

## مراجعة الوحدة السابعة Rational functions مع أسئلة امتحانات سابقة



### تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف الثاني عشر العام ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

تاريخ إضافة الملف على موقع المناهج: 2026-04-06 11:48:44

ملفات اكتب للمعلم اكتب للطالب الاختبارات الكترونية الاختبارات ا حلول ا عروض بوربوينت ا أوراق عمل  
منهج انجليزي ا ملخصات وتقارير ا مذكرات وبنوك ا الامتحان النهائي للمدرس

المزيد من مادة  
رياضيات:

### التواصل الاجتماعي بحسب الصف الثاني عشر العام



صفحة المناهج  
الإماراتية على  
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

المواد على تلغرام

### المزيد من الملفات بحسب الصف الثاني عشر العام والمادة رياضيات في الفصل الثالث

مذكرة شاملة الوحدة العاشرة الدوال المثلثية

1

مذكرة شاملة الوحدة التاسعة الدوال والعلاقات النسبية

2

مقرر الوحدات والدروس المطلوبة في الفصل الثالث منهج بريدج Bridge

3

ملزمة أوراق عمل دروس وحدات الفصل الثالث منهج ريفيل

4

حل مراجعة امتحانية وفق الهيكل الوزاري باللغة الانجليزية

5

## Chapter 7: Rational Functions

### 7.1: Multiplying and Dividing Rational Expressions

#### Lesson objectives:

1. Simplify rational expressions.
2. Simplify rational expressions by multiplying and dividing.

#### Lesson vocabulary:

- rational expressions
- Complex functions



**Solve the following equation in the simplest form :**

1)  $\frac{9}{11} = \frac{7}{8}r$

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## Example 2 Simplify by Using $-1$

Simplify  $\frac{(6x^2 - 5xy)(x + 2y)}{(x + y)(5y - 6x)}$ .

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### Check

Select the simplified form of  $\frac{(7y - 3x)(5x - 1)}{(5x^3 + x^2)(3x - 7y)}$ .

A.  $-\frac{5x - 1}{x^2(5x + 1)}$

B.  $\frac{(7y - 3x)(5x - 1)}{x^2(5x + 1)(3x - 7y)}$

C.  $\frac{1}{x^2}$

D.  $\frac{5x - 1}{5x^3 + x^2}$

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## Learn Multiplying and Dividing Rational Expressions

The method for multiplying and dividing fractions also works with rational expressions.

### Key Concept • Multiplying Rational Expressions

**Words:** To multiply rational expressions, multiply the numerators and the denominators.

**Symbols:** For all rational expressions  $\frac{a}{b}$  and  $\frac{c}{d}$  with  $b \neq 0$  and  $d \neq 0$ ,  
 $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ .

### Key Concept • Dividing Rational Expressions

**Words:** To divide rational expressions, multiply the dividend by the reciprocal of the divisor.

**Symbols:** For all rational expressions  $\frac{a}{b}$  and  $\frac{c}{d}$  with  $b \neq 0$ ,  $c \neq 0$ , and  $d \neq 0$ ,  
 $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ .

A **complex fraction** is a rational expression with a numerator and/or denominator that is also a rational expression. To simplify a complex fraction, first rewrite it as a division expression.

### Example 3 Multiply and Divide Rational Expressions

Simplify each expression.

a.  $\frac{3x}{8y} \cdot \frac{12x^2y}{9xy^3}$

b.  $\frac{10d^5}{6cd} \div \frac{30c^3d^2}{4c}$



### DIFFERENTIATION ACTIVITY



**LEVEL 1** Simplify each expression, and state when the original expression is undefined.

$\frac{x(x-3)(x+6)}{x^2+x-12}$

**LEVEL 2** Simplify each expression.

$\frac{16-c^2}{c^2+c-20}$

**LEVEL 3** Simplify each expression.

$\frac{14xy^2z^3}{21w^4x^2z} \cdot \frac{7wxyz}{12w^2y^3z}$

### **Example 4** Multiply and Divide Polynomial Expressions

**GEOMETRY** A manufacturer that creates and sells rectangular planters wants to compare the areas of the top sections of two potential sizes of planters. For some given measure  $x$ , the area for the first planter is represented by  $\frac{x^2 + 15x + 50}{x + 2}$ , and the area for the second planter is represented by  $\frac{x^2 + 30x + 200}{x + 2}$ . Write and simplify an expression that represents the ratio of the area for the first planter to that of the second.

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### Check

**BEAUTY** A producer of beauty care products wants to compare the areas of two face cream containers. For some measure  $a$ , the area for the first container is represented by  $\frac{16a^2 + 40a + 25}{3a^2 - 10a - 8}$ . The area of the second container is represented by  $\frac{4a + 5}{a^2 - 8a + 16}$ . Write and simplify an expression that represents the ratio of the area of the first container to the area of the second container.

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### Example 5 Simplify Complex Fractions

Simplify  $\frac{\frac{3x}{x-y}}{\frac{6xy}{4x^2-4y^2}}$ .

### Check

Simplify  $\frac{\frac{x^2-9y^2}{xy}}{\frac{2x+6y}{x^2}}$ .



EXIT TICKET

Simplify each expression.

$$\frac{x(x-3)(x+6)}{x^2+x-12}$$

a)  $\frac{x(x+6)}{(x+4)}$

b)  $\frac{x(x+2)}{6(x+5)}$

c)  $\frac{(x+3)(x-2)}{4}$

d)  $-\frac{x+2}{x+4}$



CHALLENGE QUESTION

Find the expression that makes the following statement true for all values of  $x$  within the domain.

$$\frac{x-6}{x+3} \cdot \frac{?}{x-6} = x-2$$

a)  $(x+3)^2$

b)  $(x-2)(x-6)$

c)  $(x+3)(x-2)$

d)  $(x-6)(x+3)$

# Chapter 7: Rational Functions

## 7.2: Adding and Subtracting Rational Expressions

### Lesson objectives:

1. Simplify rational expressions by adding and subtracting.
2. Simplify complex functions.



Simplify each expression.

$\frac{x^2 - 5x - 14}{28 + 3x - x^2}$  \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

a)  $\frac{x(x-6)}{(x+4)}$       b)  $\frac{(x+2)}{(x+5)}$  \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

c)  $\frac{x+2}{x+4}$       d)  $-\frac{x+2}{x+4}$  \_\_\_\_\_  
\_\_\_\_\_

### Learn Adding and Subtracting Rational Expressions

#### Key Concept • Adding Rational Expressions

**Words:** To add rational expressions, find the least common denominator. Rewrite each expression with the LCD. Then add.

**Symbols:** For all rational expressions  $\frac{a}{b}$  and  $\frac{c}{d}$  with  $b \neq 0$  and  $d \neq 0$ ,  
$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}.$$

#### Key Concept • Subtracting Rational Expressions

**Words:** To subtract rational expressions, find the least common denominator. Rewrite each expression with the LCD. Then subtract.

**Symbols:** For all rational expressions  $\frac{a}{b}$  and  $\frac{c}{d}$  with  $b \neq 0$  and  $d \neq 0$ ,  
$$\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad - bc}{bd}.$$

**Example 1** Add and Subtract Rational Expressions with Monomial Denominators

Simplify each expression.

a.  $\frac{7a}{4b} + \frac{4c^2}{10}$

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b.  $\frac{2x}{9y} - \frac{7y}{6z}$

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**Example 2** Add and Subtract Rational Expressions with Polynomial Denominators

Simplify  $\frac{2x + 1}{x^2 + 2x - 15} - \frac{7}{5x - 15}$ .

