق

سلسلة

الرياضيات

فئة

هيكل 11 متقدم ريفيل 2026



منصة طارق أكاديمي للرياضيات

Tarek Academy

صفحة حادي عشر (متقدم ريفيل)

الطارق

أستاذ الرياضيات

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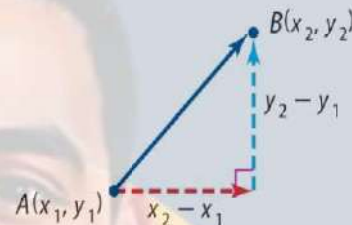
Q1-MCQ 7-2

KeyConcept Component Form of a Vector

The component form of a vector $\overrightarrow{AB}$ with initial point $A(x_1, y_1)$ and terminal point $B(x_2, y_2)$ is given by

$$\langle x_2 - x_1, y_2 - y_1 \rangle.$$

$$\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A}$$



Example 1 Express a Vector in Component Form

Find the component form of $\overrightarrow{AB}$ with initial point $A(-4, 2)$ and terminal point $B(3, -5)$.

☒ A) $(7, -7)$

B) $(-7, 7)$

C) $(7, 7)$

D) $(-7, -7)$

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{B} - \overrightarrow{A} \\ &= (3, -5) - (-4, 2) \\ &= (3 - (-4), -5 - 2) \\ &= (7, -7)\end{aligned}$$

Find the component form of $\overrightarrow{AB}$ with the given initial and terminal points.

1A. $A(-2, -7), B(6, 1)$

A) $(-8, -8)$

☒ B) $(8, 8)$

C) $(4, 6)$

D) $(-4, -6)$

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{B} - \overrightarrow{A} \\ &= (6, 1) - (-2, -7) \\ &= (6 - (-2), 1 - (-7)) \\ &= (8, 8)\end{aligned}$$

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Find the component form of $\overrightarrow{AB}$ with the given initial and terminal points.

$$\overrightarrow{B} - \overrightarrow{A} = (-9, -3) - (0, 8)$$

$$1B. A(0, 8), B(-9, -3)$$

$$(-9 - 0, -3 - 8)$$

$$= (-9, -11)$$

$$A) (9, 11)$$

$$\checkmark B) (-9, -11)$$

$$C) (-9, 11)$$

$$D) (9, -11)$$

Q2-MCQ 7-2

$$\mathbf{v} = \langle a, b \rangle$$

Component form

$$= \langle |\mathbf{v}| \cos \theta, |\mathbf{v}| \sin \theta \rangle$$

 θ , $|\mathbf{v}| \rightarrow \text{magnitude}$

Example 6 Find Component Form

Find the component form of the vector $\mathbf{v}$ with magnitude $\underline{10}$ and direction angle $\underline{120^\circ}$.

$$\checkmark A) (-5, 5\sqrt{3})$$

$$\langle 10 \cos 120, 10 \sin 120 \rangle$$

$$B) (5, -5\sqrt{3})$$

$$\langle -5, 5\sqrt{3} \rangle$$

$$C) (-5\sqrt{3}, 5)$$

$$D) (5\sqrt{3}, -5)$$



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Find the component form of v with the given magnitude and direction angle.

6A. $|v| = 8, \theta = 45^\circ$

$$\langle 8 \cos 45^\circ, 8 \sin 45^\circ \rangle$$

$$\langle 4\sqrt{2}, 4\sqrt{2} \rangle$$

A) $(4\sqrt{2}, 4\sqrt{2})$

B) $(8, 8)$

C) $(4, 4\sqrt{3})$

D) $(8\sqrt{2}, 8\sqrt{2})$

6B. $|v| = 24, \theta = 210^\circ$

$$\langle 24 \cos 210^\circ, 24 \sin 210^\circ \rangle$$

$$\langle -12\sqrt{3}, -12 \rangle$$

A) $(-12\sqrt{3}, -12)$

B) $(12\sqrt{3}, 12)$

C) $(-12, -12\sqrt{3})$

D) $(12, 12\sqrt{3})$

Q3-MCQ 7-3

$$\cos \theta = \frac{a \cdot b}{|a| |b|} \Rightarrow \theta = \cos^{-1} \left(\frac{a \cdot b}{|a| |b|} \right)$$

$$* |a| = \sqrt{x^2 + y^2}$$

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**Example 3** Find the Angle Between Two VectorsFind the angle θ between vectors u and v to the nearest tenth of a degree.

a. $u = \langle 6, 2 \rangle$ and $v = \langle -4, 3 \rangle$

A. 34.7°

B. 55.3°

☒ C. 124.7°

D. 145.3°

$$\theta = \cos^{-1} \frac{\langle 6, 2 \rangle \cdot \langle -4, 3 \rangle}{\sqrt{6^2 + 2^2} \cdot \sqrt{(-4)^2 + 3^2}}$$

$$= \cos^{-1} \left(\frac{6(-4) + 2(3)}{2\sqrt{10} \times 5} \right)$$

shift $\rightarrow \cos$

b. $u = \langle 3, 1 \rangle$ and $v = \langle 3, -3 \rangle$

A. 26.6°

☒ B. 63.4°

C. 71.6°

D. 116.6°

$$\cos^{-1} \frac{3(3) + 1(-3)}{\sqrt{3^2 + 1^2} \times \sqrt{3^2 + (-3)^2}}$$

3A. $u = \langle -5, -2 \rangle$ and $v = \langle 4, 4 \rangle$

A. 23.2°

B. 66.8°

C. 113.2°

☒ D. 156.8°

$$\theta = \cos^{-1} \left(\frac{-5(4) - 2(4)}{\sqrt{(-5)^2 + (-2)^2} \times \sqrt{4^2 + 4^2}} \right)$$

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3B. $u = \langle 9, 5 \rangle$ and $v = \langle -6, 7 \rangle$ A. 11.5° B. 78.5° ☒ C. 101.5° D. 168.5°

$$\theta = \cos^{-1} \left(\frac{9(-6) + 5(7)}{\sqrt{9^2 + 5^2} \times \sqrt{(-6)^2 + 7^2}} \right)$$

Q4-MCQ 7-5**KeyConcept** Dot Product and Orthogonal Vectors in Space

The dot product of $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$ is defined as $a \cdot b = a_1b_1 + a_2b_2 + a_3b_3$. The vectors a and b are orthogonal if and only if $a \cdot b = 0$.

Example 1 Find the Dot Product to Determine Orthogonal Vectors in SpaceFind the dot product of u and v . Then determine if u and v are orthogonal.a. $u = \langle -7, 3, -3 \rangle$, $v = \langle 5, 17, 5 \rangle$ ☒ A. 1; Not Orthogonal

B. 0; Orthogonal

C. -1; Not Orthogonal

D. 101; Not Orthogonal

$$\begin{aligned} u \cdot v &= -7(5) + 3(17) - 3(5) \\ &= 1 \end{aligned}$$

not

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b. $u = \langle 3, -3, 3 \rangle, v = \langle 4, 7, 3 \rangle \Rightarrow u \cdot v$

A. -18; Not Orthogonal

$$3(4) - 3(7) + 3(3)$$

$$= 0$$

✓ B. 0; Orthogonal

C. 42; Not Orthogonal

yes $\Rightarrow$ orthogonal

D. 12; Not Orthogonal

1A. $u = \langle 3, -5, 4 \rangle, v = \langle 5, 7, 5 \rangle \Rightarrow u \cdot v$

✓ A. 0; Orthogonal

$$3(5) - 5(7) + 4(5)$$

$$= 0$$

B. -40; Not Orthogonal

C. 10; Not Orthogonal

orthogonal

D. 70; Not Orthogonal

1B. $u = \langle 4, -2, -3 \rangle, v = \langle 1, 3, -2 \rangle \Rightarrow u \cdot v$

A. 0; Orthogonal

$$4(1) - 2(3) - 3(-2)$$

$$= 4$$

✓ B. 4; Not Orthogonal

C. 16; Not Orthogonal

Not orthogonal.

D. -8; Not Orthogonal

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Q5-MCQ 7-5

① $u \times v$

② $\text{Area} = |u \times v| = \sqrt{\dots^2 + \dots^2 + \dots^2}$

Example 5 Area of a Parallelogram in Space

Find the area of the parallelogram with adjacent sides $u = 2i + 4j - 3k$ and $v = i - 5j + 3k$.

