

أوراق عمل الوحدة السابعة Rational Functions والتاسعة Trigonometric Functions



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف الثاني عشر العام ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

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منهج انجليزي | ملخصات وتقارير | مذكرات وبنوك | الامتحان النهائي للمدرس

المزيد من مادة
رياضيات:

إعداد: ALSaedi Fatima

التواصل الاجتماعي بحسب الصف الثاني عشر العام



صفحة المناهج
الإماراتية على
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

المواد على تلغرام

المزيد من الملفات بحسب الصف الثاني عشر العام والمادة رياضيات في الفصل الثالث

حل اختبار شامل في الوحدة السابعة الدوال والعلاقات النسبية

1

اختبار شامل في الوحدة السابعة الدوال والعلاقات النسبية

2

مراجعة الوحدة السابعة Rational functions مع أسئلة امتحانات سابقة

3

مذكرة شاملة الوحدة العاشرة الدوال المثلثية

4

مذكرة شاملة الوحدة التاسعة الدوال والعلاقات النسبية

5

Name: _____ *Salama bint Butti school*

Section: _____

Ch 7: Rational Functions

Ch 9: Trigonometric Functions



This notebook is designed to help the student save time when writing questions, organize her thoughts while studying, especially in case of absence, and she should complete exercises from the textbook after each lesson, as it is her main reference.

Weeks	Monday	Tuesday	Wednesday	Thursday	Friday
Months					
1	23	24	25	26	27
Mar					
2	30	31	1 - APRIL	2	3
MAR/APR					
3	6	7	8	9	10
APR					
4	13	14	15	16	17
APR					
5	20	21	22	23	24
APR					
6	27	28	29	30	1 - MAY
APR/MAY					
7	4	5	6	7	8
MAY					
8	11	12	13	14	15
MAY					
9	18	19	20	21	22
MAY					
10	25 to 29				
MAY	-----Midterm Break-----				
11	1	2	3	4	5
JUN					
12	8	9	10	11	12
JUN					
13	15	16	17	18	19
JUN			عطلة رأس السنة الهجرية		
14	22	23	24	25	26
JUN			Finals		
15	29	30	1	2	3
JUN-JUL					العطلة الصيفية

سجل متابعة الطالبة لمادة الرياضيات Term Plan and Progress Record

LESSONS		NOTE BOOK	ALEF	H.W	LMS	QUIZZES							
1	M7L1 Multiplying and Dividing Rational Expressions					Tier 1 Mastery	20						
							19						
							18						
2	M7L2 Adding and Subtracting Rational Expressions					Tier 2 Developing	17						
							16						
							15						
							14						
3	M7L3 Graphing Reciprocal Functions					Tier 3 Emerging	13						
							12						
4	M7L4 Graphing Rational Functions						11						
5	M7L6 Solving Rational Equations and Inequalities					Tier 4 Pre-Emerging	10						
							9						
							8						
							7						
							6						
							5						
6	M9L1 Angles and Angle Measure						4						
							3						
7	M9L2 Trigonometric Functions of General Angles						2						
							1						
							0						
8	M9L3 Circular and Periodic Functions						Q1	Q2	Q3	Q4	Q5	Q6	
9	M9L4 Graphing Sine and Cosine Functions						Quizzes Mark						
							"Student's notes about her performance and the next step"						
10	M9L5 Graphing Other Trigonometric Functions						Parent's notes and signature						

قائمة تحقق الاستعداد لحصة الرياضيات - Math Preparation Checklist



<p>٣- إحضار الأدوات اللازمة للحصة (الكتاب، الدفتر، الآلة الحاسبة، الأقلام).</p> <p>Bring all the required tools for the lesson (book, notebook, calculator, pens).</p>	<p>٢- الحرص نظافة المكان من حولك، وأن تحتوي الطاولة على مادة الرياضيات فقط.</p> <p>Ensure that the area around you and your desk is clean, and that the desk contains only math materials.</p>	<p>١- الحضور في الوقت المحدد للحصة (عدم التأخر).</p> <p>Arrive to class on time</p>
<p>٦- الاستماع جيداً الى شرح المعلمة.</p> <p>Listen carefully to the teacher's explanation.</p>	<p>٥- كتابة الملاحظات المهمة.</p> <p>Write notes carefully.</p>	<p>٤- كتابة التاريخ واليوم في اعلى كل صفحة.</p> <p>Write the date and day on each page.</p>
<p>٩- عدم تناول الطعام.</p> <p>Do not eat food during the lesson.</p>	<p>٨- يمنع منعاً باتاً الأحاديث الجانبية.</p> <p>Side conversations are strictly prohibited.</p>	<p>٧- الالتزام في حل جميع التمارين المطلوبة وتسليمها في الوقت المحدد.</p> <p>Complete all required exercises and submitting them on time.</p>

Ch 7 – Rational Functions

Learning Outcomes	Tier 1 – Advanced	Tier 2 – Proficient	Tier 3 – Emerging	Tier 4 – Pre-Emerging
<p>M7L1 Multiplying and Dividing Rational Expressions</p> <ul style="list-style-type: none"> • Simplify rational expressions. • Simplify rational expressions by multiplying and dividing. 	1. Simplify Complex Fractions.	1. Simplify rational expressions by multiplying and dividing.	1. Simplify rational expressions.	1. Simplify simple rational expressions.
<p>M7L2 Adding and Subtracting Rational Expressions</p> <ul style="list-style-type: none"> • Simplify rational expressions by adding and subtracting. • Simplify Complex Fractions. 	1. Simplify Complex Fractions.	1. Simplify rational expressions by adding and subtracting.	1. Simplify rational expressions with like denominators by adding and subtracting the numerators.	1. Multiply simple expressions.
<p>M7L3 Graphing Reciprocal Functions</p> <ul style="list-style-type: none"> • Graph reciprocal functions by making tables of values. • Graph and write reciprocal function by using transformations. 	1. Write a Reciprocal Function from a Graph.	1. Graph reciprocal functions by making tables of values. 2. Graph and write reciprocal function by using transformations.	1. Limitations on the Domains of Reciprocal Functions.	1. Plot ordered pairs on the coordinate plane.
<p>M7L4 Graphing Rational Functions</p> <ul style="list-style-type: none"> • Graph and analyze rational functions with vertical and horizontal asymptotes. • Graph and analyze rational functions with oblique asymptotes. 	1. Graph and analyze rational functions with oblique asymptotes.	1. Graph and analyze rational functions with vertical and horizontal asymptotes.	1. Identify vertical asymptote and sketch basic graphs.	1. Solve simple equations.
<p>M7L6 Solving Rational Equations and Inequalities</p> <ul style="list-style-type: none"> • Solve rational equations in one variable. • Solve rational inequalities in one variable. 	1. Solve rational inequalities in one variable.	1. Solve rational equations in one variable.	1. Solve simple rational equations with one rational term.	1. Find the least common denominator.

M7L1 Multiplying and Dividing Rational Expressions

Outcomes:

1. Simplify rational expressions.
2. Simplify rational expressions by multiplying and dividing.

Keywords:

Rational equation - Rational inequality.

Starter

Simplify simple expressions.

Pre-Emerging

a. $f(x) = \frac{2x}{x}$

b. $f(x) = \frac{2x^2}{x}$

c. $f(x) = \frac{2x^2}{x^3}$

d. $f(x) = 2x^2 + x$

e. $f(x) = 2x^2 + 2x$

f. $f(x) = 2x^3 + 4x$

Learn Simplifying Rational Expressions: A rational expression is a ratio of two polynomials.

Example 1: Simplify a Rational Expression.

Simplify each expression and state when the original expression is undefined.

Emerging

2. $\frac{y^2(y+1)(y+2)}{2y(y-4)(y+2)}$

Developing

$$\frac{x^2 - 2x - 24}{2x^3 + 6x^2 - 8x}$$

Check my understanding 1:-

Simplify each expression and state when the original expression is undefined.

1. $\frac{x(x-3)(x+6)}{(x+4)(x-3)}$

$$\frac{x^2 + 2x + 1}{4x^2 + 3x - 1}$$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1
1									
Next step توجيهات للتحسين (Improvement Guidance)									

(Developing) Example 2: Simplify by Using -1.

Simplify $\frac{(6x^2 - 5xy)(x + 2y)}{(x + y)(5y - 6x)}$

7. Simplify $\frac{x^2 - 5x - 14}{28 + 3x - x^2}$

Check my understanding 2:-

Select the simplified form of $\frac{(7y - 3x)(5x - 1)}{(5x^3 + x^2)(3x - 7y)}$.

- A. $-\frac{5x - 1}{x^2(5x + 1)}$
- B. $\frac{(7y - 3x)(5x - 1)}{x^2(5x + 1)(3x - 7y)}$
- C. $\frac{1}{x^2}$
- D. $\frac{5x - 1}{5x^3 + x^2}$

(Developing) Example 3: Multiply and Divide Rational Expressions.

Simplify each expression.

a. $\frac{3x}{8y} \cdot \frac{12x^2y}{9xy^3}$

b. $\frac{10d^5}{6cd} \div \frac{30c^3d^2}{4c}$

Check my understanding 3: Simplify each expression.

15. $\frac{15a^2b^2}{21ac} \cdot \frac{14a^4c^2}{6ab^3}$

14. $\frac{9x^2yz}{5z^4} \div \frac{12x^4y^2}{50xy^4z^2}$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

(Developing) Example 4: Multiply and Divide Polynomial Expressions.

GEOMETRY A manufacturer that creates and sells rectangular planters wants to compare the areas of the top sections of two potential sizes of planters. For some given measure x , the area for the first planter is represented by $\frac{x^2+15x+50}{x+2}$, and the area for the second planter is represented by $\frac{x^2+30x+200}{x+2}$. Write and simplify an expression that represents the ratio of the area for the first planter to that of the second.

Check my understanding 4:-

17. BEAUTY A producer of beauty care products wants to compare the areas of two face cream containers. For some given measure x , the area for the first container is represented by $\frac{16a^2+40a+25}{3a^2-10a-8}$. The area for the second container is represented by $\frac{4a+5}{a^2-8a+16}$. Write and simplify an expression that represents the ratio of the area for the first container to the are of the second container.

(Mastery) Example 5: Simplify Complex Fractions

Simplify
$$\frac{\frac{3x}{x-y}}{\frac{6xy}{4x^2-4y^2}}$$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لرحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

Check my understanding 5: Simplify $\frac{\frac{x^2 - 9y^2}{xy}}{\frac{2x + 6y}{x^2}}$

Academic support (1) – Extra Practice:-

Simplify each expression and state when the original expression is undefined.

4. $\frac{(x^2 - 16x + 64)(x + 2)}{(x^2 - 64)(x^2 - 6x - 16)}$

8. $\frac{9x^2 - x^3}{x^2 - 3x - 54}$

12. $\frac{14xy^2z^3}{21w^4x^2z} \cdot \frac{7wxyz}{12w^2y^3z}$

13. $\frac{64a^2b^5}{35b^2c^3f^4} \div \frac{12a^4b^3c}{70abcf^2}$

22. $\frac{\frac{a^2 - b^2}{b^3}}{\frac{b^2 - ab}{a^2}}$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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M7L2 Adding and Subtracting Rational Expressions

Outcomes:

1. Simplify rational expressions by adding and subtracting.
2. Simplify complex fractions.

Keywords:

Complex fractions.

Starter

Pre-
Emerging

Simplify expressions.

a. $4b \cdot 10$

b. $9y \cdot 6z$

Learn Adding and Subtracting Rational Expressions

Key Concepts:

LCM

The least common factor of two or more numbers is the smallest number that is completely divisible by those numbers. Like :- LCM of 4, 8, 12 = $2 \times 2 \times 2 \times 3 = 24$, Therefore, the required least common factor 24 is the smallest number which is completely divisible by 4, 8, 12.

Adding Rational Expressions

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$$

Subtracting Rational Expressions

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - cb}{bd}$$

Example 1: Add and Subtract Rational Expressions with Monomial Denominators.

Simplify each expression.

Emerging

a. $\frac{2}{m^2p} + \frac{5}{m^2p}$

b. $\frac{12}{5y^2} - \frac{2}{5y^2}$

Developing

a. $\frac{7a}{4b} + \frac{4c^2}{10}$

b. $\frac{2x}{9y} - \frac{7y}{6z}$

Check my understanding 1: Simplify each expression.

1. $\frac{8}{3xy} - \frac{2}{3xy}$

2. $\frac{3}{8p^2r} + \frac{5}{4p^2r}$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

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Next step توجيهات للتحسين (Improvement Guidance)									

(Developing) Example 2: Add and Subtract Rational Expressions with Polynomial Denominators.

Simplify each expression.

$$\frac{2x + 1}{x^2 + 2x - 15} - \frac{7}{5x - 15}$$

$$\frac{3x}{4x^2 + 24} - \frac{2x^2}{x^4 - 1}$$

(Developing) Example 3: Use Addition and Subtraction of Rational Expressions.

PRODUCTION The rate at which some oil wells pump oil in thousands of barrels, given x years of pumping, can be given by $\frac{20}{x} + \frac{200x}{3x^2 + 20}$. Simplify the expression.

Check my understanding 2: Simplify each expression

8. $\frac{3t}{2-x} + \frac{5}{x-2}$

11. $\frac{n}{n-3} + \frac{2n+2}{n^2-2n-3}$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لـحل المسائل الرياضية/10

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Next step توجيهات للتحسين (Improvement Guidance)									

Learn Simplifying Complex Fractions

Complex fractions can be simplified by simplifying the numerator and denominator separately and then simplifying the resulting expression. You can also simplify a complex fraction by finding the LCD of all of the denominators. Then, the denominators can all be eliminated by multiplying by the LCD.

(Mastery) Example 4: Simplify Complex Fractions by Using Different LCDs.

Simplify $\frac{\frac{x}{y} + 1}{\frac{y}{x} - \frac{1}{y}}$

Check my understanding 3: Simplify $\frac{\frac{x}{xy} - 2}{2xy - \frac{3}{y}}$

A. $\frac{2xy^2 - 4xy + 6y - 3}{y^2}$

B. $\frac{1 - 2y}{2xy^2 - 3}$

C. $\frac{4xy^3 - 2xy^2 + 6y + 3}{6xy^2}$

D. 0

(Mastery) Example 5: Simplify Complex Fractions by Using the Same LCD.

Simplify $\frac{\frac{x}{y} + \frac{2}{x}}{\frac{x}{2} - \frac{y}{x}}$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

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Next step توجيهات للتحسين (Improvement Guidance)									

Check my understanding 4: Simplify $\frac{1 + \frac{b^2}{a}}{\frac{a}{2} - \frac{a}{b}}$

A. $\frac{2ab+2b^3}{a^2b-2a^2}$

B. $\frac{ab-2a+b^3-2b^2}{2b}$

C. $-\frac{ab-2a+b^3-2b^2}{a^2}$

D. $\frac{2+b^3}{a-a^2}$

Mathematics Problem-Solving Feedback Rubric روبرك التغذية الراجعة لحل المسائل الرياضية /10			
Step by step الترتيب بخطوة	1 متقدم (Advanced) 3	2 متوسط (Intermediate) 2	3 مبتدئ (Beginner) 1
فهم المسألة (Understanding the Problem)			
تنفيذ الحل (Implementing the Solution)			
التحقق من الحل وتفسيره (Verifying and Interpreting the Solution)			
التوجيهات التحسين (Improvement Guidance) → Next step			

Academic support (2) – Extra Practice:-

Simplify each expression.

