حل تدريبات الدرس الأول Operations on functions من الوحدة الرابعة منهج ريفيل

Lesson 4-1

Operations on Functions

KeyConcept Operations on Functions

Operation	Definition	Example Let $f(x) = 2x$ and $g(x) = -x + 5$.
Addition $+$	$(f + g)(x) = f(x) + g(x)$	$2x + (-x + 5) = x + 5$
Subtraction $-$	$(f - g)(x) = f(x) - g(x)$	$2x - (-x + 5) = 3x - 5$
Multiplication $\times$	$(f \cdot g)(x) = f(x) \cdot g(x)$	$2x(-x + 5) = -2x^2 + 10x$
Division $\div$	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$	$\frac{2x}{-x+5}, x \neq 5$

Example 1 Add and Subtract Functions

Given $f(x) = x^2 - 4$ and $g(x) = 2x + 1$, find each function.

a. $(f + g)(x)$

$= f(x) + g(x)$

$= (x^2 - 4) + (2x + 1)$

$= x^2 - 4 + 2x + 1 \leftarrow$

$= x^2 + 2x - 3 \leftarrow$

polynomial

Domain $(-\infty, \infty)$

All Real numbers

b. $(f - g)(x)$

$= f(x) - g(x)$

$= x^2 - 4 - (2x + 1)$

$= x^2 - 4 - 2x - 1$

$= x^2 - 2x - 5 \leftarrow$

Domain $(-\infty, \infty)$

or All Real numbers

Check

Given $f(x) = x^2 + 5x - 2$ and $g(x) = 3x - 2$, find each function.

1A. $(f + g)(x)$

$= f(x) + g(x)$

$= x^2 + 5x - 2 + 3x - 2$

$= x^2 + 8x - 4$

1B. $(f - g)(x)$

$= f(x) - g(x)$

$= x^2 + 5x - 2 - (3x - 2)$

$= x^2 + 5x - 2 - 3x + 2$

$= x^2 + 2x + 0$

$= x^2 + 2x$

$(a + b)(c + d)$

$= ac + ad + bc + bd$

Example 2 Multiply and Divide Functions

Given $f(x) = x^2 + 7x + 12$ and $g(x) = 3x - 4$, find each function. Indicate any restrictions in the domain or range.

a. $(f \cdot g)(x)$ $= f(x) \cdot g(x)$

$= (x^2 + 7x + 12)(3x - 4)$

$= 3x^3 - 4x^2 + 21x^2 - 28x + 36x - 48$

$= 3x^3 + 17x^2 + 8x - 48$

$1x^2 \cdot 3x^1 = 3x^3$

$x^2 \cdot (-4) = -4x^2$

$7x^1 \cdot 3x^1 = 21x^2$

$7x \cdot (-4) = -28x$

$12 \cdot (3x) = 36x$

$12(-4) = -48$

Domain $(-\infty, \infty)$, All Real number

b. $\left(\frac{f}{g}\right)(x)$ $= \frac{f(x)}{g(x)} = \frac{x^2 + 7x + 12}{3x - 4}$

$3x - 4 = 0$

$3x = 4$ $\quad \div 3$

$\boxed{x = \frac{4}{3}}$

$-\infty \quad \frac{4}{3} \quad \infty$

Because $x = \frac{4}{3}$ makes the denominator equal zero, $\frac{4}{3}$ is excluded from the domain of $\left(\frac{f}{g}\right)(x)$.

Domain $(-\infty, \frac{4}{3}) \cup (\frac{4}{3}, \infty)$ ✓

or $\{x \mid x \in R, x \neq \frac{4}{3}\}$ ✓

Check

Given $f(x) = x^2 - 7x + 2$ and $g(x) = x + 4$, find each function.

