

مراجعة نهاية الفصل الدراسي الثالث رياضيات



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف العاشر المتقدم ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

تاريخ إضافة الملف على موقع المناهج: 17:29:21 2026-06-15

ملفات اكتب للمعلم اكتب للطالب | اختبارات الكترونية | اختبارات | حلول | عروض بوربوينت | أوراق عمل
منهج انجليزي | ملخصات وتقارير | مذكرات وبنوك | الامتحان النهائي | للمدرس

المزيد من مادة
رياضيات:

إعداد: Nuzhat

التواصل الاجتماعي بحسب الصف العاشر المتقدم



صفحة المناهج
الإماراتية على
فيسبوك

الرياضيات

اللغة الانجليزية

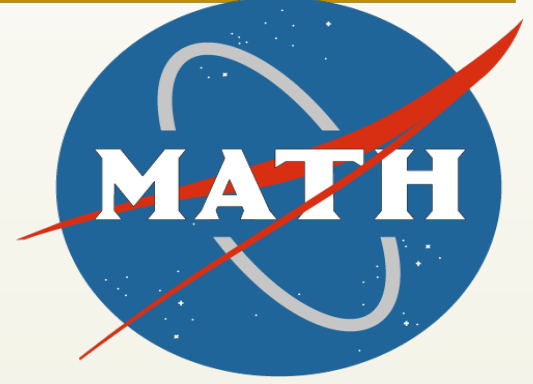
اللغة العربية

التربية الاسلامية

المواد على تلغرام

المزيد من الملفات بحسب الصف العاشر المتقدم والمادة رياضيات في الفصل الثالث

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BE

READY

END OF TERM 3

REVISION

MATHS

GRADE 10 ADV

COMPILED BY
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مدير المجمع
أمجد الجبر

وحدة الشؤون الأكاديمية
جلال عبدالرحمن صالح
حمدة العامري

EOT COVERAGE TERM -3

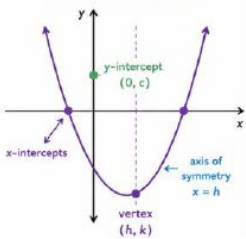
Question*	Learning Outcome/Performance Criteria**	Reference(s) in the Student Book (English Version)	
		المرجع في كتاب الطالب (النسخة الإنجليزية)	
		Example/Exercise	Page
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*	Questions might appear in a different order in the actual exam, or on the exam paper.		
*	قد تظهر الأسئلة بترتيب مختلف في الامتحان الفعلي، أو على ورقة الامتحان .		
**	As it appears in the textbook, and LMS.		
**	كما وردت في كتاب الطالب و LMS .		

1	Analyze the characteristics of the graphs of quadratic functions. Graph quadratic functions	27-32	11
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Complete parts a-c for each quadratic function.

- a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.
- b. Make a table of values that includes the vertex.
- c. Use this information to graph the function.

THE PARABOLA



STANDARD FORM

$f(x) = ax^2 + bx + c$

a = determines opening
 b = affects axis of symmetry
 c = y-intercept $(0, c)$

DIRECTION

Determined by the value of a :

$a > 0$
opens upward

$a < 0$
opens downward

27. $f(x) = 2x^2 - 6x - 9$

28. $f(x) = -3x^2 - 9x + 2$

Quadratic Function: $y =$ _____

1. y-intercept

Set $x = 0$.

$y =$ _____

2. Axis of Symmetry

$x = -\frac{b}{2a}$

$x =$ _____

3. x-coordinate of Vertex

The x-coordinate of the vertex is the axis of symmetry.

Quadratic Function: $y =$ _____

1. y-intercept

Set $x = 0$.

$y =$ _____

2. Axis of Symmetry

$x = -\frac{b}{2a}$

$x =$ _____

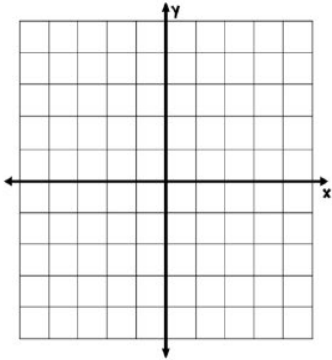
3. x-coordinate of Vertex

The x-coordinate of the vertex is the axis of symmetry.

4. Table of Values

x	y =	(x, y)

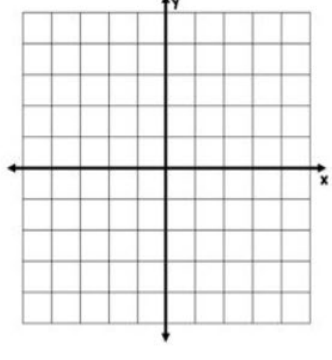
5. Graph



4. Table of Values

x	y =	(x, y)

5. Graph



29. $f(x) = -4x^2 + 5x$

30. $f(x) = 2x^2 + 11x$

Quadratic Function: $y =$ _____

1. y-intercept

Set $x = 0$.

$y =$ _____

2. Axis of Symmetry

$x = -\frac{b}{2a}$

$x =$ _____

3. x-coordinate of Vertex

The x-coordinate of the vertex is the axis of symmetry.

Quadratic Function: $y =$ _____

1. y-intercept

Set $x = 0$.

$y =$ _____

2. Axis of Symmetry

$x = -\frac{b}{2a}$

$x =$ _____

3. x-coordinate of Vertex

The x-coordinate of the vertex is the axis of symmetry.

31. $f(x) = 0.25x^2 + 3x + 4$

32. $f(x) = -0.75x^2 + 4x + 6$

Quadratic Function: $y =$ _____

1. y-intercept

Set $x = 0$.

$y =$ _____

2. Axis of Symmetry

$x = -\frac{b}{2a}$

$x =$ _____

3. x-coordinate of Vertex

The x-coordinate of the vertex is the axis of symmetry.

Quadratic Function: $y =$ _____

1. y-intercept

Set $x = 0$.

$y =$ _____

2. Axis of Symmetry

$x = -\frac{b}{2a}$

$x =$ _____

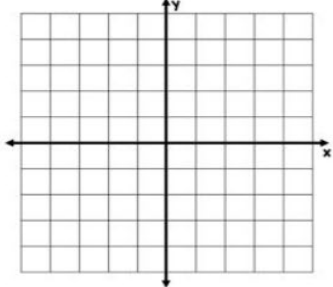
3. x-coordinate of Vertex

The x-coordinate of the vertex is the axis of symmetry.

4. Table of Values

x	y =	(x, y)

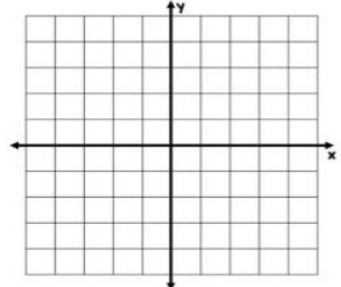
5. Graph



4. Table of Values

x	y =	(x, y)

5. Graph



Q27 $f(x) = 2x^2 - 6x - 9$

$a=2, b=-6, c=-9$
 y-intercept: set $x=0 \rightarrow y = c = -9$
 Axis of symmetry: $x = -b/2a = 6/4 = 3/2$
 Vertex y: $2(3/2)^2 - 6(3/2) - 9 = 9/2 - 9 - 9 = -27/2$
 Table: x: 0,1,2,3,4
 y: -9,-13,-13,-9,-1

ANSWER
 y-int: (0,-9) | Axis: $x = 3/2$
 | Vertex: $(3/2, -27/2)$

Q28 $f(x) = -3x^2 - 9x + 2$

$a=-3, b=-9, c=2$
 y-intercept: $c = 2$
 Axis: $x = -(-9)/(2 \cdot -3) = -3/2$
 Vertex y: $-3(9/4) + 27/2 + 2 = 35/4$
 Table: x: -3,-2,-1,0,1
 y: 2, 8, 8, 2, -10

ANSWER
 y-int: (0, 2) | Axis: $x = -3/2$
 | Vertex: $(-3/2, 35/4)$

Q29 $f(x) = -4x^2 + 5x$

$a=-4, b=5, c=0$
 y-intercept: $c = 0$
 Axis: $x = -5/(-8) = 5/8$
 Vertex y: $-4(25/64) + 5(5/8) = 25/16$
 Table: x: -1,0,1,2
 y: -9,0,1,-6

ANSWER
 y-int: (0, 0) | Axis: $x = 5/8$ |
 Vertex: $(5/8, 25/16)$

Q30 $f(x) = 2x^2 + 11x$

$a=2, b=11, c=0$
 y-intercept: $c = 0$
 Axis: $x = -11/4$
 Vertex y: $2(121/16) + 11(-11/4) = -121/8$
 Table: x: -5,-4,-3,-2,-1
 y: -5,-12,-15,-14,-9

ANSWER
 y-int: (0, 0) | Axis: $x = -11/4$
 | Vertex: $(-11/4, -121/8)$

Q31 $f(x) = 0.25x^2 + 3x + 4$

$a=0.25, b=3, c=4$
 y-intercept: $c = 4$

ANSWER

Axis: $x = -3/(0.5) = -6$
 Vertex y: $0.25(36) + 3(-6) + 4 = -5$
 Table: x: -8,-7,-6,-5,-4
 y: -4,-4.75,-5,-4.75,-4

y-int: (0, 4) | Axis: $x = -6$ |
 Vertex: $(-6, -5)$

Q32 $f(x) = -0.75x^2 + 4x + 6$

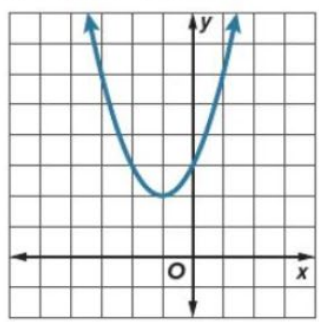
$a=-0.75, b=4, c=6$
 y-intercept: $c = 6$
 Axis: $x = -4/(-1.5) = 8/3$
 Vertex y: $-0.75(64/9) + 4(8/3) + 6 = 34/3$
 Table: x: 1,2,3,4,5
 y: 9.25,11,11.25,10,7.25

ANSWER
 y-int: (0, 6) | Axis: $x = 8/3 \approx 2.67$ | Vertex: $(8/3, 34/3)$

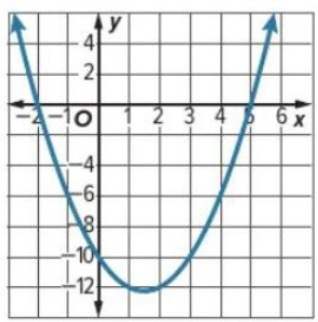
2	Solve quadratic equations by graphing	1-3	17
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Use the related graph of each equation to determine its solutions.