A. $\sqrt{146} \approx 12.1$ square units

✓ B. $\sqrt{286} \approx 16.9$ square units

C. $\sqrt{314} \approx 17.7$ square units

D. 22 square units

$$① u \times v = \begin{vmatrix} i & j & k \\ 2 & 4 & -3 \\ 1 & -5 & 3 \end{vmatrix}$$

$$= i(12 - 15) - j(6 - -3)$$

$$+ k(-10 - 4) = \langle -3, -9, -14 \rangle$$

② $\text{Area} = |u \times v| = \sqrt{(-3)^2 + (-9)^2 + (-14)^2} \approx 16.9$

5. Find the area of the parallelogram with adjacent sides $u = -6i - 2j + 3k$ and $v = 4i + 3j + k$.

A. $\sqrt{137} \approx 11.7$ square units

B. $\sqrt{465} \approx 21.6$ square units

✓ C. $\sqrt{545} \approx 23.3$ square units

D. 25 square units

$$* u \times v = \begin{vmatrix} i & j & k \\ -6 & -2 & 3 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= i(-2 - 9) - j(-6 - 12)$$

$$+ k(-18 - -8) = \langle -11, 18, -10 \rangle$$

* $\text{Area} = \sqrt{(-11)^2 + (18)^2 + (-10)^2} = \sqrt{545} \approx 23.3$

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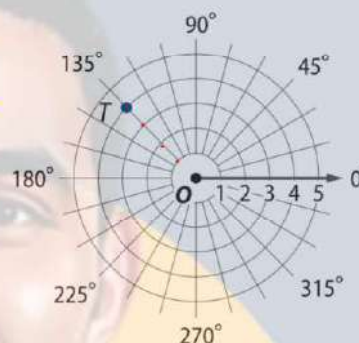
AD Reveal



Q6-MCQ 8-1

Example 3 Multiple Representations of Polar Coordinates

Find four different pairs of polar coordinates that name point T if $-360^\circ \leq \theta \leq 360^\circ$.



$$(4, 135^\circ), (4, -225^\circ), (-4, 315^\circ), (-4, -45^\circ) \quad \text{A}$$

$$(4, 135^\circ), (4, 225^\circ), (-4, 45^\circ), (-4, -315^\circ) \quad \text{B}$$

$$(4, 135^\circ), (4, -135^\circ), (-4, 135^\circ), (-4, -135^\circ) \quad \text{C}$$

$$(4, 135^\circ), (4, -225^\circ), (-4, 135^\circ), (-4, -45^\circ) \quad \text{D}$$

$$* (4, 135) \quad \checkmark$$

$$\theta - 360$$

$$\theta + 360$$

$$* (4, 135 - 360) = (4, -225) \quad \checkmark$$

$$* (-r, \theta \pm 180)$$

$$(-4, 135 - 180) = (-4, -45) \quad \checkmark$$

$$* (-4, 135 + 180) = (-4, 315) \quad \checkmark$$



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Find three additional pairs of polar coordinates that name the given point if $-360^\circ \leq \theta \leq 360^\circ$ or $-2\pi \leq \theta \leq \pi$.

3A. $(5, 240^\circ)$ $(5, 240^\circ), (5, -240^\circ), (-5, 60^\circ), (-5, -60^\circ)$ (A) $(5, 240^\circ), (5, 120^\circ), (-5, 300^\circ), (-5, -60^\circ)$ (B) $(5, 240^\circ), (5, -120^\circ), (-5, 60^\circ), (-5, -300^\circ)$ (C) ✓ $(5, 240^\circ), (5, -120^\circ), (-5, 120^\circ), (-5, -240^\circ)$ (D)

$$*(5, 240 - 360) = (5, -120)$$

$$*(-r, \theta - 180) = (-5, 240 - 180) = (-5, 60) *$$

$$*(-r, \theta + 180) = (-5, 240 + 180) = (-5, 420) \text{ Big}$$

$$(-5, 420 - 360) = (-5, 60) * (-5, 60 - 360) = (-5, -300)$$

Find three additional pairs of polar coordinates that name the given point if $-360^\circ \leq \theta \leq 360^\circ$ or $-2\pi \leq \theta \leq \pi$.

$$*(2, \frac{\pi}{6} - 2\pi) = (2, -\frac{11\pi}{6})$$

3B. $(2, \frac{\pi}{6})$

Radian

$$+ \frac{2\pi}{1} = \frac{2\pi}{1}$$

$$- \frac{\pi}{1} = -\pi$$
 $(2, \frac{\pi}{6}), (2, -\frac{5\pi}{6}), (-2, \frac{7\pi}{6}), (-2, -\frac{\pi}{6})$ (A) $(2, \frac{\pi}{6}), (2, -\frac{7\pi}{6}), (-2, \frac{5\pi}{6}), (-2, -\frac{11\pi}{6})$ (B) $(2, \frac{\pi}{6}), (2, -\frac{\pi}{6}), (-2, \frac{5\pi}{6}), (-2, -\frac{5\pi}{6})$ (C) $(2, \frac{\pi}{6}), (2, -\frac{11\pi}{6}), (-2, \frac{7\pi}{6}), (-2, -\frac{5\pi}{6})$ (D) ✓

$$*(-2, \frac{\pi}{6} + \pi) = (-2, \frac{7\pi}{6})$$

$$= (-2, \frac{7\pi}{6})$$

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Q7 -MCQ 8.1

Find the distance between each pair of points. (Example 5)

30. $(2, 30^\circ), (5, 120^\circ)$

a) 4.0

☒ b) 5.4

c) 6.0

d) 7.0

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_2 - \theta_1)}$$

$$\sqrt{2^2 + 5^2 - 2(2)(5) \cos(120 - 30)} = 5.39$$

Find the distance between each pair of points. (Example 5)

31. $(3, \frac{\pi}{2}), (8, \frac{4\pi}{3})$

A) $\sqrt{73 - 12\sqrt{3}}$

B) $\sqrt{73 + 12\sqrt{3}}$

☒ C) $\sqrt{73 - 24\sqrt{3}}$

D) $\sqrt{73}$

$$\sqrt{3^2 + 8^2 - 2(3)(8) \cos(\frac{4\pi}{3} - \frac{\pi}{2})} = \sqrt{73 - 24\sqrt{3}}$$

32. $(6, 45^\circ), (-3, 300^\circ)$

33. $(7, -\frac{\pi}{3}), (1, \frac{2\pi}{3})$

$$\sqrt{4^2 + 1^2 - 2(1)(4) \cos(\frac{2\pi}{3} - -\frac{\pi}{3})} = 9$$

a) 6.0

b) 7.0

☒ c) 8.0

d) 9.0

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Q8-MCQ 8-2

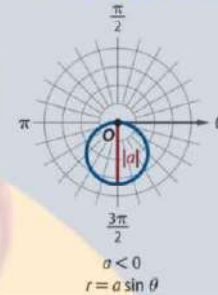
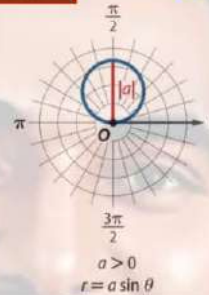
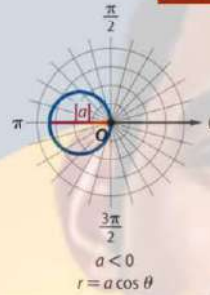
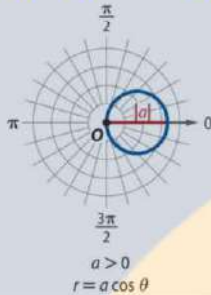
Concept Summary Special Types of Polar Graphs

sin + → up

Circles

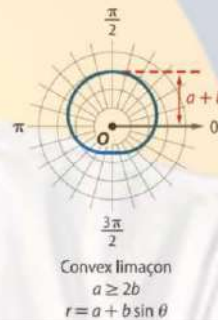
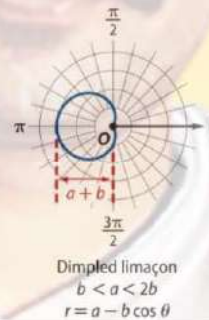
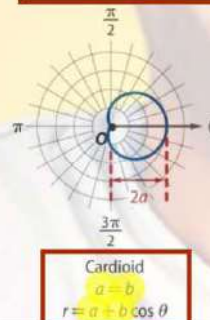
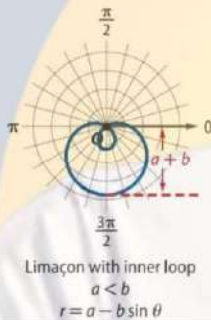
$$r = a \cos \theta \text{ or } r = a \sin \theta$$

cos < + Right
left
sin - → down.