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**Check**

Simplify  $\frac{3x}{4x^2 + 4} - \frac{2x^2}{x^4 - 1}$ .

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## DIFFERENTIATION ACTIVITY



**LEVEL 1** Simplify each expression.

$$\frac{3}{x} + \frac{5}{y}$$

**LEVEL 2** Simplify each expression.

$$\frac{12}{5y^2} - \frac{2}{5yz}$$

**LEVEL 3** Simplify each expression.

$$\frac{3}{w-3} - \frac{2}{w^2-9}$$

## Learn Simplifying Complex Fractions

Complex fractions can be simplified by simplifying the numerator and denominator separately and then simplifying the resulting expression. You can also simplify a complex fraction by finding the LCD of all of the denominators. Then, the denominators can all be eliminated by multiplying by the LCD.

**Example 4** Simplify Complex Fractions by Using Different LCDs

Simplify  $\frac{\frac{x}{y} + 1}{\frac{y}{x} - \frac{1}{y}}$ .

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**Check**

Simplify  $\frac{\frac{x}{xy} - 2}{2xy - \frac{3}{y}}$ .

A.  $\frac{2xy^2 - 4xy^3 + 6y - 3}{y^2}$

B.  $\frac{1 - 2y}{2xy^2 - 3}$

C.  $\frac{4xy^3 - 2xy^2 - 6y + 3}{6xy^2}$

D. 0

**Example 5** Simplify Complex Fractions by Using the Same LCD

Simplify  $\frac{\frac{x}{y} + \frac{2}{x}}{\frac{x}{2} - \frac{y}{x}}$ .

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## Check

Simplify  $\frac{1 + \frac{b^2}{a}}{\frac{a}{2} - \frac{a}{b}}$ .

A.  $\frac{2ab + 2b^3}{a^2b - 2a^2}$

B.  $\frac{ab - 2a + b^3 - 2b^2}{2b}$

C.  $-\frac{ab - 2a + b^3 - 2b^2}{a^2}$

D.  $\frac{2 + b^3}{a - a^2}$



EXIT TICKET

Simplify each expression.  $\frac{\frac{2}{a-1} + \frac{3}{a-4}}{\frac{6}{a^2-5a+4}}$

a)  $\frac{5a - 36}{60}$

b)  $\frac{5a - 6}{a^2}$

c)  $\frac{5a + 36}{60a^2}$

d)  $\frac{5a - 11}{6}$



CHALLENGE QUESTION

Find the slope of the line that passes through each pair of points.

$A\left(\frac{2}{p}, \frac{1}{2}\right)$  and  $B\left(\frac{1}{3}, \frac{3}{p}\right)$

a)  $-\frac{2}{3}$

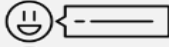
b)  $\frac{6}{5}$

c)  $-\frac{3}{2}$

d)  $\frac{11}{6}$



## TEACHER'S FEEDBACK:



NEXT STEP:

Solve the following questions showing full steps

1 Simplify each expression.

$$\frac{(x^2 - 9)(x^2 - z^2)}{4(x + z)(x - 3)}$$

a)  $\frac{x(x - 6)}{(x + 4)}$

b)  $\frac{x(x + 2)}{6(x + 5)}$

c)  $\frac{(x + 3)(x - z)}{4}$

d)  $-\frac{x + 2}{x + 4}$

2 Simplify each expression.

$$\frac{x(x - 3)(x + 6)}{x^2 + x - 12}$$

a)  $\frac{x(x + 6)}{(x + 4)}$

b)  $\frac{x(x + 2)}{6(x + 5)}$

c)  $\frac{(x + 3)(x - 2)}{4}$

d)  $-\frac{x + 2}{x + 4}$

3

Hachi needs to buy fencing for her rectangular garden.

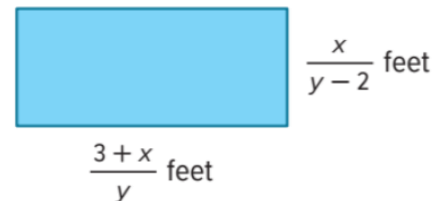
Write an expression, in simplest form, that represents the number of feet of fencing Hachi needs.

a)  $\frac{2xy + (6 + 2y)(y - 2)}{y(y - 2)}$

b)  $\frac{2xy + (6 + 2x)(y - 2)}{y(y - 2)}$

c)  $\frac{2xy + (6 + 2x)(y + 2)}{y(y - 2)}$

d)  $\frac{2xy + (6 + 2x)(y - 2)}{(y - 2)}$



## Chapter 7: Rational Functions

### 7.3: Graphing Reciprocal Functions

#### Lesson objectives:

1. Graph Reciprocal Functions by making tables of values.
2. Graph and write reciprocal functions by using transformations.

#### Lesson vocabulary:

- Reciprocal functions.
- Vertical asymptote.
- Horizontal asymptote.
- Hyperbola.



MULTIPLE CHOICE Add. (Lesson 7-2)

$$\frac{x^2 - 1}{x^2 - 3x - 10} + \frac{x + 3}{x + 2}$$

A.  $\frac{2x + x + 2}{(x - 4)(x + 2)}$

B.  $\frac{3x^2 + 20x + 32}{(x + 2)(3x + 10)}$

C.  $\frac{x^2 + x + 2}{(x + 2)(x - 5)}$

D.  $\frac{2x^2 - 2x - 16}{(x - 5)(x + 2)}$

## Learn Graphing Reciprocal Functions

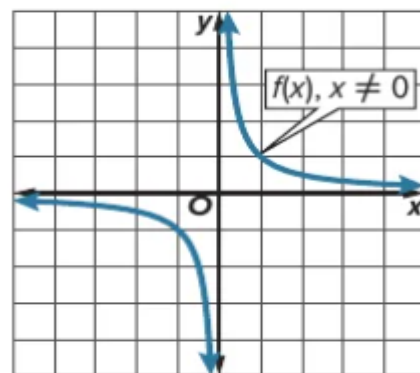
A **reciprocal function** has an equation of the form  $f(x) = \frac{n}{b(x)}$ , where  $n$  is a real number and  $b(x)$  is a linear expression that cannot equal 0.

The parent function of a reciprocal function is  $f(x) = \frac{1}{x}$ . A **vertical asymptote** is a vertical line that a graph approaches. A **horizontal asymptote** is a horizontal line that a graph approaches. Because the function  $f(x) = \frac{1}{x}$  is not defined when  $x = 0$ , there is a vertical asymptote at  $x = 0$ . The type of graph formed by a reciprocal function is called a **hyperbola**.

The domain of a function is limited to values for which the function is defined. Values for which the function is not defined are called **excluded values**.

### Key Concept • Reciprocal Functions

<b>Parent function</b>	$f(x) = \frac{1}{x}$
<b>Type of graph</b>	hyperbola
<b>Domain and range</b>	all nonzero real numbers
<b>Asymptotes</b>	$x = 0$ and $f(x) = 0$
<b>Intercepts</b>	none
<b>Not defined</b>	$x = 0$



A reciprocal function has two asymptotes, which are lines that a graph approaches. The vertical asymptote is determined by the excluded value of  $x$ , and the horizontal asymptote is determined by the value that is undefined for  $f(x)$ .