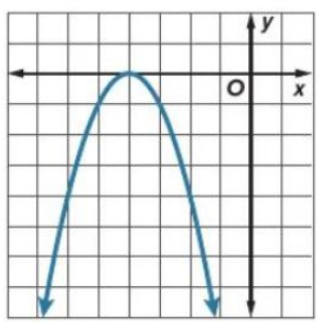
1. $x^2 + 2x + 3 = 0$



2. $x^2 - 3x - 10 = 0$



3. $-x^2 - 8x - 16 = 0$



Q1 $x^2 + 2x + 3 = 0$ $D = b^2 - 4ac = 4 - 12 = -8$ $D < 0 \rightarrow$ graph does not cross x-axis No real solutions		ANSWER No real solutions (graph never crosses x-axis)
Q2 $x^2 - 3x - 10 = 0$ $D = 9 + 40 = 49$ ($D > 0$) $x = (3 \pm 7)/2$ $x = 5$ or $x = -2$ Graph crosses x-axis at -2 and 5		ANSWER $x = -2$ and $x = 5$
Q3 $-x^2 - 8x - 16 = 0$ Multiply by -1: $x^2 + 8x + 16 = 0$ $(x + 4)^2 = 0 \rightarrow x = -4$ Graph touches x-axis at one point: $x = -4$		ANSWER $x = -4$ (double/repeated root)

3	Perform operations with complex numbers	29-32	25
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Simplify.

29. $(6 - 3i) + (4 - 2i)$

30. $(-11 + 4i) - (1 - 5i)$

31. $(2 + i)(3 - i)$

32. $(5 - 2i)(4 - i)$

Q29 $(6 - 3i) + (4 - 2i)$	
Add real parts: $6+4 = 10$ Add imaginary parts: $-3+(-2) = -5$ $= 10 - 5i$	ANSWER $10 - 5i$
Q30 $(-11 + 4i) - (1 - 5i)$	
Real: $-11-1 = -12$ Imaginary: $4-(-5) = 9 = -12 + 9i$	ANSWER $-12 + 9i$
Q31 $(2 + i)(3 - i)$	
$= 6-2i+3i-i^2$ $i^2 = -1$ so $-i^2 = +1 = 6+i+1 = 7 + i$	ANSWER $7 + i$
Q32 $(5 - 2i)(4 - i)$	
$= 20-5i-8i+2i^2 = 20-13i+2(-1) = 18 - 13i$	ANSWER $18 - 13i$

4	Perform operations with pure imaginary numbers	7-12	25
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Simplify.

7. $\sqrt{-23} \cdot \sqrt{-46}$

8. $\sqrt{-6} \cdot \sqrt{-3}$

9. $\sqrt{-5} \cdot \sqrt{-10}$

10. $(3i)(-2i)(5i)$

11. i^{11}

12. $4i(-6i)^2$

Q7 $\sqrt{-23} \cdot \sqrt{-46}$	
$= i\sqrt{23} \cdot i\sqrt{46}$ $= i^2 \cdot \sqrt{(23 \times 46)} = -\sqrt{1058}$ $1058 = 529 \times 2, \sqrt{1058} = 23\sqrt{2} = -23\sqrt{2}$	ANSWER -23√2
Q8 $\sqrt{-6} \cdot \sqrt{-3}$	
$= i\sqrt{6} \cdot i\sqrt{3} = i^2\sqrt{18}$ $= -\sqrt{18} = -\sqrt{(9 \times 2)}$ $= -3\sqrt{2}$	ANSWER -3√2
Q9 $\sqrt{-5} \cdot \sqrt{-10}$	
$= i\sqrt{5} \cdot i\sqrt{10} = i^2\sqrt{50}$ $= -\sqrt{50} = -\sqrt{(25 \times 2)} = -5\sqrt{2}$	ANSWER -5√2
Q10 $(3i)(-2i)(5i)$	
$= (3 \times -2 \times 5) \cdot i^3 \quad i^3 = i^2 \cdot i = -i$ $= -30 \cdot (-i) = 30i$	ANSWER 30i
Q11 i^{11}	
Powers of i repeat every 4: $i^1=i, i^2=-1, i^3=-i, i^4=1$ $11 = 4 \times 2 + 3 \rightarrow i^{11} = i^3 = -i$	ANSWER -i
Q12 $4i(-6i)^2$	
$(-6i)^2 = 36i^2 = 36(-1) = -36$ $4i \times (-36) = -144i$	ANSWER -144i

5	Complete the square in the case of a trinomial that is not a perfect square	19-24	39
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Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square trinomial.

19. $x^2 + 10x + c$
20. $x^2 - 14x + c$
21. $x^2 + 24x + c$

22. $x^2 + 5x + c$
23. $x^2 - 9x + c$
24. $x^2 - x + c$

Q19 $x^2 + 10x + c$

Rule: $c = (b/2)^2$
 $c = (10/2)^2 = 5^2 = 25$
 $x^2+10x+25 = (x+5)^2$

ANSWER
 $c = 25 \rightarrow (x + 5)^2$

Q20 $x^2 - 14x + c$

$c = (-14/2)^2 = (-7)^2 = 49$
 $x^2-14x+49 = (x-7)^2$

ANSWER
 $c = 49 \rightarrow (x - 7)^2$

Q21 $x^2 + 24x + c$

$c = (24/2)^2 = 12^2 = 144$
 $x^2+24x+144 = (x+12)^2$

ANSWER
 $c = 144 \rightarrow (x + 12)^2$

Q22 $x^2 + 5x + c$

$c = (5/2)^2 = 25/4$
 $x^2+5x+25/4 = (x+5/2)^2$

ANSWER
 $c = 25/4 \rightarrow (x + 5/2)^2$

Q23 $x^2 - 9x + c$

$c = (-9/2)^2 = 81/4$
 $x^2-9x+81/4 = (x-9/2)^2$

ANSWER
 $c = 81/4 \rightarrow (x - 9/2)^2$

Q24 $x^2 - x + c$

$c = (-1/2)^2 = 1/4$
 $x^2-x+1/4 = (x-1/2)^2$

ANSWER
 $c = 1/4 \rightarrow (x - 1/2)^2$

6	Solve quadratic equations by using the Quadratic Formula	24-30	48
		36-38	

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

24. $x^2 - 8x + 16 = 0$

25. $x^2 - 11x - 26 = 0$

26. $3x^2 - 2x = 0$

27. $20x^2 + 7x - 3 = 0$

28. $5x^2 - 6 = 0$

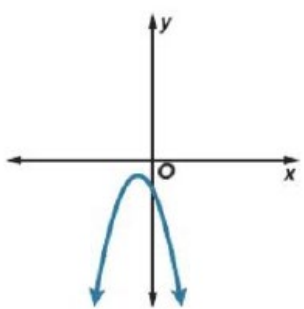
29. $x^2 - 6 = 0$

30. $x^2 + 8x + 13 = 0$

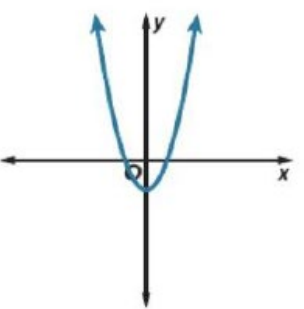
Q24 $x^2 - 8x + 16 = 0$	
$D = (-8)^2 - 4(1)(16) = 64 - 64 = 0$ $x = 8/2 = 4$ (double root)	ANSWER $D = 0 \rightarrow 1$ real (double) root: $x = 4$
Q25 $x^2 - 11x - 26 = 0$	
$D = 121 + 104 = 225$ $x = (11 \pm 15)/2$ $x = 13$ or $x = -2$	ANSWER $D = 225 \rightarrow 2$ distinct real roots: $x = 13$ and $x = -2$
Q26 $3x^2 - 2x = 0$	
$D = 4 - 0 = 4$ Factor: $x(3x - 2) = 0$ $x = 0$ or $x = 2/3$	ANSWER $D = 4 \rightarrow 2$ distinct real roots: $x = 0$ and $x = 2/3$
Q27 $20x^2 + 7x - 3 = 0$	
$D = 49 + 240 = 289$ $x = (-7 \pm 17)/40$ $x = 10/40 = 1/4$ or $x = -24/40 = -3/5$	ANSWER $D = 289 \rightarrow x = 1/4$ and $x = -3/5$
Q28 $5x^2 - 6 = 0$	
$D = 0 + 120 = 120$ $x^2 = 6/5$ $x = \pm\sqrt{6/5} = \pm\sqrt{30}/5$	ANSWER $D = 120 \rightarrow x = \pm\sqrt{30}/5$
Q29 $x^2 - 6 = 0$	
$D = 0 + 24 = 24$ $x^2 = 6 \rightarrow x = \pm\sqrt{6}$	ANSWER $D = 24 \rightarrow x = \pm\sqrt{6}$
Q30 $x^2 + 8x + 13 = 0$	
$D = 64 - 52 = 12$ $x = (-8 \pm 2\sqrt{3})/2 = -4 \pm \sqrt{3}$	ANSWER $D = 12 \rightarrow x = -4 \pm \sqrt{3}$

REGULARITY Describe the discriminant of the related equation of each graph. Then determine the type and number of roots.