Limaçons

$$r = a \pm b \cos \theta \text{ or } r = a \pm b \sin \theta \text{ where } a \text{ and } b \text{ are both positive}$$



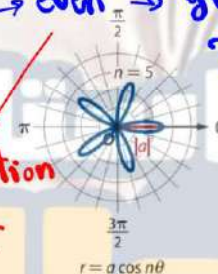
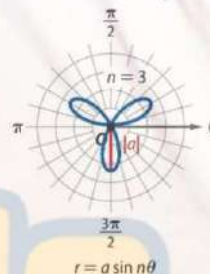
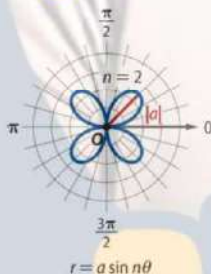
n → petals

Roses

$$r = a \cos n\theta \text{ or } r = a \sin n\theta \text{ where } n \geq 2 \text{ is an integer}$$

The rose has n petals if n is odd and $2n$ petals if n is even.

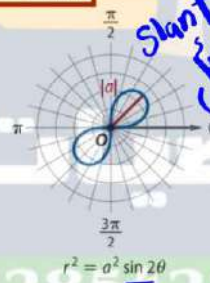
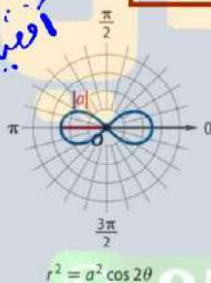
* $n \rightarrow$ odd → Same
* $n \rightarrow$ even → graph $2 \times n$



equation
 $\frac{n}{2}$

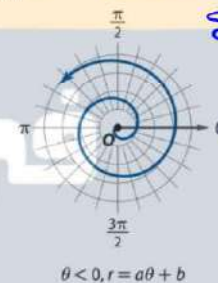
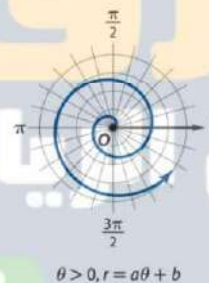
Lemniscates (LEM-nis-keyts)

$$r^2 = a^2 \cos 2\theta \text{ or } r^2 = a^2 \sin 2\theta$$



Spirals of Archimedes (ahr-kuh-MEE-deez)

$$r = a\theta + b$$



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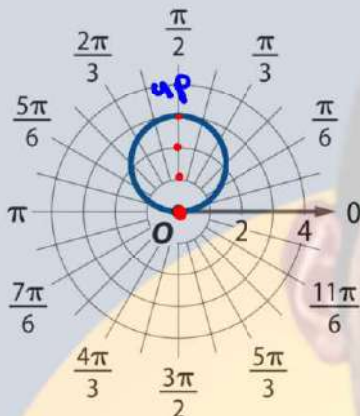
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AD Reveal



Write an equation for each graph.

35.



a) $r = 3\sin(\theta)$

b) $r = 3\cos(\theta)$

c) $r = -3\cos(\theta)$

d) $r = -3\sin(\theta)$

up $\rightarrow$ Sin

circle

$$y = a \sin \theta$$

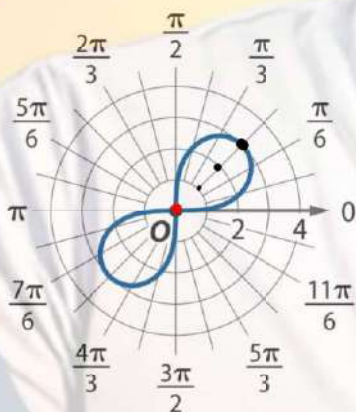


height

$$|a| = 3$$

$$3 \sin \theta$$

36.



a) $r^2 = 3\sin(\theta)$

b) $r = 4\cos(2\theta)$

c) $r^2 = 9\cos(\theta)$

d) $r^2 = 9\sin(2\theta)$

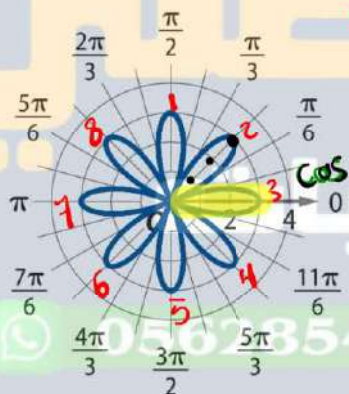
leminiscate.

$$r^2 = a^2 \sin^2 \theta$$

$$a = 3 \rightarrow a^2 = 3^2 = 9$$

$$r^2 = 9 \sin 2\theta$$

37.



rose

a) $r = 3\sin(4\theta)$

b) $r = 3\cos(4\theta)$

c) $r = -3\cos(8\theta)$

d) $r = -3\sin(8\theta)$

$$|a| = 3$$

$$r = a \cos n\theta$$

$$n = \frac{\text{even}}{2} = \frac{8}{2} = 4$$

$$r = 3 \cos 4\theta$$

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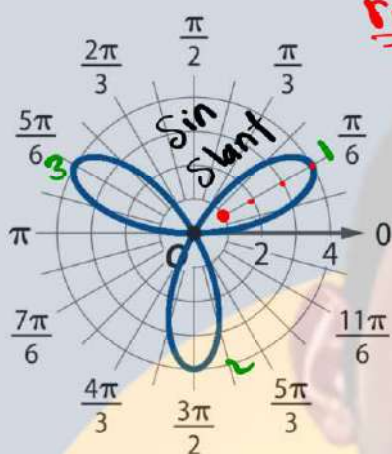
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38.

rose $3 \rightarrow \text{odd} \rightarrow n \text{ same}$

a) $r = 4 \sin(3\theta)$

$|n = 3|$

b) $r = 4 \cos(3\theta)$

$|a = 4|$

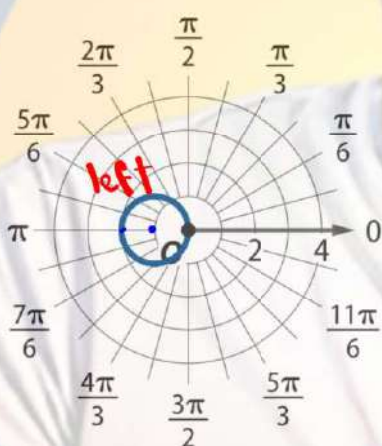
c) $r = 3 \cos(4\theta)$

d) $r = 3 \sin(4\theta)$

$$r = a \sin n\theta$$

$$= 4 \sin 3\theta$$

39.

circle $\rightarrow$ left $\rightarrow - \rightarrow \cos$

a) $r = 2 \sin(\theta)$

$r = -a \cos \theta$

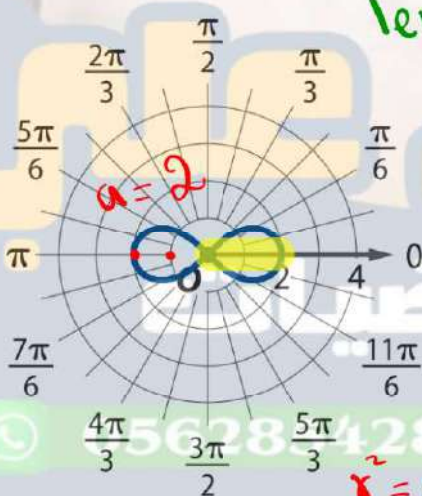
b) $r = -2 \cos(\theta)$

$r = -2 \cos \theta$

c) $r = 2 \cos(\theta)$

d) $r = 3 \sin(\theta)$

40.

lemniscates $\rightarrow$ Horizontal

a) $r^2 = 16 \cos(2\theta)$

 $\rightarrow \cos$

b) $r = 4 \sin(2\theta)$

$$a^2 = 2^2$$

$$= 4$$

c) $r^2 = 16 \sin(2\theta)$

d) $r^2 = 4 \cos(2\theta)$

$$r^2 = a^2 \cos 2\theta \rightarrow 4 \cos 2\theta$$

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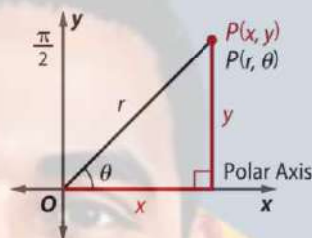
Q9-MCQ 8-3

KeyConcept Convert Polar to Rectangular Coordinates

If a point P has polar coordinates (r, θ) , then the rectangular coordinates (x, y) of P are given by

$$x = r \cos \theta \quad \text{and} \quad y = r \sin \theta.$$

That is, $(x, y) = (r \cos \theta, r \sin \theta)$.



