

منهج ريفيل ملخص مراجعة تبسيط العبارات النسبية وحل المعادلات الكسرية، وتحليل الدوال النسبية



تم تحميل هذا الملف من موقع المناهج الإماراتية

موقع المناهج ← المناهج الإماراتية ← الصف الثاني عشر العام ← رياضيات ← الفصل الثالث ← ملفات متنوعة ← الملف

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ملفات اكتب للمعلم اكتب للطالب | اختبارات الكترونية | اختبارات | حلول | عروض بوربوينت | أوراق عمل
منهج انجليزي | ملخصات وتقارير | مذكرات وبنوك | الامتحان النهائي | للمدرس

المزيد من مادة
رياضيات:

التواصل الاجتماعي بحسب الصف الثاني عشر العام



صفحة المناهج
الإماراتية على
فيسبوك

الرياضيات

اللغة الانجليزية

اللغة العربية

التربية الاسلامية

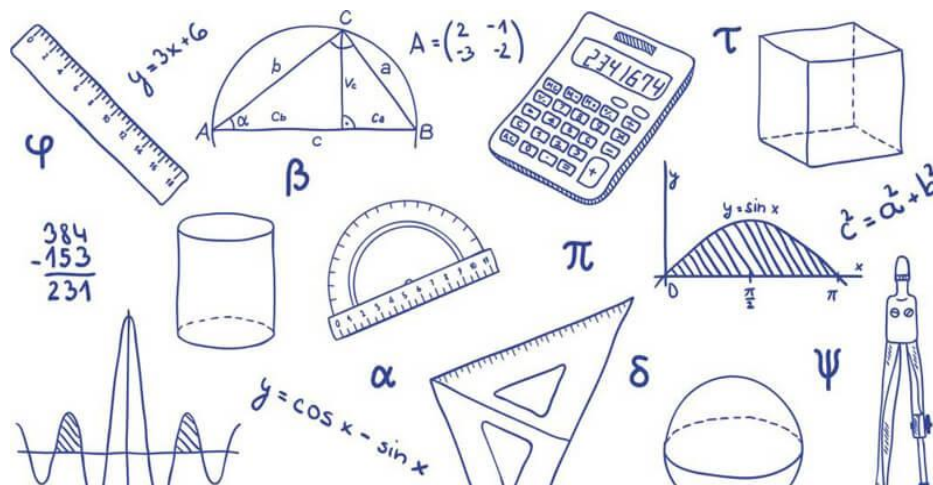
المواد على تلغرام

المزيد من الملفات بحسب الصف الثاني عشر العام والمادة رياضيات في الفصل الثالث

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MATH 12 GEN

T3



Khaled alabdullah



1	أبسط الكسور النسبية
	Simplify rational expressions.

Example 2 & Check	312
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Example 2 Simplify by Using -1

Simplify $\frac{(6x^2 - 5xy)(x + 2y)}{(x + y)(5y - 6x)}$.

Select the simplified form of $\frac{(7y - 3x)(5x - 1)}{(5x^3 + x^2)(3x - 7y)}$.

- A. $-\frac{5x - 1}{x^2(5x + 1)}$
- B. $\frac{(7y - 3x)(5x - 1)}{x^2(5x + 1)(3x - 7y)}$
- C. $\frac{1}{x^2}$
- D. $\frac{5x - 1}{5x^3 + x^2}$

Example 3 Multiply and Divide Rational Expressions

Simplify each expression.

a. $\frac{3x}{8y} \cdot \frac{12x^2y}{9xy^3}$

b. $\frac{10d^5}{6cd} \div \frac{30c^3d^2}{4c}$

Simplify $\frac{\frac{3x}{x-y}}{\frac{6xy}{4x^2-4y^2}}.$

Simplify $\frac{\frac{x^2-9y^2}{xy}}{\frac{2x+6y}{x^2}}.$

Simplify each expression.

a. $\frac{7a}{4b} + \frac{4c^2}{10}$

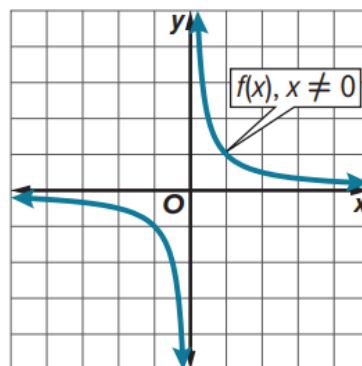
b. $\frac{2x}{9y} - \frac{7y}{6z}$

Simplify $\frac{2x + 1}{x^2 + 2x - 15} - \frac{7}{5x - 15}$.