2A. $(f \cdot g)(x)$

2B. $\left(\frac{f}{g}\right)(x)$ $= \frac{f(x)}{g(x)}$

$= \frac{x^2 - 7x + 2}{x + 4}$

$x + 4 = 0$

$x = -4$

Domain All Real number except $x = -4$

or

$\{x \mid x \in R, x \neq -4\}$

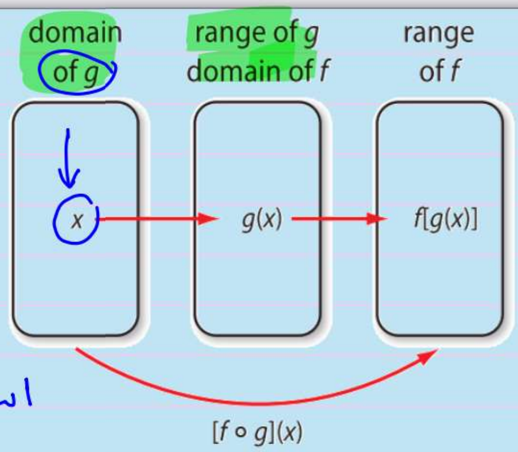
or $(-\infty, -4) \cup (-4, \infty)$

KeyConcept Composition of Functions

Words Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composition function $f \circ g$ can be described by

$$[f \circ g](x) = f[g(x)].$$

Model



$[f \circ g](x) = f[g(x)]$

استبدال كل x في $f(x)$ بـ الدالة $g(x)$

Example 4 Compose Functions by Using Ordered Pairs

Given f and g, find $[f \circ g](x)$ and $[g \circ f](x)$. State the domain and range for each.

$f = \{(1, 12), (10, 11), (0, 13), (9, 7)\}$ $g = \{(4, 1), (5, 0), (13, 9), (12, 10)\}$

Domain $\{1, 10, 0, 9\}$ Range $\{12, 11, 13, 7\}$

Domain of $g(x) = \{4, 5, 13, 12\}$

Range of $g(x) = \{1, 0, 9, 10\}$

Part A Find $[f \circ g](x)$ and $[g \circ f](x)$.

$[f \circ g](x) = f(g(x))$

$g(4) = 1$

$f(g(4)) = f(1) = 12$ (4, 12)

$g(5) = 0$

$f(g(5)) = f(0) = 13$ (5, 13)

$g(13) = 9$

$f(g(13)) = f(9) = 7$ (13, 7)

$g(12) = 10$

$f[g(12)] = f(10) = 11$ (12, 11)

Domain $(f \circ g)(x) = \{4, 5, 13, 12\}$

Range $= \{12, 13, 7, 11\}$

$(g \circ f)(x) = g[f(x)]$

$f(1) = 12$

$g[f(1)] = g(12) = 10$ $\boxed{(1, 10)}$

$f(10) = 11$

$g[f(10)] = g(11)$ undefined

$f(0) = 13$

$g[f(0)] = g(13) = 9$ $\boxed{(0, 9)}$

$f(9) = 7$

$g[f(9)] = g(7)$ undefined

Because 11 and 7 are not in the domain of g, $g \circ f$ is undefined for $x = 11$ and $x = 7$. So, $g \circ f = \{(1, 10), (0, 9)\}$.

$g \circ f$
Domain $\{1, 0\}$
Range $\{10, 9\}$

Part B State the domain and range.

$[f \circ g](x)$: The domain is the x-coordinates of the composed function, so D = $\{4, 5, 13, 12\}$. The range is the y-coordinates of the composed function, so R = $\{12, 13, 7, 11\}$.

$[g \circ f](x)$: The domain is the x-coordinates of the composed function, so D = $\{1, 0\}$. The range is the y-coordinates of the composed function, so R = $\{10, 9\}$

Example 5 Compose Functions

Given $f(x) = 2x - 5$ and $g(x) = 3x$, find $[f \circ g](x)$ and $[g \circ f](x)$. State the domain and range for each.

Part A Find $[f \circ g](x)$ and $[g \circ f](x)$.

$[f \circ g](x) = f(g(x))$
$= f(3x)$
$= 2(3x) - 5$
$= 6x - 5$

Domain
All Real numbers

$[g \circ f](x) = g(f(x))$
$= g(2x-5)$
$= 3(2x-5)$
$= 6x - 15$

Domain All Real numbers

Part B State the domain and range.

Because $[f \circ g](x)$ and $[g \circ f](x)$ are both linear functions with nonzero slopes, D = {all real numbers} and R = {all real numbers} for both functions.