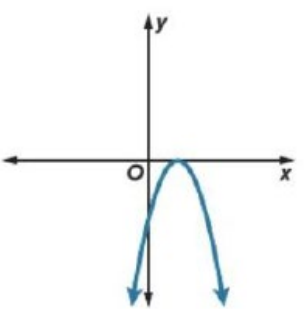
36.



37.



38.



Q36 Graph 36 — discriminant & roots?

Graph opens down
Vertex sits exactly ON the x-axis
Touches at one point only → $D = 0$

ANSWER
 $D = 0 \rightarrow$ One real (double) root. Graph touches x-axis at one point.

Q37 Graph 37 — discriminant & roots?

Graph opens up
Crosses x-axis at TWO points
 $D > 0$

ANSWER
 $D > 0 \rightarrow$ Two distinct real roots. Graph crosses x-axis at two points.

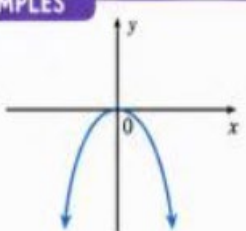
Q38 Graph 38 — discriminant & roots?

Graph opens down
Vertex is BELOW the x-axis
Never crosses x-axis → $D < 0$

ANSWER
 $D < 0 \rightarrow$ No real roots (2 complex). Graph does not touch x-axis.

EXAMPLES

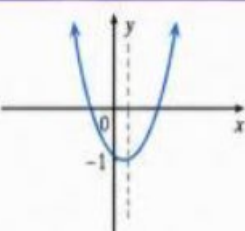
36.



- Opens down → $a < 0$
- Vertex at $(0, 0) \rightarrow h = 0, k = 0$
- Axis of symmetry: $x = 0$
- y-intercept: $(0, 0) \rightarrow c = 0$
- Equation: $y = a(x - 0)^2 + 0 = ax^2$
Take $a = -1$ (any negative works) → $y = -x^2$
- Standard form: $y = -x^2 \rightarrow a = -1, b = 0, c = 0$
- Discriminant: $D = b^2 - 4ac = 0^2 - 4(-1)(0) = 0$

$D = 0 \rightarrow$ One real root (double root)
The graph touches the x-axis at one point.

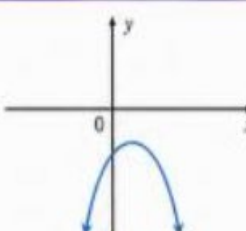
37.



- Opens up → $a > 0$
- Vertex at $(0, -1) \rightarrow h = 0, k = -1$
- Axis of symmetry: $x = 0$
- y-intercept: $(0, -1) \rightarrow c = -1$
- Equation: $y = a(x - 0)^2 - 1 = ax^2 - 1$
Take $a = 1 \rightarrow y = x^2 - 1$
- Standard form: $y = x^2 - 1 \rightarrow a = 1, b = 0, c = -1$
- Discriminant: $D = 0^2 - 4(1)(-1) = 4 > 0$

$D > 0 \rightarrow$ Two distinct real roots
The graph crosses the x-axis at two points.

38.



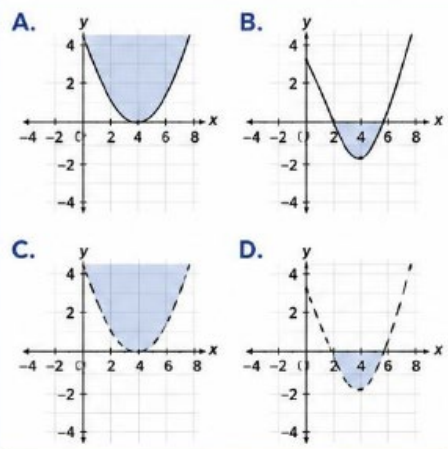
- Opens down → $a < 0$
- Vertex at $(0, -2) \rightarrow h = 0, k = -2$
- Axis of symmetry: $x = 0$
- y-intercept: $(0, -2) \rightarrow c = -2$
- Equation: $y = a(x - 0)^2 - 2 = ax^2 - 2$
Take $a = -1 \rightarrow y = -x^2 - 2$
- Standard form: $y = -x^2 - 2 \rightarrow a = -1, b = 0, c = -2$
- Discriminant: $D = 0^2 - 4(-1)(-2) = -8 < 0$

$D < 0 \rightarrow$ No real roots
The graph does not touch or cross the x-axis.

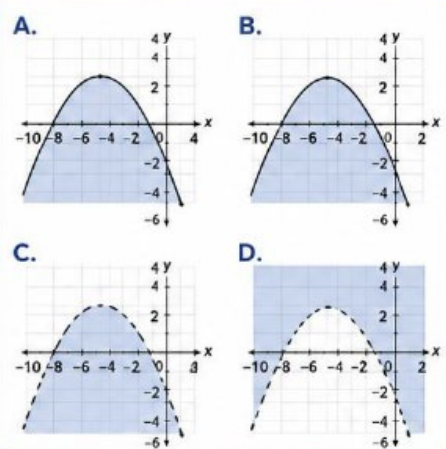
7	Solve quadratic inequalities in one variable	13-18	55
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Solve each inequality by graphing.

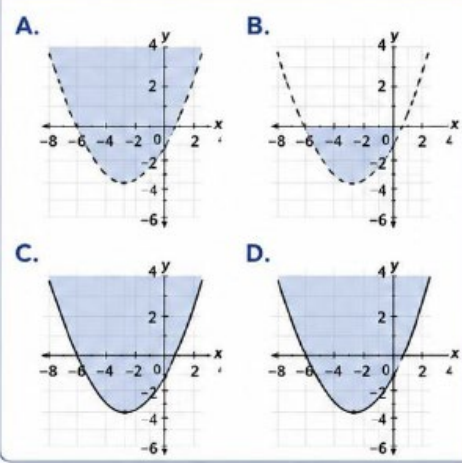
13. $x^2 - 6x + 9 \leq 0$



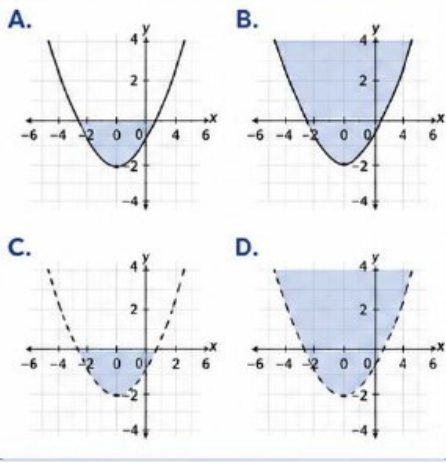
14. $-x^2 - 4x + 32 \geq 0$



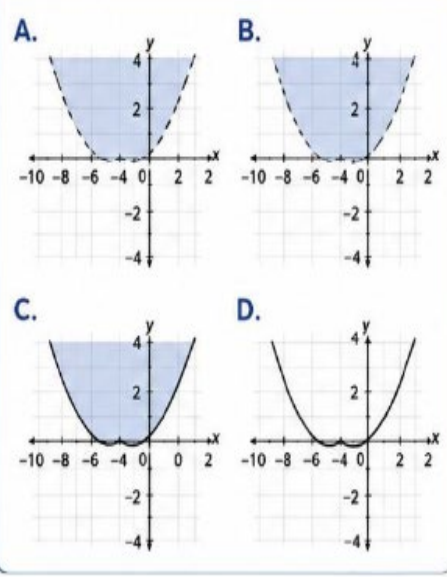
15. $\frac{3}{4}x^2 + \frac{3}{4}x - 10 > 5$