Example 1 Polar Coordinates to Rectangular Coordinates

Find the rectangular coordinates for each point with the given polar coordinates.

Radian

a. $P\left(4, \frac{\pi}{6}\right)$

r θ

A	$(\sqrt{2}, -\sqrt{2})$
☒ B	$(2\sqrt{3}, 2)$
C	$(3, -3\sqrt{3})$
D	$\left(-\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

$$* x = r \cos \theta = 4 \cos \frac{\pi}{6} = 2\sqrt{3}$$

$$* y = r \sin \theta = 4 \sin \frac{\pi}{6} = 2$$

b. $Q(-2, 135^\circ)$

r θ

☒ A	$(\sqrt{2}, -\sqrt{2})$
B	$(2\sqrt{3}, 2)$
C	$(3, -3\sqrt{3})$
D	$\left(-\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

$$* x = r \cos \theta = -2 \cos 135^\circ = \sqrt{2}$$

$$* y = -2 \sin 135^\circ = -\sqrt{2}$$

c. $V(3, -120^\circ)$

$$x = 3 \cos(-120) = -\frac{3}{2}$$

$$y = 3 \sin(-120) = -\frac{3\sqrt{3}}{2}$$

A	$(\sqrt{2}, -\sqrt{2})$
B	$(2\sqrt{3}, 2)$
C	$(3, -3\sqrt{3})$
☒ D	$(-\frac{3}{2}, -\frac{3\sqrt{3}}{2})$

1A. $R(-6, -120^\circ)$

$$* x = r \cos \theta = -6 \cos(-120) = 3$$

$$* y = r \sin \theta = -6 \sin(-120) = 3\sqrt{3}$$

A	$(\sqrt{2}, -\sqrt{2})$
B	$(2\sqrt{3}, 2)$
☒ C	$(3, 3\sqrt{3})$
D	$(-\frac{3}{2}, -\frac{3\sqrt{3}}{2})$

1B. $S(5, \frac{\pi}{3})$ Radian

$$* x = 5 \cos \frac{\pi}{3} = \frac{5}{2}$$

$$* y = 5 \sin \frac{\pi}{3} = \frac{5\sqrt{3}}{2}$$

A	$(-\frac{3\sqrt{2}}{2}, -\frac{2\sqrt{3}}{2})$
B	$(2\sqrt{3}, 2)$
C	$(3, -3\sqrt{3})$
☒ D	$(\frac{5}{2}, \frac{5\sqrt{3}}{2})$

1C. $T(-3, 45^\circ)$

$$x = -3 \cos 45 = -\frac{3\sqrt{2}}{2}$$

$$y = -3 \sin 45 = -\frac{3\sqrt{2}}{2}$$

☒ A	$(-\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2})$
B	$(2\sqrt{3}, 2)$
C	$(3, -3\sqrt{3})$
D	$(\frac{5}{2}, \frac{5\sqrt{3}}{2})$

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* Types of equations *

* Parabola :-

Vertex

(h, k)

$$* y = x^2$$

$$* y = a(x-h)^2 + k$$

$$* y = ax^2 + bx + c$$

$$* x = y^2$$

$$* x = a(y-k)^2 + h$$

$$* x = ay^2 + by + c$$

* Circle :- $* (x-h)^2 + (y-k)^2 = r^2$ } Center (h, k)
 $* x^2 + y^2 = r^2$ } $r = \sqrt{r^2}$

* ellipse ; $* \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$$* \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

* hyperbola :- $* \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$$* \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Q10-MCQ 8-3

Example 4 Rectangular Equations to Polar Equations

Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.

a. $(x - 4)^2 + y^2 = 16$

$(r \cos \theta - 4)^2 + (r \sin \theta)^2 = 16$ circle

$r^2 \cos^2 \theta - 8r \cos \theta + 16 + r^2 \sin^2 \theta = 16$
 $r \rightarrow \text{alone}$

$r^2 (\cos^2 \theta + \sin^2 \theta) = 16 - 16 + 8r \cos \theta$

$r^2 \cdot 1 = 8r \cos \theta$

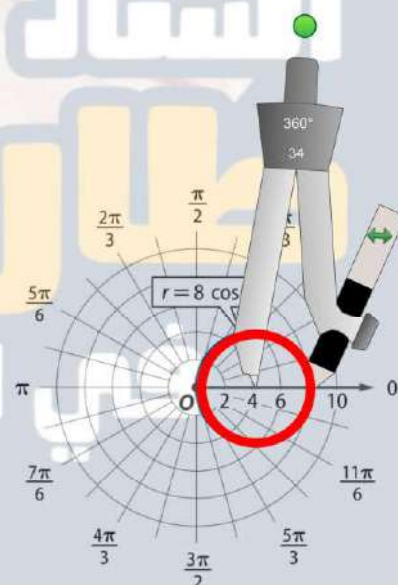
$|r = 8 \cos \theta| \rightarrow \text{circle} \quad |r = a \cos \theta|$

+ Right

graph of circle

$r = \sqrt{16} = 4$

A	Circle; $r = 8 \cos \theta$
B	Circle; $r = 4 \cos \theta$
C	Parabola; $r = \tan \theta \sec \theta$
D	Hyperbola; $r = \sec^2 \theta$



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b. $y = x^2$

parabola

$$r \sin \theta = (r \cos \theta)^2$$

$$\cancel{r} \sin \theta = \cancel{r}^2 \cos^2 \theta$$

$$\sin \theta = r \cos^2 \theta$$

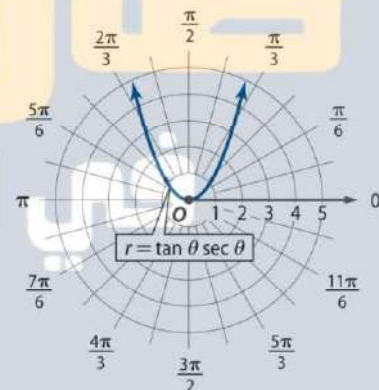
 $r \rightarrow$ alone

$$r = \frac{\sin \theta}{\cos^2 \theta}$$

$$r = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta}$$

$$r = \tan \theta \cdot \sec \theta$$

A	Circle; $r = 8 \cos \theta$
B	Circle; $r = 4 \cos \theta$
C	Parabola; $r = \tan \theta \sec \theta$
D	Hyperbola; $r = \sec^2 \theta$



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4A. $x^2 + (y - 3)^2 = 9$ Circle

$$(r \cos \theta)^2 + (r \sin \theta - 3)^2 = 9$$

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta - 6r \sin \theta + 9 = 9$$