6. $\frac{7}{4gh} + \frac{3}{4h^2}$

5. $\frac{12}{5y^2} - \frac{2}{5yz}$

7. $\frac{3}{w-3} - \frac{2}{w^2-9}$

Challenging question

16. $\frac{\frac{2}{x-3} + \frac{3x}{x^2-9}}{\frac{3}{x+3} - \frac{4x}{x^2-9}}$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية |/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

M7L3 Graphing Reciprocal Functions

Outcomes:

1. Graph reciprocal functions by making tables of values.
2. Graph and write reciprocal functions by using transformations.

Keywords:

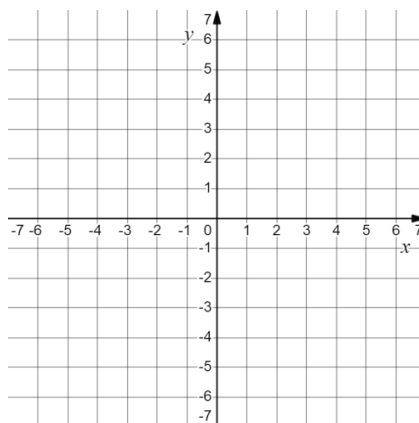
Reciprocal function – Vertical asymptote – Horizontal asymptote – Hyperbola – Excluded values.

Starter

Pre-Emerging

Plot ordered pairs on the coordinate plane.

1. (0, 2)
2. (-1, 5)
3. (-6, -3)
4. (3, 5)



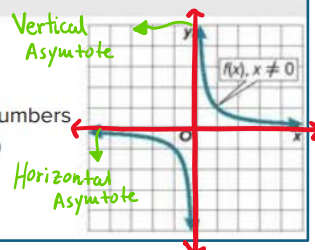
Learn Graphing Reciprocal Functions

A **reciprocal function** has an equation of the form $f(x) = \frac{n}{b(x)}$, where n is a real number and b is a linear expression that cannot equal 0.

A reciprocal function has two asymptotes, which are lines that a graph approaches. The **vertical asymptote** is determined by the excluded value of x , and the **horizontal asymptote** is determined by the value that is undefined for $f(x)$.

Key Concept • Reciprocal Functions

Parent function	$f(x) = \frac{1}{x}$
Type of graph	hyperbola
Domain and range	all nonzero real numbers
Asymptotes	$x = 0$ and $f(x) = 0$
Intercepts	none
Not defined	$x = 0$



Horizontal asymptote: For $f(x) = \frac{n}{b(x)}$, the horizontal asymptote is $f(x) = 0$. For a reciprocal function of the form $f(x) = \frac{n}{b(x)} + k$, where k is a constant, the horizontal asymptote is $f(x) = k$.

Vertical asymptote: For $f(x) = \frac{n}{b(x)}$, the vertical asymptote is $b(x) = 0$. For example, $f(x) = \frac{1}{x}$, the vertical asymptote is $x = 0$.

(Emerging) Example 1: Limitations on the Domains of Reciprocal Functions.

Determine the excluded value of x for each function.

a. $g(x) = \frac{6}{x}$

b. $g(x) = \frac{2}{x-7}$

c. $g(x) = \frac{-5}{3x+4}$

Check my understanding 1: Determine the excluded value of x for each function.

1. $f(x) = \frac{5}{x}$

2. $g(x) = \frac{-2}{x+2}$

5. $f(x) = \frac{5}{2x+3}$

(Developing) Example 2: Graph a Reciprocal Function by Using a Table.

Consider $g(x) = \frac{1}{2x-5} + 2$, identify the key features and graph the function.

1. Vertical asymptote:

2. Domain:

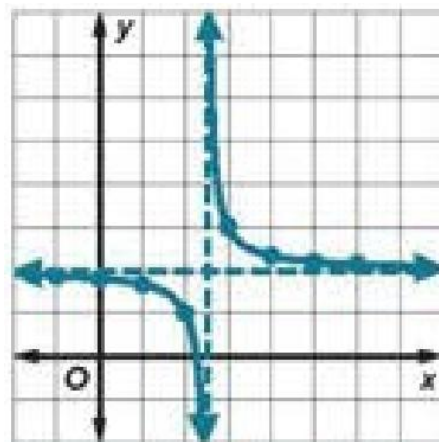
3. Horizontal asymptote:

4. Range:

5. X-intercept:

6. Y-intercept:

7. Table then graph:



Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لـحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying and Interpreting the Solution)		
	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1
1									
Next step توجيهات للتحسين (Improvement Guidance)									

Check my understanding 2:-

8. Consider $f(x) = -\frac{1}{x} + 4$, identify the key features and graph the function.

1. Vertical asymptote:

2. Domain:

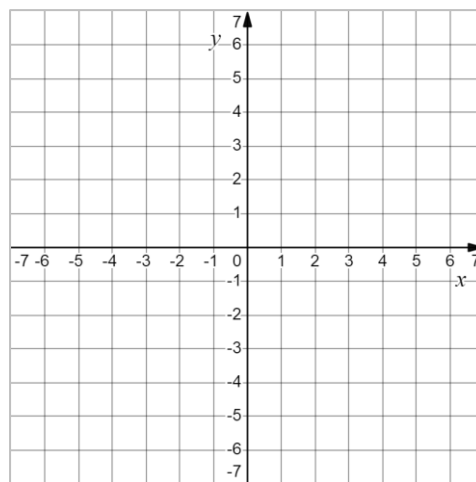
3. Horizontal asymptote:

4. Range:

5. X-intercept:

6. Y-intercept:

7. Table then graph:

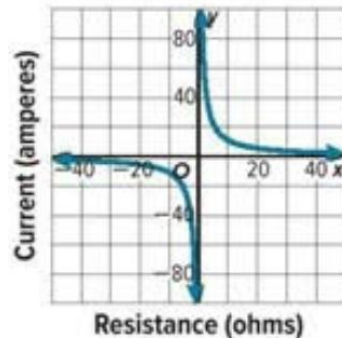


Mathematics Problem-Solving Feedback Rubric روبرك التغذية الراجعة لحل المسائل الرياضية								/10
الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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Next step توجيهات للتحسين (Improvement Guidance)									

(Mastery) Example 3: Graph a Reciprocal Function by Using a Table.

ELECTRICITY Ohm's law states that electric circuits operate according to the equation $I = \frac{V}{R}$ where I is the electrical current in amperes, V is the electromotive force in volts, and R is the resistance of the circuit in ohms. Analyze the electrical current of a 110-volt curling iron in relation to the resistance of its circuit.

- domain:
- range:
- intercepts:
- positive:
- negative:
- symmetry:
- end behavior:

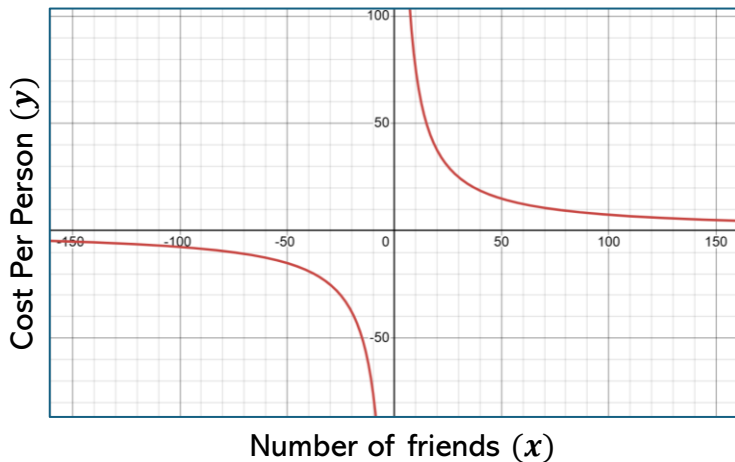


Check my understanding 3:

CABIN RENTAL A group of friends plans to rent a luxury apartment in Burj Khalifa. The apartment costs 750 AED to rent for the weekend. Let x be the number of friends who are sharing the cost and let y be the cost for each person. Graph a function representing the average cost per person to rent the apartment. Analyze the key features of the graph.

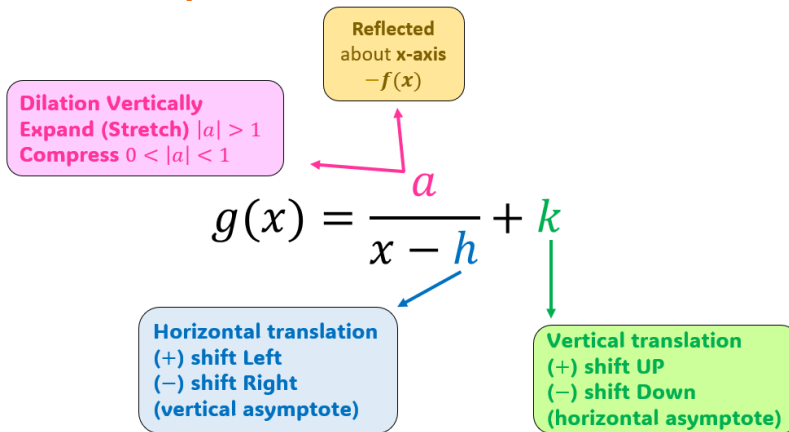
$$f(x) = \frac{750}{x}$$

- domain:
- range:
- intercepts:
- positive:
- negative:
- symmetry:
- end behavior:



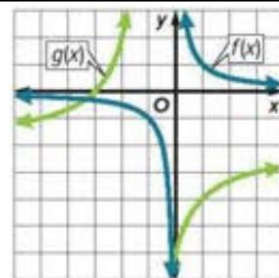
Mathematics Problem-Solving Feedback Rubric روبرك التغذية الراجعة لحل المسائل الرياضية								/10
الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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Next step توجيهات للتحسين (Improvement Guidance)									

Learn Transformations of Reciprocal Functions



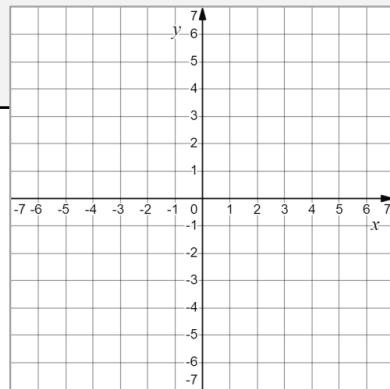
(Developing) Example 4: Graph a Transformation of a Reciprocal Function.

Graph $g(x) = \frac{-4}{x+1} - 2$. State the domain and range.



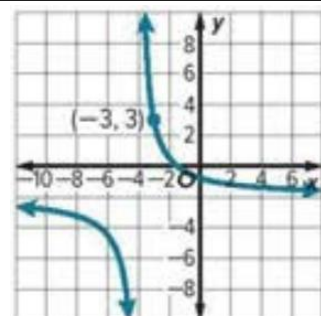
Check my understanding 4:

Graph $g(x) = \frac{1}{x-1} + 3$. State the domain and range.



(Mastery) Example 5: Write a Reciprocal Function from a Graph.

Identify the values of a , h and k . Then write a function for the graph $g(x) = \frac{a}{x-h} + k$.

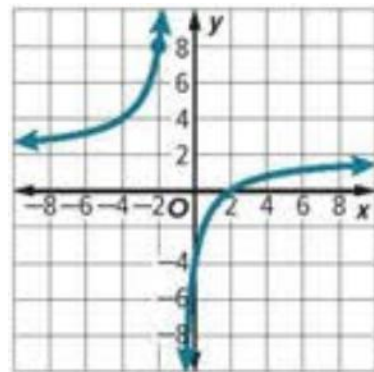


Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لـحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

Check my understanding 5:

Identify the values of a , h and k . Then write a function for the graph $g(x) = \frac{a}{x-h} + k$.



Academic support (3) – Extra Practice:-

Determine the excluded value of x for each function.

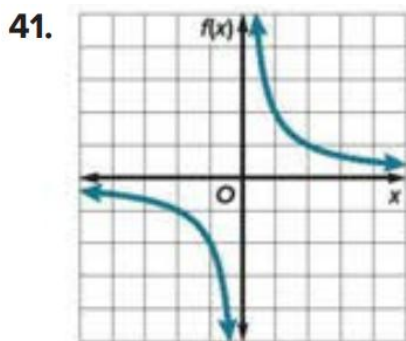
3. $f(x) = \frac{10}{x-3}$

6. $g(x) = \frac{5}{7x-9}$

Identify key features: vertical, horizontal asymptote, domain and range, then describe the transformation for each function.

10. $f(x) = \frac{6}{x} - 3$

Identify the values of a , h and k . Then write a function for the graph $g(x) = \frac{a}{x-h} + k$.



Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لـحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

M7L4 Graphing Rational Functions

Outcomes:

1. Graph and analyze rational functions with vertical and horizontal asymptotes.
2. Graph and analyze rational functions with oblique asymptotes.

Keywords:

Rational function – Oblique asymptote – Point discontinuity.

Starter

Pre-
Emerging

Solve each equation.

1. $x + 3 = 0$

2. $x^3 = 0$

3. $4x = 2$

4. $3x + 1 = 0$

Learn Graphing Rational Functions with Vertical and Horizontal Asymptotes

A rational function has an equation of the form $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions and $b(x) \neq 0$.

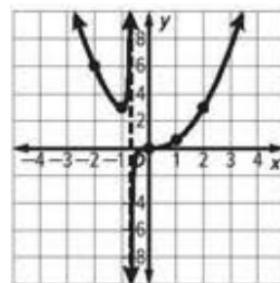
Key Concept • Vertical and Horizontal Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

- $f(x)$ has a vertical asymptote whenever $b(x) = 0$.
- $f(x)$ has at most one horizontal asymptote.
 - If the degree of $a(x)$ is greater than the degree of $b(x)$, there is no horizontal asymptote.
 - If the degree of $a(x)$ is less than the degree of $b(x)$, the horizontal asymptote is the line $y = 0$.
 - If the degree of $a(x)$ equals the degree of $b(x)$, the horizontal asymptote is the line $y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}$.

(Emerging - Developing) Example 1: Graph with No Horizontal Asymptotes.

Graph $f(x) = \frac{x^3}{x + \frac{2}{3}}$



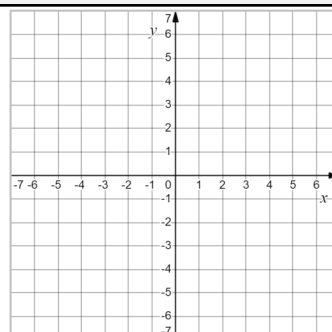
Check my understanding 1:-

Consider $g(x) = \frac{(0.5x - 1)^3}{x}$.

Part A There is a zero at $x = \underline{\quad? \quad}$.

Part B There is a vertical asymptote at $x = \underline{\quad? \quad}$.

Part C Graph the function.



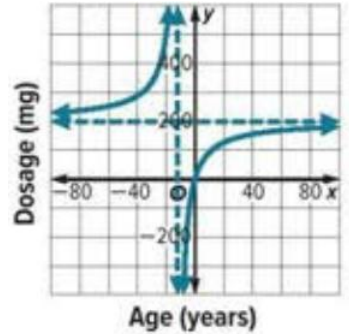
Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

(Developing) Example 2: Use Graphs of Rational Functions.