For a reciprocal function in the form  $f(x) = \frac{n}{b(x)}$ , the horizontal asymptote is  $f(x) = 0$  because there is no value of  $x$  that will result in  $f(x) = 0$ . For a reciprocal function of the form  $f(x) = \frac{n}{b(x)} + k$ , where  $k$  is a constant, the horizontal asymptote is  $f(x) = k$ .

**Example 1** Limitations on the Domains of Reciprocal Functions

Determine the excluded value of  $x$  for each function.

a.  $g(x) = \frac{6}{x}$

b.  $g(x) = \frac{2}{x-7}$

c.  $g(x) = \frac{-5}{3x+4}$



**DIFFERENTIATION ACTIVITY**

Determine the excluded value of  $x$  for each function.

**LEVEL 1**  $f(x) = \frac{5}{x}$



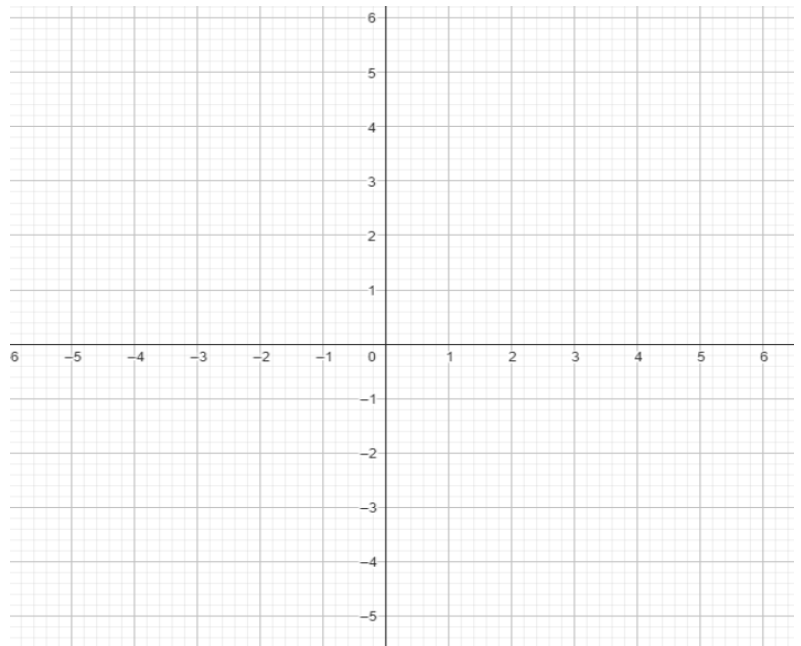
**LEVEL 2**  $f(x) = \frac{10}{x-3}$

**LEVEL 3**  $g(x) = \frac{5}{7x-9}$

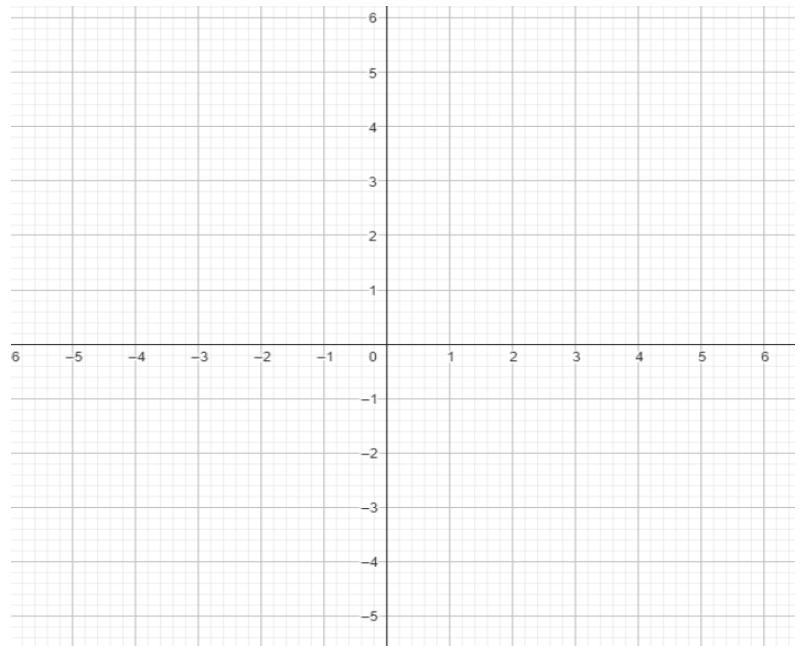
**Example 2** Graph a Reciprocal Function by Using a Table

Identify the asymptotes, domain, and range of each function. Then graph the function and identify its intercepts.

1  $f(x) = \frac{1}{x - 1}$




2  $g(x) = \frac{1}{2x - 5} + 2$




## Key Concept • Transformations of Reciprocal Functions

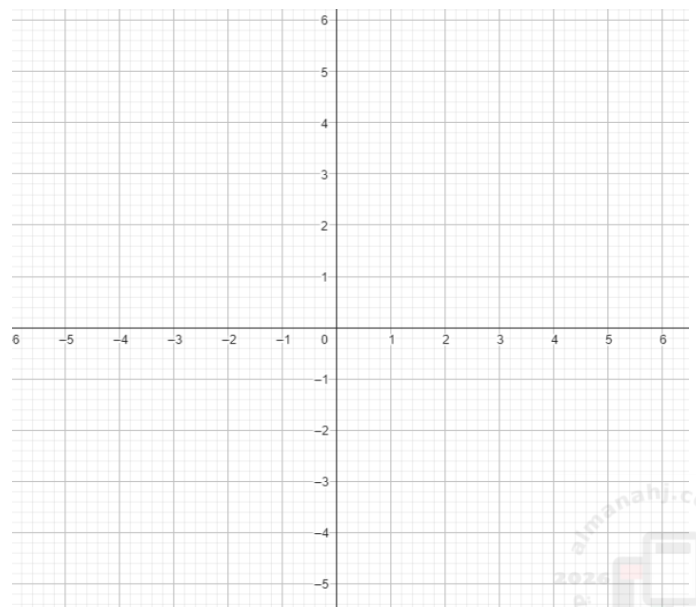
$$g(x) = \frac{a}{x-h} + k$$

<b><math>h</math> – horizontal translation</b>	If $h > 0$ , the graph of $f(x)$ is translated $h$ units right. If $h < 0$ , the graph of $f(x)$ is translated $ h $ units left. The <i>vertical asymptote</i> is at $x = h$ .
<b><math>k</math> – vertical translation</b>	If $k > 0$ , the graph of $f(x)$ is translated $k$ units up. If $k < 0$ , the graph of $f(x)$ is translated $ k $ units down. The <i>horizontal asymptote</i> is at $f(x) = k$ .
<b><math>a</math> – orientation and shape</b>	If $ a  > 1$ , the graph is stretched vertically. If $0 <  a  < 1$ , the graph is compressed vertically.

## Check

Consider  $g(x) = \frac{3}{x} + 1$ . Identify features of  $g(x)$ .

Vertical Asymptote	
Horizontal Asymptote	
Domain	
Range	
y- intercept	
x- intercept	


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### Example 5 Write a Reciprocal Function from a Graph

Identify the values of  $a$ ,  $h$ , and  $k$ . Then write a function for the graph  $g(x) = \frac{a}{x-h} + k$ .

