أوراق عمل مراجعة الوحدة الخامسة functions Exponential الدوال الأسية منهج ريفيل

المزيد من الملفات بحسب الصف الثاني عشر العام والمادة رياضيات في الفصل الأول	
1	الخطة الفصلية للدروس المقررة في الفصل منهج ريفيل
2	مراجعة الوحدة الأولى functions Quadratic منهج ريفيل
3	مراجعة الدرس الخامس القانون العام والمميز من الوحدة الأولى منهج بريدج
4	مراجعة الدرس الرابع حل المعادلات التربيعية بالتحليل إلى العوامل من الوحدة الأولى منهج بريدج
5	مراجعة الدرس الثالث حل المعادلات التربيعية بإكمال المربع من الوحدة الأولى منهج بريدج

Lesson 5-1

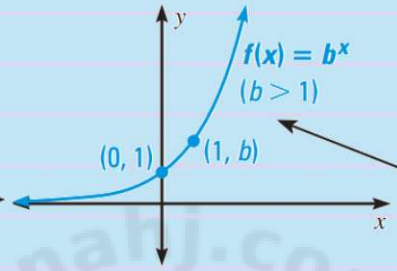
Graphing Exponential Functions

Learn Graphing Exponential Growth Functions

KEY CONCEPT

Parent Function for Exponential Growth Functions

The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base b. The general shape of the graph of $f(x) = b^x$ is shown below.



The *x*-axis is an asymptote of the graph. An asymptote is a line that a graph approaches more and more closely.

The graph rises from left to right, passing through the points (0, 1) and (1, *b*).

The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

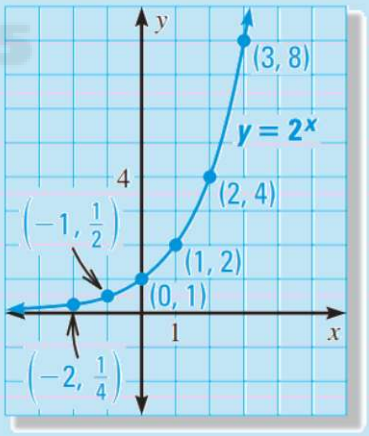
EXAMPLE 1 Graph $y = b^x$ for $b > 1$

Graph $y = 2^x$.

Solution

STEP 1 **Make** a table of values.

x	−2	−1	0	1	2	3
y						



Domain

Range

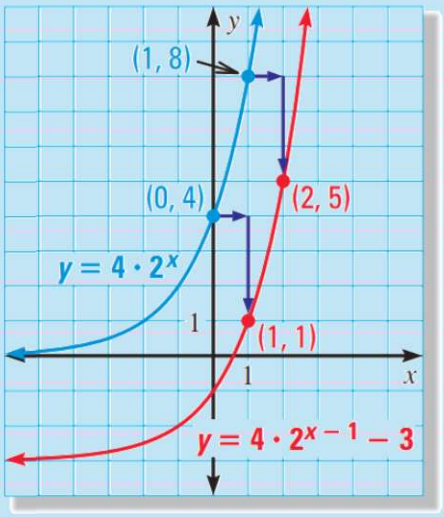
y-intercept

Asymptote

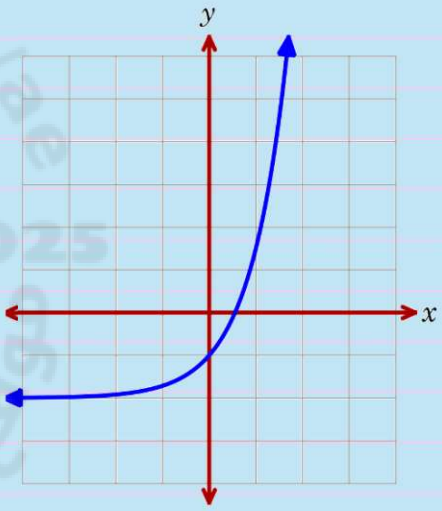
End behavior.

EXAMPLE 2 Graph $y = ab^{x-h} + k$ for $b > 1$

Graph $y = 4 \cdot 2^{x-1} - 3$. State the domain and range.