PHARMACY Young’s Rule can be used to estimate the dosage of medicine to give a child if you know the adult dosage of the same medicine. Young’s rule can be written as $y = \frac{Ax}{x+12}$, where A is the adult dosage in milligrams 200mg and x is the age of the child in years.

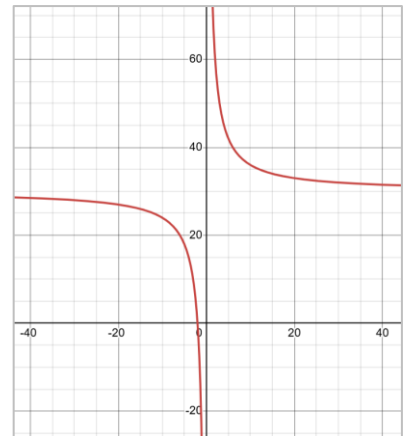
- Find vertical asymptote: _____.
- Find horizontal asymptote: _____.
- Find x- intercept: _____.
- Find y-intercept: _____.
- Find the dosage for a 12-year-old child if the adult dosage is 200mg.
_____.



Check my understanding 2:

5. INTERNET An Internet service provider charges customers a 60AED installation fee plus 30AED per month for Internet service. A function that models the average monthly cost is $f(x) = \frac{60+30x}{x}$, where x is the number of months.

- Find vertical asymptote: _____.
- Find horizontal asymptote: _____.
- Find x- intercept: _____.
- Find y-intercept: _____.
- Find the average monthly cost to a customer that has Internet service for 8 months.
_____.



Critical Thinking Question:

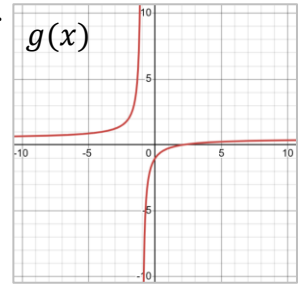
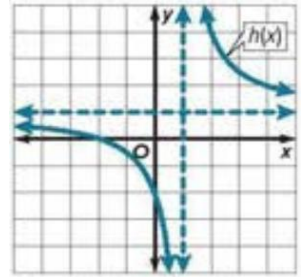
REGULARITY Describe how you use the degrees of the numerator and denominator to learn about a function’s asymptotes.

Mathematics Problem-Solving Feedback Rubric روبرك التغذية الراجعة لحل المسائل الرياضية								/10
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1	3	2	1	3	2	1	3	2	1
Next step توجيهات للتحسين (Improvement Guidance)									

(Developing - Mastery) Example 3: Compare Rational Functions.

Consider $g(x) = \frac{x-2}{2x+2}$, and rational function $h(x)$ shown in the graph.

- Find the zeros of $g(x)$: _____.
- Find vertical asymptote of $g(x)$: _____.
- Find horizontal asymptote of $g(x)$: _____.
- Which function has the greater y-intercept? _____.
- Compare the asymptotes of $g(x)$ and $h(x)$:

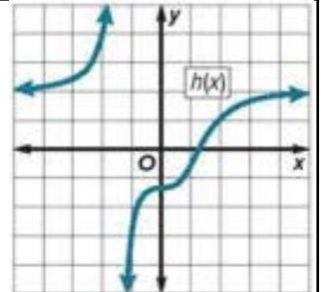
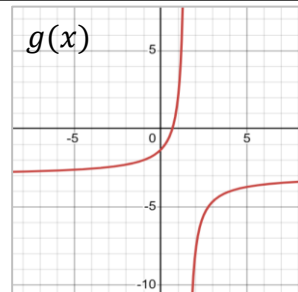


Check my understanding 3:

Consider $g(x) = \frac{-6x+4}{2x-3}$ and the graph of $h(x)$. Which function has the higher horizontal asymptote?

Which function has the higher horizontal asymptote?

- _____ has the higher horizontal asymptote.



Learn Graphing Rational Functions with Oblique Asymptotes

An oblique asymptote, or slant asymptote, is neither horizontal nor vertical.

Key Concept • Oblique Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1 and $b(x) \neq 0$, then $f(x)$ has an oblique asymptote if the degree of $a(x)$ minus the degree of $b(x)$ equals 1.

The equation of the asymptote is $f(x) = \frac{a(x)}{b(x)}$ with no remainder.

In some cases, graphs of rational functions may have point discontinuity, which looks like a hole in the graph. This is because the function is undefined at that point. If the original function is undefined for $x = a$ but the related rational expression of the function in simplest form is defined for $x = a$, then there is a point discontinuity or hole in the graph at $x = a$.

Key Concept • Point Discontinuity

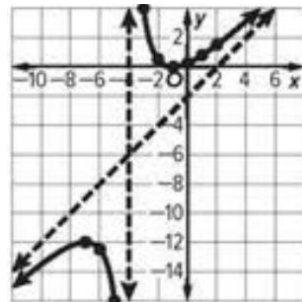
If $f(x) = \frac{a(x)}{b(x)}$, $b(x) \neq 0$, and $x - c$ is a factor of both $a(x)$ and $b(x)$, then there is a point discontinuity at $x = c$.

Mathematics Problem-Solving Feedback Rubric روبرك التغذية الراجعة لحل المسائل الرياضية								/10
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Next step توجيهات للتحسين (Improvement Guidance)									

(Mastery) Example 4: Graph with Oblique Asymptotes.

Consider $f(x) = \frac{x^2+2x+1}{x+4}$.

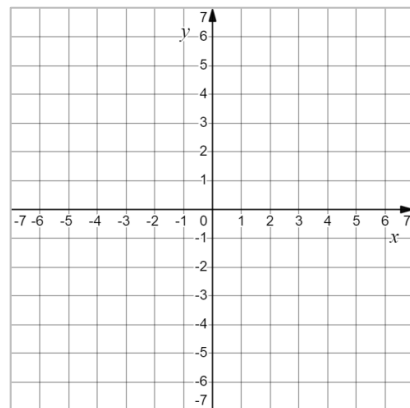
- Find the zeros of $f(x)$: _____.
- Find vertical asymptote of $f(x)$: _____.
- Find horizontal asymptote of $f(x)$: _____.
- Find oblique asymptote of $f(x)$: _____.



Check my understanding 4:

Find the asymptotes of $f(x) = \frac{2x^2+3x+2}{2x+1}$

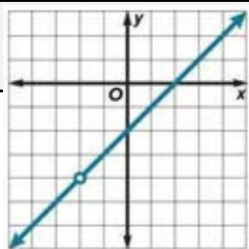
- Find the zeros of $f(x)$: _____.
- Find vertical asymptote of $f(x)$: _____.
- Find oblique asymptote of $f(x)$: _____.



(Emerging) Example 5: Graph with Oblique Asymptotes.

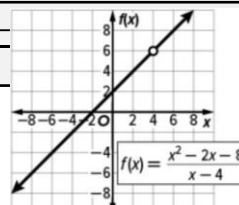
Graph each function. Find the point discontinuity.

$f(x) = \frac{x^2 - 4}{x + 2}$



21. $f(x) = \frac{(x-4)(x^2-4)}{x^2-6x+8}$

Check my understanding 5: 17. Find the point discontinuity of $f(x) = \frac{x^2-2x-8}{x-4}$

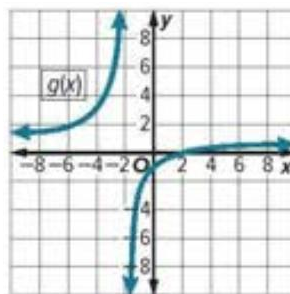
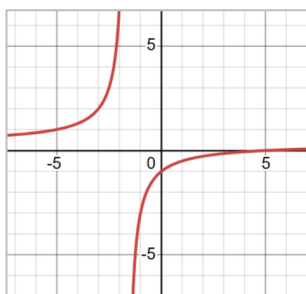


Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية

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1									
Next step توجيهات للتحسين (Improvement Guidance)									

Academic support (4) – Extra Practice:-

8. Consider $f(x) = \frac{x-5}{3x+5}$, and rational function $g(x)$ shown in the graph.



Which function has the greater y-intercept? _____.

- Compare the asymptotes of $f(x)$ and $g(x)$:

14. Consider $f(x) = \frac{2x^2+7x}{x-2}$, Find the following.

- Find the zeros of $f(x)$: _____.

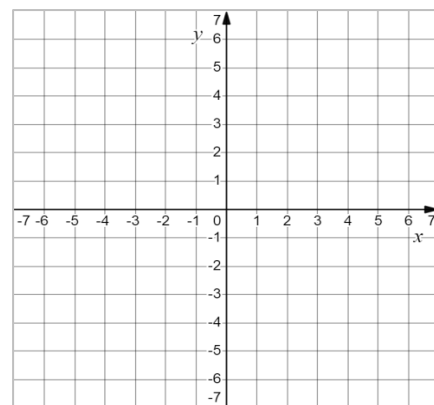
- Find x-intercept: _____.

- Find y-intercept: _____.

- Find vertical asymptote of $f(x)$: _____.

- Find horizontal asymptote of $f(x)$: _____.

- Find oblique asymptote of $f(x)$: _____



18. Find the point of discontinuity of $f(x) = \frac{x^2+4x-12}{x-2}$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

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1									
Next step توجيهات للتحسين (Improvement Guidance)									

M7L6 Solving Rational Equations and Inequalities

Outcomes:

1. Solve rational equations in one variable.
2. Solve rational inequalities in one variable.

Keywords:

Rational equation – Rational inequality.

Starter

Pre-Emerging

Find the least common denominator.

1. between 2 and 7.

2. between 12 and 48.

3. between $\frac{4}{5}$ and $\frac{1}{3}$.

Learn Solving Rational Equations

A rational equation contains at least one rational expression. To solve these equations, it is often easier to first eliminate the fractions. You can eliminate the fractions by multiplying each side of the equation by the least common denominator (LCD). Solving rational equations in this way can yield results that are not solutions of the original equation. You can identify these extraneous solutions by substituting each result into the original equation to see if it makes the equation true.

Example 1: Solve a Rational Equation.

Solve each equation.

Emerging

$$\frac{9}{x} = \frac{12}{5}$$

Developing

$$\frac{7}{12} + \frac{9}{x-4} = \frac{55}{48}$$

Check my understanding 1:

Solve each equation.

1. $\frac{2x+3}{x+1} = \frac{3}{2}$

$$\frac{5}{6} - \frac{2}{4x+1} = \frac{x}{3}$$

- | | |
|--------------------|-------------------|
| A. $-\frac{67}{4}$ | B. $-\frac{1}{4}$ |
| C. $\frac{1}{4}$ | D. $\frac{1}{2}$ |
| E. $\frac{7}{4}$ | F. 3 |

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

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1									
Next step توجيهات للتحسين (Improvement Guidance)									

(Developing) Example 2: Solve a Rational Equation with an Extraneous Solution.

Solve each equation.

9. $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$

10. $\frac{12p+12}{p^2+7p+12} - \frac{3}{p+3} = \frac{5}{p+4}$

$\frac{2m}{m-4} - \frac{m^2+7m+4}{3m^2-18m+24} = \frac{4m}{3m-6}$

Check my understanding 2:

6. $\frac{2}{a+2} + \frac{10}{a+5} = \frac{36}{a^2+7a+10}$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1
1									
Next step توجيهات للتحسين (Improvement Guidance)									

(Mastery) Example 3: Mixture Problem.

FIRST AID Rubbing alcohol, a commonly used first aid antiseptic, typically contains 70% isopropyl alcohol. Suppose you are adding a 50% isopropyl alcohol liquid to 200 milliliters of a liquid that is 91% isopropyl alcohol. How much of a 50% isopropyl alcohol liquid should be added to create rubbing alcohol that is 70% isopropyl alcohol?

$$\text{Percentage of isopropyl alcohol in liquid} = \frac{\text{amount of isopropyl alcohol}}{\text{total amount of liquid}}$$

(Mastery) Example 4: Distance Problem.

BATS Mexican free-tailed bats have an average flight speed of 25 miles per hour. Suppose it takes one of the bats 5 hours to fly 121.8 miles round trip one night. Assuming that the bat flew at a constant speed, determine the speed of the wind.

$$t = \frac{d}{r}$$

(Mastery) Example 5: Work Problem.

AGRICULTURE If it takes a 24-row planter 10 hours to plant a field and 6 hours if a 16-row planter is also used, how long would it take to plant the field if only the 16-row planter were used?

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

Check my understanding 3:

13. SOLUTION Evita adds a 75% acid solution to 8 milliliters of solution that is 15% acid.

a. Write a function that represents the percent of acid in the resulting solution, where x is the amount of 75% acid solution added.

b. How much 75% acid solution should be added to create a solution that is 50% acid?

Check my understanding 4:

14. FLIGHT TIME The distance between John F. Kennedy International Airport and Los Angeles International Airport is about 2500 miles. Let S be the airspeed of a jet, which is the speed at which the jet would be traveling in still air. The ground speed, which indicates how fast the jet is traveling over the land surface, is affected by the airspeed and the wind. Traveling in the same direction as the wind increases the ground speed; and traveling in the opposite direction decreases ground speed.

a. Suppose the current wind speed is 100 miles per hour at the altitude at which the jet will fly. Write an equation for S if it takes 2 hours and 5 minutes longer to fly between New York and Los Angeles against the wind versus flying with the wind.

b. Solve the equation in part a for S .

Check my understanding 5:

15. CONSTRUCTION It takes Rosita 32 hours to drywall a basement by herself and 18 hours if Paola helps her. How long would it take Paola to drywall the basement by herself? Round your answer to the nearest hour.

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

Academic support (5) – Extra Practice:-

Solve each equation.