Simplify $\frac{3x}{4x^2 + 4} - \frac{2x^2}{x^4 - 1}$.

Key Concept • Reciprocal Functions

Parent function	$f(x) = \frac{1}{x}$
Type of graph	hyperbola
Domain and range	all nonzero real numbers
Asymptotes	$x = 0$ and $f(x) = 0$
Intercepts	none
Not defined	$x = 0$



7	تحديد خصائص دوال المقلوب	(14,18)	441		
	Determine properties of reciprocal functions.			(23,24)	334

Determine the values of x for which $f(x)$ is undefined.

23. $f(x) = \frac{2}{-2x + 5}$

24. $f(x) = \frac{-12}{-3x - 7}$

8	تمثيل تحويلات دوال المقلوب بيانياً	مثال 3	443	
	Graph transformations of reciprocal functions.			Example 4 & Check 331 & 332

Graph $g(x) = \frac{-4}{x+1} - 2$. State the domain and range.

Graph $g(x) = \frac{1}{x-1} + 3$.

9	تمثيل الدوال النسبية ذات خطوط التقارب الأفقية والرأسية بيانياً	المفهوم الأساسي • خطوط التقارب الأفقية والرأسية	449	
	Graph rational functions with vertical and horizontal asymptotes.		Key Concept • Vertical and Horizontal Asymptotes	337

Key Concept • Vertical and Horizontal Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

- $f(x)$ has a vertical asymptote whenever $b(x) = 0$.
- $f(x)$ has at most one horizontal asymptote.
 - If the degree of $a(x)$ is greater than the degree of $b(x)$, there is no horizontal asymptote.
 - If the degree of $a(x)$ is less than the degree of $b(x)$, the horizontal asymptote is the line $y = 0$.
 - If the degree of $a(x)$ equals the degree of $b(x)$, the horizontal asymptote is the line $y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}$.

10	تمثيل الدوال النسبية ذات خطوط التقارب الأفقية والرأسية بيانياً Graph rational functions with vertical and horizontal asymptotes.	مثال 1	450		
				Example 1	337 & 338

Example 1 Graph with No Horizontal Asymptotes

Graph $f(x) = \frac{x^3}{x + \frac{2}{3}}$.

Check

Consider $g(x) = \frac{(0.5x - 1)^3}{x}$.

Part A There is a zero at $x = \underline{\hspace{1cm}}?$

Part B There is a vertical asymptote at $x = \underline{\hspace{1cm}}?$

Part C Graph the function.

11	تمثيل الدوال النسبية ذات خط التقارب المائل ونقطة الانفصال بيانياً Graph rational functions with oblique asymptotes and point discontinuity.	المفهوم الأساسي • خط التقارب المائل	451		
				Key Concept • Oblique Asymptotes	340

Key Concept • Oblique Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1 and $b(x) \neq 0$, then $f(x)$ has an oblique asymptote if the degree of $a(x)$ minus the degree of $b(x)$ equals 1. The equation of the asymptote is $f(x) = \frac{a(x)}{b(x)}$ with no remainder.

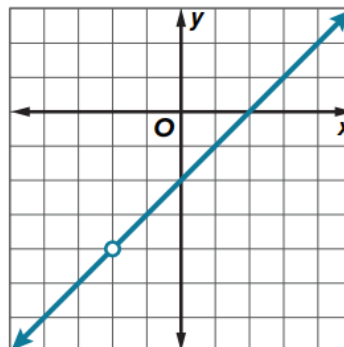
Example 5 Graph with Point Discontinuity

Graph $f(x) = \frac{x^2 - 4}{x + 2}$. Find the point discontinuity.

Notice that $\frac{x^2 - 4}{x + 2} = \frac{(x + 2)(x - 2)}{x + 2}$ or $x - 2$.

However, because the denominator of the original function cannot be 0, there is a discontinuity at $x + 2 = 0$ or $x = -2$.

Therefore, the graph of $f(x) = \frac{x^2 - 4}{x + 2}$ is the graph of $f(x) = x - 2$ with a hole or point discontinuity at $x = -2$.