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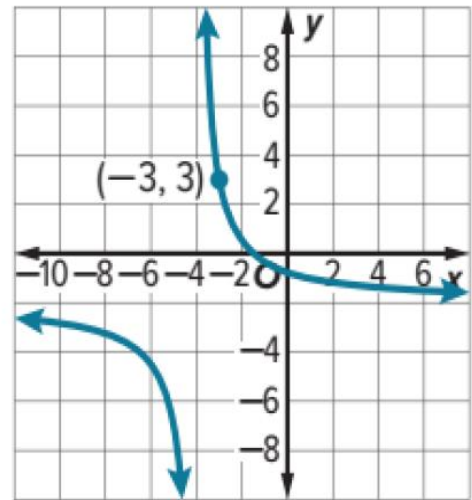
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### Check

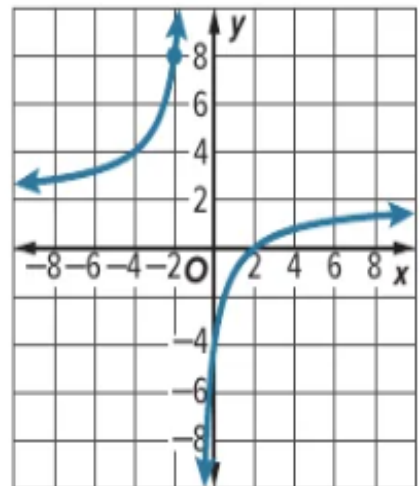
Identify the values of  $a$ ,  $h$ , and  $k$  for the graph.

$$a = \underline{\quad}, h = \underline{\quad}, k = \underline{\quad}$$

Write a function for the graph

$$g(x) = \frac{a}{x-h} + k.$$

$$g(x) = \underline{\hspace{2cm}}$$





## DIFFERENTIATION ACTIVITY

○	LEVEL 1
★	
○	LEVEL 2
★	★
○	LEVEL 3
★	★

**LEVEL 1** State the domain and range of  $g(x) = \frac{1}{x-1} + 3$ .

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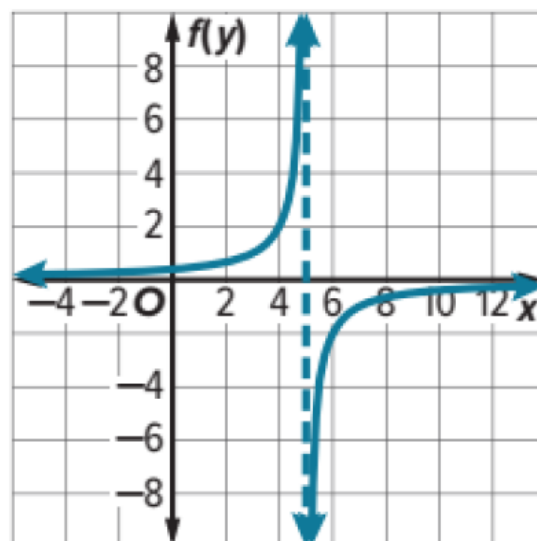
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**LEVEL 2** Identify the values of  $a$ ,  $h$ , and  $k$ . Then write a function for the graph  $g(x) = \frac{a}{x-h} + k$ .



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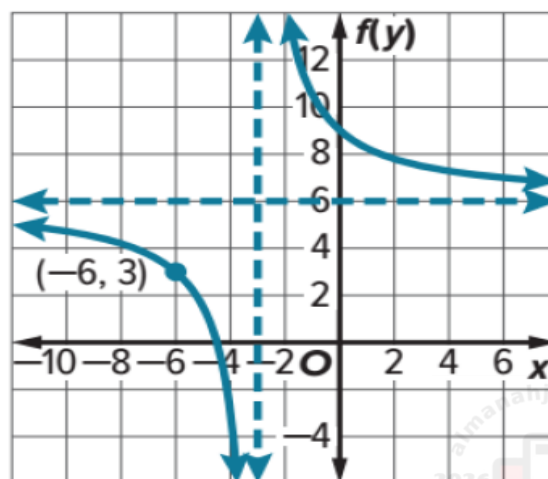
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**LEVEL 3** Identify the values of  $a$ ,  $h$ , and  $k$ . Then write a function for the graph  $g(x) = \frac{a}{x-h} + k$ .



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EXIT TICKET

Determine the excluded value of  $x$  for each function.

$$f(x) = \frac{2}{3x - 18}$$

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CHALLENGE  
QUESTION

**WHICH ONE DOESN'T BELONG?** Find the function that does not belong. Justify your conclusion.

$$f(x) = \frac{3}{x+1}$$

$$g(x) = \frac{x+2}{x^2+1}$$

$$h(x) = \frac{5}{x^2+2x+1}$$

$$j(x) = \frac{20}{x-7}$$

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## Chapter 7: Rational Functions

### 7.4: Graphing Rational Functions

#### Lesson objectives:

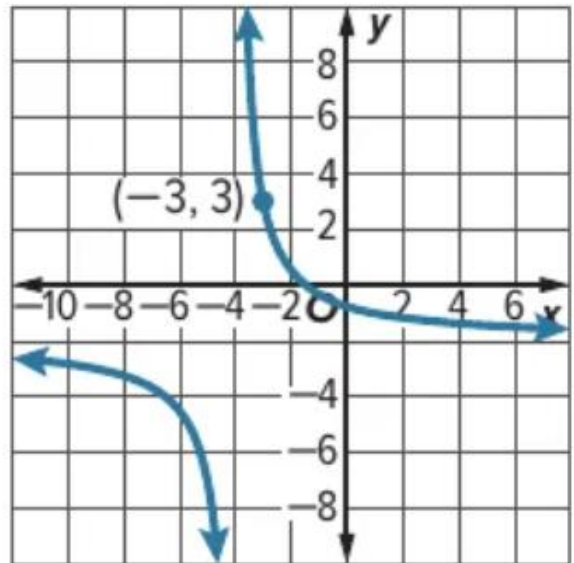
1. Graph and analyze rational functions with vertical and horizontal asymptotes.
2. Graph and analyze rational functions with oblique asymptotes.

#### Lesson vocabulary:

- Rational function.
- Oblique asymptote.
- Point discontinuity



Determine the domain and the range of  $g(x)$



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### Key Concept • Vertical and Horizontal Asymptotes

If  $f(x) = \frac{a(x)}{b(x)}$ ,  $a(x)$  and  $b(x)$  are polynomial functions with no common factors other than 1, and  $b(x) \neq 0$ , then:

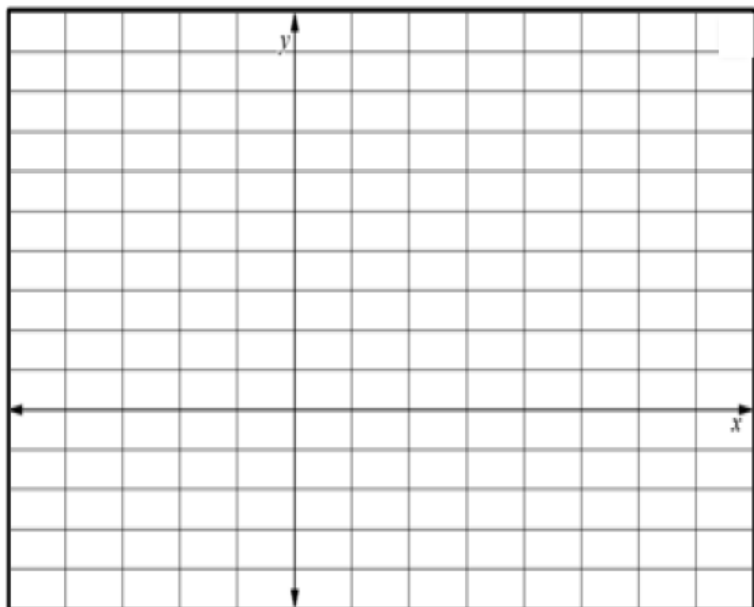
- $f(x)$  has a vertical asymptote whenever  $b(x) = 0$ .
- $f(x)$  has at most one horizontal asymptote.
  - If the degree of  $a(x)$  is greater than the degree of  $b(x)$ , there is no horizontal asymptote.
  - If the degree of  $a(x)$  is less than the degree of  $b(x)$ , the horizontal asymptote is the line  $y = 0$ .
  - If the degree of  $a(x)$  equals the degree of  $b(x)$ , the horizontal asymptote is the line  $y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}$ .

### Example 1 Graph with no horizontal asymptotes

Graph  $g(x) = \frac{x^3}{x + \frac{2}{3}}$

Find the zeros

Find the Asymptotes



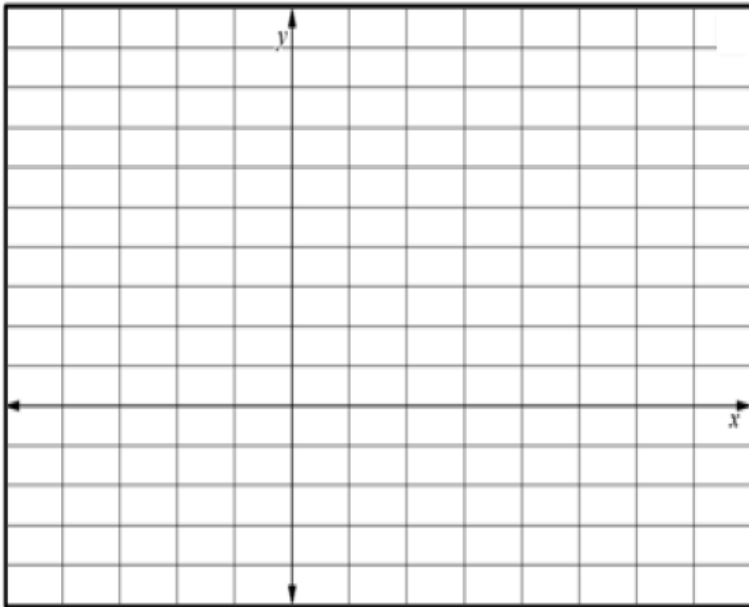
$x$	$g(x) = \frac{x^3}{x + \frac{2}{3}}$	$(x, y)$

CHECK:

Graph  $g(x) = \frac{(0.5x - 1)^3}{x}$ .

Find the zeros

Find the Asymptotes



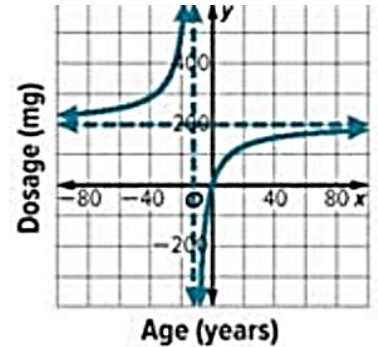
$x$	$g(x) = \frac{(0.5x - 1)^3}{x}$	$(x, y)$

## Example 2 Use Graphs of Rational Functions

PHARMACY Young's Rule can be used to estimate the dosage of medicine to give a child if you know the adult dosage of the same medicine. Young's rule can be written as  $y = \frac{Ax}{x+12}$  where  $A$  is the adult dosage in milligrams and  $x$  is the age of the child in years.

Graph the function for an adult dose of **200 mg**.

- Find vertical asymptote: \_\_\_\_\_.
- Find horizontal asymptote: \_\_\_\_\_.
- Find x- intercept: \_\_\_\_\_.
- Find y-intercept: \_\_\_\_\_.
- Find the dosage for a 12-year-old child if the adult dosage is 200mg.

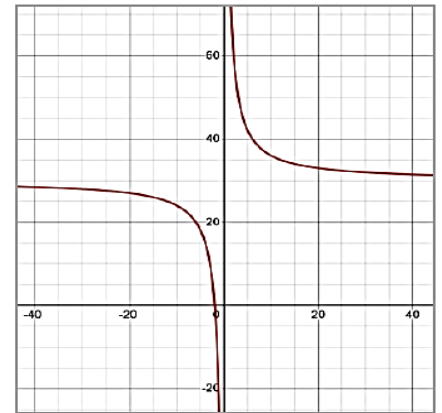


### CHECK:

**INTERNET** An Internet service provider charges customers a \$60 installation fee plus \$30 per month for Internet service. A function that models the average monthly cost is

$$f(x) = \frac{(60 + 30x)}{x} \quad \text{where } x \text{ is the number of months.}$$

- Find vertical asymptote: \_\_\_\_\_.
- Find horizontal asymptote: \_\_\_\_\_.
- Find x- intercept: \_\_\_\_\_.
- Find y-intercept: \_\_\_\_\_.
- Find the average monthly cost to a customer that has Internet service for 8 months.



### Critical Thinking Question:

**REGULARITY** Describe how you use the degrees of the numerator and denominator to learn about a function's asymptotes.

**Vertical asymptotes occurs at** \_\_\_\_\_

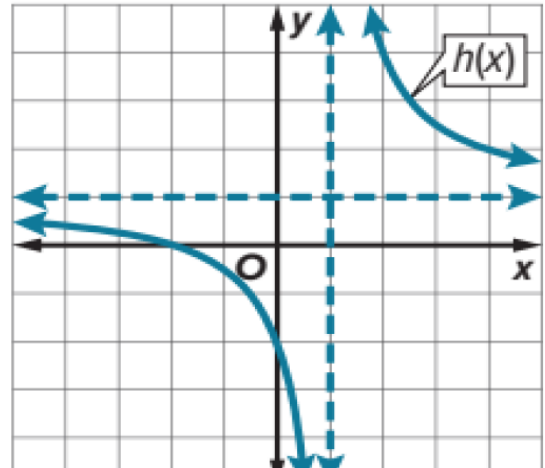
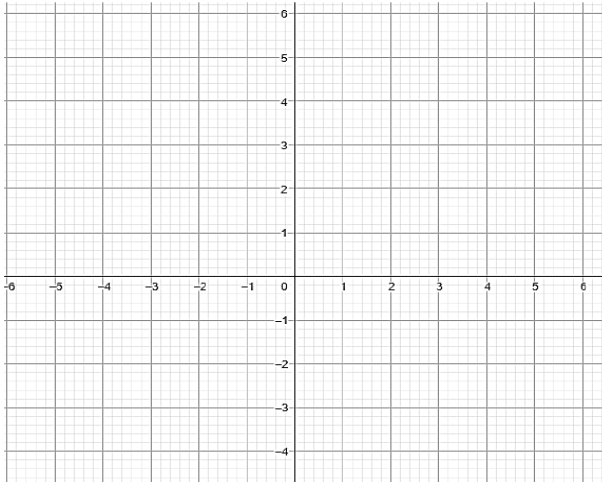
**Horizontal asymptotes :** \_\_\_\_\_

$\frac{p(x)}{q(x)}$	$p(x) > q(x)$	$p(x) < q(x)$	$p(x) = q(x)$
<b>Number of Horizontal asymptotes</b>	_____	_____	_____
<b>example</b>	graph of $y = \frac{x^2}{(x+1)}$	graph of $y = \frac{(x+1)}{x^2}$	graph of $y = \frac{(x+1)}{(x-2)}$

### Example 3 Compare Rational Functions

Consider  $g(x) = \frac{x-2}{2x+2}$  and rational function  $h(x)$  shown in the graph.