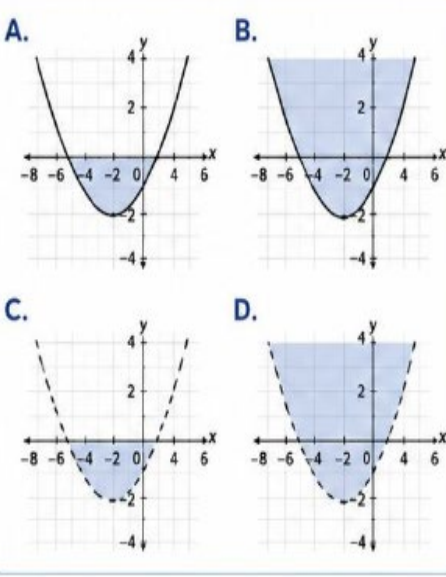
16. $x^2 - x - 6 \leq 0$



17. $x^2 + 8x + 16 \geq 0$



18. $x^2 - 2x - 24 \leq 0$



Q13 $x^2 - 6x + 9 \leq 0$

$(x-3)^2 \leq 0$

A square is always ≥ 0 , equals 0 only at $x=3$

ANSWER

Answer: A $\rightarrow$ Solution: $x = 3$ only

Solution: $\{x = 3\}$

Q14 $-x^2 - 4x + 32 \geq 0$

Multiply by -1 (flip sign): $x^2 + 4x - 32 \leq 0$

Factor: $(x+8)(x-4) \leq 0$

Solution: $-8 \leq x \leq 4$

ANSWER

Answer: B $\rightarrow -8 \leq x \leq 4$

Q15 $(3/4)x^2 + (3/4)x - 10 > 5$

$(3/4)x^2 + (3/4)x - 15 > 0$

Multiply by $4/3$: $x^2 + x - 20 > 0$

Factor: $(x+5)(x-4) > 0$

Solution: $x < -5$ or $x > 4$

ANSWER

Answer: A $\rightarrow x < -5$ or $x > 4$

Q16 $x^2 - x - 6 \leq 0$

$(x-3)(x+2) \leq 0$

Roots at $x=-2$ and $x=3$

Solution: $-2 \leq x \leq 3$

ANSWER

Answer: D $\rightarrow -2 \leq x \leq 3$

Q17 $x^2 + 8x + 16 \geq 0$

$(x+4)^2 \geq 0$

A perfect square is always ≥ 0

True for ALL real x

ANSWER

Answer: C $\rightarrow$ All real numbers $(-\infty, +\infty)$

Q18 $x^2 - 2x - 24 \leq 0$

$(x-6)(x+4) \leq 0$

Roots at $x=-4$ and $x=6$

Solution: $-4 \leq x \leq 6$

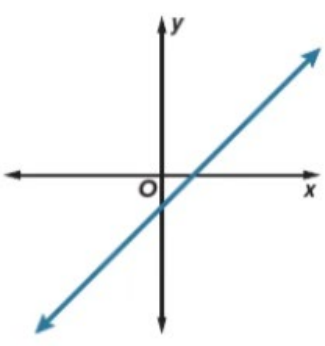
ANSWER

Answer: D $\rightarrow -4 \leq x \leq 6$

8	Graph polynomial functions and locate their zeros	15-20	80
		23-26	81

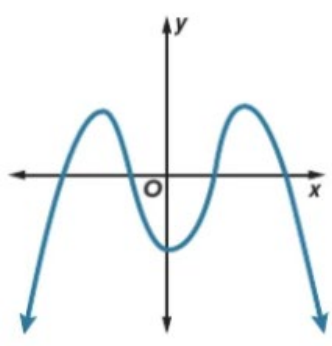
Use the graph to state the number of real zeros of the function.

15.



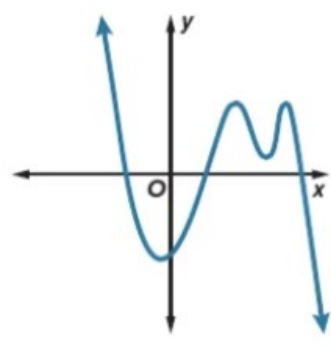
ANSWER
 1 real zero

16.



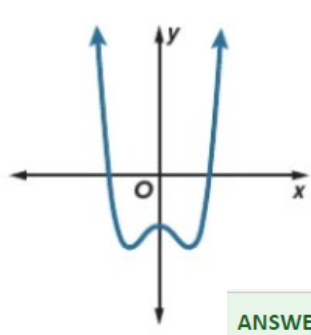
ANSWER
 4 real zeros

17.



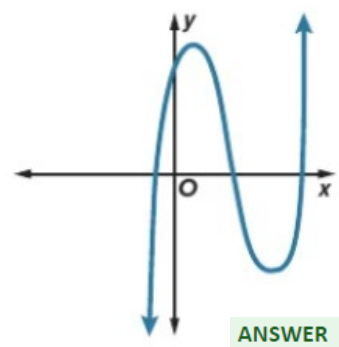
ANSWER
 3 real zeros

18.



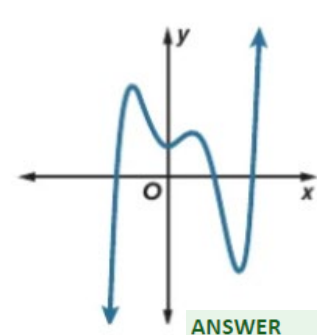
ANSWER
 4 real zeros

19.



ANSWER
 3 real zeros

20.



ANSWER
 3 real zeros

Describe the end behavior, state the degree and leading coefficient of each polynomial. If the function is not a polynomial, explain why.

23. $f(x) = -5x^4 + 3x^2$

Leading term: $-5x^4$
 Degree 4 (even), LC = -5 (negative)
 Even degree + negative LC:
 $x \rightarrow +\infty: f \rightarrow -\infty$ and $x \rightarrow -\infty: f \rightarrow -\infty$

24. $g(x) = 2x^5 + 6x^4$

Leading term: $2x^5$
 Degree 5 (odd), LC = 2 (positive)
 $x \rightarrow +\infty: f \rightarrow +\infty$ and $x \rightarrow -\infty: f \rightarrow -\infty$

25. $g(x) = 8x^4 + 5x^5$

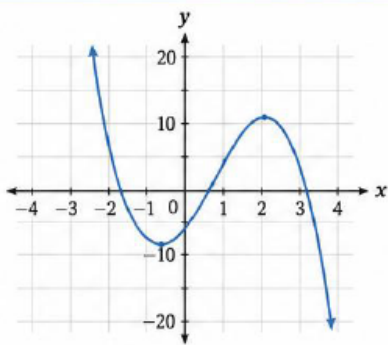
Reorder: $5x^5 + 8x^4$
 Leading term: $5x^5$
 Degree 5 (odd), LC = 5 (positive)
 $x \rightarrow +\infty: f \rightarrow +\infty$ and $x \rightarrow -\infty: f \rightarrow -\infty$

26. $h(x) = 9x^6 - 5x^7 + 3x^2$

Reorder: $-5x^7 + 9x^6 + 3x^2$
 Leading term: $-5x^7$
 Degree 7 (odd), LC = -5 (negative)
 $x \rightarrow +\infty: f \rightarrow -\infty$ and $x \rightarrow -\infty: f \rightarrow +\infty$

9	Find the relative maxima and minima of polynomial functions	5-8	89
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5. $f(x) = -2x^3 + 12x^2 - 8x$



- | | |
|----|---|
| A. | Max at $x \approx -1$,
Min at $x \approx 2$ |
| B. | Max at $x \approx 1$,
Min at $x \approx -1$ |
| C. | Max at $x \approx 2$,
Min at $x \approx 0$ |
| D. | Max at $x \approx -2$,
Min at $x \approx 1$ |

IMPORTANT NOTES

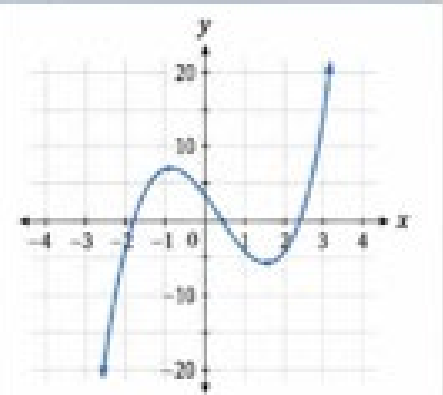
- Relative extrema can only occur at critical numbers.
- Always check the behavior of $f'(x)$ around each critical number.
- Endpoints of an interval must be checked separately for absolute maxima and minima.

Q5 $f(x) = -2x^3 + 12x^2 - 8x$

STEP 1: Look at the graph carefully.
STEP 2: Find where graph changes UP $\rightarrow$ DOWN (peak = MAX).
The graph rises, reaches a PEAK at approximately $x = 2$.
 $\rightarrow$ Relative MAXIMUM at $x \approx 2$
STEP 3: Find where graph changes DOWN $\rightarrow$ UP (valley = MIN).
The graph dips to a VALLEY at approximately $x = 0$.
 $\rightarrow$ Relative MINIMUM at $x \approx 0$
Match to option C: Max $x \approx 2$, Min $x \approx 0$ ✓

ANSWER
Answer: C $\rightarrow$ Relative
Max at $x \approx 2$, Relative
Min at $x \approx 0$

6. $f(x) = 2x^3 - 4x^2 - 3x + 4$

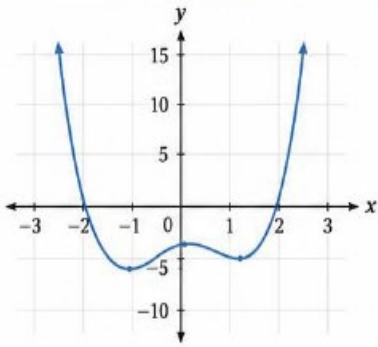


Q6 $f(x) = 2x^3 - 4x^2 - 3x + 4$

STEP 1: Examine the graph of $f(x) = 2x^3 - 4x^2 - 3x + 4$.
STEP 2: Locate the local PEAK (change UP $\rightarrow$ DOWN).
Graph peaks at approximately $x = -0.3$ (closest to -1 in options).
 $\rightarrow$ Relative MAXIMUM at $x \approx -1$
STEP 3: Locate the local VALLEY (change DOWN $\rightarrow$ UP).
Graph dips to valley at approximately $x = 1.6$ (closest to 2 in options).
 $\rightarrow$ Relative MINIMUM at $x \approx 2$
Match to option B: Max $x \approx -1$, Min $x \approx 2$ ✓