Consider $f(x) = \frac{(x+3)^2}{x+3}$. Find the point discontinuity. Then graph the function.

$f(x) = \frac{(x+3)^2}{x+3}$ has a point discontinuity at $x = \underline{\quad? \quad}$.

Example 1 Solve a Rational Equation

Solve $\frac{7}{12} + \frac{9}{x-4} = \frac{55}{48}$.

Select the solution(s) of $\frac{5}{6} - \frac{2}{4x+1} = \frac{x}{3}$.

Example 2 Solve a Rational Equation with an Extraneous Solution

Solve $\frac{2m}{m-4} - \frac{m^2 + 7m + 4}{3m^2 - 18m + 24} = \frac{4m}{3m-6}$.

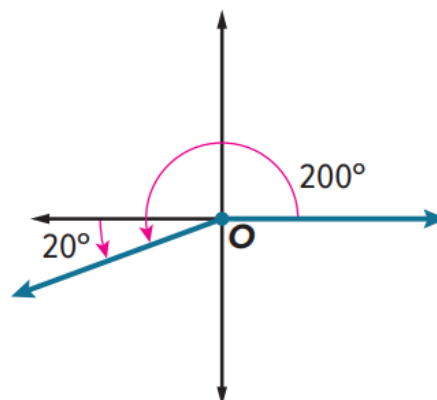
Solve $\frac{3x}{x-4} - \frac{x^2 - 7x - 4}{x^2 - 16} = \frac{5}{x+4}$.

Example 1 Draw an Angle in Standard Position

Draw an angle in standard position that measures 200° .

Because the measure of the angle is positive, rotate the terminal side counterclockwise.

The negative x-axis represents 180° .
Because $200^\circ = 180^\circ + 20^\circ$, draw the terminal side 20° counterclockwise past the negative x-axis.

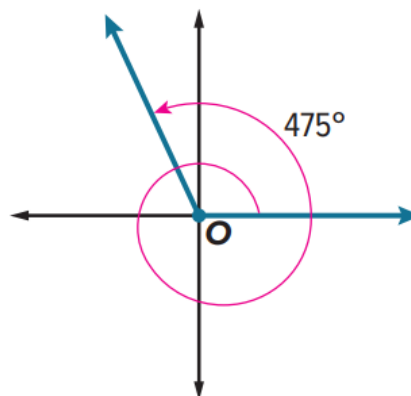


Example 2 Draw an Angle with More Than One Rotation

Draw an angle in standard position that measures 475° .

$$475^\circ = 360^\circ + 115^\circ$$

The terminal side of the angle will make more than one rotation counterclockwise. First, make a full rotation starting at the positive x-axis. Continue the rotation. Draw the terminal side of the angle 115° counterclockwise past the positive x-axis.



Draw an angle in standard position that measures -400° .

Example 3 Identify Coterminal Angles

Find an angle with a positive measure and an angle with a negative measure that are coterminal with a 35° angle.

positive angle: $35^\circ + 360^\circ = 395^\circ$

negative angle: $35^\circ - 360^\circ = -325^\circ$

Check

Find an angle with a positive measure and an angle with a negative measure that are coterminal with a -50° angle.

positive angle: $\underline{\quad? \quad}^\circ$

negative angle: $\underline{\quad? \quad}^\circ$

Example 4 Convert Degrees to Radians

Rewrite -100° in radians.

$$-100^\circ = -100^\circ \cdot \frac{\pi \text{ radians}}{180^\circ}$$

Multiply by conversion factor.

$$= \frac{-100\pi}{180} \text{ or } -\frac{5\pi}{9} \text{ radians}$$

Simplify.

Check

Rewrite -220° in radians. Write the solution as a fraction in simplest form.

_____ radians

Example 5 Convert Radians to Degrees

Rewrite $\frac{11\pi}{4}$ in degrees.

$$\frac{11\pi}{4} = \frac{11\pi}{4} \text{ radians} \cdot \frac{180^\circ}{\pi \text{ radians}}$$

Multiply by conversion factor.

$$= \frac{11\pi}{4} \text{ radians} \cdot \frac{180^\circ}{\cancel{\pi} \text{ radians}}$$

Cancel units.