#### Part A Graph $g(x)$ .



- Find the zeros of  $g(x)$ : \_\_\_\_\_.
- Find vertical asymptote of  $g(x)$ : \_\_\_\_\_.
- Find horizontal asymptote of  $g(x)$ : \_\_\_\_\_.


- Which function has the greater y-intercept? \_\_\_\_\_
- Compare the asymptotes of  $g(x)$  and  $h(x)$ :  
 \_\_\_\_\_  
 \_\_\_\_\_

## Graphing Rational Functions with Oblique Asymptotes

An **oblique asymptote**, or slant asymptote, is neither horizontal nor vertical.

### Key Concept • Oblique Asymptotes

If  $f(x) = \frac{a(x)}{b(x)}$ , where  $a(x)$  and  $b(x)$  are polynomial functions with no  $b(x)$  common factors other than  $1$  and  $b(x) \neq 0$ , then  $f(x)$  has an oblique asymptote if the degree of  $a(x)$  minus the degree of  $b(x)$  equals  $1$ .

The equation of the asymptote is  $f(x) = \frac{a(x)}{b(x)}$  with no remainder.

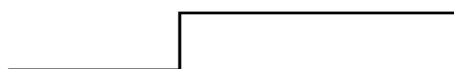
### Key Concept • Point Discontinuity

If  $f(x) = \frac{a(x)}{b(x)}$ ,  $b(x) \neq 0$ , and  $x - c$  is a factor of both  $a(x)$  and  $b(x)$ , then there is a point discontinuity at  $x = c$ .

### Example 4 Graph Oblique Asymptote

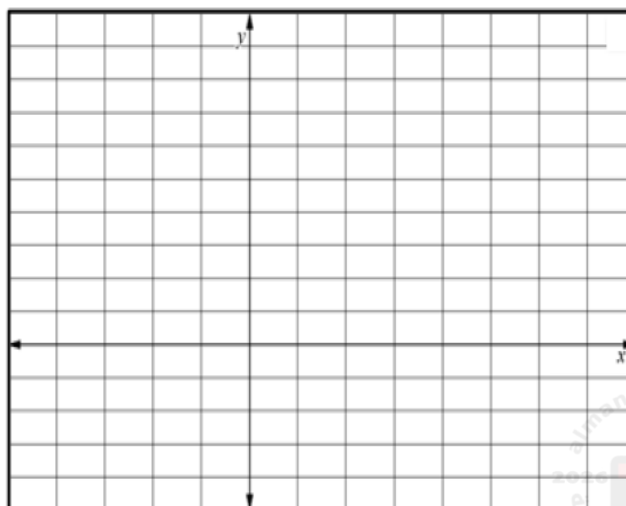
Find the zeros and asymptotes of each function. Then graph each function.  $f(x) = \frac{x^2 + 2x + 1}{x + 4}$

- Find the zeros of  $f(x)$ : \_\_\_\_\_.
- Find vertical asymptote of  $f(x)$ : \_\_\_\_\_.
- Find horizontal asymptote of  $f(x)$ : \_\_\_\_\_.
- Find oblique asymptote of  $f(x)$ : \_\_\_\_\_.



Graph  $f(x)$ .

$x$	$f(x) = \frac{x^2 + 2x + 1}{x + 4}$	$(x, y)$





## DIFFERENTIATION ACTIVITY



### LEVEL 1

Find the zeros and asymptotes of each function.

$$f(x) = \frac{(x - 4)^2}{x + 2}$$

- a) zero at  $x = 4$  vertical asymptote  $x = -2$
- b) zero at  $x = 4$  vertical asymptote  $x = 2$
- c) zero at  $x = -4$  vertical asymptote  $x = -2$
- d) zero at  $x = -4$  vertical asymptote  $x = -2$

### LEVEL 2

Find the asymptotes of each function.

$$f(x) = \frac{(x + 3)^2}{x - 5}$$

- a) vertical asymptote  $x = -5$  and oblique asymptote at  $y = x + 11$
- b) vertical asymptote  $x = 5$  and oblique asymptote at  $y = x + 11$
- c) vertical asymptote  $x = 5$  and oblique asymptote at  $y = x - 11$
- d) vertical asymptote  $x = 5$  and oblique asymptote at  $y = 2x$

### LEVEL 3

Find the zeros and asymptotes of each function.

$$f(x) = \frac{6x^2 + 4x + 2}{x + 2}$$

- a) oblique at  $y = x - 8$ , vertical asymptote  $x = -2$
- b) oblique at  $y = 6x - 8$ , vertical asymptote  $x = -2$
- c) oblique at  $y = 6x + 8$  vertical asymptote  $x = -2$
- d) oblique at  $y = 6x - 2$  vertical asymptote  $x = -2$

### Example 5 Graph with point of Discontinuity

Graph  $f(x) = \frac{x^2 - 4}{x + 2}$ . Find the point of discontinuity

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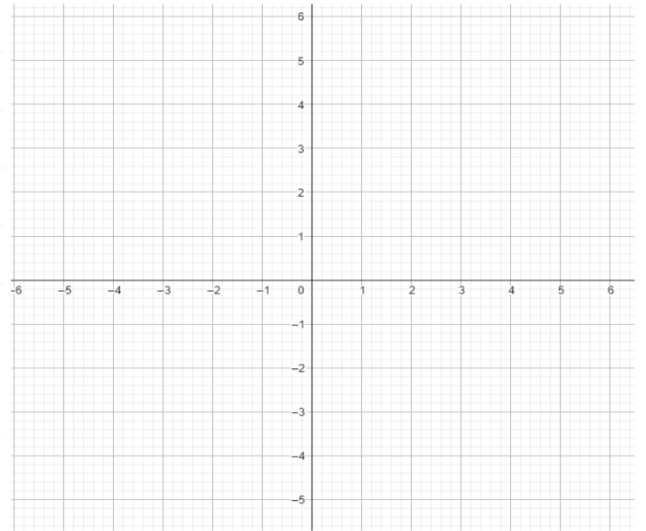
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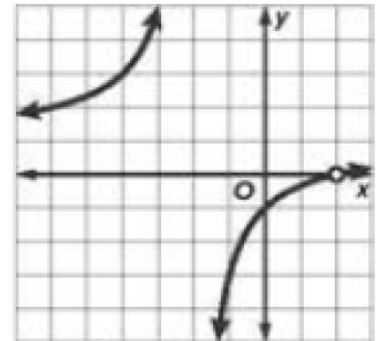
EXIT TICKET

Consider the graph of

$$f(x) = \frac{x^2 - 4x + 4}{x^2 - 4}$$

Find any asymptotes.