ANSWER
Answer: B $\rightarrow$ Relative
Max at $x \approx -1$, Relative
Min at $x \approx 2$

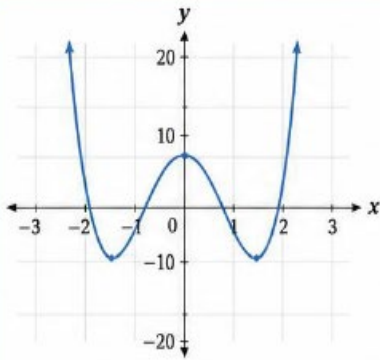
7. $f(x) = x^4 + 2x - 1$



STEP 1: $f(x) = x^4 + 2x - 1$ is a degree-4 (even) polynomial.
STEP 2: Both ends of the graph go UP ($\rightarrow +\infty$).
This means the graph has a U-like shape overall.
There is NO relative maximum (graph goes to $+\infty$ on both sides).
STEP 3: Find the VALLEY (lowest local point).
The graph has one valley at approximately $x \approx -0.79$.
Closest option: $x \approx -0.5$ (option D).
 $\rightarrow$ Relative MINIMUM at $x \approx -0.5$

ANSWER
Answer: D $\rightarrow$ No
relative max; Relative
Min at $x \approx -0.5$

8. $f(x) = x^4 + 8x^2 - 12$



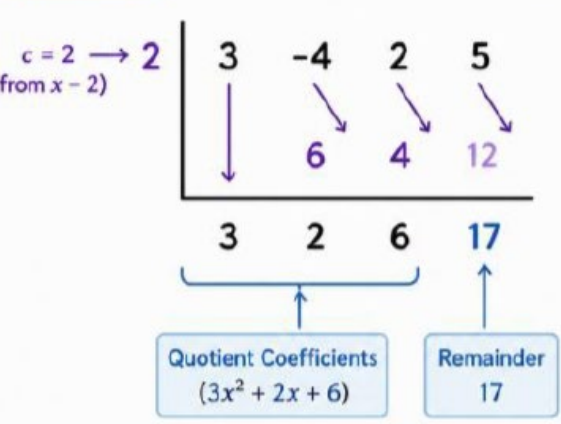
Q8 $f(x) = x^4 + 8x^2 - 12$

STEP 1: $f(x) = x^4 + 8x^2 - 12$ is an EVEN function (symmetric about y-axis).
The graph looks like a W-shape.
STEP 2: Locate the local PEAK (MAX) at center.
The middle 'hump' of the W is at $x = 0$.
 $\rightarrow$ Relative MAXIMUM at $x \approx 0$
STEP 3: Locate the two symmetric VALLEYS (MIN).
The W has two dips, symmetric about y-axis at $x \approx \pm 1$.
 $\rightarrow$ Relative MINIMUM at $x \approx \pm 1$
Match to option B: Max $x \approx 0$, Min $x \approx \pm 1$ ✓

ANSWER
Answer: B $\rightarrow$ Relative
Max at $x \approx 0$, Relative
Min at $x \approx \pm 1$

10	Divide polynomials using synthetic division	11-16	105
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EXAMPLE: Divide $3x^3 - 4x^2 + 2x + 5$ by $(x - 2)$.



- STEPS:**
- 1 Write the value c on the left.
 - 2 Write the coefficients of the dividend on the top row.
 - 3 Bring down the first coefficient.
 - 4 Multiply by c and write the product under the next coefficient.
 - 5 Add and bring down. Repeat until the end.

SO REMEMBER:

- The bottom row (except the last number) gives the quotient.
- The last number is the remainder.

Therefore: $3x^3 - 4x^2 + 2x + 5 = (x - 2)(3x^2 + 2x + 6) + 17$

Simplify using synthetic division.

11. $(3v^2 - 7v - 10)(v - 4)^{-1}$

$c = 4$ | coefficients: $[3, -7, -10]$

4	3	-7	-10
		12	20

3 5 10
 Quotient: $3v+5$, Remainder: 10

ANSWER
 $3v + 5 + 10/(v - 4)$

12. $(3t^4 + 4t^3 - 32t^2 - 5t - 20)(t + 4)^{-1}$

$c = -4$ | $[3, 4, -32, -5, -20]$

-4	3	4	-32	-5	-20
		-12	32	0	20

3 -8 0 -5 0
 Quotient: $3t^3-8t^2-5$, Remainder: 0

ANSWER
 $3t^3 - 8t^2 - 5$

13. $\frac{y^3 + 6}{y + 2}$

Missing terms $\rightarrow$ use 0: $[1, 0, 0, 6]$

-2	1	0	0	6
		-2	4	-8

1 -2 4 -2
 Quotient: y^2-2y+4 , Remainder: -2

ANSWER
 $y^2 - 2y + 4 - 2/(y + 2)$

14. $\frac{2x^3 - x^2 - 18x + 32}{2x - 6}$

Factor 2: $(2x-6) = 2(x-3)$, $c=3$

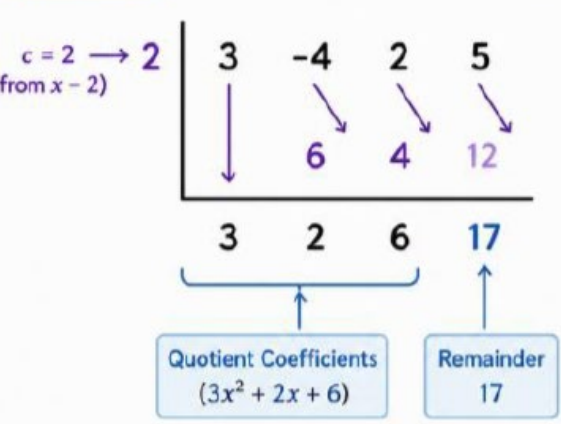
3	2	-1	-18	32
		6	15	-9

2 5 -3 23
 Divide by 2: $x^2+(5/2)x-3/2$, rem 23

ANSWER
 $x^2 + (5/2)x - (3/2) + 23/(2x-6)$

10	Divide polynomials using synthetic division	11-16	105
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EXAMPLE: Divide $3x^3 - 4x^2 + 2x + 5$ by $(x - 2)$.



- STEPS:**
- 1 Write the value c on the left.
 - 2 Write the coefficients of the dividend on the top row.
 - 3 Bring down the first coefficient.
 - 4 Multiply by c and write the product under the next coefficient.
 - 5 Add and bring down. Repeat until the end.

SO REMEMBER:

- The bottom row (except the last number) gives the quotient.
- The last number is the remainder.

Therefore: $3x^3 - 4x^2 + 2x + 5 = (x - 2)(3x^2 + 2x + 6) + 17$

15. $(4p^3 - p^2 + 2p) \div (3p - 1)$

16. $(3c^4 + 6c^3 - 2c + 4)(c + 2)^{-1}$

Q15 $(4p^3 - p^2 + 2p) \div (3p - 1)$

Use long division (non-monic divisor):
 $4p^3 \div 3p = (4/3)p^2$
 $(4/3)p^2(3p-1) = 4p^3 - (4/3)p^2$
 Remainder: $(1/3)p^2 + 2p$
 $(1/3)p^2 \div 3p = (1/9)p$
 Remainder: $(19/9)p \rightarrow$ gives $19/27$ rem $19/27$

ANSWER
 $(4/3)p^2 + (1/9)p + 19/27 + (19/27)/(3p-1)$

Q16 $(3c^4 + 6c^3 - 2c + 4)(c + 2)^{-1}$

Missing $c^2 \rightarrow$ use 0: $[3, 6, 0, -2, 4], c = -2$

-2	3	6	0	-2	4
	-6	0	0	4	
	3	0	0	-2	8

Quotient: $3c^3 - 2$, Remainder: 8

ANSWER
 $3c^3 - 2 + 8/(c + 2)$

11	Divide Polynomials	1-4	105
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Simplify each expression.

1. $\frac{15y^3 + 6y^2 + 3y}{3y}$

ANSWER
 $5y^2 + 2y + 1$

2. $(4f^5 - 6f^4 + 12f^3 - 8f^2)(4f^2)^{-1}$

ANSWER
 $f^3 - (3/2)f^2 + 3f - 2$

3. $(6j^2k - 9jk^2) \div (3jk)$

ANSWER
 $2j - 3k$

4. $(4a^2h^2 - 8a^3h + 3a^4) \div (2a^2)$

ANSWER
 $2h^2 - 4ah + (3/2)a^2$

HOW TO DO IT

- 1 Divide the **first term** of the polynomial by the single term.
- 2 Divide **each remaining term** by the same single term.
- 3 Simplify each result using the rules of exponents and arithmetic.
- 4 Write the simplified terms in order.

REMEMBER

You are not finding a "quotient" and "remainder" here—this is different from long division. You are simply dividing **each term**.



TIPS

- Remember the rule: $x^a \div x^b = x^{a-b}$
- Any number divided by a variable stays as a fraction.
- Keep track of signs!

12	Use the Binomial Theorem to write and find the coefficients of specified terms in binomial expansions	9-12&22,26	111&112
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Expand each binomial.