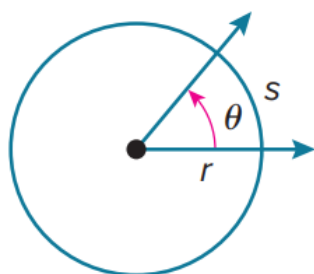
$$= \frac{1980^\circ}{4} \text{ or } 495^\circ$$

Simplify.

Key Concept • Arc Length

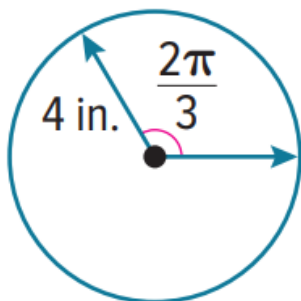
Words: For a circle with radius r and central angle θ (in radians), the arc length s equals the product of r and θ .

Symbols: $s = r\theta$

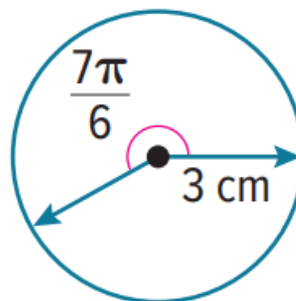


Find the length of each arc. Round to the nearest tenth.

63.



64.



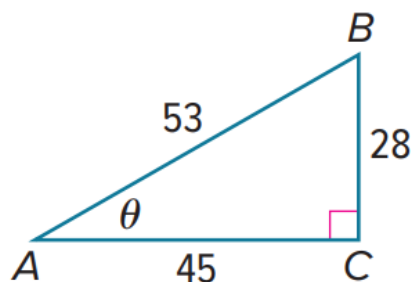
Example 1 Evaluate Trigonometric Functions

Find the exact values of the six trigonometric functions for angle θ .

leg opposite θ : $BC = 28$

leg adjacent θ : $AC = 45$

hypotenuse: $AB = 53$



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{28}{53}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{45}{53}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{28}{45}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{53}{28}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{53}{45}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{45}{28}$$

Find the exact values of the six trigonometric functions for angle θ .

$$\sin \theta = \frac{?}{?}$$

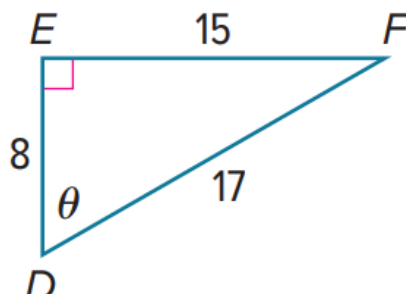
$$\cos \theta = \frac{?}{?}$$

$$\tan \theta = \frac{?}{?}$$

$$\csc \theta = \frac{?}{?}$$

$$\sec \theta = \frac{?}{?}$$

$$\cot \theta = \frac{?}{?}$$



Example 2 Find Trigonometric Ratios

If $\cos A = \frac{9}{13}$, find the exact values of the five remaining trigonometric functions for A .

If $\sec B = \frac{11}{3}$, find the exact values of the five remaining trigonometric functions for B .

Example 5 Find Reference Angles

Sketch each angle. Then find the measure of its reference angle.

a. 155°

b. $-\frac{8\pi}{3}$

Find the reference angle θ' of an angle that measures $\frac{23\pi}{6}$.

Example 6 Use a Reference Angle to Find a Trigonometric Value

Find the exact value of $\tan \frac{7\pi}{4}$.

Part A Find the measure of the reference angle.

The terminal side of $\frac{7\pi}{4}$ lies in Quadrant IV.

$$\theta' = 2\pi - \theta$$

$$= 2\pi - \frac{7\pi}{4} \text{ or } \frac{\pi}{4}$$

Find the reference angle θ' .

$$\theta = \frac{7\pi}{4}$$

Part B Use the reference angle to evaluate the function.

$$\tan \frac{7\pi}{4} = -\tan \frac{\pi}{4}$$

$$= -\tan 45^\circ$$

$$= -1$$

The tangent function is negative in Quadrant IV.

$$\frac{\pi}{4} \text{ radians} = 45^\circ$$

$$\tan 45^\circ = 1$$

Check

Find the exact value of $\sec 225^\circ$.