Identify the value of k and write a function for the graph of $j(x) = f(x) + k$ as it relates to $f(x) = 3.5^x$.



EXPONENTIAL GROWTH MODELS When a real-life quantity increases by a fixed percent each year (or other time period), the amount y of the quantity after t years can be modeled by the equation

$$A(t) = a(1 + r)^t$$

where a is the initial amount and r is the percent increase expressed as a decimal. Note that the quantity $1 + r$ is the growth factor.

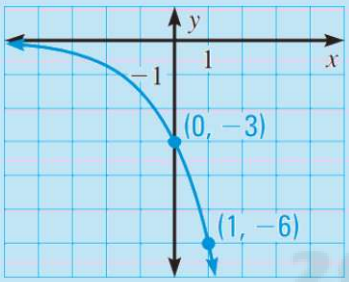
EXAMPLE 4

SUBSCRIPTIONS Subscriptions to an online arts and crafts club have been increasing by 20% every year. The club began with 40 members. Make a graph of the number of subscribers over the first 5 years of the club's existence. About how many subscriptions are there after Year 4?

MATCHING GRAPHS **Match the function with its graph.**

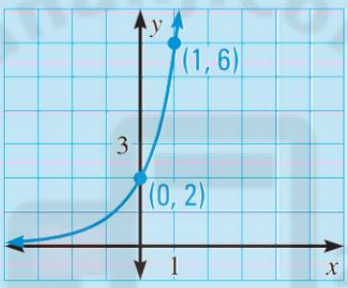
3. $y = 3 \cdot 2^x$

A.



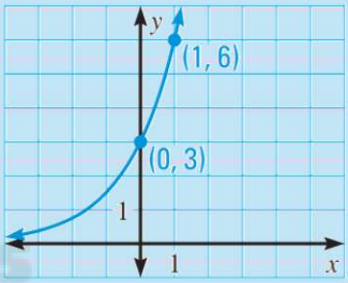
4. $y = -3 \cdot 2^x$

B.

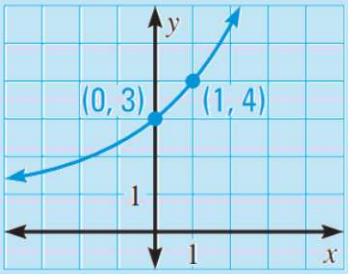


5. $y = 2 \cdot 3^x$

C.



★ **MULTIPLE CHOICE** The graph of which function is shown?



Ⓐ $f(x) = 2(1.5)^x - 1$

Ⓑ $f(x) = 2(1.5)^x + 1$

Ⓒ $f(x) = 3(1.5)^x - 1$

Ⓓ $f(x) = 3(1.5)^x + 1$

★ **MULTIPLE CHOICE** The student enrollment E of a high school was 1310 in 1998 and has increased by 10% per year since then. Which exponential growth model gives the school's student enrollment in terms of t, where t is the number of years since 1998?

Ⓐ $E = 0.1(1310)^t$

Ⓑ $E = 1310(0.1)^t$

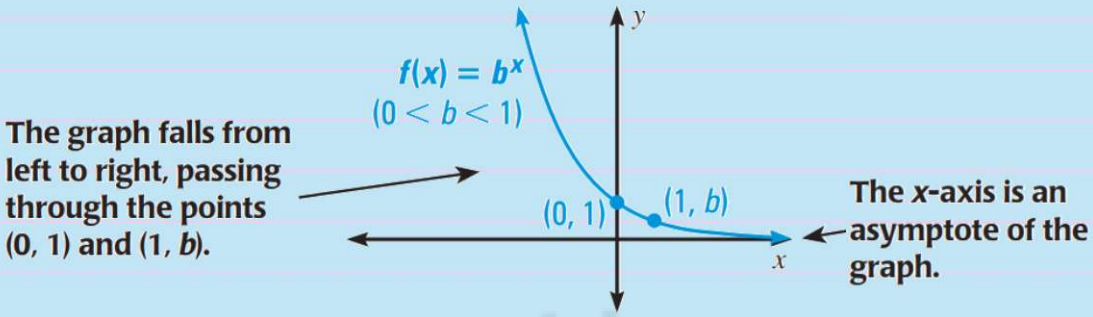
Ⓒ $E = 1.1(1310)^t$

Ⓓ $E = 1310(1.1)^t$

KEY CONCEPT For Your Notebook

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The general shape of the graph of $f(x) = b^x$ is shown below.