9. $(3x + 4y)^5$

ANSWER
 $243x^5 + 1620x^4y + 4320x^3y^2 + 5760x^2y^3 + 3840xy^4 + 1024y^5$

10. $(2c - 2d)^7$

ANSWER
 $128c^7 - 896c^6d + 2688c^5d^2 - 4480c^4d^3 + 480c^3d^4 - 2688c^2d^5 + 896cd^6 - 128d^7$

11. $(8h - 3j)^4$

ANSWER
 $4096h^4 - 6144h^3j + 3456h^2j^2 - 864hj^3 + 81j^4$

12. $(4a + 3b)^6$

ANSWER
 $4096a^6 + 18432a^5b + 34560a^4b^2 + 34560a^3b^3 + 19440a^2b^4 + 5832ab^5 + 729b^6$

22. **REGULARITY** Use Pascal's Triangle to find the fourth term in the expansion of $(2x + 7)^6$. Why is it the same as the fourth term in the expansion of $(7 + 2x)^6$?

ANSWER
 4th term = $54880x^3$. Same because $(7+2x)^6 = (2x+7)^6$ — addition is commutative.

26. **PERSEVERE** Find the sixth term of the expansion of $(\sqrt{a} + \sqrt{b})^{12}$.

Use General term formula
 $T_{r+1} = {}^nC_r a^{n-r} b^r$ Put $r = 5$
 $T_6 = C(12,5) \cdot (a^{1/2})^{(12-5)} \cdot (b^{1/2})^5$
 $= C(12,5) \cdot a^{7/2} \cdot b^{5/2}$
 $C(12,5) = 792$
 $= 792 \cdot a^{7/2} \cdot b^{5/2}$

ANSWER
 $792 \cdot a^{7/2} \cdot b^{5/2}$

13	Use Pascal's Triangle to write binomial expansions	1-6	111
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PASCAL'S TRIANGLE

Helpful for keeping coefficients aligned when working with powers of x .

Row n contains the coefficients for $(x + a)^n$.

Use row n to align terms from x^n to x^0 .

		1				
	1		1			
	1	2	1			
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	

Use Pascal's triangle to expand each binomial.

1. $(x - y)^3$
2. $(a + b)^4$

ANSWER

$x^3 - 3x^2y + 3xy^2 - y^3$

ANSWER

$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

3. $(g - h)^4$
4. $(m + 1)^4$

ANSWER

$g^4 - 4g^3h + 6g^2h^2 - 4gh^3 + h^4$

ANSWER

$m^4 + 4m^3 + 6m^2 + 4m + 1$

5. $(y - z)^6$
6. $(d + 2)^8$

ANSWER

$y^6 - 6y^5z + 15y^4z^2 - 20y^3z^3 + 15y^2z^4 - 6yz^5 + z^6$

ANSWER

$d^8+16d^7+112d^6+448d^5+1120d^4+1792d^3+1792d^2+1024d+256$

14	Graph polynomial functions and locate their zeros	Example 1	119
		1-6	121

Example 1 Solve a Polynomial Equation by Graphing

Use a graphing calculator to solve $x^4 + 3x^2 - 5 = -4x^3$ by graphing.

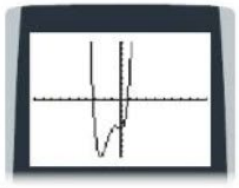
Step 1 Find a related function. Write the equation with 0 on the right.

$x^4 + 3x^2 - 5 = -4x^3$	Original equation
$x^4 + 3x^2 - 5 + 4x^3 = -4x^3 + 4x^3$	Add $4x^3$ to each side.
$x^4 + 4x^3 + 3x^2 - 5 = 0$	Simplify.

A related function is $f(x) = x^4 + 4x^3 + 3x^2 - 5$.

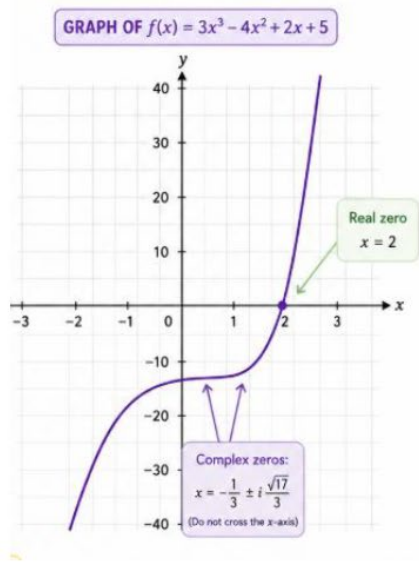
Step 2 Graph the related function.

Enter the equation in the **Y =** list and graph the function.



Step 3 Find the zeros.

Use the **zero** feature from the **CALC** menu.
 The real zeros are about -3.22 and 0.84 .



Use a graphing calculator to solve each equation by graphing. If necessary, round to the nearest hundredth.

- | | | |
|-------------------------------------|----------------------------|---------------------|
| 1. $\frac{2}{3}x^3 + x^2 - 5x = -9$ | 2. $x^3 - 9x^2 + 27x = 20$ | 3. $x^3 + 1 = 4x^2$ |
|-------------------------------------|----------------------------|---------------------|

ANSWER
 Answer: B → $x \approx -2.68, 0.68, 3.00$

ANSWER
 Answer: A → $x \approx 1.00, 3.27, 7.73$

ANSWER
 Answer: B → $x \approx -1.73, 0.27, 1.46$

- | | | |
|----------------------------|--|---------------------|
| 4. $x^6 - 15 = 5x^4 - x^2$ | 5. $\frac{1}{2}x^5 = \frac{1}{5}x^2 - 2$ | 6. $x^8 = -x^7 + 3$ |
|----------------------------|--|---------------------|

ANSWER
 Answer: B → $x \approx -2.00, -0.94, 0.94, 2.00$

ANSWER
 Answer: A → $x \approx -2.04, -0.40, 1.37$

ANSWER
 Answer: B → $x \approx -0.76, 0.53$

15	Factorize polynomials	1-10	127
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COMMON FACTORISATION METHODS				
1. HIGHEST COMMON FACTOR (HCF) Factor out the greatest common factor of all terms. Example: $12x^3 + 18x^2 - 6x$ $= 6x(2x^2 + 3x - 1)$	2. FACTORISING TRINOMIALS $x^2 + bx + c$ Find two numbers that multiply to c and add to b . Rewrite and factor by grouping. Example: $x^2 + 7x + 12$ $= x^2 + 3x + 4x + 12$ $= x(x + 3) + 4(x + 3)$ $= (x + 3)(x + 4)$	3. DIFFERENCE OF TWO SQUARES Use $a^2 - b^2 = (a - b)(a + b)$. Example: $9x^2 - 16$ $= (3x)^2 - 4^2$ $= (3x - 4)(3x + 4)$	4. SUM AND DIFFERENCE OF TWO CUBES Use the special formulas. Sum of two cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ Difference of two cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ Example: $x^3 + 8$ $= x^3 + 2^3$ $= (x + 2)(x^2 - 2x + 4)$ $x^3 - 27$ $= x^3 - 3^3$ $= (x - 3)(x^2 + 3x + 9)$	5. FACTORISING BY GROUPING Group terms and factor out the common factor from each group. Example: $ax + ay + bx + by$ $= a(x + y) + b(x + y)$ $= (a + b)(x + y)$

Factor completely. If the polynomial is not factorable, write *prime*.

1. $8c^3 - 27d^3$

ANSWER
 $(2c - 3d)(4c^2 + 6cd + 9d^2)$

2. $64x^4 + xy^3$

ANSWER
 $x(4x + y)(16x^2 - 4xy + y^2)$

3. $a^8 - a^2b^6$

ANSWER
 $a^2(a+b)(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$

4. $x^6y^3 + y^9$

ANSWER
 $y^3(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

5. $18x^6 + 5y^6$

ANSWER
Prime

6. $w^3 - 2y^3$

ANSWER
Prime

7. $gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2$ 8. $12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$

ANSWER
 $(g - 3h + 6f)(x + y)(x - y)$

ANSWER
 $(2a - 3b + 4c)(6x^2 - 5y^2)$

9. $a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$ 10. $8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$

ANSWER
 $(a - b)(a^2 + ab + b^2)(x - 8)^2$

ANSWER
 $(x + 5)^2(2x - y)(4x^2 + 2xy + y^2)$

16	Find all of the rational zeros of a polynomial function	22-27	128
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Solve each equation.

22. $x^4 + 6x^2 + 5 = 0$

ANSWER
 $x = \pm i$ and $x = \pm i\sqrt{5}$ (no real solutions)

23. $x^4 - 3x^2 - 10 = 0$

ANSWER
 $x = \pm\sqrt{5}$ (real) and $x = \pm i\sqrt{2}$ (complex)



24. $4x^4 - 14x^2 + 12 = 0$

ANSWER
 $x = \pm\sqrt{2}$ and $x = \pm\sqrt{6}/2$

25. $9x^4 - 27x^2 + 20 = 0$

ANSWER
 $x = \pm\sqrt{15}/3$ and $x = \pm 2\sqrt{3}/3$

26. $4x^4 - 5x^2 - 6 = 0$

ANSWER
 $x = \pm\sqrt{2}$ and $x = \pm i\sqrt{3}/2$

27. $24x^4 + 14x^2 - 3 = 0$

ANSWER
 $x = \pm\sqrt{6}/6$ and $x = \pm i\sqrt{3}/2$

17	Use factor theorem to determine factors of polynomials.	31-38	140
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STEP 1: USE SYNTHETIC DIVISION

Test possible zeros using synthetic division.
 $f(-1) = 0$, so $(x + 1)$ is a factor.