Example 1 Find Sine and Cosine Given a Point on the Unit Circle

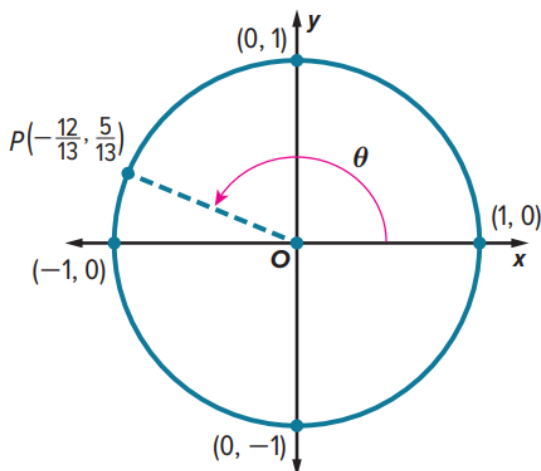
The terminal side of θ in standard position intersects the unit circle at $P\left(-\frac{12}{13}, \frac{5}{13}\right)$.

Find $\cos \theta$ and $\sin \theta$.

$$P\left(-\frac{12}{13}, \frac{5}{13}\right) = P(\cos \theta, \sin \theta)$$

$$\cos \theta = -\frac{12}{13}$$

$$\sin \theta = \frac{5}{13}$$



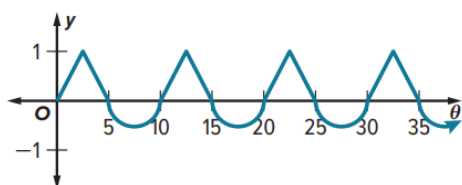
Check

The terminal side of θ in standard position intersects the unit circle at $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$. Find $\cos \theta$ and $\sin \theta$. Write the solutions as decimals.

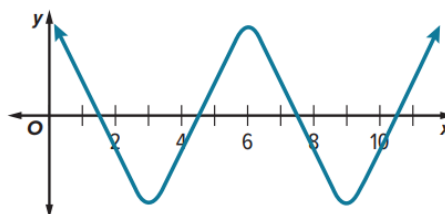
$$\cos \theta = \underline{\quad ? \quad} \qquad \sin \theta = \underline{\quad ? \quad}$$

Determine the period of the function.

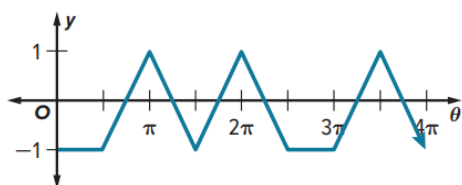
13.



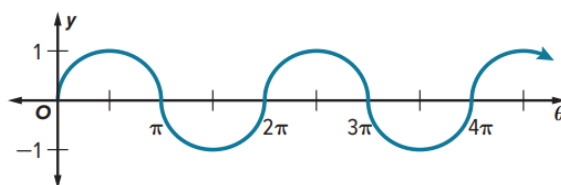
14.



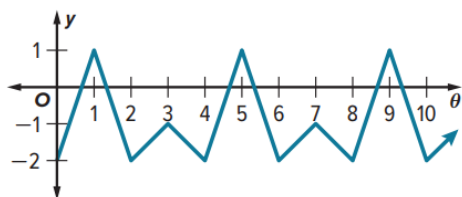
15.



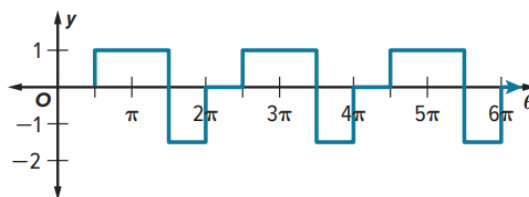
16.



17.



18.



Example 5 Evaluate Trigonometric Expressions

Find the exact value of $\cos \frac{10\pi}{3}$.

$$\begin{aligned}\cos \frac{10\pi}{3} &= \cos \left(\frac{4\pi}{3} + \frac{6\pi}{3} \right) & \frac{4\pi}{3} + \frac{6\pi}{3} &= \frac{10\pi}{3} \\ &= \cos \frac{4\pi}{3} & \cos (x + 2\pi) &= \cos x \\ &= -\frac{1}{2} & \text{Use the unit circle.}\end{aligned}$$

Check

Find the exact value of each expression.