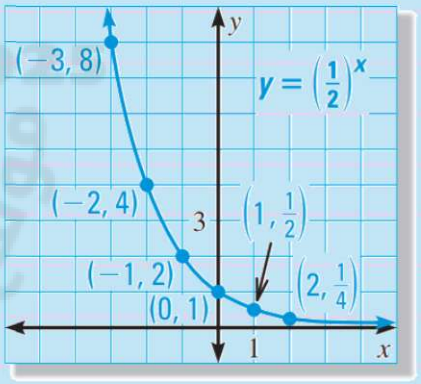
The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

EXAMPLE 1 Graph $y = b^x$ for $0 < b < 1$

Graph $y = \left(\frac{1}{2}\right)^x$.

Solution

x	−3	−2	−1	0	1	2
y						



Domain

Range

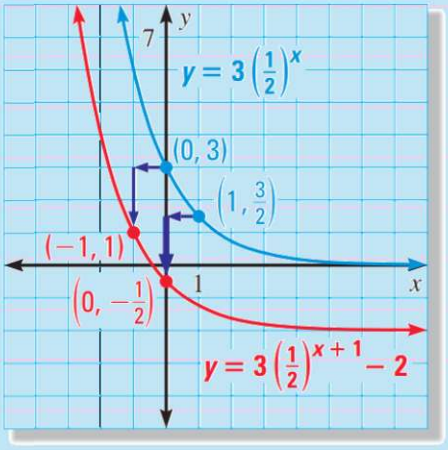
y-intercept

Asymptote

End behavior.

EXAMPLE 2 Graph $y = ab^{x-h} + k$ for $0 < b < 1$

Graph $y = 3\left(\frac{1}{2}\right)^{x+1} - 2$. State the domain and range.



EXPONENTIAL DECAY MODELS When a real-life quantity decreases by a fixed percent each year (or other time period), the amount y of the quantity after t years can be modeled by the equation

$$A(t) = a(1 - r)^t$$

where a is the initial amount and r is the percent decrease expressed as a decimal. Note that the quantity $1 - r$ is the decay factor.

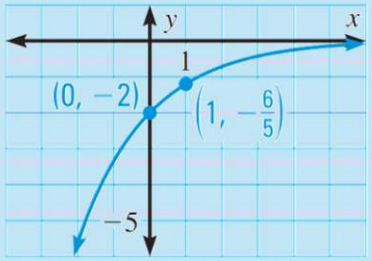
EXAMPLE 3 Solve a multi-step problem

SNOWMOBILES A new snowmobile costs \$4200. The value of the snowmobile decreases by 10% each year.

- Write an exponential decay model giving the snowmobile's value y (in dollars) after t years. Estimate the value after 3 years.

SNOWMOBILE The value of a snowmobile has been decreasing by 7% each year since it was new. After 3 years, the value is \$3000. Find the original cost of the snowmobile.

★ **MULTIPLE CHOICE** The graph of which function is shown?

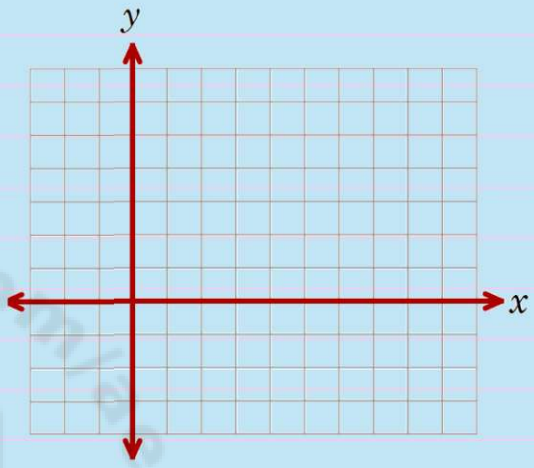


Ⓐ $y = 2\left(-\frac{3}{5}\right)^x$ Ⓑ $y = -2\left(\frac{3}{5}\right)^x$

Ⓒ $y = -2\left(\frac{2}{5}\right)^x$ Ⓓ $y = 2\left(-\frac{2}{5}\right)^x$

Graph the function. State the domain and range.