-1	1	-4	1	6
	↓			
	1	-5	6	0

Quotient:
 $x^2 - 5x + 6$

Remainder 0
 $\Rightarrow (x + 1)$ is a factor

STEP 2: WRITE THE FACTORS AND USE TRINOMIALS

We have one linear factor $(x + 1)$ and a quadratic factor $x^2 - 5x + 6$.
Factor the quadratic using trinomials that multiply to 6 and add to -5 .

$x^2 - 5x + 6 = (x - 2)(x - 3)$

$-2 + -3 = -5$ $(-2)(-3) = 6$

Factors:
 $(x + 1), (x - 2), (x - 3)$

STEP 3: WRITE THE FACTORED FORM

Combine all the factors.
 $f(x) = (x + 1)(x - 2)(x - 3)$

Check (optional):
 $(x + 1)(x - 2)(x - 3) = x^3 - 4x^2 + x + 6$ ✓

★ **KEY TAKEAWAY:**

- Use the Factor Theorem to find linear factors $(x - c)$.
- Divide using synthetic division.
- Factor any remaining quadratic using trinomials.
- Write the factored form as a product of all factors.

31. $x^3 + 2x^2 - x - 2; x + 1$

32. $x^3 + x^2 - 5x + 3; x - 1$

ANSWER
 $(x + 1)(x + 2)(x - 1)$

ANSWER
 $(x - 1)^2(x + 3)$

33. $x^3 + 3x^2 - 4x - 12; x + 3$

34. $x^3 - 6x^2 + 11x - 6; x - 3$

ANSWER
 $(x + 3)(x + 2)(x - 2)$

ANSWER
 $(x - 3)(x - 2)(x - 1)$

17	Use factor theorem to determine factors of polynomials.	31-38	140
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35. $x^3 + 2x^2 - 33x - 90; x + 5$

ANSWER
 $(x + 5)(x + 3)(x - 6)$

36. $x^3 - 6x^2 + 32; x - 4$

ANSWER
 $(x - 4)^2(x + 2)$

37. $x^3 - x^2 - 10x - 8; x + 2$

$(x + 2)(x + 1)(x - 4)$

38. $x^3 - 19x + 30; x - 2$

ANSWER
 $(x - 2)(x - 3)(x + 5)$

**KEY TAKEAWAY:**

- Use the Factor Theorem to find linear factors $(x - c)$.
- Divide using synthetic division.
- Factor any remaining quadratic using trinomials.
- Write the factored form as a product of all factors.

18	Determine the number and type of roots for a polynomial equation	17-25	149
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Example 4

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

17. $g(x) = 3x^3 - 4x^2 - 17x + 6$			
	Positive	Negative	Imaginary
A.	2	1	0
B.	1	2	0
C.	3	0	0
D.	1	1	1

ANSWER
Answer: A → Pos: 2, Neg: 1, Imag: 0

18. $h(x) = 4x^3 - 12x^2 - x + 3$			
	Positive	Negative	Imaginary
A.	2	1	0
B.	1	2	0
C.	3	0	0
D.	1	1	1

ANSWER
Answer: A → Pos: 2, Neg: 1, Imag: 0

19. $f(x) = x^3 - 8x^2 + 2x - 4$			
	Positive	Negative	Imaginary
A.	2	1	0
B.	1	2	0
C.	0	3	0
D.	1	1	1

Answer: → Pos: 3 or 1, Neg: 0, Imag: 0 or 2

20. $p(x) = x^3 - x^2 + 4x - 6$			
	Positive	Negative	Imaginary
A.	2	0	1
B.	1	1	1
C.	0	2	1
D.	1	2	0

Answer: → Pos: 3 or 1, Neg: 0, Imag: 0 or 2

D.	1	1	1
21. $q(x) = x^4 + 7x^2 + 3x - 9$			
	Positive	Negative	Imaginary
A.	2	2	0
B.	1	3	0
C.	4	0	0
D.	2	0	2

Answer: → Pos: 1, Neg: 1, Imag: 2

D.	1	2	0
22. $f(x) = x^4 - x^3 - 5x^2 + 6x + 1$			
	Positive	Negative	Imaginary
A.	2	2	0
B.	1	3	0
C.	0	2	2
D.	2	0	2

Answer: A → Pos: 2, Neg: 2, Imag: 0

23. $f(x) = x^4 - 5x^3 + 2x^2 + 5x + 7$			
	Positive	Negative	Imaginary
A.	4	0	0
B.	3	1	0
C.	2	2	0
D.	1	1	2

Answer: C → Pos: 2 or 0, Neg: 2, Imag: 0 or 2

24.	$f(x) = 2x^3 - 7x^2 - 2x + 12$		
	Positive	Negative	Imaginary
A.	2	1	0
B.	1	2	0
C.	3	0	0
D.	1	1	1

Answer: A → Pos: 2 or 0, Neg: 1, Imag: 0 or 2

25. $f(x) = -3x^5 + 5x^4 + 4x^2 - 8$			
	Positive	Negative	Imaginary
A.	2	2	1
B.	1	1	3
C.	3	1	1
D.	0	2	3

Answer: → Pos: 2 or 0, Neg: 1, Imag: 2 or 4

26. $f(x) = x^4 - 2x^2 - 5x + 19$			
	Positive	Negative	Imaginary
A.	2	0	2
B.	1	1	2
C.	0	2	2
D.	3	1	0

Answer: A → Pos: 2 or 0, Neg: 2 or 0, Imag: 0, 2, or 4

19	Find the composition of functions	21-30	165
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If $f(x) = 3x$, $g(x) = x + 4$, and $h(x) = x^2 - 1$, find each value.

21. $f[g(1)]$
22. $g[h(0)]$
23. $g[f(-1)]$

24. $h[f(5)]$
25. $g[h(-3)]$
26. $h[f(10)]$

27. $f[h(8)]$
28. $[f \circ (h \circ g)](1)$
29. $[f \circ (g \circ h)](-2)$

30. $h[f(-6)]$

WHAT IS COMPOSITION?

The composition of two functions f and g is a new function.

The composition $f \circ g$ means "do g first, then f ."

The composition $g \circ f$ means "do f first, then g ."

x
(input)

$\xrightarrow{g}$

$g(x)$

$\xrightarrow{f}$

$f(g(x))$
(output)

19	Find the composition of functions	21-30	165
----	-----------------------------------	-------	-----

If $f(x) = 3x$, $g(x) = x + 4$, and $h(x) = x^2 - 1$, find each value.
 Given: $f(x) = 3x$, $g(x) = x + 4$, $h(x) = x^2 - 1$

Q21 $f[g(1)]$

$g(1) = 1+4 = 5$
 $f(5) = 3 \times 5 = 15$

ANSWER
15

Q22 $g[h(0)]$

$h(0) = 0-1 = -1$
 $g(-1) = -1+4 = 3$

ANSWER
3

Q23 $g[f(-1)]$

$f(-1) = -3$
 $g(-3) = -3+4 = 1$

ANSWER
1

Q24 $h[f(5)]$

$f(5) = 15$
 $h(15) = 225-1 = 224$

ANSWER
224

Q25 $g[h(-3)]$

$h(-3) = 9-1 = 8$
 $g(8) = 8+4 = 12$

ANSWER
12

Q26 $h[f(10)]$

$f(10) = 30$
 $h(30) = 900-1 = 899$

ANSWER
899

Q27 $f[h(8)]$

$h(8) = 64-1 = 63$
 $f(63) = 3 \times 63 = 189$

ANSWER
189

Q28 $[f \circ (h \circ g)](1)$

$g(1)=5 \rightarrow h(5)=24 \rightarrow f(24)=72$

ANSWER
72

Q29 $[f \circ (g \circ h)](-2)$

$h(-2)=3 \rightarrow g(3)=7 \rightarrow f(7)=21$

ANSWER
21

Q30 $h[f(-6)]$

$f(-6)=-18 \rightarrow h(-18)=324-1=323$

ANSWER
323

20	Find the inverse of a function or relation. Determine whether two functions or relations are inverses	17-22	172
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Determine whether each pair of functions are inverse functions. Write yes or no.

17. $f(x) = x - 1$
 $g(x) = 1 - x$

18. $f(x) = 2x + 3$
 $g(x) = \frac{1}{2}(x - 3)$

1 ALGEBRAIC METHOD (COMPOSITION)

Check both $f(g(x)) = x$ and $g(f(x)) = x$.

EXAMPLE:

Determine whether $f(x) = 3x - 2$ and $g(x) = \frac{x+2}{3}$ are inverses.

Step 1: Find $f(g(x))$.

$$f(g(x)) = f\left(\frac{x+2}{3}\right) = 3\left(\frac{x+2}{3}\right) - 2$$
$$= (x+2) - 2 = x \quad \checkmark$$

Step 2: Find $g(f(x))$.

$$g(f(x)) = g(3x - 2) = \frac{(3x - 2) + 2}{3}$$
$$= \frac{3x}{3} = x \quad \checkmark$$

Conclusion: Since $f(g(x)) = x$ and $g(f(x)) = x$, f and g are inverses.

19. $f(x) = 5x - 5$
 $g(x) = \frac{1}{5}x + 1$

20. $f(x) = 2x$
 $g(x) = \frac{1}{2}x$

21. $h(x) = 6x - 2$
 $g(x) = \frac{1}{6}x + 3$

22. $f(x) = 8x - 10$
 $g(x) = \frac{1}{8}x + \frac{5}{4}$

Determine whether each pair of functions are inverse functions. Write yes or no.