$$\sin \frac{8\pi}{3} = \underline{\quad? \quad}$$

$$\cos \frac{5\pi}{6} = \underline{\quad? \quad}$$

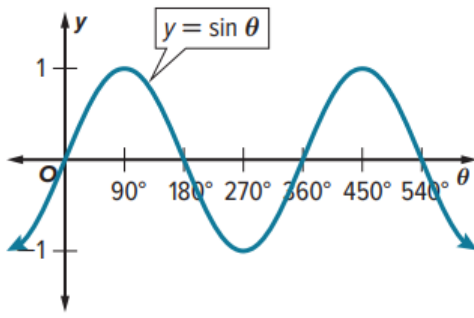
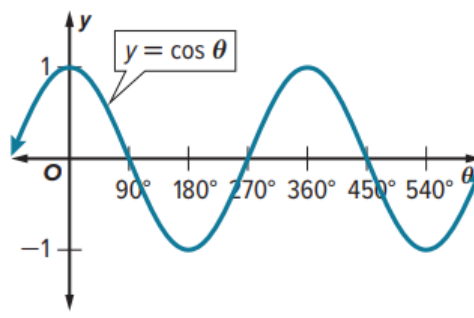
$$\sin \frac{21\pi}{4} = \underline{\quad? \quad}$$

$$\cos \frac{11\pi}{3} = \underline{\quad? \quad}$$

$$\cos 60^\circ = \underline{\quad? \quad}$$

$$\sin 600^\circ = \underline{\quad? \quad}$$

Key Concept • Sine and Cosine Functions

Parent	$y = \sin x$	$y = \cos x$
Graph		
Domain	all real numbers	all real numbers
Range	$\{y \mid -1 \leq y \leq 1\}$	$\{y \mid -1 \leq y \leq 1\}$
Amplitude	1	1
Midline	$y = 0$	$y = 0$
Period	360°	360°
Oscillation	between -1 and 1	between -1 and 1

Find the amplitude and period of each function.

5. $y = 2 \cos \theta$

6. $y = 2 \sin \theta$

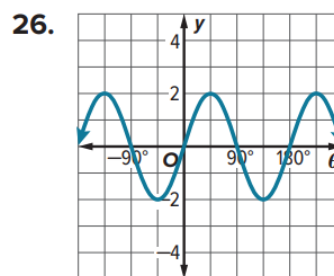
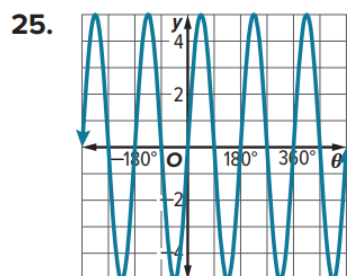
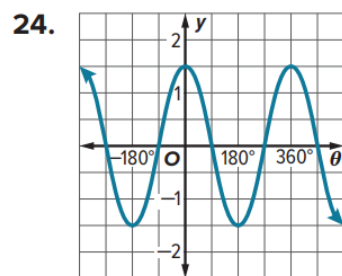
7. $y = \cos \frac{1}{2}\theta$

8. $y = \frac{3}{4} \cos \theta$

9. $y = \frac{1}{2} \sin 2\theta$

10. $y = 3 \cos 2\theta$

REGULARITY For each graph, identify the period and write an equation.

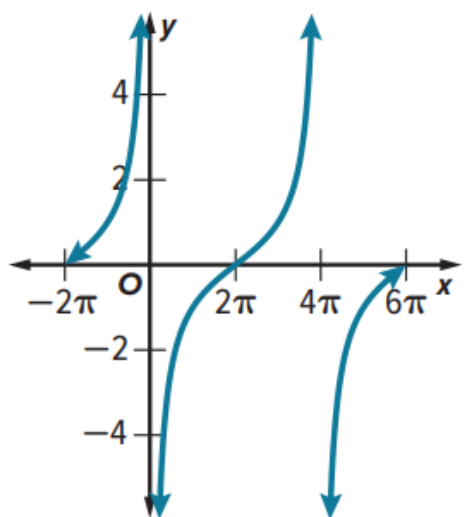


Example 1 Graph a Tangent Function with a Dilation

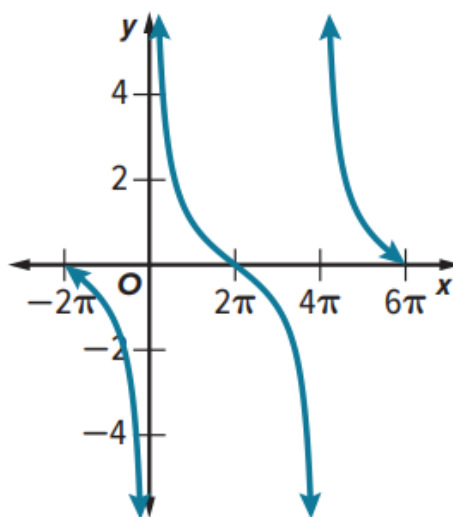
Find the period, asymptotes, x -intercepts, midline, and transformations of $y = \tan 3x$. Then graph the function.