- a) Vertical Asymptote:  $x = -2, x = 2$   
Horizontal Asymptote:  $y = 1$
- b) Vertical Asymptote:  $x = 4$   
Horizontal Asymptote:  $y = 1$
- c) Vertical Asymptote:  $x = -2$   
Horizontal Asymptote:  $y = 1$
- d) Vertical Asymptote:  $x = 2$   
Horizontal Asymptote: *none*

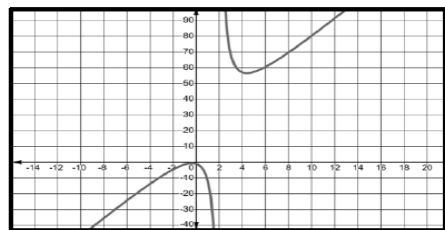


CHALLENGE QUESTION

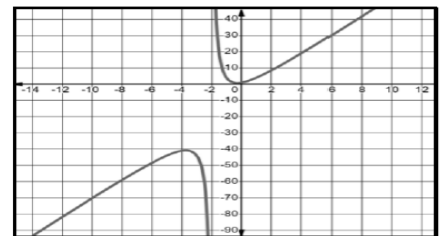
Which graph represents the function

$$f(x) = \frac{6x^2 + 4x + 2}{x + 2}$$

a)



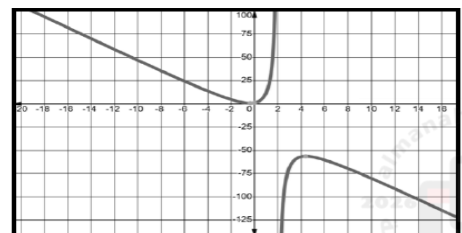
b)

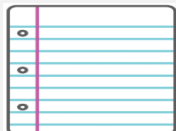


c)

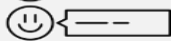
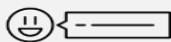


d)





## TEACHER'S FEEDBACK:



NEXT STEP:

- 1 Find the zeros and asymptotes of each function.

$$f(x) = \frac{3x^2 + 8}{2x - 1}$$

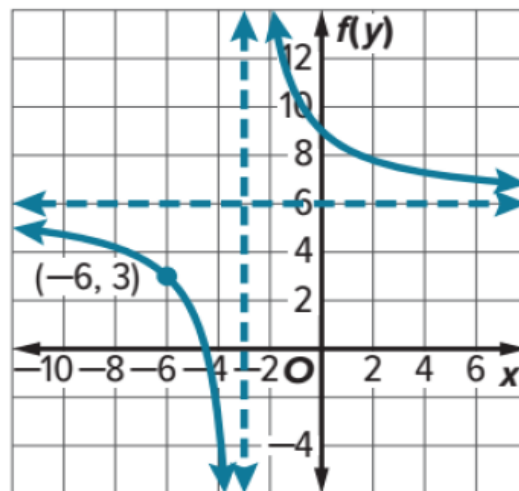
a) zero at  $x = -\frac{8}{3}$  vertical asymptote  $x = \frac{1}{2}$

b) zero at  $x = -3$  vertical asymptote  $x = -2$

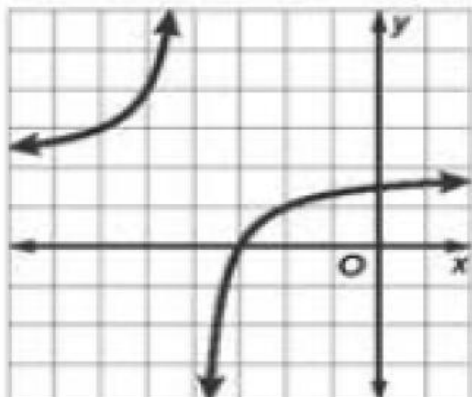
c) no zeros and vertical asymptote  $x = \frac{1}{2}$

d) zero at  $x = 3$  vertical asymptote  $x = 2$

- 2 Identify the values of  $a$ ,  $h$ , and  $k$ . Then write a function for the graph  $g(x) = \frac{a}{x-h} + k$ .



3  $f(x) = \frac{2x + 6}{x + 4}$



Describe the end behavior.

- a) As  $x \rightarrow \infty, f(x) \rightarrow \infty$   
As  $x \rightarrow -\infty, f(x) \rightarrow -\infty$
- b) As  $x \rightarrow \infty, f(x) \rightarrow 2$   
As  $x \rightarrow -\infty, f(x) \rightarrow 2$
- c) As  $x \rightarrow \infty, f(x) \rightarrow 4$   
As  $x \rightarrow -\infty, f(x) \rightarrow 4$
- d) As  $x \rightarrow \infty, f(x) \rightarrow 0$   
As  $x \rightarrow -\infty, f(x) \rightarrow 0$

## Chapter 7: Rational Functions

### 7.6 Solving Rational equations and inequalities

#### Lesson objectives:

1. Solve rational equations in one variable.
2. Solve rational inequalities in one variable.

#### Lesson vocabulary:

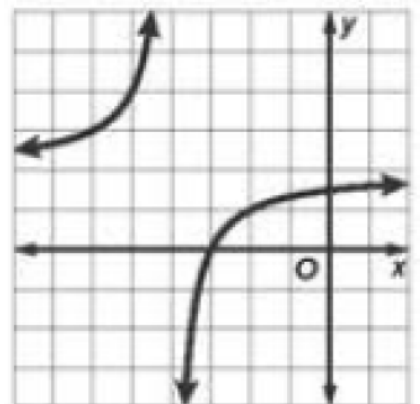
- Rational equations.
- rational inequality.



$$f(x) = \frac{2x + 6}{x + 4}$$

Find the domain.

- a)  $\{x|x \neq -3\}$
- b)  $\{x|x \neq 2\}$
- c)  $\{x|x \neq -4\}$
- d) *all real numbers*



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## Learn Solving Rational Equations

A **rational equation** contains at least one rational expression. To solve these equations, it is often easier to first eliminate the fractions. You can eliminate the fractions by multiplying each side of the equation by the least common denominator (LCD). Solving rational equations in this way can yield results that are not solutions of the original equation. You can identify these extraneous solutions by substituting each result into the original equation to see if it makes the equation true.

There are three types of problems that are commonly solved by using rational equations: mixture problems, uniform motion problems, and work problems.

### Example 1 Solve a Rational Equation

Solve  $\frac{7}{12} + \frac{9}{x-4} = \frac{55}{48}$ .

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### Check

Select the solution(s) of  $\frac{5}{6} - \frac{2}{4x+1} = \frac{x}{3}$ .

A.  $-\frac{67}{4}$

B.  $-\frac{1}{4}$

C.  $\frac{1}{4}$

D.  $\frac{1}{2}$

E.  $\frac{7}{4}$

F. 3

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**Example 2** Solve a Rational Equation with an Extraneous Solution

Solve  $\frac{2m}{m-4} - \frac{m^2 + 7m + 4}{3m^2 - 18m + 24} = \frac{4m}{3m-6}$ .

When solving a rational equation, any possible solution that results in a zero in the denominator must be excluded from the list of solutions.

**Check**

Solve  $\frac{3x}{x-4} - \frac{x^2 - 7x - 4}{x^2 - 16} = \frac{5}{x+4}$ .