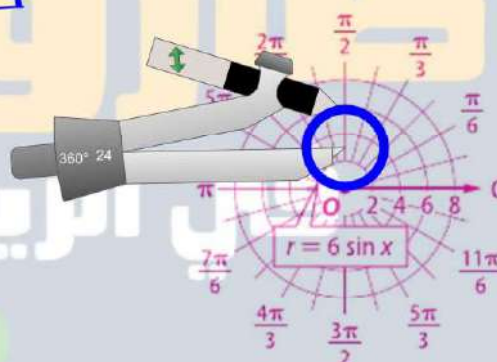
$$r^2 (\cos^2 \theta + \sin^2 \theta) = \cancel{9} - 9 + 6r \sin \theta$$

$$r^2 \cdot 1 = \cancel{6r} \sin \theta$$

$$r = 6 \sin \theta \rightarrow \text{Circle}$$

 $(+) \rightarrow \text{up}$

$$\text{* graph } r = \sqrt{9} = 3$$



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4B. $x^2 - y^2 = 1$

hyperbola

$$(r \cos \theta)^2 - (r \sin \theta)^2 = 1$$

$$r^2 \cos^2 \theta - r^2 \sin^2 \theta = 1$$

$$r^2 (\cos^2 \theta - \sin^2 \theta) = 1$$

$$r^2 \cos 2\theta = 1$$

$$r^2 = \frac{1}{\cos 2\theta}$$

$$| r^2 = \sec 2\theta |$$

A	Hyperbola; $r^2 \cos 2\theta = 1$
B	Circle; $r = 6 \sin \theta$
C	Parabola; $r = \tan \theta \sec \theta$
D	Hyperbola; $r = \sec^2 \theta$

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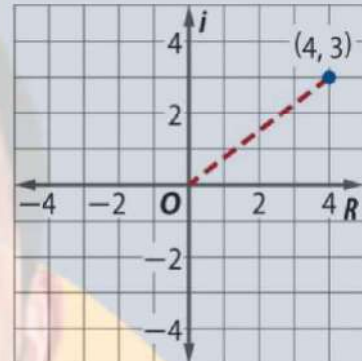
Q11-MCQ 8.5**Example 1 Graphs and Absolute Values of Complex Numbers**

Graph each number in the complex plane, and find its absolute value.

a. $z = 4 + 3i$

$$\text{Point} = (4, 3)$$

$$|z| = \sqrt{4^2 + 3^2} = 5$$

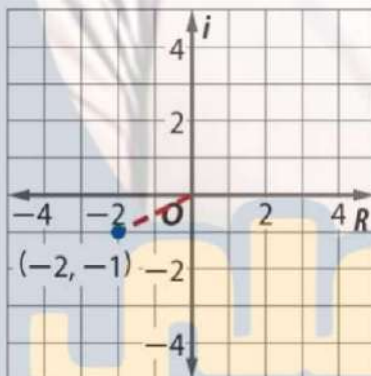


- A) A point in the first quadrant whose coordinates are $(4, 3)$; absolute value: $|z| = 5$
- B) A point in the first quadrant whose coordinates are $(3, 4)$; absolute value: $|z| = 25$
- C) A point in the second quadrant whose coordinates are $(-4, 3)$; absolute value: $|z| = 5$
- D) A point on the real axis at $(4, 0)$; absolute value: $|z| = 7$

Example 1 Graphs and Absolute Values of Complex Numbers

Graph each number in the complex plane, and find its absolute value.

b. $z = -2 - i$



$$\text{Point} = (-2, -1)$$

$$|z| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$$

- A) A point in the second quadrant whose coordinates are $(-2, 1)$; absolute value: $|z| = \sqrt{3}$
- B) A point in the third quadrant whose coordinates are $(-2, -1)$; absolute value: $|z| = \sqrt{5}$
- C) A point in the third quadrant whose coordinates are $(-2, -1)$; absolute value: $|z| = 5$
- D) A point in the fourth quadrant whose coordinates are $(2, -1)$; absolute value: $|z| = \sqrt{5}$

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$$1A. 5 + 2i$$

$$* r = \sqrt{5^2 + 2^2} \\ = \sqrt{29} \approx 5.39$$

$$* \theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{2}{5}\right) = 21.80$$

$$z = r (\cos \theta + i \sin \theta) \\ = 5.39 (\cos 21.80 + i \sin 21.80)$$

$$A) z \approx 5.39(\cos 0.38 + i \sin 0.38)$$

$$\checkmark B) z \approx 5.39(\cos 21.80^\circ + i \sin 21.80^\circ)$$

$$C) z \approx 2.65(\cos 0.38 + i \sin 0.38)$$

$$D) z \approx 5.39(\cos 1.19 + i \sin 1.19)$$

$$1B. -3 + 4i$$

$$* \text{Point} = (-3, 4)$$

A) A point in the first quadrant whose coordinates are (3, 4); absolute value: $|z| = 5$

$\checkmark$ B) A point in the second quadrant whose coordinates are $(-3, 4)$; absolute value: $|z| = 5$

C) A point in the second quadrant whose coordinates are $(-3, 4)$; absolute value: $|z| = 25$

D) A point in the third quadrant whose coordinates are $(-3, -4)$; absolute value: $|z| = 5$

$$* |z| = \sqrt{(-3)^2 + 4^2} \\ = 5$$



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Q12-MCQ 8.5

KeyConcept Product and Quotient of Complex Numbers in Polar Form

Given the complex numbers $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$:

Product Formula

$$z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$$

$$* r_1 \cdot r_2$$

$$* \theta_1 + \theta_2$$

Quotient Formula

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)], \text{ where } z_2 \text{ and } r_2 \neq 0$$

$$* \frac{r_1}{r_2}$$

$$* \theta_1 - \theta_2$$

Example 4 Product of Complex Numbers in Polar Form

Find $2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right) \cdot 4\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$ in polar form. Then express the product in rectangular form.

A	$4\sqrt{3} - 4i$
B	$3\sqrt{3} - 4i$
C	$-3.88 + 14.49i$
D	$3.11 + 11.59i$

$$* r_1 \cdot r_2 = 2 \times 4 = 8 \quad \#$$

$$* \theta_1 + \theta_2 = \frac{5\pi}{3} + \frac{\pi}{6} = \frac{11\pi}{6} \quad \#$$

$$z = 8 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$$

rectangular $\rightarrow$ mode $\rightarrow 2$
 Radian $\rightarrow$ Shift $\rightarrow$ Mode $\rightarrow 1 \rightarrow 2$

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Find each product in polar form. Then express the product in rectangular form.

4A. $3\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) \cdot 5\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

A	$4\sqrt{3} - 4i$
B	$3\sqrt{3} - 4i$
C	$-3.88 + 14.49i$
D	$3.11 + 11.59i$

$$* r_1 \cdot r_2 = 3 \times 5 = \boxed{15}$$

$$* \theta_1 + \theta_2 = \frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$$

$$* Z = 15 \left(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12} \right)$$

rectangular $\begin{cases} \rightarrow \text{mode} \rightarrow 2 \\ \rightarrow \text{radian} \end{cases}$

4B. $-6\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) \cdot 2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

A	$4\sqrt{3} - 4i$
B	$3\sqrt{3} - 4i$
C	$-3.88 + 14.49i$
D	$3.11 + 11.59i$

$$* r_1 \cdot r_2 = -6 \times 2 = \boxed{-12}$$

$$* \theta_1 + \theta_2 = \frac{3\pi}{4} + \frac{2\pi}{3} = \frac{17\pi}{12}$$

$$Z = -12 \left(\cos \frac{17\pi}{12} + i \sin \frac{17\pi}{12} \right)$$

rectangular $\begin{cases} \rightarrow \text{mode} \rightarrow 2 \\ \rightarrow \text{radian} \end{cases}$

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Q13-MCQ 9.1

Example 1 Identify Arithmetic Sequences

Determine whether each sequence is arithmetic.

a. 5, -6, -17, -28, ...

$$d = -6 - 5 = -11$$

$$d = -17 - (-6) = -11$$

$$d = -28 - (-17) = -11$$

$$\rightarrow d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

A) ~~متتالية حسابية~~: $d = -11$ | Yes, arithmetic; $d = -11$

B) ~~متتالية حسابية~~: $d = 11$ | Yes, arithmetic; $d = 11$

C) ~~متتالية حسابية~~: $d = -6$ | Yes, arithmetic; $d = -6$

D) ليست متتالية حسابية | Not an arithmetic sequence

Example 1 Identify Arithmetic Sequences

Determine whether each sequence is arithmetic.

b. -4, 12, 28, 42, ...