2. $\frac{-12}{y} = y - 7$

3. $\frac{9}{x-7} - \frac{7}{x-6} = \frac{13}{x^2 - 13x + 42}$

7. $\frac{x}{2x-1} + \frac{3}{x+4} = \frac{21}{2x^2 + 7x - 4}$

12. $\frac{8}{t^2 - 9} + \frac{4}{t + 3} = 1$

Mathematics Problem-Solving Feedback Rubric روبرك التغذية الراجعة لحل المسائل الرياضية									التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)			
الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)					
1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1			
Next step توجيهات للتحسين (Improvement Guidance)										27		

Ch 9 – Trigonometric Functions

Learning Outcomes	Tier 1 – Advanced	Tier 2 – Proficient	Tier 3 – Emerging	Tier 4 – Pre-Emerging
M9L1 – Angles and Angle Measure <ul style="list-style-type: none"> • Draw angles in standard position and identify coterminal angles. • Convert between degrees and radian measures and find arc lengths by using central angles. 	1. Convert between degree and radian measures and apply them to basic arc length problems.	1. Draw an Angle with More Than One Rotation.	1. Draw angles in standard position on the coordinate plane. 2. Find and write positive and negative coterminal angles. 3. Convert angle measures between degrees and radians.	1. Identify the vertex, initial side, and terminal side of an angle. 2. Recognize degree and radian as units used to measure angles.
M9L2 – Trigonometric Functions of General Angles <ul style="list-style-type: none"> • Find values of trigonometric functions for acute angles. • Find values of trigonometric functions of general angles. • Find values of trigonometric functions by using reference angles. 	1. Use Trigonometric Functions.	1. Find values of trigonometric functions of general angles. 2. Find values of trigonometric functions by using reference angles.	1. Find values of trigonometric functions for acute angles.	1. Use the Pythagorean Theorem to find the value of r .
M9L3 – Circular and Periodic Functions <ul style="list-style-type: none"> • Find values of trigonometric functions given a point on a unit circle or the measure of a special angle. • Find values of trigonometric functions that model periodic events. 	1. Find values of trigonometric functions that model periodic events.	1. Find Trigonometric Values of Special Angles. 2. Identify the Period of a Function.	1. Find Sine and Cosine Given a Point on the Unit Circle.	1. Find the exact value of each trigonometric function.
M9L4 – Graphing Sine and Cosine Functions <ul style="list-style-type: none"> • Graph and analyze sine and cosine functions. • Model periodic real-world situations with sine and cosine functions. 	1. Model periodic real-world situations with sine and cosine functions.	1. Graph and analyze sine and cosine functions.	1. Identify the Amplitude, Midline, and Period of a Graph.	1. Complete the table of $y = \sin x$.
M9L5 – Graphing Other Trigonometric Functions <ul style="list-style-type: none"> • Graph and analyze tangent functions. • Graph and analyze reciprocal trigonometric functions. 	1. Graph and analyze reciprocal trigonometric functions.	1. Graph a Tangent Function with a Dilation and a Reflection.	1. Find Domain, Range, Amplitude, Period, x-Intercepts, and Midline of a basic tangent function.	1. Determine the vertical asymptote(s) for each function.

M9L1 Angles and Angle Measure

Outcomes:

1. Draw angles in standard position and identify coterminal angles.
2. Convert between degrees and radian measures and find arc lengths by using central angles.

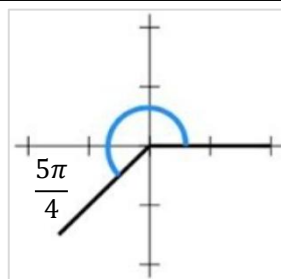
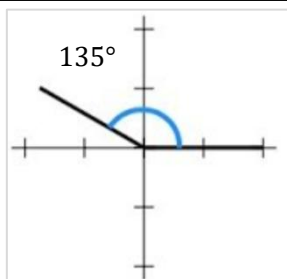
Keywords:

Standard position – Initial side – Terminal side – Coterminal angles – Radian – Central angle of a circle.

Starter

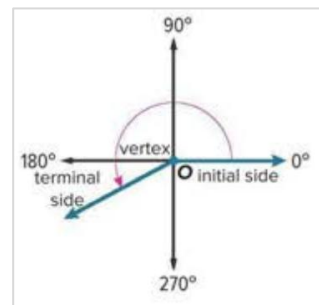
Pre-Emerging

Identify the vertex, initial side, and terminal side of the angle. What unit is used to measure this angle?



Learn Angles in Standard Position

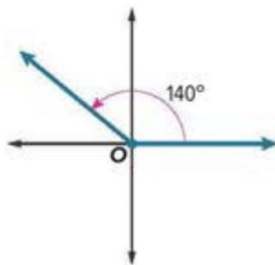
- An angle on the coordinate plane is in standard position if the vertex is at the origin and one ray is on the positive x -axis. The initial side of the angle is the ray that is fixed on the x -axis. The terminal side is the ray that rotates about the center.
- An angle in standard position is measured by the amount of rotation in degrees from the initial side to the terminal side. Angle measures can be positive or negative.



Key Concept – Angle Measures

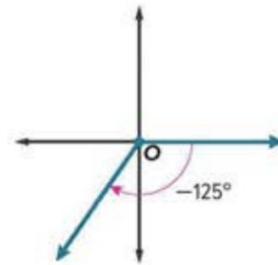
Positive Angle Measures

If the measure of an angle is positive, the terminal side is rotated counterclockwise.



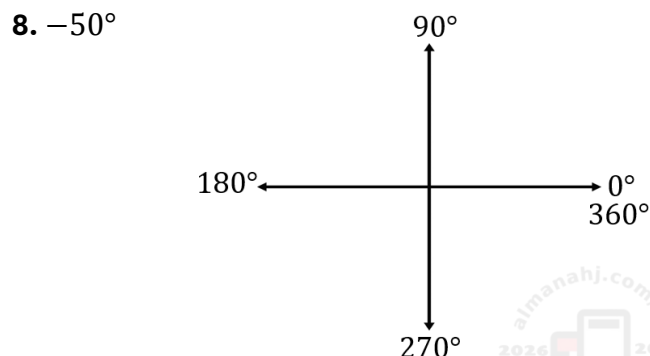
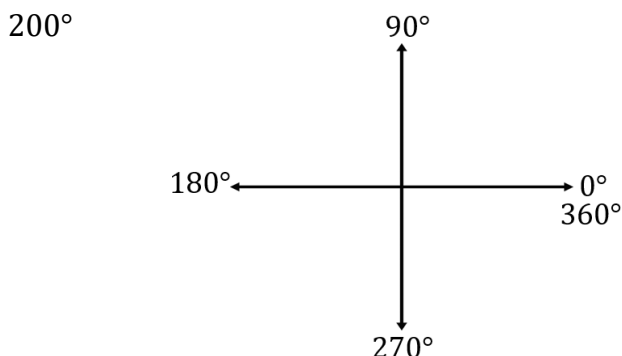
Negative Angle Measures

If the measure of an angle is negative, the terminal side is rotated clockwise.



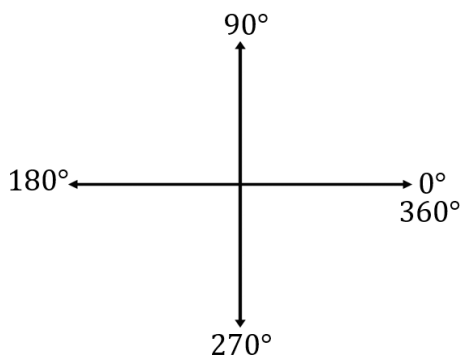
(Emerging) Example 1: Draw an Angle in Standard Position.

Draw an angle with the given measure in standard position.

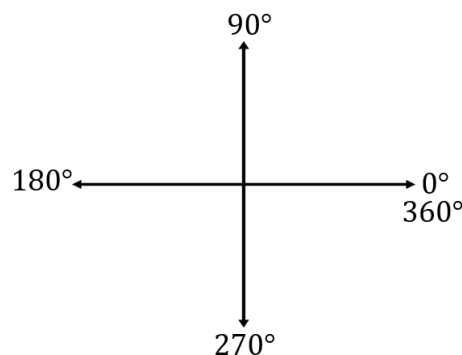


Check my understanding 1: Draw an angle with the given measure in standard position.

13. 135°



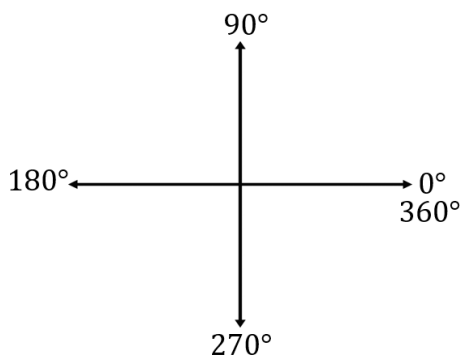
-200°



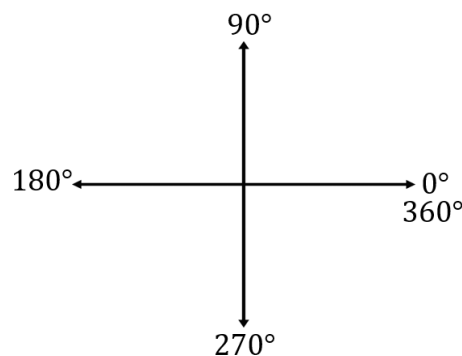
(Developing) Example 2: Draw an Angle with More Than One Rotation.

Draw an angle with the given measure in standard position.

475°

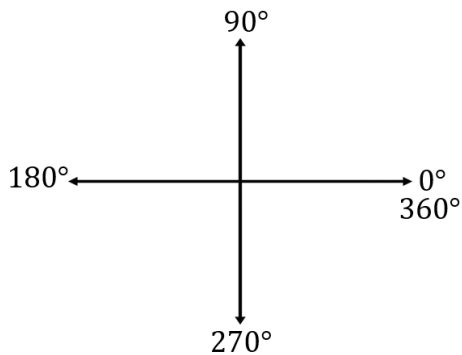


15. -560°

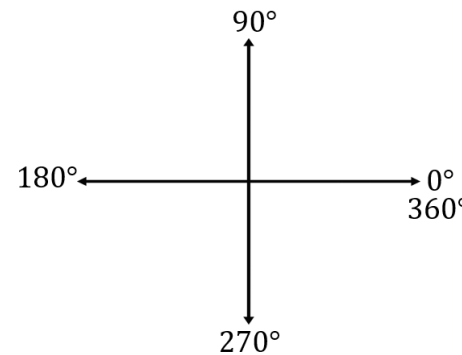


Check my understanding 2: Draw an angle with the given measure in standard position.

6. 390°



14. -450°



(Emerging) Example 3: Identify Coterminal Angles.

Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.

35°

30. $-\frac{3\pi}{2}$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

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1									
Next step توجيهات للتحسين (Improvement Guidance)									

Check my understanding 3: Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.

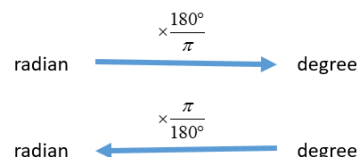
17. -75°

28. $\frac{2\pi}{5}$

Learn Degrees and Radians

- Angles can also be measured in units called radians, which are based on arc length. A radian is the measure of an angle θ in standard position with a terminal side that intercepts an arc with the same length as the radius of the circle. The circumference of a circle is $2\pi r$ and an angle that measures 1 radian has an arc length of r . So, one complete revolution around a circle equals 2π radians.

Radian-Degree Conversion



(Emerging) Example 4: Convert Degrees to Radians.

Rewrite -100° in radians.

(Emerging) Example 5: Convert Radians to Degrees.

Rewrite $\frac{11\pi}{4}$ in radians.

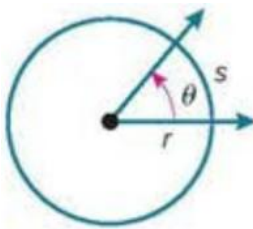
Check my understanding 4: Rewrite each degree measure in radians and each radian measure in degrees.

40. 210°

44. $\frac{7\pi}{6}$

A central angle of a circle has its vertex at the center of the circle and sides that are radii. Central angles intercept an arc of the circle. The endpoints of the intercepted arc lie on the rays of the central angle. If you know the measure of a central angle and the radius of the circle, you can find the length of the arc that is intercepted by the angle.

Key Concept – Arc Length

Words	Symbols	
For a circle with radius r and central angle θ (in radians), the arc lengths equals the product of r and θ .	$s = r\theta$	

Mathematics Problem-Solving Feedback Rubric روبرك التغذية الراجعة لحل المسائل الرياضية									/10
الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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Next step توجيهات للتحسين (Improvement Guidance)									

(Mastery) Example 6: Find Arc Length.

TRAFFIC A traffic circle, or roundabout, is a circular roadway at the intersection of two or more streets that allows cars to travel through more continuously than a traffic light or stop sign. The diameter of a traffic circle is 160 feet. How far does a car travel in the roundabout if it goes three-fourths of the way around?

Check my understanding 5:

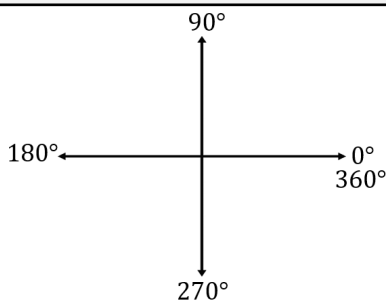
52. A traffic roundabout has a diameter of 200 meters. How far does an automobile travel in the roundabout if it goes one-fourth of the way around?

Step by step	1 التقديم Advanced	2 التوسيط Intermediate	3 النهج Beginner /10
فهم المسألة (Understanding the Problem)				
تحليل المسألة (Implementing the Solution)				
التحقق من الحل والتأكد (Verifying and Interpreting the Solution)				
التعميم (Improvement Guidance) + Next step				

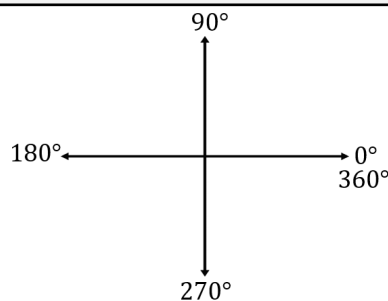
Academic support (6) – Extra Practice:-

Draw angles in standard position and identify coterminal angles.

11. 305°



9. -420°



Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.

27. 110°

31. $\frac{2\pi}{3}$

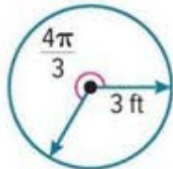
Rewrite each degree measure in radians and each radian measure in degrees.

57. -820°

58. 4π

Find the length of each arc. Round to the nearest tenth.

65.



M9L2 – Trigonometric Functions of General Angles

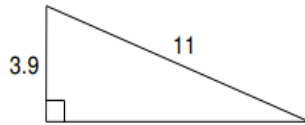
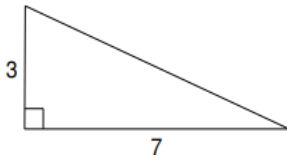
Outcomes:	1. Find values of trigonometric functions for acute angles.
	2. Find values of trigonometric functions of general angles.
	3. Find values of trigonometric functions by using reference angles.
Keywords:	Trigonometry – Trigonometric ratio – Trigonometric function – Sine – Cosine – Tangent – Cosecant – Secant – Cotangent – Quadrantal angle – Reference angle.

Starter

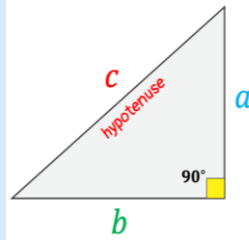
Prior knowledge

Pre-Emerging

Find each missing length to the nearest tenth.



PYTHAGOREAN THEOREM



$$c^2 = a^2 + b^2$$

$$\star c = \sqrt{a^2 + b^2}$$

$$\star a = \sqrt{c^2 - b^2}$$

$$\star b = \sqrt{c^2 - a^2}$$

Learn Trigonometric Functions in Right Triangles

- **Trigonometry** is the study of relationships among the angles and sides of triangles. A **trigonometric ratio** compares the lengths of two sides of a right triangle. A **trigonometric function** relates the angles of a triangle to the lengths of its sides.