Check

For each pair of functions, find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

A. $f(x) = \{(3, -2), (-1, -5), (4, 7), (10, 8)\}$, $g(x) = \{(4, 3), (2, -1), (9, 4), (3, 10)\}$

$[f \circ g](x) = f(g(x))$

$g(4) = 3 \Rightarrow f(g(4)) = f(3) = -2$ $(4, -2)$
$g(2) = -1 \Rightarrow f(g(2)) = f(-1) = -5$ $(2, -5)$
$g(9) = 4 \Rightarrow f(g(9)) = f(4) = 7$ $(9, 7)$
$g(3) = 10 \Rightarrow f(g(3)) = f(10) = 8$ $(3, 8)$

Domain of $(f \circ g)(x)$ $\{4, 2, 9, 3\}$
Range of $(f \circ g)(x) = \{-2, -5, 7, 8\}$

واجب
$[g \circ f](x) = ?$

B. $f(x) = x^2 + 2$ and $g(x) = x - 6$

$[f \circ g](x) = f(g(x))$
$= f(x - 6)$
$= (x-6)^2 + 2$
$= x^2 - 12x + 36 + 2$
$= x^2 - 12x + 38$

$(x-6)^2 = (x-6)(x-6)$
$= x^2 - 6x - 6x + 36$
$= x^2 - 12x + 36$

$(a-b)^2 = a^2 - 2ab + b^2$
$(x-6)^2 = x^2 - 12x + 36$

Domain All Real numbers

Given $f(x) = -x + 1$ and $g(x) = 2x^3 - x$, find $[f \circ g](x)$ and $[g \circ f](x)$. State the domain and range for each.

$[f \circ g](x) =$?

$[g \circ f](x) =$?

Domain of $[f \circ g](x)$: ?

Domain of $[g \circ f](x)$: ?

Range of $[f \circ g](x)$: ?

Range of $[g \circ f](x)$: ?

$(g \circ f)(x) = g[f(x)]$
$= g(-x+1)$
$= 2(-x+1)^3 - (-x+1)$
$= 2(-x+1)^3 + x - 1$

Domain All Real number

All Real number

Real-World Example 4 Use Composition of Functions

SHOPPING A new car dealer is discounting all new cars by 12%. At the same time, the manufacturer is offering a AED 1500 rebate on all new cars. Mr. Ahmed is buying a car that is priced AED 24,500. Will the final price be lower if the discount is applied before the rebate or if the rebate is applied before the discount?

discount $= \frac{12}{100} \times x = 0.12x$

The original price is discounted by 12%. $d(x) = x - 0.12x = 0.88x$

There is a AED 1500 rebate on all new cars. $r(x) = x - 1500$

If the discount is applied *before* the rebate, then the final price of Mr. Ahmed's new car is represented by $[r \circ d](24{,}500)$.

$[r \circ d](x) = r[d(x)] = r(0.88x)$
$= (0.88x) - 1500$
$= 0.88x - 1500$

$[r \circ d](24500) = 0.88(24500) - 1500$
$= 20060$

If the rebate is given *before* the discount is applied, then the final price of Mr. Ahmed's car is represented by $[d \circ r](24{,}500)$.

$[d \circ r](x) = d[r(x)]$

$[d \circ r](x) = d[r(x)]$
$= d[x - 1500]$
$= 0.88(x - 1500)$

$(d \circ r)(24500) = 0.88(24500 - 1500)$
$= 20240$

$[r \circ d](24{,}500) =$ 20060 and $[d \circ r](24{,}500) =$ 20240. So, the final price of the car is less when the discount is applied before the rebate.

Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 2x$

$g(x) = -4x + 5$

$$(f+g)(x) = f(x)+g(x) = (2x)+(-4x+5) = 2x-4x+5 = -2x+5$$

$$(f-g)(x) = f(x)-g(x) = (2x)-(-4x+5) = 2x+4x-5 = 6x-5$$

$$(f\cdot g)(x) = f(x)\cdot g(x) = (2x)\cdot(-4x+5) = -8x^2+10x$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x}{-4x+5} = \frac{2x}{-4x+5},\ x \neq \frac{5}{4}$$

===

4. $f(x) = x^2$ $g(x) = x - 5$

$(f+g)(x) = f(x) + g(x)$
$= x^2 + x - 5$
Domain All Real numbers $(-\infty,\infty)$

$(f-g)(x) = f(x) - g(x)$
$= x^2 - (x-5)$
$= x^2 - x + 5$ $(-\infty,\infty)$

$(f\cdot g)(x) = f(x)\cdot g(x)$
$= x^2(x-2)$
$= x^3 - 2x^2$ $(-\infty,\infty)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \frac{x^2}{x-5}$

$x-5=0 \Rightarrow x=5$

Domain All Real numbers except $x=5$

or $(-\infty,5)\cup(5,\infty)$

or $\{x \mid x\in R,\ x\neq 5\}$

6. $f(x) = 3x^2 - 4$ $g(x) = x^2 - 8x + 4$

☆ $(f-g)(x) = f(x) - g(x)$
$= 3x^2 - 4 - (x^2 - 8x + 4)$
$= 3x^2 - 4 - x^2 + 8x - 4$
$= 2x^2 + 8x - 8$
Domain All Real Numbers

☆ $(f\cdot g)(x) = f(x)\cdot g(x)$
$= (3x^2-4)(x^2-8x+4)$
$= 3x^4 - 24x^3 + 12x^2 - 4x^2 + 32x - 16$
$= 3x^4 - 24x^3 + 8x^2 + 32x - 16$
Domain All Real numbers

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \frac{3x^2-4}{x^2-8x+4}$

$x^2 - 8x + 4 = 0$
$x = 4+2\sqrt{3}$
$x = 4-2\sqrt{3}$

$\{x \mid x\in R,\ x\neq 4\pm 2\sqrt{3}\}$
Domain

7. FINANCE Trevon opens a checking account that he only uses to pay fixed bills, which are expenses that are the same each month, such as car loans or rent. The checking account has an initial balance of \$1750 and Trevon deposits \$925 each month. The balance of the account can be modeled by $a(x) = 1750 + 925x$, where x is the number of months since the account was opened. The total of Trevon's fixed bills is modeled by $b(x) = 840x$. Define and graph the function that represents the account balance after he pays his bills.

a. Identify and write a new function to represent the account balance.

b. Graph the combined function.

Solution:

a. The account balance after paying his bills is represented by subtracting the bill function from the account balance function.

$$(a-b)(x) = a(x) - b(x)$$

b. Use the slope, 85, and y-intercept, 1750, of the new function to graph the account balance after paying bills function.

For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each.

9. $f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\}$
$g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\}$

10. $f = \{(-7, 0), (4, 5), (8, 12), (-3, 6)\}$
$g = \{(6, 8), (-12, -5), (0, 5), (5, 1)\}$

Find $[f \circ g](x)$ and $[g \circ f](x)$. State the domain and range for each.

14. $f(x) = -3x$
$g(x) = -x + 8$

15. $f(x) = x^2 + 6x - 2$
$g(x) = x - 6$

If $f(x) = 3x$, $g(x) = x + 4$, and $h(x) = x^2 - 1$, find each value.

21. $f[g(1)]$

$g(1) = 1+4 = 5$

$f(g(1)) = f(5)$

$= 3(5)$

$= 15$

22. $g[h(0)]$

$h(0) = 0^2 - 1$

$= -1$

$g[h(0)] = g(-1)$

$= (-1)+4$

$= 3$

23. $g[f(-1)]$

$f(-1) = 3(-1)$

$= -3$

$g[f(-1)] = g(-3)$

$= -3+4$

$= 1$

24. $h[f(5)]$

25. $g[h(-3)]$

26. $h[f(10)]$

27. $f[h(8)]$

28. $[f \circ (h \circ g)](1)$

$f[h(g(1))]$

$g(1) = 1+4 = 5$

$f[h(5)]$

$h(5) = 5^2 - 1 = 24$

$f[24] = 3(24) = 72$

29. $[f \circ (g \circ h)](-2)$

30. $h[f(-6)]$

31. $f[h(0)]$

32. $f[g(7)]$