17. $f(x) = x - 1$
 $g(x) = 1 - x$ **no**

18. $f(x) = 2x + 3$
 $g(x) = \frac{1}{2}(x - 3)$ **yes**

19. $f(x) = 5x - 5$
 $g(x) = \frac{1}{5}x + 1$ **yes**

20. $f(x) = 2x$
 $g(x) = \frac{1}{2}x$ **yes**

21. $h(x) = 6x - 2$
 $g(x) = \frac{1}{6}x + 3$ **no**

22. $f(x) = 8x - 10$
 $g(x) = \frac{1}{8}x + \frac{5}{4}$ **yes**

21	Simplify expressions in exponential or radical form	1-12	179
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Simplify.

1. $\pm\sqrt{121x^4y^{16}}$ $\pm 11x^2y^8$

2. $\pm\sqrt{225a^{16}b^{36}}$ $\pm 15a^8b^{18}$

4. $-\sqrt{16c^4d^2}$ $-4c^2|d|$

5. $-\sqrt{81a^{16}b^{20}c^{12}}$ $-9a^8b^{10}c^6$

7. $\sqrt[4]{16(x-3)^{12}}$ $2|(x-3)^3|$

8. $\sqrt[8]{x^{16}y^8}$ $x^2|y|$

10. $\sqrt[6]{x^{18}}$ $|x^3|$

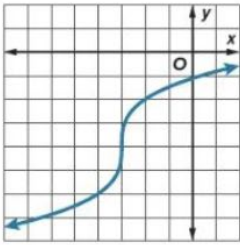
11. $\sqrt[4]{a^{12}}$ $|a^3|$

Example 7 Write a Radical Function

Write a radical function for the graph of $g(x)$.

Step 1 Identify the index.

Because the domain and range for $g(x)$ is all real numbers, the index is odd. This function can be represented by $g(x) = a\sqrt[3]{x-h} + k$.



Step 2 Identify any transformations.

The function has been translated 3 units left and 4 units down.

Therefore, $h = -3$ and $k = -4$. To find the value of a , use a point as well as the values of h and k .

$g(x) = a\sqrt[3]{x-h} + k$ Cube root function
 $-2 = a\sqrt[3]{-2 - (-3)} + (-4)$ $h = -3, k = -4, (x, g(x)) = (-2, -2)$
 $-2 = a \cdot 1 - 4$ Simplify.
 $a = 2$ Simplify.

Step 3 Write the function.

Substitute the values of a , h , and k to write the function. The graph is represented by $g(x) = 2\sqrt[3]{x+3} - 4$.

Step 3 Write the function.

Substitute the values of a , h , and k to write the function. The graph is represented by $g(x) = 2\sqrt[3]{x+3} - 4$.

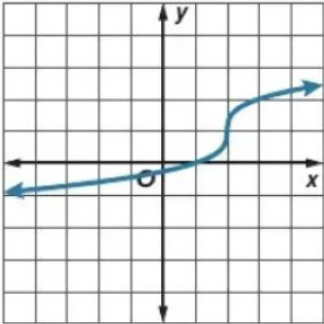
$y = a\sqrt[3]{b(x-h)} + k$

a : vertical stretch
reflection b : horizontal stretch
r compression h : horizontal
shift k : vertical
shift

Transformation	How it affects the graph	Example	Graph feature changes
$a\sqrt[3]{x}$	Vertical stretch by $ a $. Reflect in x -axis if $a < 0$.	$y = 2\sqrt[3]{x}$ $y = -\sqrt[3]{x}$	Range: $y \geq 0$ or $y \leq 0$
$\sqrt[3]{bx}$	Horizontal compression by $b > 1$. Horizontal stretch by $0 < b < 1$.	$y = \sqrt[3]{4x}$ $y = \sqrt[3]{\frac{1}{4}x}$	Domain: $x \geq 0$
$\sqrt[3]{x-h}$	Shift right h units (left if $h < 0$).	$y = \sqrt[3]{x-3}$	Domain: $x \geq h$ Start: $(h, 0)$
$\sqrt[3]{x} + k$	Shift up k units (down if $k < 0$).	$y = \sqrt[3]{x} - 2$	Range: $y \geq k$ Start: $(0, k)$
$a\sqrt[3]{b(x-h)} + k$	Combination of all transformations.	$y = -2\sqrt[3]{3(x-1)} + 4$	

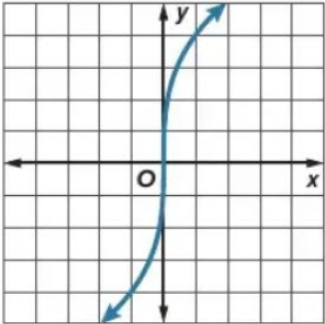
Write a radical function for each graph.

27.



$f(x) = \sqrt[3]{x-2} + 1$

28.



$f(x) = 4\sqrt[3]{x}$

23	Add, subtract, multiply, and divide radical expressions	11-16	199
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Simplify.

11. $\sqrt{2} + \sqrt{8} + \sqrt{50}$

1

ADDITION

Add like radicals (same index and radicand).

RULE: $a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$

12. $\sqrt{12} - 2\sqrt{3} + \sqrt{108}$

2

SUBTRACTION

Subtract like radicals (same index and radicand).

RULE: $a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}$

13. $8\sqrt{5} - \sqrt{45} - \sqrt{80}$

14. $2\sqrt{48} - \sqrt{75} - \sqrt{12}$

15. $\sqrt{28x} - \sqrt{14} + \sqrt{63x}$

16. $\sqrt{135} + 5\sqrt{10d} - 3\sqrt{60}$

Simplify.

11. $\sqrt{2} + \sqrt{8} + \sqrt{50}$ $8\sqrt{2}$

12. $\sqrt{12} - 2\sqrt{3} + \sqrt{108}$ $6\sqrt{3}$

13. $8\sqrt{5} - \sqrt{45} - \sqrt{80}$ $\sqrt{5}$

14. $2\sqrt{48} - \sqrt{75} - \sqrt{12}$ $\sqrt{3}$

15. $\sqrt{28x} - \sqrt{14} + \sqrt{63x}$ $5\sqrt{7x} - \sqrt{14}$

16. $\sqrt{135} + 5\sqrt{10d} - 3\sqrt{60}$ $5\sqrt{10d} - 3\sqrt{15}$

24	Simplify radical expressions	5-10	199
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Simplify.

5. $\sqrt[4]{64a^4b^4}$

6. $\sqrt[3]{-8d^2f^5}$

5. $\sqrt[4]{64a^4b^4}$ $2|ab|\sqrt[4]{4}$

6. $\sqrt[3]{-8d^2f^5}$ $-2f\sqrt[3]{d^2f^2}$

7. $\sqrt{\frac{25}{36}r^2t}$

8. $\sqrt{\frac{192k^4}{64}}$

7. $\sqrt{\frac{25}{36}r^2t}$ $\frac{5}{6}|r|\sqrt{t}$

8. $\sqrt{\frac{192k^4}{64}}$ $k^2\sqrt{3}$

9. $\sqrt[5]{\frac{3072h^8}{243f^5}}$

10. $\sqrt[3]{\frac{432n^{12}}{64q^6}}$

9. $\sqrt[5]{\frac{3072h^8}{243f^5}}$ $\frac{4h \cdot \sqrt[5]{3h^3}}{3f}$

10. $\sqrt[3]{\frac{432n^{12}}{64q^6}}$ $\frac{3n^4 \cdot \sqrt[3]{2}}{2q^2}$

KEY PROPERTIES OF RADICALS ($a \geq 0, b \geq 0, n \geq 2$)					
1 PRODUCT PROPERTY $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ Multiply under the same radical. Example: $\sqrt{3} \cdot \sqrt{12} = \sqrt{3 \cdot 12}$ $= \sqrt{36} = 6$	2 QUOTIENT PROPERTY $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ ($b \neq 0$) Divide under the same radical. Example: $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}}$ $= \sqrt{9} = 3$	3 POWER PROPERTY $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$ A radical to the same power as its index eliminates the radical. Example: $(\sqrt[3]{7})^3 = 7$	4 ROOT OF A POWER $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ ($a \geq 0$) Bring the exponent out as a fraction. Example: $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$	5 EXTRACT PERFECT n^{th} POWERS $\sqrt[n]{a^m b} = a^{\frac{m}{n}} \sqrt[n]{b}$ ($a \geq 0$) Take out perfect n^{th} powers from under the radical. Example: $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$	6 CHANGE OF INDEX $\sqrt[n]{a} = \sqrt[mn]{a^m}$ ($a \geq 0$) Convert to an equivalent radical with a different index. Example: $\sqrt{2} = \sqrt[4]{2^2}$

25	Solve radical equations	1-6	207
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Solve each equation.

1. $5\sqrt{j} = 1$

2. $\sqrt{b - 5} = 4$

3. $\sqrt{3n + 1} = 5$

4. $2 + \sqrt{3p + 7} = 6$

5. $\sqrt{k - 4} - 1 = 5$

6. $5 = \sqrt{2g - 7}$

Example 1

Solve each equation.

1. $5\sqrt{j} = 1$ $\frac{1}{25}$

2. $\sqrt{b - 5} = 4$ 21

3. $\sqrt{3n + 1} = 5$ 8

4. $2 + \sqrt{3p + 7} = 6$ 3

5. $\sqrt{k - 4} - 1 = 5$ 40

6. $5 = \sqrt{2g - 7}$ 16