Key Concept – Trigonometric Functions in Right Triangles

If θ is the measure of an acute angle of a right triangle, then the trigonometric functions involving the **opposite side opp**, the **adjacent side adj**, and the **hypotenuse hyp** are defined as follows.

sine: $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

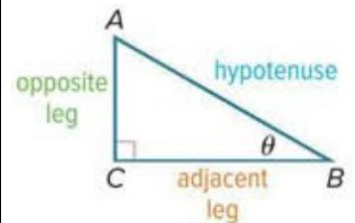
cosecant: $\csc \theta = \frac{\text{hyp}}{\text{opp}}$

cosine: $\cos \theta = \frac{\text{adj}}{\text{hyp}}$

secant: $\sec \theta = \frac{\text{hyp}}{\text{adj}}$

tangent: $\tan \theta = \frac{\text{opp}}{\text{adj}}$

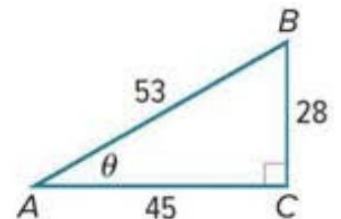
cotangent: $\cot \theta = \frac{\text{adj}}{\text{opp}}$



(Emerging) Example 1: Evaluate Trigonometric Functions.

Find the exact values of the six trigonometric functions for angle θ .

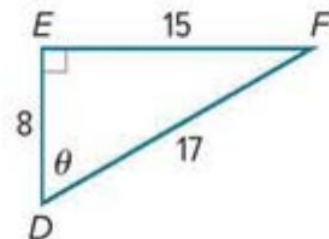
- | | |
|------------------------------|------------------------------|
| • $\sin \theta = \text{---}$ | • $\csc \theta = \text{---}$ |
| • $\cos \theta = \text{---}$ | • $\sec \theta = \text{---}$ |
| • $\tan \theta = \text{---}$ | • $\cot \theta = \text{---}$ |



Check my understanding 1:

Find the exact values of the six trigonometric functions for angle θ .

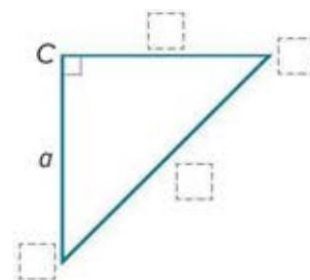
- $\sin \theta = \text{---}$
- $\cos \theta = \text{---}$
- $\tan \theta = \text{---}$
- $\csc \theta = \text{---}$
- $\sec \theta = \text{---}$
- $\cot \theta = \text{---}$



(Developing) Example 2: Find Trigonometric Ratios.

If $\cos A = \frac{9}{13}$, find the exact values of the five remaining trigonometric functions for A .

- $\sin \theta = \text{---}$
- $\cos \theta = \text{---}$
- $\tan \theta = \text{---}$
- $\csc \theta = \text{---}$
- $\sec \theta = \text{---}$
- $\cot \theta = \text{---}$



If $\sec B = \frac{11}{3}$, find the exact values of the five remaining trigonometric functions for B .

- $\sin \theta = \text{---}$
- $\cos \theta = \text{---}$
- $\tan \theta = \text{---}$
- $\csc \theta = \text{---}$
- $\sec \theta = \text{---}$
- $\cot \theta = \text{---}$

Check my understanding 2:

7. If $\tan A = \frac{8}{15}$, find the exact values of the five remaining trigonometric functions for A .

- $\sin \theta = \text{---}$
- $\cos \theta = \text{---}$
- $\tan \theta = \text{---}$
- $\csc \theta = \text{---}$
- $\sec \theta = \text{---}$
- $\cot \theta = \text{---}$

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لـحل المسائل الرياضية

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

Learn Trigonometric Functions of General Angles

- You can find values of trigonometric functions for angles greater than 90° or less than 0° when given a point on the terminal side of the angle.

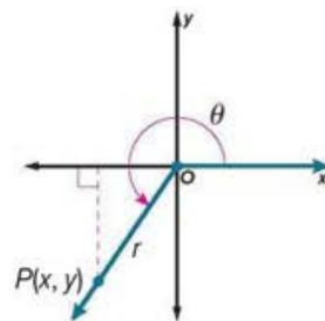
Key Concept – Trigonometric Functions of General Angles

Let θ be an angle in standard position, and let $P(x, y)$ be a point on its terminal side. By the Pythagorean Theorem, $r = \sqrt{x^2 + y^2}$ where r is the distance from the origin to point P along the terminal side. Using the coordinates of point P and r , the six trigonometric functions of θ are defined below.

$$\sin \theta = \frac{y}{r} \qquad \csc \theta = \frac{r}{y}, y \neq 0$$

$$\cos \theta = \frac{x}{r} \qquad \sec \theta = \frac{r}{x}, x \neq 0$$

$$\tan \theta = \frac{y}{x}, x \neq 0 \qquad \cot \theta = \frac{x}{y}, y \neq 0$$

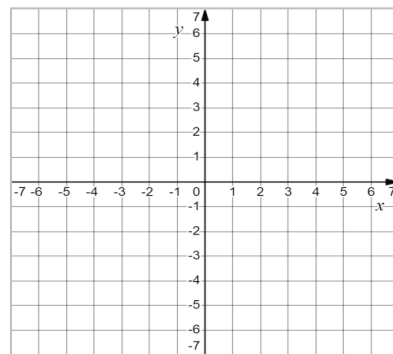


- If the terminal side of angle θ in standard position coincides with one of the axes, the angle is called a quadrantal angle.
- Since r is the distance from the origin to point $P(x, y)$ along the terminal side:
 - if $\theta = 0^\circ$ or 360° , then $r = x$.
 - if $\theta = 90^\circ$, then $r = y$.
 - if $\theta = 180^\circ$, then $r = |x|$.
 - if $\theta = 270^\circ$, then $r = |y|$.

(Developing) Example 3: Evaluate Trigonometric Functions Given a Point.

The terminal side of θ in standard position contains the point $(-6, 4)$. Find the exact values of the six trigonometric functions of θ .

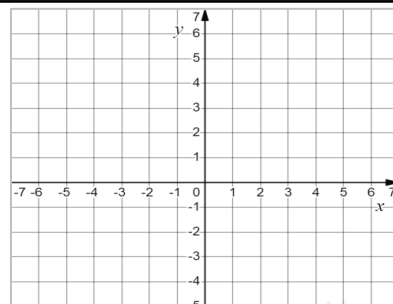
- $\sin \theta = \underline{\hspace{2cm}}$
- $\csc \theta = \underline{\hspace{2cm}}$
- $\cos \theta = \underline{\hspace{2cm}}$
- $\sec \theta = \underline{\hspace{2cm}}$
- $\tan \theta = \underline{\hspace{2cm}}$
- $\cot \theta = \underline{\hspace{2cm}}$



(Developing) Example 4: Evaluate Trigonometric Functions of Quadrantal Angles.

The terminal side of θ in standard position contains the point $P(-5, 0)$. Find the exact values of the six trigonometric functions of θ .

- $\sin \theta = \underline{\hspace{2cm}}$
- $\csc \theta = \underline{\hspace{2cm}}$
- $\cos \theta = \underline{\hspace{2cm}}$
- $\sec \theta = \underline{\hspace{2cm}}$
- $\tan \theta = \underline{\hspace{2cm}}$
- $\cot \theta = \underline{\hspace{2cm}}$

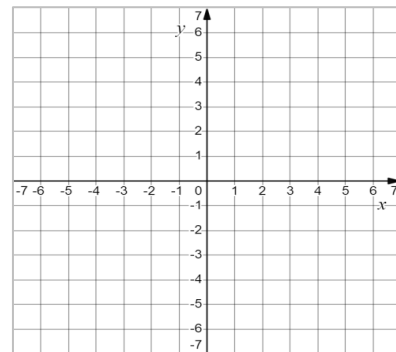


Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره والتحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1
1									
Next step توجيهات للتحسين (Improvement Guidance)									

Check my understanding 3:

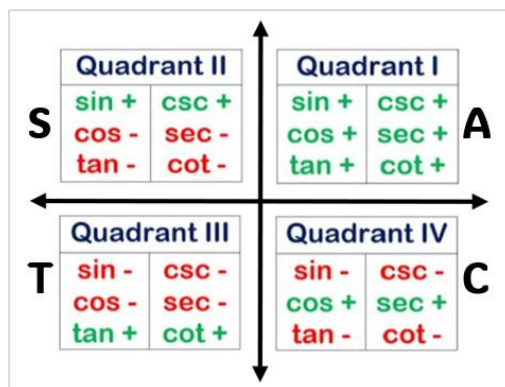
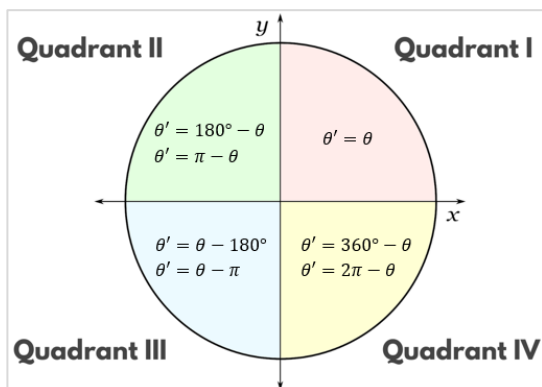
14. The terminal side of θ in standard position contains the point (3,4). Find the exact values of the six trigonometric functions of θ .



- $\sin \theta = \underline{\hspace{2cm}}$
- $\cos \theta = \underline{\hspace{2cm}}$
- $\tan \theta = \underline{\hspace{2cm}}$
- $\csc \theta = \underline{\hspace{2cm}}$
- $\sec \theta = \underline{\hspace{2cm}}$
- $\cot \theta = \underline{\hspace{2cm}}$

Learn Trigonometric Functions with Reference Angles

- For a nonquadrantal angle θ in standard position, its reference angle is the acute angle θ' formed by the terminal side and the x -axis. The rules for finding the measures of reference angles vary depending on the quadrant in which the terminal side is located.
- You can use reference angles to evaluate trigonometric functions for any angle θ . The sign of the function is determined by the quadrant in which the terminal side of θ lies.



(Developing) Example 5: Find Reference Angles.

Sketch each angle. Then find the measure of its reference angle.

<p>a. 155°</p>	<p>38. -210°</p>	<p>b. $-\frac{8\pi}{3}$</p>	<p>c. $\frac{23\pi}{6}$</p>
----------------------------------	------------------------------------	--	--

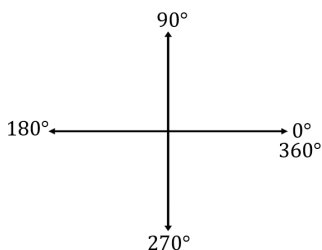
Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1
Next step توجيهات للتحسين (Improvement Guidance)									

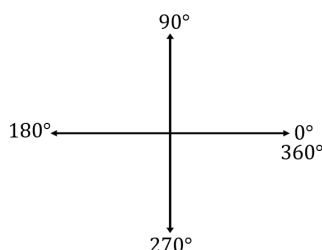
Check my understanding 4:

Sketch each angle. Then find the measure of its reference angle.

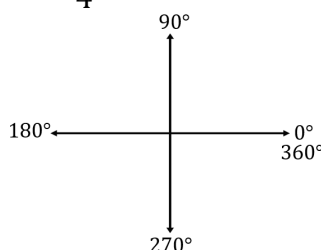
29. 230°



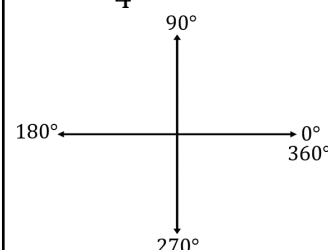
-40°



33. $\frac{7\pi}{4}$



39. $-\frac{7\pi}{4}$



(Emerging) Example 6: Use a Reference Angle to Find a Trigonometric Value.

Find the exact value of each trigonometric function.

$\tan \frac{7\pi}{4}$

$\sec 225^\circ$

Check my understanding 5:

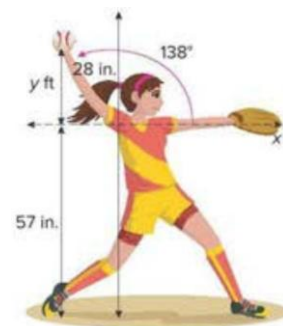
Find the exact value of each trigonometric function.

44. $\sin(-150^\circ)$

43. $\csc \frac{\pi}{4}$

(Mastery) Example 7: Use Trigonometric Functions.

SOFTBALL During a windmill pitch in fastpitch softball, a pitcher's arm makes a complete clockwise rotation before releasing the ball. Suppose a pitcher has an arm length of 28 inches, and the axis from which the pitcher's arm swings is at her shoulder height of 57 inches. What is the height of the ball when the pitcher's arm is in the position shown?



Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

Check my understanding 6:

47. **ROOFING** A roofer rests a ladder of a height of 12 feet against a building so that the base of the ladder is x feet from the bottom of the side of the building, forming a 71.6° angle with the ladder and ground. Find the distance from the bottom of the ladder to the side of the building.

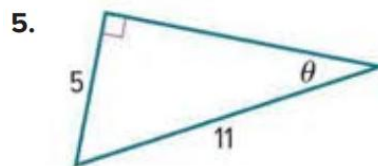
a. Write an equation that can be used to find the distance from the bottom of the ladder to the side of the building.

b. How far is the bottom of the ladder from the side of the building? Round to the nearest tenth if necessary.

Academic support (7) – Extra Practice:-

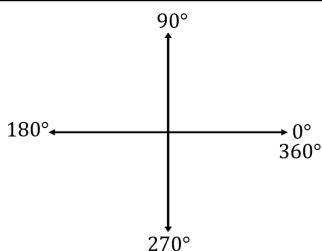
Find the exact values of the six trigonometric functions for angle θ .

- $\sin \theta =$ _____
- $\csc \theta =$ _____
- $\cos \theta =$ _____
- $\sec \theta =$ _____
- $\tan \theta =$ _____
- $\cot \theta =$ _____

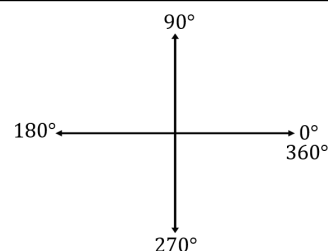


Sketch each angle. Then find the measure of its reference angle.

30. 205°



32. $-\frac{\pi}{6}$



The exact value of each trigonometric function.

53. $\cos(270^\circ)$

57. $\cot(-\pi)$

In a right triangle, $\angle A$ is acute.

51. If $\sin B = \frac{3}{8}$, what is $\tan B$?

50. If $\tan A = \frac{7}{12}$, what is $\cos A$?

Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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1									
Next step توجيهات للتحسين (Improvement Guidance)									

M9L3 – Circular and Periodic Functions

Outcomes:	1. Find values of trigonometric functions given a point on a unit circle or the measure of a special angle.
	2. Find values of trigonometric functions that model periodic events.
Keywords:	Unit circle – Circular function – Periodic function – Cycle – Period.

Starter

Find the exact value of each trigonometric function.

1. $\tan(2\pi) =$

2. $\sec(213^\circ) =$

3. $\cos(0) =$

Learn Circular Functions

- A unit circle is a circle with a radius of 1 unit centered at the origin on the coordinate plane. Notice that on a unit circle, the radian measure of a central angle $\theta = \frac{s}{1}$ or s , so the radian measure of an angle is the length of the arc on the unit circle subtended by the angle.
- You can use a point P on the unit circle to generalize sine and cosine functions by applying the definitions of trigonometric functions in right triangles.
- Both $\cos \theta = x$ and $\sin \theta = y$ are functions of θ . Because they are defined using a unit circle, they are circular functions, which describe a point on a circle as the function of an angle defined in radians.
- The unit circle is commonly used to show the exact values of $\cos \theta$ and $\sin \theta$ for special angles. The cosine values are the x -coordinates of the points where the terminal sides of the angles intersect the unit circle, and the sine values are the y -coordinates.