$$y = \left(\frac{2}{3}\right)^{x-4} - 1$$



★ **MULTIPLE CHOICE** What is the asymptote of the graph of $y = \left(\frac{1}{2}\right)^{x-2} + 3$?

Ⓐ $y = -3$ Ⓑ $y = -2$ Ⓒ $y = 2$ Ⓓ $y = 3$

Determine whether each function represents *exponential growth* or *exponential decay*.

a. $f(x) = 5^x$

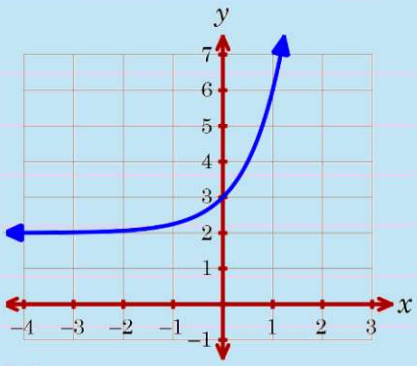
b. $g(x) = \left(\frac{2}{7}\right)^x$

c. $h(x) = \left(\frac{4}{3}\right)^x$

d. $j(x) = 1.05^x$

e. $k(x) = 0.85^x$

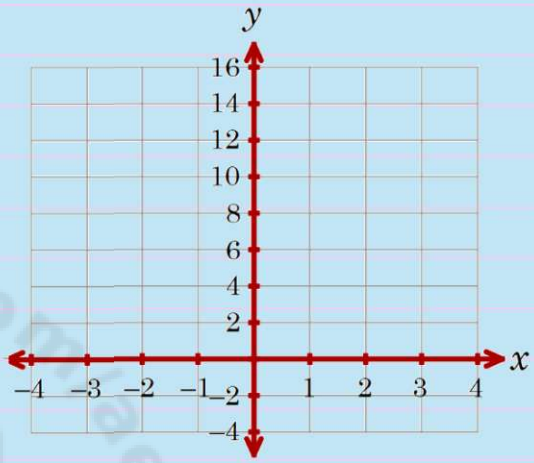
Consider $f(x) = \begin{cases} \left(\frac{1}{2}\right)^x \text{ for } x < -1 \\ 2x + 4 \text{ for } x \geq -1 \end{cases}$ and $g(x)$ shown in the graph.



Part A Graph $f(x)$.

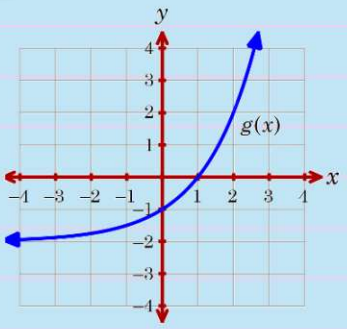
Part B Which function has the lesser relative minimum?

Part C Compare the y-intercepts and end behavior of $f(x)$ and $g(x)$.

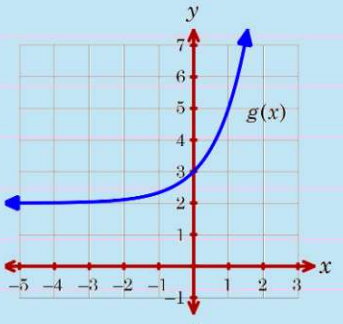


Identify the value of k and write a function $g(x)$ for each graph as it relates to $f(x)$.