A) ~~متتالية حسابية~~: $d = 16$ | Yes, arithmetic; $d = 16$

$$d = 12 - (-4) = 16$$

B) ~~متتالية حسابية~~: $d = 14$ | Yes, arithmetic; $d = 14$

$$d = 28 - 12 = 16$$

C) ~~ليست متتالية حسابية~~ | Not an arithmetic sequence

$$d = 42 - 28 = 14$$

D) ~~متتالية حسابية~~: $d = 12$ | Yes, arithmetic; $d = 12$



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1A. 7, 12, 16, 20, ...

$$d = 12 - 7 = 5$$

$$d = 16 - 12 = 4$$

$$5 \neq 4$$

A) $d = 5$ | Yes, arithmetic; $d = 5$ B) ~~ليست متتالية حسابية~~ | Not an arithmetic sequenceC) $d = 4$ | Yes, arithmetic; $d = 4$ D) $d = 3$ | Yes, arithmetic; $d = 3$

1B. -6, 3, 12, 21, ...

$$d = 3 - -6 = 9$$

$$d = 12 - 3 = 9$$

$$d = 21 - 12 = 9$$

A) $d = -9$ | Yes, arithmetic; $d = -9$ B) $d = 6$ | Yes, arithmetic; $d = 6$ C) $d = 9$ | Yes, arithmetic; $d = 9$ D) ~~ليست متتالية حسابية~~ | Not an arithmetic sequence

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Q14-MCQ 9.1

Find the next three terms of each geometric sequence. Then graph the sequence.

14. 8, 12, 18, 27, ...

$$* r = \frac{a_2}{a_1} = \frac{12}{8} = 1.5$$

* Next $\rightarrow$ last $\times r$

$$27 \times 1.5 = 40.5, \quad 40.5 \times 1.5 = 60.75$$

☒ A) 40.5, 60.75, 91.125

B) 36, 45, 54

C) 37.5, 48, 59.5

D) 38, 49, 61

15. 8, 16, 32, 64, ...

$$* r = \frac{16}{8} = 2$$

* Next : last $\times r$

$$64 \times 2 = 128$$

$$128 \times 2 = 256$$

☒ A) 128, 256, 512

B) 96, 128, 160

C) 80, 96, 112

D) 100, 200, 400

16. 250, 50, 10, 2, ...

$$* r = \frac{50}{250} = 0.2$$

* Next : $2 \times 0.2 = 0.4$

A) -6, -14, -22

☒ B) 0.4, 0.08, 0.016

C) 0.5, 0.1, 0.02

D) 0.2, 0.02, 0.002

$$0.4 \times 0.2 = 0.08$$

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17. $9, -3, 1, -\frac{1}{3}, \dots$

$$* r = \frac{-3}{9} = -\frac{1}{3}$$

$$* \text{Next} :- -\frac{1}{3} \times -\frac{1}{3} = \frac{1}{9}$$

$$\frac{1}{9} \times -\frac{1}{3} = -\frac{1}{27}$$

A) $\frac{1}{9}, -\frac{1}{27}, \frac{1}{81}$

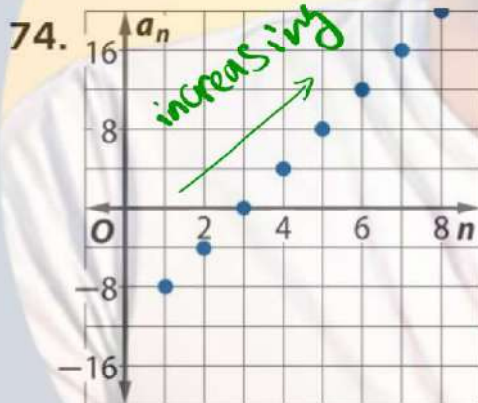
B) $-\frac{1}{6}, \frac{1}{9}, -\frac{1}{12}$

C) $-\frac{4}{3}, -\frac{7}{3}, -\frac{10}{3}$

D) $-\frac{1}{9}, \frac{1}{27}, -\frac{1}{81}$

Q15-MCQ 9-2

Determine whether each sequence is convergent or divergent.

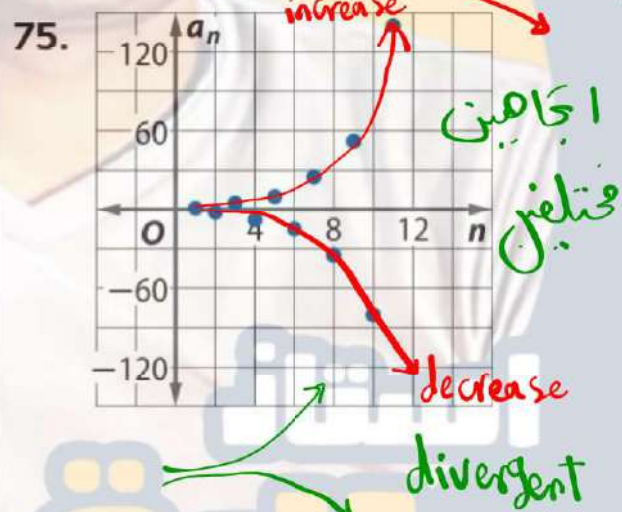


A) Convergent, approaches 0

B) ☒ Divergent

C) Convergent, approaches 16

D) Convergent, approaches 8



A) Convergent, approaches 120

B) Convergent, approaches -60

C) ☒ Divergent

D) Convergent, approaches 0

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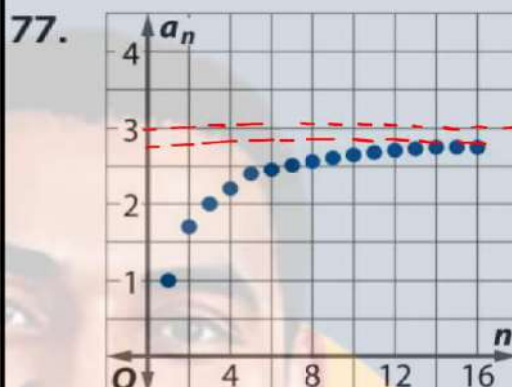
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- A) Divergent
B) Convergent, approaches 4
C) Convergent, approaches -14
D) Convergent, approaches 0



- A) Convergent, approaches 2.75
B) Divergent
C) Convergent, approaches 4
D) Convergent, approaches 1

Q16-MCQ 9-2

57. $-2 - 1 + 0 + 1 + 2 + 3 + 4 + 5; \quad n = 1 \dots 8$

Question: Which of the following correctly represents the series in sigma notation starting with a lower bound of $n = 1$?

- A) $\sum_{n=1}^8 (n-3) \rightarrow (1-3) = -2, (2-3) = -1, (3-3) = 0$
B) $\sum_{n=1}^8 (n-2)$
C) $\sum_{n=1}^7 (n-3)$
D) $\sum_{n=1}^8 (-2n)$
- 8 terms $\rightarrow$ 8
ن = 1 عند Counting

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58. $\frac{1}{20} + \frac{1}{25} + \frac{1}{30} + \frac{1}{35} + \frac{1}{40} + \frac{1}{45}; n = 4 \rightarrow 5 \rightarrow 6 \rightarrow \dots 9$

Question: Which of the following correctly represents the series in sigma notation starting with a lower bound of $n = 4$?

- A) $\sum_{n=4}^9 \frac{1}{5n}$ $= \frac{1}{5(4)} = \frac{1}{20}, \frac{1}{5(5)} = \frac{1}{25}, \frac{1}{5(6)} = \frac{1}{30}$
 B) $\sum_{n=4}^9 \frac{1}{5n+5}$
 C) $\sum_{n=4}^9 \frac{1}{5n-5}$
 D) $\sum_{n=4}^{10} \frac{1}{5n}$
- not counting $\leftarrow n \neq 1$ نكسر
 last-First + 1 = $9 - 4 + 1 = 6$ terms
 $\rightarrow 10 - 4 + 1 = 7$

59. $8 + 27 + 64 + \dots + 1000; n = 2$

$8, 27, 64, 125, 216, 343, 512, 729, 1000$

Question: Which of the following correctly represents the finite series in sigma notation starting with a lower bound of $n = 2$?

- A) $\sum_{n=2}^{10} n^2 \rightarrow 2^2 = 4$
 B) $\sum_{n=2}^{10} n^3 \rightarrow 2^3 = 8, 3^3 = 27, 4^3 = 64$ ✓
 C) $\sum_{n=2}^{100} 8n$
 D) $\sum_{n=2}^9 n^3 \rightarrow$
- last-First + 1 = $10 - 2 + 1 = 9$ terms ✓

60. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{128}; n = 1 \rightarrow \text{Counting}$

Question: Which of the following correctly represents the finite series in sigma notation starting with a lower bound of $n = 1$?