Key Concept • Functions on a Unit Circle

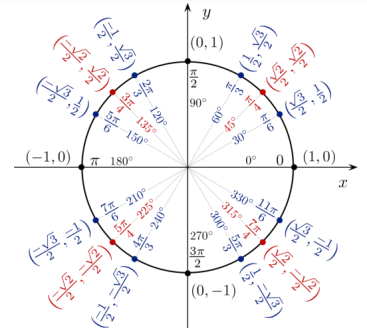
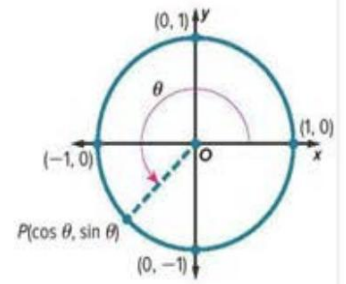
Words: If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos \theta = x$ and $\sin \theta = y$.

Symbols: $P(x, y) = P(\cos \theta, \sin \theta)$

Example:

If $\theta = \frac{5\pi}{4}$,

$P(x, y) = P\left(\cos \frac{5\pi}{4}, \sin \frac{5\pi}{4}\right)$.



(Emerging) Example 1: Find Sine and Cosine Given a Point on the Unit Circle.

The terminal side of θ in standard position intersects the unit circle at $P\left(-\frac{12}{13}, \frac{5}{13}\right)$. Find $\cos \theta$ and $\sin \theta$.

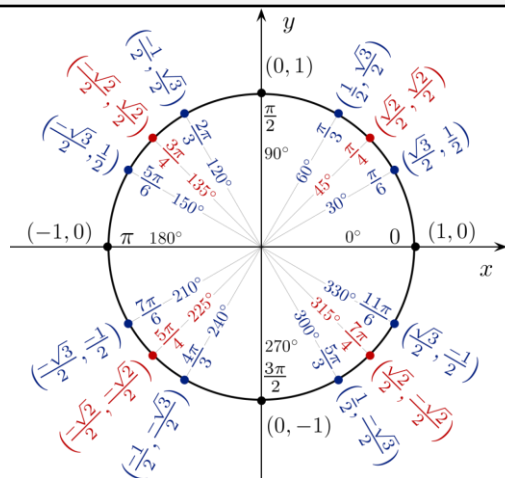
Check my understanding 1: The terminal side of θ in standard position intersects the unit circle at $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$. Find $\cos \theta$ and $\sin \theta$. Write the solutions as decimals.

Mathematics Problem-Solving Feedback Rubric روبرك التغذية الراجعة لحل المسائل الرياضية								/10
الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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Next step توجيهات للتحسين (Improvement Guidance)									

(Developing) Example 2: Find Trigonometric Values of Special Angles.

Find the exact values of the six trigonometric functions for an angle $\frac{5\pi}{4}$ that measures radians.

- $\sin \theta =$ _____
- $\cos \theta =$ _____
- $\tan \theta =$ _____
- $\csc \theta =$ _____
- $\sec \theta =$ _____
- $\cot \theta =$ _____



Check my understanding 2:

Find the exact values of the six trigonometric functions for an angle $\frac{4\pi}{3}$ that measures radians.

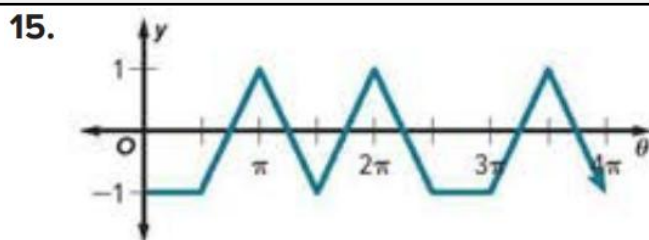
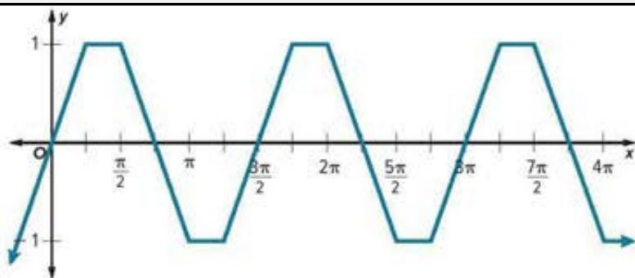
- $\sin \theta =$ _____
- $\cos \theta =$ _____
- $\tan \theta =$ _____
- $\csc \theta =$ _____
- $\sec \theta =$ _____
- $\cot \theta =$ _____

Learn Periodic Functions

- A periodic function has y -values that repeat at regular intervals. One complete pattern of a periodic function is called a cycle, and the horizontal length of one cycle is called the period.

(Developing) Example 3: Identify the Period of a Function.

Determine the period of the function.



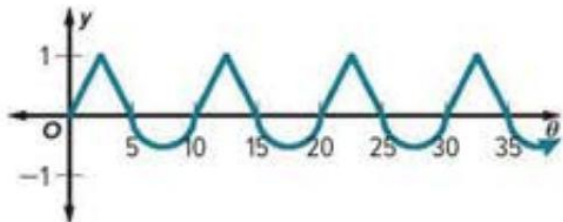
Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية/10

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
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Next step توجيهات للتحسين (Improvement Guidance)									

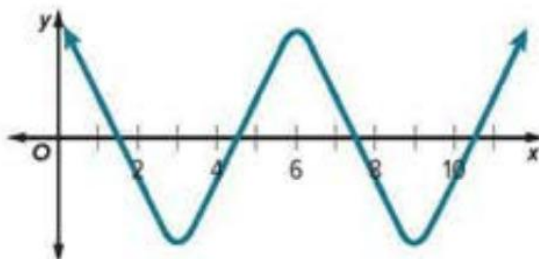
Check my understanding 3:

Determine the period of the function.

13.



14.

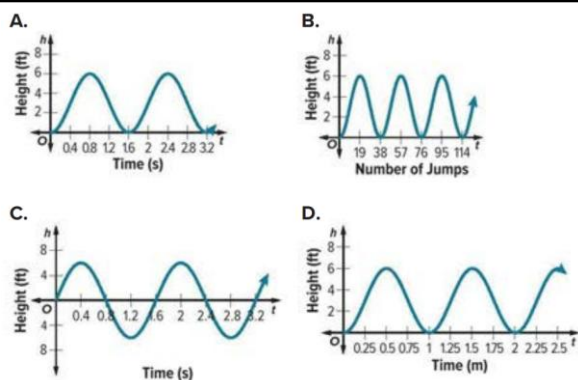


(Mastery-Apply) Example 4: Graph Periodic Functions.

CAROUSELS Abu Dhabi’s Park Carousel is considered the fastest carousel in the world, taking riders for a spin at 18 miles per hour. The indoor carousel has a diameter of 50 feet and can complete about 10 rotations per minute. The distance of a rider d from the front wall of the building varies periodically as a function of time t . Identify the period of the function. Then graph the function. Assume that a rider begins at the point closest to the wall, 20 feet from the wall.

Check my understanding 4:

In 2016, Sultan set the record for the fewest pogo stick jumps in one minute, when he jumped up and down only 38 times in one minute. Sultan’s height off the ground h while jumping is a function of time t . Suppose that at the highest point of each jump, Sultan was 6 feet off the ground. Select the graph of the function.



Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لاجل المسائل الرياضية/10

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1									
Next step توجيهات للتحسين (Improvement Guidance)									

(Emerging) Example 5: Evaluate Trigonometric Expressions.

Find the exact value of $\cos \frac{10\pi}{3}$.

Check my understanding 5:

Find the exact value of each expression.

$\sin \frac{8\pi}{3} = \underline{\quad ? \quad}$

$\cos \frac{5\pi}{6} = \underline{\quad ? \quad}$

$\sin \frac{21\pi}{4} = \underline{\quad ? \quad}$

$\cos \frac{11\pi}{3} = \underline{\quad ? \quad}$

$\cos 60^\circ = \underline{\quad ? \quad}$

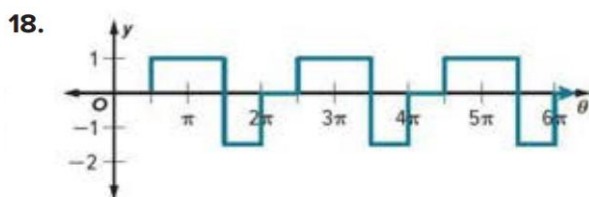
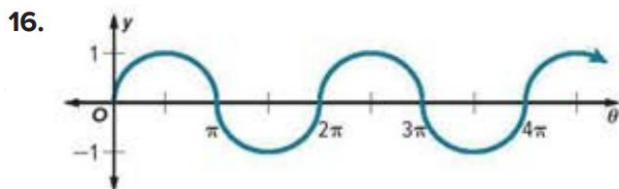
$\sin 600^\circ = \underline{\quad ? \quad}$

Academic support (8) – Extra Practice:-

5. The terminal side of θ in standard position intersects the unit circle at $P\left(\frac{1}{6}, -\frac{\sqrt{35}}{6}\right)$. Find $\cos \theta$ and $\sin \theta$.

7. Find the exact values of the six trigonometric functions for an angle $\frac{\pi}{6}$ that measures radians.

Determine the period of the function.



Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لـحل المسائل الرياضية

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1									
Next step توجيهات للتحسين (Improvement Guidance)									

M9L4 – Graphing Sine and Cosine Functions

Outcomes:

1. Graph and analyze sine and cosine functions.
2. Model periodic real-world situations with sine and cosine functions.

Keywords:

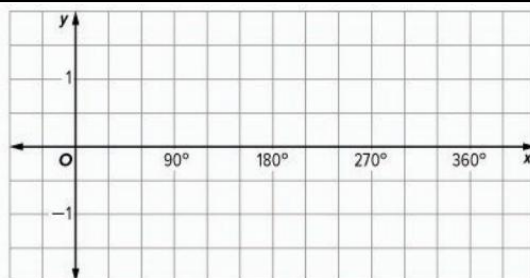
Oscillation - Midline - Amplitude - Sinusoidal function - Frequency.

Starter

Pre-Emerging

Complete the table of values for the function $y = \sin x$. Then sketch a graph of the sine function.

x	0°	90°	120°	180°	270°	360°
$\sin x$						



Learn Graphing Sine and Cosine Functions

- Like other functions you have studied, trigonometric functions can be graphed on the coordinate plane. Sine and cosine functions are oscillating functions. The **oscillation** of a function refers to how much the graph of the function varies between its extreme values as it approaches positive or negative infinity. The **midline** is the line about which a graph oscillates, so it is halfway between the maximum and minimum. The **amplitude** of the graph of a sine or cosine function equals half the difference between the maximum and minimum values of the function.

Sine and Cosine Functions

Function	$y = a \sin bx$	$y = a \cos bx$
Parent graph		
Domain	All real numbers	All real numbers
Range	$\{y \mid y_{Min} \leq y \leq y_{Max}\}$	$\{y \mid y_{Min} \leq y \leq y_{Max}\}$
Amplitude	$ a $ OR $\frac{y_{Max} - y_{Min}}{2}$	$ a $ OR $\frac{y_{Max} - y_{Min}}{2}$
Midline	$y = 0$	$y = 0$
Period	$\frac{360^\circ}{b}$	$\frac{360^\circ}{b}$
Oscillation	Between Minimum point and Maximum point	Between Minimum point and Maximum point

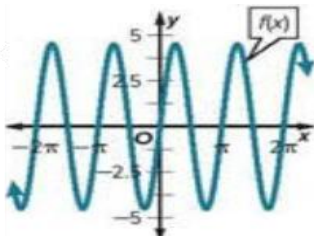
(Emerging) Example 1: Identify the Amplitude and Period from a Graph.

Identify the amplitude, midline, and period of $f(x)$.

The amplitude is _____

The midline is at $y =$ _____

The period is _____



(Emerging) Example 2: Identify the Amplitude and Period from an Equation.

Identify the amplitude and period of $f(x) = 3 \cos 5x$

The amplitude is _____

The period is _____

Check my understanding 1:

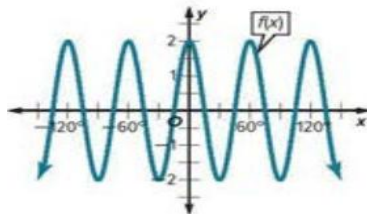
Identify the amplitude, midline, and period of $f(x)$.

Identify the amplitude and period of $f(x) = 7 \sin 8x$

The amplitude is _____

The midline is at $y =$ _____

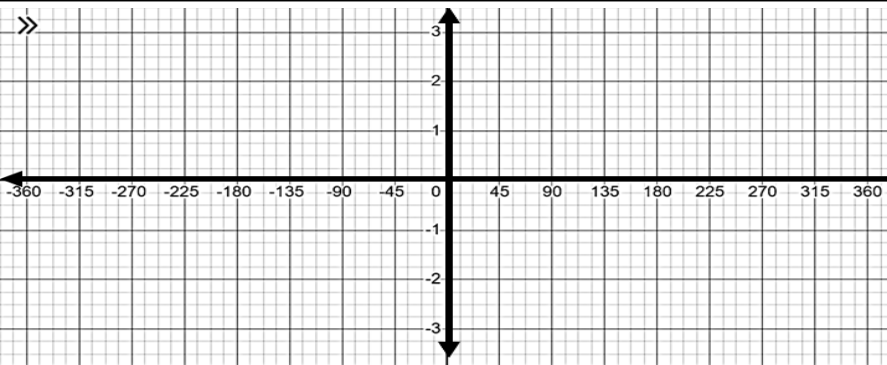
The period is _____



The amplitude is _____

The period is _____

(Developing) Example 3: Graph a Sine Function. Graph $y = 0.5 \sin 4x$



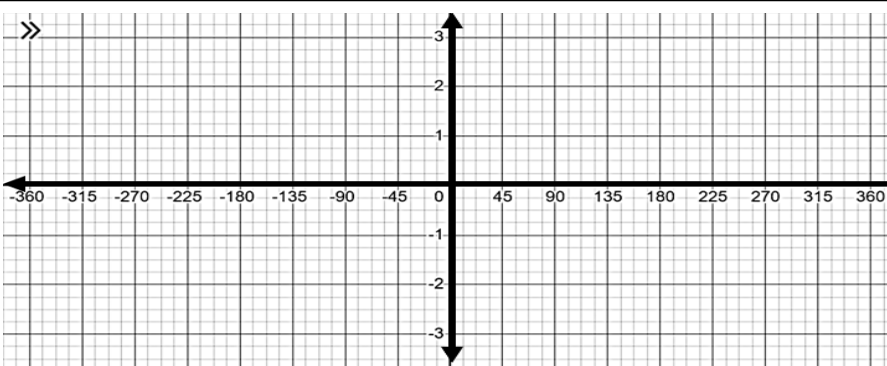
Graph using calculator table:-

Start: 0

End: $\frac{360}{b}$ OR $\frac{2\pi}{b}$

Step: $\frac{90}{b}$ OR $\frac{\pi}{b}$

(Developing) Example 4: Graph a Cosine Function. Graph $y = 2 \cos 3x$

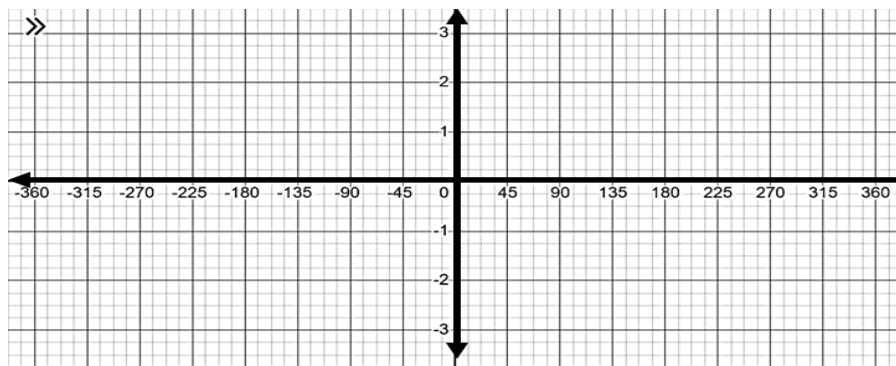


Mathematics Problem-Solving Feedback Rubric | روبرك التغذية الراجعة لحل المسائل الرياضية

الترتيب Step by step	فهم المسألة (Understanding the Problem)			تنفيذ الحل (Implementing the Solution)			التحقق من الحل وتفسيره (Verifying the Solution and Interpreting the Solution)		
	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1	متقدم (Advanced) 3	متوسط (Intermediate) 2	مبتدئ (Beginner) 1
1									
Next step توجيهات للتحسين (Improvement Guidance)									

Check my understanding 2:

13. Graph $y = \sin 4\theta$



Graph using calculator table:-

Start: 0

End: $\frac{360}{b}$ OR $\frac{2\pi}{b}$

Step: $\frac{90}{b}$ OR $\frac{\pi}{b}$

(Mastery) Example 5: Characteristics of the Sine and Cosine Functions.