Part B Select the graph of $y = \tan 0.25x$.

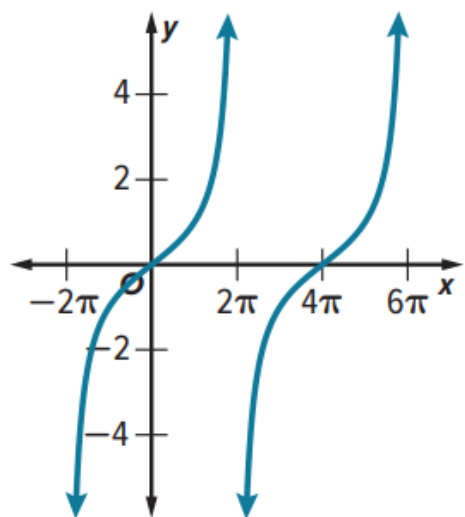
A.



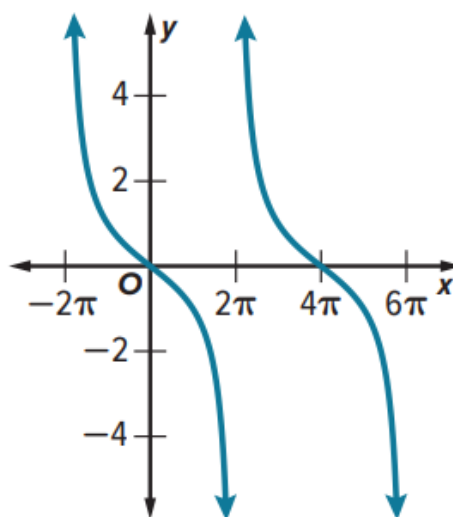
B.



C.



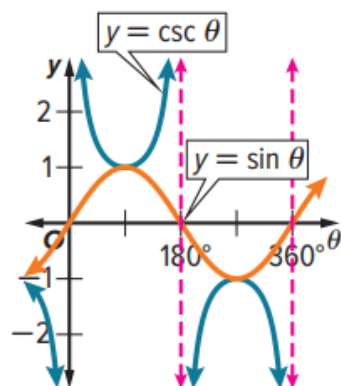
D.



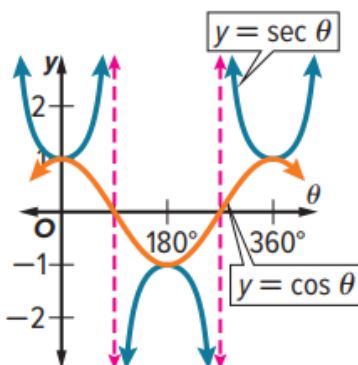
Key Concept • Graphs of Reciprocal Functions

Parent Function

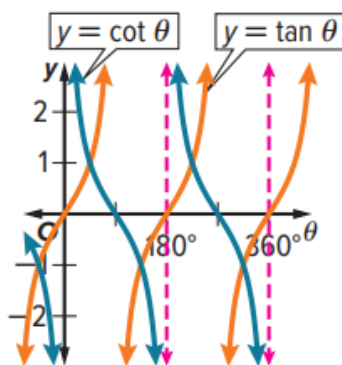
$y = \csc x$



$y = \sec x$



$y = \cot x$



Domain

$\{x \mid x \neq 180n^\circ, n \text{ is an integer}\}$

$\{x \mid x \neq (90 + 180n)^\circ, n \text{ is an integer}\}$

$\{x \mid x \neq 180n^\circ, n \text{ is an integer}\}$

Range

$\{y \mid y \leq -1 \text{ or } y \geq 1\}$

$\{y \mid y \leq -1 \text{ or } y \geq 1\}$

all real numbers

Amplitude

undefined

undefined

undefined

Period

 360° 360° 180°

Midline

$y = 0$

$y = 0$

$y = 0$