- A) $\sum_{n=1}^7 \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^1 = \frac{1}{2}, \left(\frac{1}{2}\right)^2 = \frac{1}{4}, \left(\frac{1}{2}\right)^3 = \frac{1}{8}$
 B) $\sum_{n=1}^7 \frac{1}{2n}$
 C) $\sum_{n=1}^6 \left(\frac{1}{2}\right)^n$
 D) $\sum_{n=1}^7 \left(\frac{1}{2}\right)^{n-1}$
- $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128}$
 7 terms

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61. $-8 + 16 - 32 + 64 - 128 + 256 - 512$; $n = 3 \neq 1$

Question: Which of the following correctly represents the alternating series in sigma notation starting with a lower bound of $n = 3$?

- A) $\sum_{n=3}^9 (-2)^n = (-2)^3 = -8, (-2)^4 = 16, (-2)^5 = -32$
- B) $\sum_{n=3}^9 -1 \cdot 2^n$
- C) $\sum_{n=3}^9 (-1)^{n+1} 2^n$
- D) $\sum_{n=3}^{10} (-2)^n$
- last - First + 1 = 9 - 3 + 1 = 7 terms

62. $8\left(-\frac{1}{3}\right) + 8\left(\frac{1}{9}\right) + 8\left(-\frac{1}{27}\right) + \dots + 8\left(-\frac{1}{243}\right)$; $n = 1$

Question: Which of the following correctly represents the alternating series in sigma notation starting with a lower bound of $n = 1$?

- A) $\sum_{n=1}^5 8\left(-\frac{1}{3}\right)^n \rightarrow 8\left(-\frac{1}{3}\right) = 8\left(-\frac{1}{3}\right), 8\left(-\frac{1}{3}\right)^2 = 8\left(\frac{1}{9}\right)$
- B) $\sum_{n=1}^5 8\left(\frac{1}{3}\right)^n$
- C) $\sum_{n=1}^5 8(-3)^n$
- D) $\sum_{n=1}^6 8\left(-\frac{1}{3}\right)^n \rightarrow 8\left(-\frac{1}{3}\right) + 8\left(\frac{1}{9}\right) + 8\left(-\frac{1}{27}\right) + 8\left(\frac{1}{81}\right) + 8\left(-\frac{1}{243}\right)$
- Counting
- 5 terms



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Q17-MCQ 9-3

Key Concept n th Term of an Arithmetic Sequence

The n th term a_n of an arithmetic sequence in which the first term is a_1 and the common difference is d is given by the following formula, where n is any natural number.

$$a_n = a_1 + (n - 1)d$$

Example 1 Find the n th Term

Find the 12th term of the arithmetic sequence 9, 16, 23, 30,

93 (A)

$$a_1 = 9, \quad d = 16 - 9 = 7, \quad n = 12$$

86 (B)

$$a_{12} = a_1 + (n-1)d$$

77 (C)

$$= 9 + (12 - 1)(7)$$

100 (D)

Find the indicated term of each arithmetic sequence.

1A. $a_1 = -4, d = 6, n = 9$

50 (A)

$$a_9 = -4 + (9-1)(6)$$

44 (B)

$$a_9 = 44$$

38 (C)

56 (D)

1B. a_{20} for $a_1 = 15, d = -8$

-145 (A)

$$a_{20} = 15 + (20-1)(-8)$$

-153 (B)

$$= -137$$

-137 (C)

-160 (D)

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Q18-MCQ 9-3

$$57. \sum_{k=1}^{16} (4k - 2)$$

إجمالي

Question: Find the sum of the given arithmetic series.

- A) 496
- ☒ B) 512
- C) 62
- D) 1,024

$$58. \sum_{k=4}^{13} (4k + 1)$$

Question: Find the sum of the given arithmetic series.

- ☒ A) 350
- B) 315
- C) 70
- D) 385

$$59. \sum_{k=5}^{16} (2k + 6)$$

Question: Find the sum of the given arithmetic series.

- A) 324
- B) 270
- ☒ C) 342
- D) 162

$$60. \sum_{k=0}^{12} (-3k + 2)$$

Question: Find the sum of the given arithmetic series.

- A) -192
- B) -240
- ☒ C) -208
- D) 26



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Q19-MCQ 9-4

Example 3 Find Geometric Means

Find three geometric means between 2 and 1250.

6, 18, 54 (A)

10, 50, 250 (B)

8, 32, 128 (C)

12, 72, 432 (D)

Handwritten solution for finding geometric means:

Diagram showing a sequence: $2, _, _, _, 1250$. Arrows point from 2 to $a_1 = 2$ and from 1250 to $a_5 = 1250$. A box contains $n = 5$.

Formula: $a_n = a_1 \cdot r^{n-1}$

Equation: $1250 = 2 \cdot r^4$ (with a circled 4). Arrows point to "Shift" and "Solve".

Result: $r = \pm 5$ (boxed). A note "even $\pm$ " points to the result.

② means First $\times r$

$$\rightarrow 2 \times \pm 5 = \pm 10$$

$$\rightarrow \pm 10 \times \pm 5 = 50$$

$$\rightarrow 50 \times \pm 5 = \pm 250$$

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3. Find four geometric means between 0.5 and 512.

2, 8, 32, 128 (A)

1, 2, 4, 8 (B

1.5, 4.5, 13.5, 40.5 (C

2.5, 12.5, 62.5, 312.5 (D

$\bar{5}_{12} = 0.\bar{5}$ $\bar{5} \rightarrow \text{odd}$

Shift $\rightarrow$ solve
 $\rightarrow$ =

$$\boxed{r = H}$$

② means

First yr

$$0.5 \times 4 = \boxed{2}$$

$$2 + 4 = \boxed{8}$$

$$8 \times 4 = \boxed{32}$$

$$32 \times H = \overline{128}$$

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Q20-MCQ 9-4

Example 6 Find the First Term of a Series

Find a_1 in a geometric series for which $S_n = 13,116$, $n = 7$, and $r = 3$.

A	$a_1 = 4$
B	$a_1 = 6$
C	$a_1 = 12$
D	$a_1 = 18$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$13116 = \frac{a_1(1 - 3^7)}{1 - 3}$$

shift $\rightarrow$ solve $\rightarrow$ =

6. Find a_1 in a geometric series for which $S_n = -26,240$, $n = 8$, and $r = -3$.

A	$a_1 = -16$
B	$a_1 = 8$
C	$a_1 = 16$
D	$a_1 = 32$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$-26240 = \frac{a_1(1 - (-3)^8)}{1 - (-3)}$$

shift $\rightarrow$ solve $\rightarrow$ =

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Q21-MCQ 9.5

Example 3 Infinite Series in Sigma Notation

Find $\sum_{k=1}^{\infty} 18\left(\frac{4}{5}\right)^{k-1}$.

$\hookrightarrow r = \frac{4}{5} < 1$
has a sum
 $\infty = 99$ دو!

A	90
B	72
C	48
D	Not exist

3. Find $\sum_{k=1}^{\infty} 12\left(\frac{3}{4}\right)^{k-1}$.

$\hookrightarrow r = \frac{3}{4} < 1$
Has a sum
 $\infty = 99$ دو!

A	18
B	72
C	48
D	Not exist

Q22-MCQ 9.6

Example 2 Write Recursive Formulas

Write a recursive formula for each sequence.

a. 2, 10, 18, 26, 34, ...