SPRINGS An object on a spring oscillates according to the function $y = 40 \cos \pi t$, where y is the distance in centimeters above its equilibrium position at time t in seconds.

Part A Find the period and frequency, and describe them in the context of the situation.

Part B Identify the domain and range in the context of the situation.

Period =

Domain =

Frequency =

Range =

Check my understanding 3:

ELECTRONICS The electric power delivered for household use in the United States is 110 volts. The function $y = 100\sqrt{2} \sin 120\pi t$ represents the effective current, where t is the time in seconds.

Part A Find the period and frequency, and describe them in the context of the situation.

Part B Identify the domain and range.

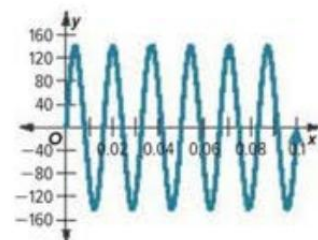
Period =

Domain =

Frequency =

Range =

The current completes _____ cycle(s) per second, and will reach the maximum every _____ second(s).



(Mastery) Example 6: Characteristics of the Sine and Cosine Functions.

ELECTRICITY The voltage supplied by an electrical outlet can be represented by a periodic function. The voltage oscillates between -120 and 120 volts, with a frequency of 50 cycles per second. Write and graph a function for the voltage v as a function of time t .

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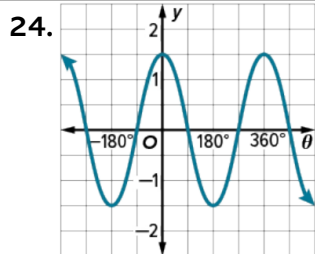
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Check my understanding 4:

20. FERRIS WHEELS A Ferris wheel at a state fair has a diameter of 65 feet and makes 4 complete revolutions each minute. Santiago boards a car of the Ferris wheel at the car’s lowest point, and he rides for 2 minutes. Write and graph a trigonometric function that **models** his height above or below the axle of the Ferris wheel θ minutes after the ride starts.



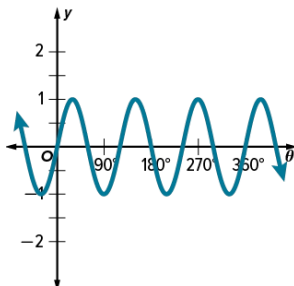
Challenging questions: For each graph, identify the period and write an equation.



Academic support (9) – Extra Practice:-

3. Identify the amplitude, midline, and period of $f(x)$.

The amplitude is _____
The midline is at $y =$ _____
The period is _____



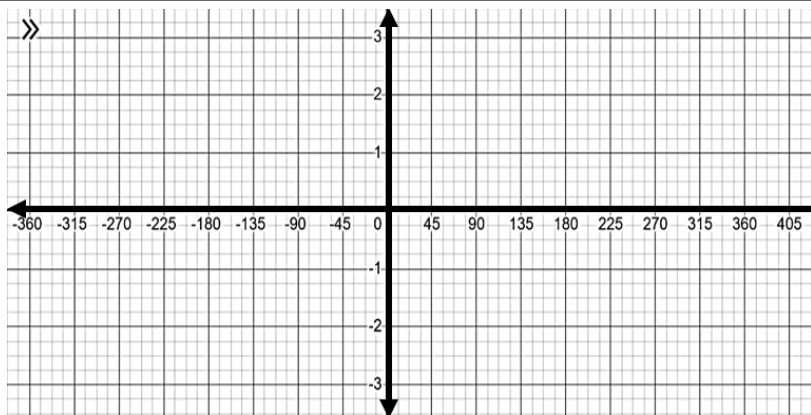
9. Identify the amplitude and period of $f(x) = \frac{1}{2} \sin 2\theta$

The amplitude is _____
The period is _____

Find the amplitude and period of each function. Then graph the function.

12. $y = \cos 3\theta$

The amplitude is _____
The period is _____



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M9L5 – Graphing Other Trigonometric Functions

Outcomes:

1. Graph and analyze tangent functions.
2. Graph and analyze reciprocal trigonometric functions.

Keywords:

Reciprocal trigonometric functions.

Starter

Pre-Emerging

Determine the vertical asymptote(s) for each function.

1. $f(x) = \frac{3}{x}$

2. $f(x) = \frac{6}{x-11}$

3. $f(x) = \frac{9}{4x-10}$

4. $f(x) = \frac{x+2}{x^2-36}$

Learn Graphing Tangent Functions

- Because $\tan x = \frac{\sin x}{\cos x}$, the tangent function is undefined when $\cos x = 0$, which occurs at $x = (90 + 180n)^\circ$, where n is an integer. As a result, the graph of the tangent function has asymptotes where $x = (90 + 180n)^\circ$.

Graphs of Tangent Functions

Function	$y = a \tan bx$	<p>Graph</p>
Domain	$\{x \mid x \neq (90 + 180n)^\circ, n \text{ is an integer}\}$	
Range	All real numbers	
Amplitude	undefined	
Period	$\frac{180^\circ}{ b }$ OR $\frac{\pi}{ b }$	
Number of x-intercepts in one cycle	$\frac{180n}{ b }$	
Midline	$y = 0$	
Asymptotes	$x = \frac{(90+180n)^\circ}{ b }$, where n is an integer.	

$$y = a \tan bx$$

Reflected about the x -axis $-f(x)$

Vertical Dilation
Stretch: $|a| > 1$
Compress: $0 < |a| < 1$

Horizontal Dilation
Stretch: $0 < |b| < 1$
Compress: $|b| > 1$

(Emerging-Developing) Example 1: Graph a Tangent Function with a Dilation.

Find the period, asymptotes, x -intercepts, midline, and transformations of $y = \tan 3x$. Then graph the function.

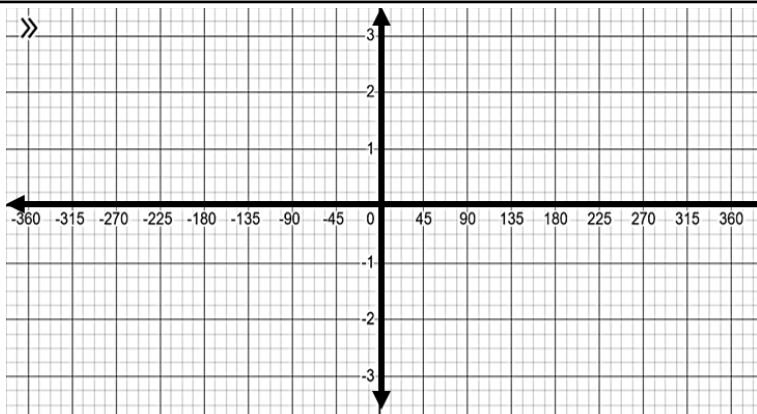
Period:

Asymptotes:

x -intercepts:

Midline:

Transformations:



(Emerging-Developing) Example 2: Graph a Tangent Function with a Dilation and a Reflection.

Find the period, asymptotes, x -intercepts, midline, and transformations of $y = -\frac{1}{3} \tan 2x$. Then graph the function.

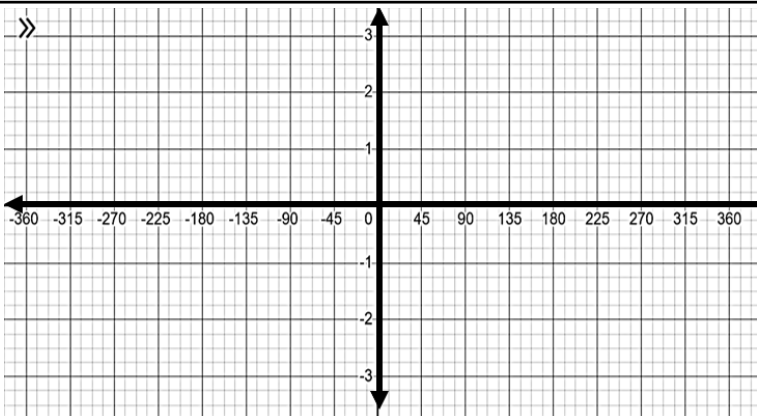
Period:

Asymptotes:

x -intercepts:

Midline:

Transformations:



Check my understanding 1:

4. Find the period, asymptotes, x -intercepts, midline, and transformations of $y = \frac{1}{2} \tan x$. Then graph the function.

Period:

Asymptotes:

x -intercepts:

Midline:

Transformations:



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Learn Graphing Reciprocal Trigonometric Functions

- The reciprocal trigonometric functions, cosecant, secant, and cotangent, can be expressed as

$\frac{1}{\sin x}$, $\frac{1}{\cos x}$, and $\frac{1}{\tan x}$, respectively. As a result, the graphs of the reciprocal trigonometric functions have asymptotes when the corresponding sine, cosine, or tangent function equals 0 and the reciprocal function is undefined.

Graphs of Reciprocal Functions

Function	$y = a \csc bx$	$y = a \sec bx$	$y = a \cot bx$
Graph			
Domain	$\{x \mid x \neq 180n^\circ, n \text{ is an integer}\}$	$\{x \mid x \neq (90 + 180n)^\circ, n \text{ is an integer}\}$	$\{x \mid x \neq 180n^\circ, n \text{ is an integer}\}$
Range	$\{y \mid y \leq -1 \text{ or } y \geq 1\}$	$\{y \mid y \leq -1 \text{ or } y \geq 1\}$	all real numbers
Amplitude	undefined	undefined	undefined
Period	$\frac{360^\circ}{ b }$	$\frac{360^\circ}{ b }$	$\frac{180^\circ}{ b }$
Midline	$y = 0$	$y = 0$	$y = 0$

(Developing) Example 3: Graph a Cosecant Function.

Find the period, asymptotes, relative extrema, and midline of $y = \csc 0.5x$. Then graph the function.

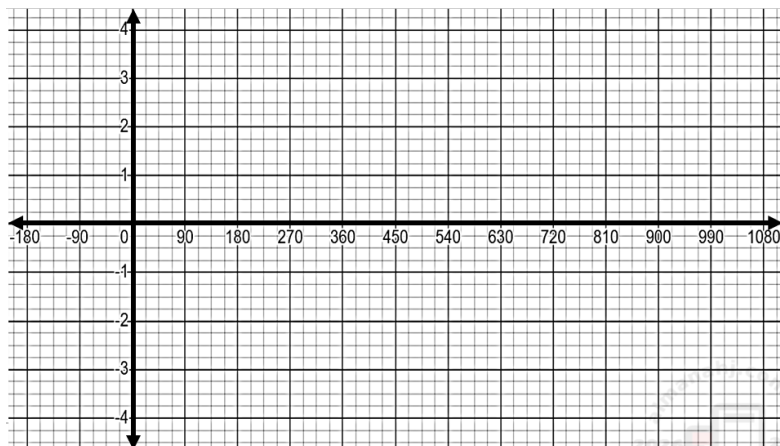
Period:

Asymptotes:

relative maxima:

relative minima:

Midline:



(Developing) Example 3: Graph a Secant Function.

13. Find the period, asymptotes, relative extrema, and midline of $y = 3\sec x$. Then graph the function.

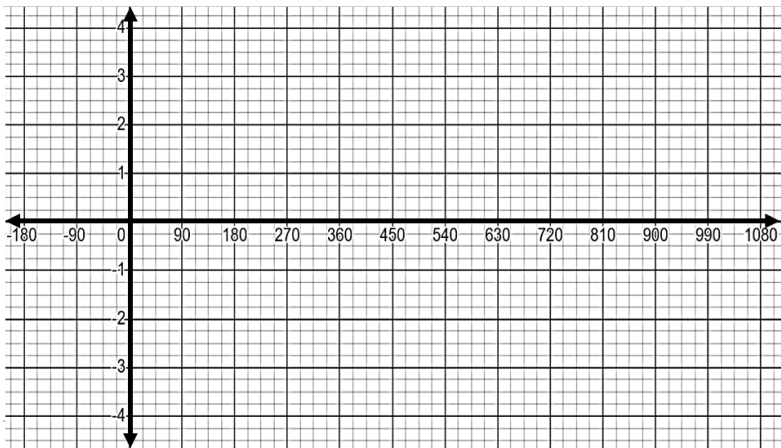
Period:

Asymptotes:

relative maxima:

relative minima:

Midline:



Check my understanding 2:

9. Find the period, asymptotes, relative extrema, and midline of $y = \frac{1}{2}\csc x$. Then graph the function.

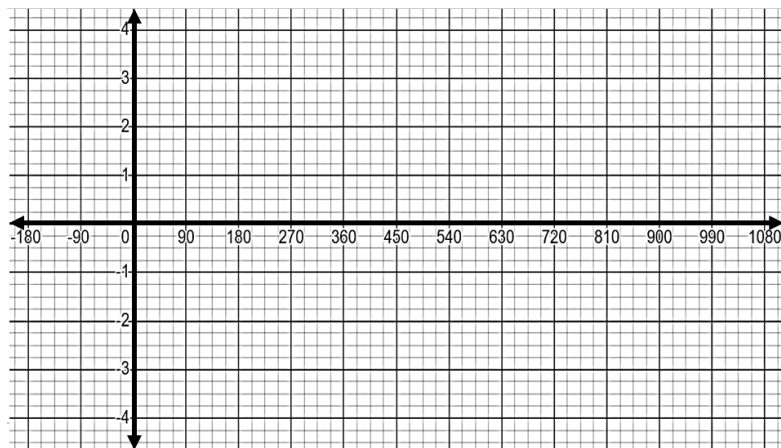
Period:

Asymptotes:

relative maxima:

relative minima:

Midline:



(Developing) Example 4: Graph a Cotangent Function.