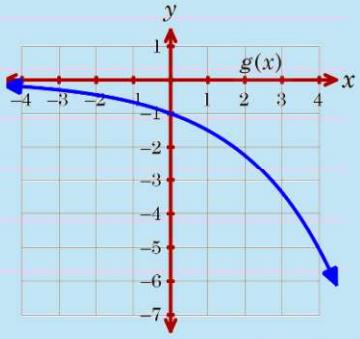
11. $f(x) = 2^x$, $g(x) = f(x) + k$



12. $f(x) = 3^x$, $g(x) = f(x) + k$



13. $f(x) = \left(\frac{3}{2}\right)^x$, $g(x) = k \cdot f(x)$



Lesson 5-2

Solving Exponential Equations and Inequalities

Learn Solving Exponential Equations

KEY CONCEPT

Property of Equality for Exponential Equations

Algebra If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.

Example If $3^x = 3^5$, then $x = 5$. If $x = 5$, then $3^x = 3^5$.

Solve each equation.

1. $25^{2x+3} = 25^{5x-9}$

2. $9^{8x-4} = 81^{3x+6}$

3. $4^{x-5} = 16^{2x-31}$

4. $4^{3x-3} = 8^{4x-4}$

5. $9^{-x+5} = 27^{6x-10}$

6. $125^{3x-4} = 25^{4x+2}$

Solve each equation.

27. $\left(\frac{1}{2}\right)^{4x+1} = 8^{2x+1}$

28. $\left(\frac{1}{5}\right)^{x-5} = 25^{3x+2}$

37. $\left(\frac{25}{81}\right)^{2x+1} = \left(\frac{729}{125}\right)^{-3x+1}$

GUIDED PRACTICE

Solve the equation.

1. $9^{2x} = 27^{x-1}$

2. $100^{7x+1} = 1000^{3x-2}$

3. $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$

KEY CONCEPT

Compound Interest

Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years is given by this equation:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

7. INTEREST *Bianca invested* $5000 *in an account that pays* 5% *annual interest.*

a. *Write a function that represents the value in Bianca's account* y *after* x *years.*

b. *After how many years will the value in Bianca's account be* $25,000$*? Round to the nearest tenth if necessary.*

FINANCE *If Aisha deposits* $500 *in a checking account that pays* 0.9% *annual interest compounded twice a month, what will be her account balance in* 25 *years? Round to the nearest cent.*

11. BANKING *Siobhan deposits* $1200 *into a savings account that pays* 5.2% *annual interest compounded monthly. What will be the balance after* 4 *years? Round to the nearest cent.*

12. INVESTING *Nancy deposits* $2500 *into an investing account that pays* 6.1% *annual interest compounded quarterly. What will be the balance after* 10 *years? Round to the nearest cent.*

FINANCE *You deposit* \$2000 *in an account that pays* 4% *annual interest. Find the balance after 3 years if the interest is compounded daily.*

KEY CONCEPT

Words Let $b > 1$. Then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.

Example If $2^x > 2^6$, then $x > 6$. If $x > 6$, then $2^x > 2^6$.

This property also holds true for $\leq$ and $\geq$.

Solve $27^{2x+6} \geq 81^{x-5}$.

Solve $125^{x+2} \leq 25^{4x-7}$.

15. $\left(\frac{1}{36}\right)^{6x-3} > 6^{3x-9}$

16. $64^{4x-8} < 256^{2x+6}$

17. $\left(\frac{1}{27}\right)^{3x+13} \leq 9^{5x-\frac{1}{2}}$

18. $\left(\frac{1}{9}\right)^{2x+7} \leq 27^{6x-12}$

19. $\left(\frac{1}{8}\right)^{-2x-6} > \left(\frac{1}{32}\right)^{-x+11}$

20. $9^{9x+1} < \left(\frac{1}{243}\right)^{-3x+5}$

29. $\left(\frac{1}{64}\right)^{c-2} < 32^{2c}$

30. $\left(\frac{1}{8}\right)^{3x+4} = \left(\frac{1}{4}\right)^{-2x+4}$

32. $\left(\frac{1}{9}\right)^{3t+5} \leq \left(\frac{1}{243}\right)^{t-6}$

41. $25^{4t+1} \geq 125^{2t}$

Challenge *Solve for x:* $16^{18} + 16^{18} + 16^{18} + 16^{18} = 4^x$