$d = 10 - 2 = 8$, $18 - 10 = 8$, $26 - 18 = 8$

$a_n = a_{n-1} + d \rightarrow a_n = a_{n-1} + 8$

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A	$a_1 = 2; a_n = 2a_{n-1} + 8$
B	$a_1 = 2; a_n = a_{n-1} + 8$
C	$a_1 = 2; a_n = 2a_{n-1} - 8$
D	$a_1 = 2; a_n = a_{n-1} + 10$

b. 16, 56, 196, 686, 2401, ...

$$a_1 = 16, a_n = 3.5 \cdot a_{n-1} \quad (A)$$

$$a_1 = 16, a_n = a_{n-1} + 40 \quad (B)$$

$$a_1 = 16, a_n = 2.5 \cdot a_{n-1} \quad (C)$$

$$a_1 = 3.5, a_n = 16 \cdot a_{n-1} \quad (D)$$

$$r = \frac{56}{16} = 3.5$$

* Geometric

$$a_n = r \cdot a_{n-1}$$

$$a_n = 3.5 a_{n-1}$$

c. $r = 3$ و $a_4 = 108$

$$a_1 = 4, a_n = 3 \cdot a_{n-1} \quad (A)$$

$$a_1 = 3, a_n = 4 \cdot a_{n-1} \quad (B)$$

$$a_1 = 4, a_n = a_{n-1} + 3 \quad (C)$$

$$a_1 = 12, a_n = 3 \cdot a_{n-1} \quad (D)$$

 $r \rightarrow$ Geometric

$$a_n = r \cdot a_{n-1}$$

$$a_n = 3 a_{n-1}$$

$$a_4 = 108 \rightarrow a_3 = \frac{a_4}{r} = \frac{108}{3} = 36$$

$$a_2 = \frac{a_3}{r} = \frac{36}{3} = 12, a_1 = \frac{a_2}{r} = \frac{12}{3} = 4$$

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2A. 8, 20, 50, 125, 312.5, ..
or

$$r = \frac{20}{8} = 2.5$$

A	$a_1 = 8; a_n = 2a_{n-1}$
B	$a_1 = 8; a_n = 2.5a_{n-1}$
C	$a_1 = 8; a_n = 4a_{n-1}$
D	$a_1 = 8; a_n = a_{n-1} + 16$

$$a_n = r \cdot a_{n-1} \\ = 2.5 a_{n-1}$$

2B. 8, 17, 26, 35, 44, ...

A	$a_1 = 8; a_n = 2a_{n-1}$
B	$a_1 = 8; a_n = 2a_{n-1} + 9$
C	$a_1 = 8; a_n = a_{n-1} + 9$
D	$a_1 = 8; a_n = a_{n-1}$

$$d = 17 - 8 = 9 \\ 26 - 17 = 9 \quad \text{arithmetic}$$

$$a_n = a_{n-1} + d$$

$$a_n = a_{n-1} + 9$$

2C. $a_3 = 16$, $r = 4$

→ geometric

A	$a_1 = 1; a_n = 2a_{n-1}$
B	$a_1 = 1; a_n = 4a_{n-1}$
C	$a_1 = 1; a_n = a_{n-1} + 4$
D	$a_1 = 1; a_n = 2a_{n-1} + 4$

$$a_n = r \cdot a_{n-1}$$

$$|a_n = 4a_{n-1}|$$

$$a_3 = 16$$

$$a_2 = \frac{16}{r} = \frac{16}{4} = 4$$

$$a_1 = \frac{a_2}{r} = \frac{4}{4} = 1$$

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Q23-MCQ 9.6

Example 4 Iterate a Function

Find the first three iterates x_1 , x_2 , and x_3 of $f(x) = 5x + 4$ for an initial value of $x_0 = 2$.

10, 54, 274 (A)

14, 74, 374 (B) ✓

14, 64, 324 (C)

2, 14, 74 (D)

$$x_1 = 5(2) + 4 = 14$$

$$x_2 = 5(14) + 4 = 74$$

$$x_3 = 5(74) + 4 = 374$$

4. Find the first three iterates x_1 , x_2 , and x_3 of $f(x) = -3x + 8$ for an initial value of $x_0 = 6$.

-10, -22, 74 (A)

-10, 38, -106 (B) ✓

6, -10, 38 (C)

-18, 62, -178 (D)

$$x_1 = -3(6) + 8 = -10$$

$$x_2 = -3(-10) + 8 = 38$$

$$x_3 = -3(38) + 8 = -106$$



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Q24-MCQ 9.7

Example 4 Determine a Single Term

Find the fifth term of $(y + z)^{11}$. $\rightarrow n$

$$462y^6z^5 \text{ (A)}$$

$$330y^4z^7 \text{ (B)}$$

$$330y^7z^4 \text{ (C)}$$

$$462y^7z^4 \text{ (D)}$$

$$* r = p - 1 = 5 - 1 = \boxed{4}$$

$$* n = 11$$

$${}^nC_r (F)^{n-r} (S)^r = \#$$

$${}^{11}C_4 (y)^{11-4} (z)^4$$

$$11 \rightarrow \text{Shift} \rightarrow \div \rightarrow 4 \quad 330 y^7 z^4$$

4. Find the sixth term of $(c + d)^{10}$. $\rightarrow n$

$$252c^5d^5 \text{ (A)}$$

$$210c^6d^4 \text{ (B)}$$

$$252c^6d^4 \text{ (C)}$$

$$210c^5d^5 \text{ (D)}$$

$$* r = p - 1 = 6 - 1 = \boxed{5}$$

$$* n = 10$$

$${}^nC_r (F)^{n-r} (S)^r$$

$${}^{10}C_5 (c)^{10-5} (d)^5$$

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Q25-MCQ 9.8 مثال مهارة دوافعاً على

Find a counterexample to disprove each statement.

6. $3^n + 1$ is divisible by 4.

$$n = 2$$

$$3^2 + 1 = 10$$

$$\frac{10}{4} = 2.5$$

not divisible

a) $n = 1$

b) $n = 3$

c) $n = 2$

d) $n = 5$

Question 7

Find a counterexample to disprove the statement:

" $2^n + 3^n$ is divisible by 4, where n is a positive integer."

$$a) n = 2 \rightarrow 2^2 + 3^2 = 13 \rightarrow \frac{13}{4} = 3.25 \text{ not}$$

$$b) n = 1 \rightarrow 2^1 + 3^1 = 5 \rightarrow \frac{5}{4} = 1.25$$

c) $n = 3$

$$n = 0, 2, 3, \dots$$

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Question 21

$$n = 1, 2, 3, \dots$$

Find a counterexample to disprove the statement:

" $1 + 2 + 3 + \dots + n = n^2$, where n is a positive integer."

a) $n = 1$

$$\rightarrow 1 = 1^2 \rightarrow 1 = 1 \checkmark$$

b) $n = 3$

$$\rightarrow 3 = 3^2 \rightarrow 3 \neq 9 \alpha$$

c) $n = 2$

$$\rightarrow 2 = 2^2 \rightarrow 2 \neq 4 \alpha$$

d) There is no counterexample

Question 22

Find a counterexample to disprove the statement:

" $1 + 8 + 27 + \dots + n^3 = (2n + 2)^2$, where n is a positive integer."

a) $n = 1$

$$\rightarrow 1, (2(1) + 2)^2$$

b) $n = 2$

c) $n = 3 \rightarrow 1 + 8 + 27 = 36$

d) $n = 4$

$$(2(3) + 2)^2 = 64$$

$$|36 \neq 64|$$

Question 23

Find a counterexample to disprove the statement:

" $n^2 - n + 15$ is prime, where n is a positive integer."

a) $n = 3$

b) $n = 5$

c) $n = 1$

$$\rightarrow 1^2 - 1 + 15 = 15 \text{ not prime}$$

d) $n = 2$

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Question 24

$$P = 2, 3, 5, 7, 11, 13, 17, 19, 23$$

Find a counterexample to disprove the statement:

" $n^2 + n + 23$ is prime, where n is a positive integer."

a) $n = 23$

b) $n = 1 \rightarrow 1^2 + 1 + 23 = 25$ not prime.

c) $n = 2$

d) $n = 3$

Prime: numbers that divide by

Self and 1

عددها لا تقبل القسمة على نفسها

و 1

25 not prime

$$① - \frac{25}{25} = 1$$

$$② - \frac{25}{1} = 25$$

$$③ - \frac{25}{5} = 5$$

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IN

Math

$[a + b]$
 $\pi = 3.14$
 $A = \frac{ab + c}{d}$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $a^2 + b^2 = c^2, c = \sqrt{a^2 + b^2}$
 $c^2 + a^2 = b^2, c^2 - b^2 = a^2$
 $a^2 = 2a$
 $f(a + b) = c$
 $(x + y)^2 - (x - y)$
 $+ 2c = 1$
 ψ
 z
 8
 $MATH$
 $\frac{a}{c} = \frac{HB}{a}$
 $Me =$
 90°

خاص بالمنصة



خاص بجميع الجروبات و القنوات



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