Find the period, asymptotes, x -intercepts and midline of $y = -4\cot 2x$. Then graph the function.

Note:-

Asymptotes: $\frac{180n}{|b|}$

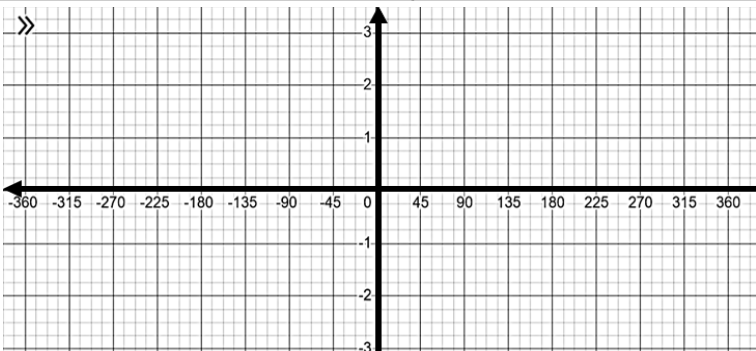
x -intercepts: $\frac{(90+180n)^\circ}{|b|}$

Period:

Asymptotes:

x -intercepts:

Midline:



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Next step توجيهات للتحسين (Improvement Guidance)									

(Developing) Example 4: Graph a Cotangent Function.

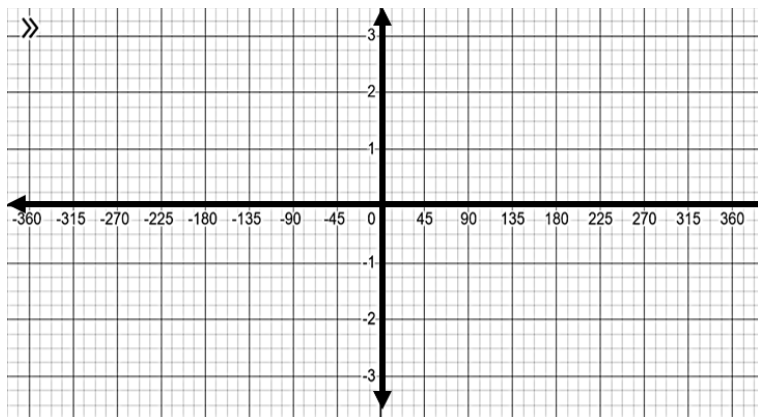
10. Find the period, asymptotes, x -intercepts and midline of $y = \cot 2x$. Then graph the function.

Period:

Asymptotes:

x -intercepts:

Midline:



Check my understanding 3:

12. Find the period, asymptotes, x -intercepts and midline of $y = -2 \cot x$. Then graph the function.

Note:-

Asymptotes: $\frac{180n}{|b|}$

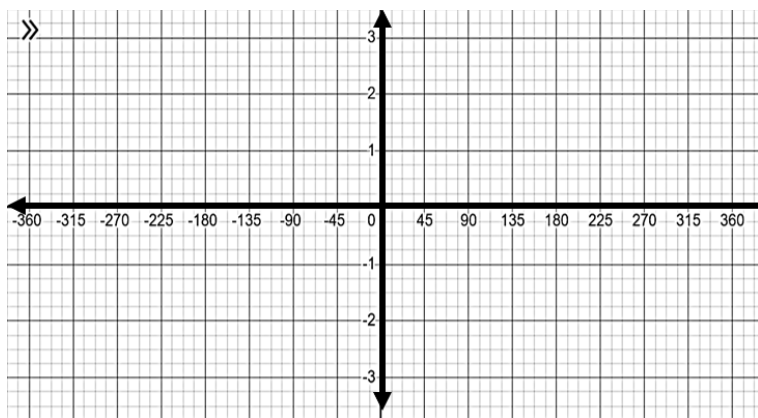
x -intercepts: $\frac{(90+180n)^\circ}{|b|}$

Period:

Asymptotes:

x -intercepts:

Midline:



State the period, asymptotes, and relative extrema for each function, where $0^\circ \leq x \leq 360^\circ$.

1. $y = -6 \csc x$

Period:

Asymptotes:

relative maxima:

relative minima:

2. $y = \sec 2x$

Period:

Asymptotes:

relative maxima:

relative minima:

3. $y = \cot -3x$

Period:

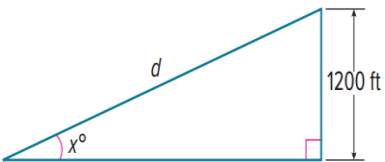
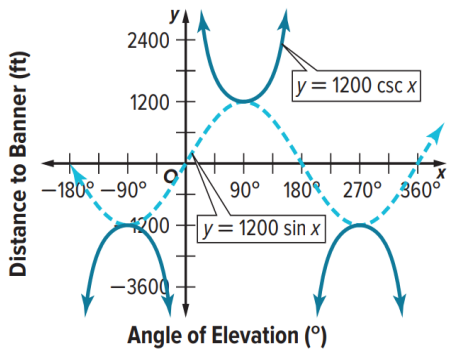
Asymptotes:

x -intercepts:

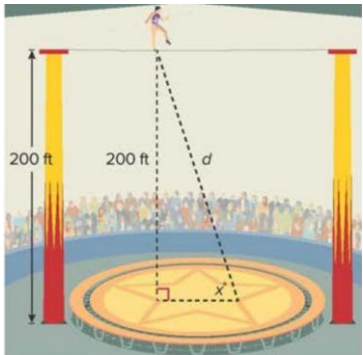
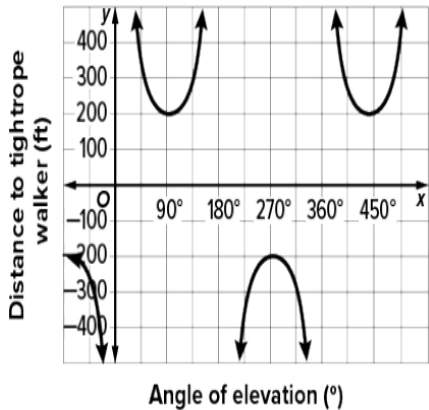
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(Mastery) Example 5: Apply a Reciprocal Trigonometric Function.

ADVERTISING Suppose a banner towing plane is approaching a music festival at an elevation of 1200 feet above the crowd at the festival. The plane will eventually fly directly over the crowd. Let d be the distance in feet the banner is from the music festival, and let x be the angle of elevation to the banner from the perspective of the crowd. Write a function that relates the distance as a function of an angle x . Then, graph the function and analyze its key features.

Part A Write a function.	Part B Graph the function.	Part C Analyze the graph.
		<p>Domain:</p> <p>Range:</p> <p>x-Intercept:</p> <p>y-Intercept:</p> <p>Relative minimum:</p> <p>Relative maximum:</p> <p>Increasing:</p> <p>Decreasing:</p>

Check my understanding 4: 16. Suppose an acrobat is walking from a high wire that is attached on its ends to two different towers at a height of 200 feet above the floor, as shown. The acrobat will eventually walk directly over the location of a camera in the floor. Let d be the distance in feet the acrobat is from the camera.

Part A Write a function.	Part B Graph the function.	Part C Analyze the graph.
		<p>Domain:</p> <p>Range:</p> <p>x-Intercept:</p> <p>y-Intercept:</p> <p>Relative minimum:</p> <p>Relative maximum:</p> <p>Increasing:</p> <p>Decreasing:</p>

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Academic support (10) – Extra Practice:-

3. Find the period, asymptotes, x -intercepts, midline, and transformations of $y = \tan 2x$. Then graph the function.

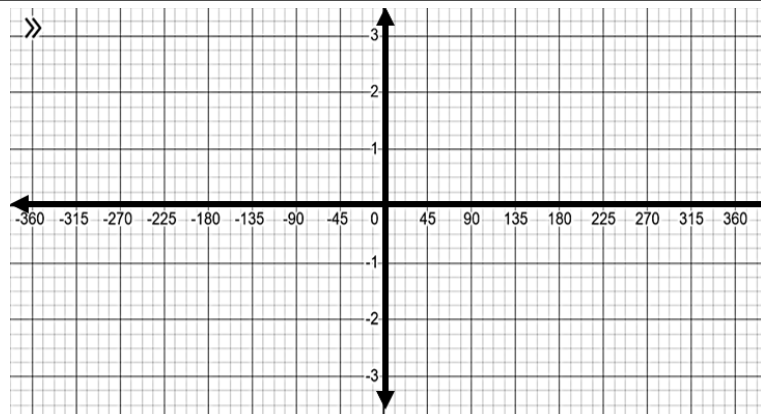
Period:

Asymptotes:

x -intercepts:

Midline:

Transformations:



14. Find the period, asymptotes, relative extrema, and midline of $y = \sec \frac{1}{3}x$. Then graph the function.

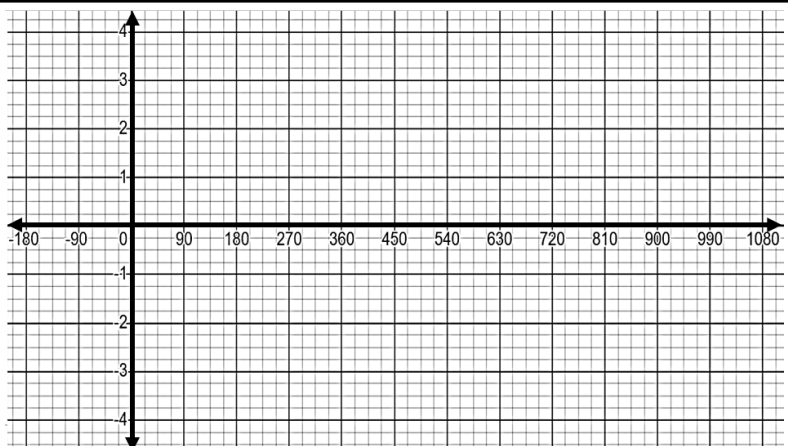
Period:

Asymptotes:

relative maxima:

relative minima:

Midline:



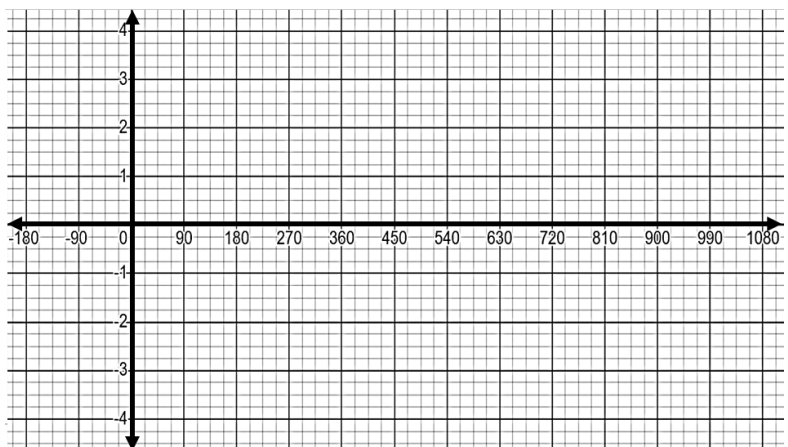
11. Find the period, asymptotes, x -intercepts and midline of $y = \cot \frac{1}{2}x$. Then graph the function.

Period:

Asymptotes:

x -intercepts:

Midline:



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توجيهات للتحسين (Improvement Guidance)	نقاط القوة (Strengths)	المستوى	المعيار (Criterion)
Continue analyzing the given data carefully and ensure no information is overlooked. استمري في تحليل المعطيات بدقة وتأكدي من عدم إغفال أي معلومة.	You clearly identified the given data and the required outcome and understood the relationship between them. حددت المعطيات والمطلوب وفهمت العلاقة بدقة.	Tier 1 Mastery	فهم المسألة (Understanding the Problem)
Review the problem twice before solving it to ensure a precise understanding of the required outcome. راجعي المسألة مرتين قبل الحل للتأكد من فهم المطلوب بشكل دقيق.	You identified most of the given data and the required outcome with slight confusion. حددت معظم المعطيات والمطلوب مع بعض الالتباس.	Tier 2 Developing	
Try identifying key terms in the problem to help you understand it. حاولي تحديد الكلمات المفتاحية في المسألة للمساعدة في فهمها.	You found difficulty identifying the given data or required outcome. واجهت صعوبة في تحديد المعطيات أو المطلوب.	Tier 3 Emerging	
Use a diagram or a table to clarify the relationship between the given data. استخدمي رسماً أو جدولاً لتوضيح العلاقة بين المعطيات.	Difficulty in identifying data or required outcome. صعوبة في تحديد المعطيات والمطلوب.	Tier 4 Pre-Emerging	
Maintain accuracy in calculations and ensure you review your steps. حافظي على الدقة في الحسابات وتأكدي من مراجعة خطواتك.	You performed accurate calculations and followed steps correctly. نفذت الخطوات والعمليات بدقة.	Tier 1 Mastery	تنفيذ الحل (Implementing the Solution)
Check each solving step after performing an operation to avoid minor mistakes. تحققي من خطوات الحل بعد كل عملية لتجنب الأخطاء البسيطة.	You made minor calculation errors or unclear steps. بعض الأخطاء البسيطة ظهرت في الحسابات أو عرض الخطوات.	Tier 2 Developing	
Try breaking down calculations into smaller steps to simplify execution. حاولي تبسيط العمليات الحسابية وتقسيمها إلى خطوات صغيرة لتسهيل التنفيذ.	Steps partially correct but incomplete. خطوات صحيحة جزئياً لكنها ناقصة.	Tier 3 Emerging	
Use colours or symbols to organize the solution steps. استخدمي الألوان أو الرموز لتنظيم خطوات الحل.	Difficulty executing even basic steps. صعوبة في تنفيذ الخطوات الأساسية.	Tier 4 Pre-Emerging	
Keep verifying the reasonableness of the solution and interpreting its results. استمري في التحقق من معقولية الحل وتفسير نتائجه.	You verified correctness and interpreted the solution well. تحققت من الحل وفسرته بشكل صحيح.	Tier 1 Mastery	التحقق من الحل وتفسيره (Verifying and Interpreting the Solution)
Try explaining the solution to someone else to clarify your understanding. حاولي شرح الحل لشخص آخر لتوضيح فهمك له.	You checked the solution but interpretation incomplete. تحققت من الحل دون تفسير كامل.	Tier 2 Developing	
Compare your answer to the given data to verify its reasonableness. قارني إجابتك بالمعطيات الأولية للتحقق من معقوليتها.	You attempted verification when prompted. حاولت التحقق من الحل عند التوجيه.	Tier 3 Emerging	
Use the strategy “What do I know? What do I need to find.” استخدمي أسلوب: "ماذا أعرف؟ ماذا أحتاج أن أجد؟"	You did not verify correctness of the solution. لم تتحقي من صحة الإجابة.	Tier 4 Pre-Emerging	