## DIFFERENTIATION ACTIVITY

○	LEVEL 1	★
○	LEVEL 2	★ ★
○	LEVEL 3	★ ★ ★

**LEVEL 1** Solve:  $\frac{2x+3}{x+1} = \frac{3}{2}$

**LEVEL 2** Solve:  $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$

**LEVEL 3** Solve:  $\frac{12p+12}{p^2+7p+12} - \frac{3}{p+3} = \frac{5}{p+4}$



EXIT TICKET

**Solve:**  $\frac{8}{t^2 - 9} + \frac{4}{t + 3} = 1$

a)  $t = 5$  \_\_\_\_\_

b)  $t = 1$  \_\_\_\_\_

c)  $t = -1, t = 5$  \_\_\_\_\_

d)  $t = -1$  \_\_\_\_\_



CHALLENGE  
QUESTION

**Solve:**  $\frac{3x}{x - 4} - \frac{x^2 - 7x - 4}{x^2 - 16} = \frac{5}{x + 4}$

a)  $x = -4$  \_\_\_\_\_

b)  $x = -3$  \_\_\_\_\_

c)  $x = -4, x = -3$  \_\_\_\_\_

d) No solution \_\_\_\_\_

## Past years MOE exam questions - Practice

Q1 The expression  $\frac{7x - 3}{x + 4}$  is equivalent to which of the following

- a)  $7 - \frac{3}{x + 4}$
- b)  $7 - \frac{31}{x + 4}$
- c)  $-\frac{3}{4}$
- d)  $\frac{7 - 3}{4}$

Q2 Simplify the expression  $\frac{x}{x - 4} - \frac{3}{4 - x}$

- a)  $\frac{x + 3}{x - 4}$
- b)  $\frac{x - 3}{x - 4}$
- c)  $\frac{x - 3}{x + 4}$
- d)  $\frac{x - 3}{x^2 - 16}$

Q3 Simplify the expression  $\frac{3x^2 + x - 2}{2x^2 - 3x - 5}$

- a)  $\frac{x - 2}{2x - 5}$
- b)  $\frac{x + 1}{2x - 5}$
- c)  $\frac{3x - 2}{x + 1}$
- d)  $\frac{3x - 2}{2x - 5}$

Q4 Simplify the expression

$$\frac{6c}{5d} \times \frac{15cd^2}{8a}$$

- a)  $\frac{90c^2d^2}{40ad}$
- b)  $\frac{9c^2d}{4a}$
- c)  $\frac{9cd^2}{4ad}$
- d)  $\frac{9cd}{4ad}$

Q5 Simplify the expression

$$\frac{5m^2n - 15mn^3}{5mn}$$

- a)  $m - 15mn^2$
- b)  $m - 10n^2$
- c)  $\frac{m - n^2}{5n}$
- d)  $m - 3n^2$

Q6 Simplify the expression

$$\frac{a^2x - b^2x}{a - b}$$

- a)  $x^2(a - b)$
- b)  $x(a + b)$
- c)  $x(a - b)$
- d)  $\frac{a^2x}{b}$

Q7 Simplify the expression

$$\frac{4x}{x^2 + 9x + 18} + \frac{5}{x + 6}$$

- a)  $\frac{4x + 5}{(x + 3)(x + 6)}$
- b)  $\frac{9x + 15}{(x + 3)(x + 6)}$
- c)  $\frac{3(3x + 1)}{(x + 3)(x + 6)}$
- d)  $\frac{4x + 5}{x^2 + 10x + 24}$

**Q8** Identify a vertical asymptote of the function

$$y = \frac{5}{x-4} + 2$$

- a)  $y = -2$
- b)  $x = -2$
- c)  $x = 4$
- d)  $y = 4$

**Q9** Identify a vertical asymptote of the function

$$y = \frac{3}{x-1} - 2$$

- a)  $y = -2$
- b)  $x = -2$
- c)  $x = 1$
- d)  $y = 1$

**Q10** Identify a vertical asymptote of the function

$$y = \frac{3}{x+1} + 2$$

- a)  $y = -1$
- b)  $x = -1$
- c)  $x = 2$
- d)  $y = 4$

**Q11** Find the range of the function

$$g(x) = \frac{5}{3(x-1)} - 2$$

- a)  $\{g(x) | g(x) \neq -2\}$
- b)  $\{g(x) | g(x) \neq -1\}$
- c)  $\{g(x) | g(x) \neq 1\}$
- d)  $\{g(x) | g(x) \neq 2\}$

**Q12** Determine the value of  $x$  for which is not defined

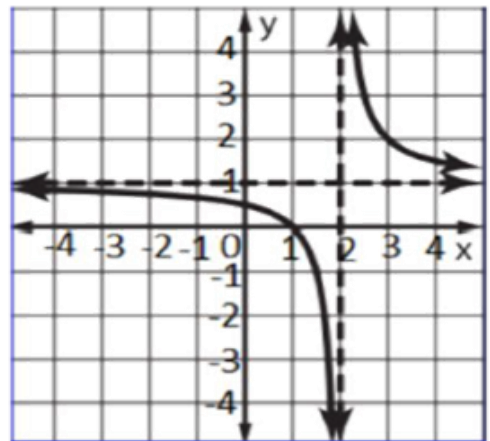
$$f(x) = \frac{4}{3x - 6}$$

- a)  $x = 4$
- b)  $x = 6$
- c)  $x = 3$
- d)  $x = 2$

**Q13** What is the equation of the function shown in the graph

$$f(x) = \frac{a}{x - h} + k$$

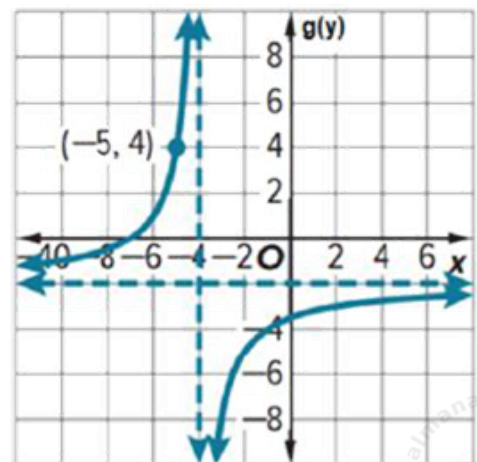
- a)  $f(x) = \frac{1}{x + 2} + 1$
- b)  $f(x) = \frac{1}{x - 2} + 1$
- c)  $f(x) = \frac{1}{x - 1} + 2$
- d)  $f(x) = \frac{1}{x + 1} + 2$



**Q14** What is the equation of the function shown in the graph

$$f(x) = \frac{a}{x - h} + k$$

- a)  $f(x) = \frac{-6}{x + 2} - 4$
- b)  $f(x) = \frac{-6}{x + 4} - 2$
- c)  $f(x) = \frac{6}{x + 4} - 2$
- d)  $f(x) = \frac{-4}{x + 2} - 6$



**Q15** Solve the equation

$$\frac{2}{x+1} = \frac{4}{x}$$

- a)  $x = 2$
- b)  $x = \frac{1}{2}$
- c)  $x = -2$
- d)  $x = -\frac{1}{2}$

**Q16** Solve the equation

$$\frac{3}{n-5} = \frac{8}{n}$$

- a)  $x = 4$
- b)  $x = 0$
- c)  $x = 5$
- d)  $x = 8$