ملزمة أوراق عمل دروس وحدات الفصل الثالث منهج ريفيل

THE FEATURED PROGRAM EDUCATION

البرنامج المميز

12 GENERAL

MATH ENG

MATH TERM 3

Mr. Ahmed Ata
The Featured Program

2025-2026

البرنامج المميز طريقك للتميز

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UAE - ABU DHABI

THE FEATURED PROGRAM EDUCATION

البرنامج المميز

MATH ENG

12 GENERAL

LESSON 7-1

Multiplying and Dividing Rational Expressions

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Lesson (7-1)

Multiplying and Dividing Rational Expressions

Outcomes

1. Simplify rational expressions
2. Simplify rational expressions by multiplying and dividing

Simplify each expression, and state when the original expression is undefined.

1. $\dfrac{x^2 - 2x - 24}{2x^3 + 6x^2 - 8x}$

2. $\dfrac{x^2 + 2x + 1}{4x^2 + 3x - 1}$,

3. $\dfrac{x(x - 3)(x + 6)}{x^2 + x - 12}$

4 $\frac{y^2(y^2+3y+2)}{2y(y-4)(y+2)}$

Simplify each expression.

5 $\frac{(6x^2-5xy)(x+2y)}{(x+y)(5y-6x)}$.

6 $\frac{(7y-3x)(5x-1)}{(5x^3+x^2)(3x-7y)}$.

7 $\frac{x^2-5x-14}{28+3x-x^2}$

8 $\dfrac{9x^2 - x^3}{x^2 - 3x - 54}$

9 $\dfrac{3x}{8y} \cdot \dfrac{12x^2y}{9xy^3}$

10 $\dfrac{15a^2b^2}{21ac} \cdot \dfrac{14a^4c^2}{6ab^3}$

11 $\dfrac{10d^5}{6cd} \div \dfrac{30c^3d^2}{4c}$

12 $\frac{9x^2yz}{5z^4} \div \frac{12x^4y^2}{50xy^4z^2}$

13 $\dfrac{\frac{3x}{x-y}}{\frac{6xy}{4x^2-4y^2}}$.

14 $\dfrac{\frac{x^2-9y^2}{xy}}{\frac{2x+6y}{x^2}}$.

15 $\dfrac{\dfrac{a^2-b^2}{b^3}}{\dfrac{b^2-ab}{a^2}}$

16 **Simplify each expression.** $\dfrac{x(x-3)(x+6)}{x^2+x-12}$

$a)\ \dfrac{x(x+6)}{(x+4)}$ $b)\ \dfrac{x(x+2)}{6(x+5)}$

$c)\ \dfrac{(x+3)(x-2)}{4}$ $d)\ -\dfrac{x+2}{x+4}$

17 **Simplify each expression.** $\dfrac{(x^2-9)(x^2-z^2)}{4(x+z)(x-3)}$

$a)\ \dfrac{x(x-6)}{(x+4)}$ $b)\ \dfrac{x(x+2)}{6(x+5)}$

$c)\ \dfrac{(x+3)(x-z)}{4}$ $d)\ -\dfrac{x+2}{x+4}$

18 **Simplify each expression.** $\dfrac{x^2(x+2)(x-4)}{6x(x^2+x-20)}$

$a)\ \dfrac{x(x-6)}{(x+4)}$ $\quad$ $b)\ \dfrac{x(x+2)}{6(x+5)}$

$c)\ \dfrac{(x+3)(x-2)}{4}$ $\quad$ $d)\ -\dfrac{x+2}{x+4}$

19 **Simplify each expression.** $\dfrac{x^2-5x-14}{28+3x-x^2}$

$a)\ \dfrac{x(x-6)}{(x+4)}$ $\quad$ $b)\ \dfrac{(x+2)}{(x+5)}$

$c)\ \dfrac{x+2}{x+4}$ $\quad$ $d)\ -\dfrac{x+2}{x+4}$

20 **Simplify each expression.** $\dfrac{9x^2-x^3}{x^2-3x-54}$

$a)\ \dfrac{x}{x+6}$ $\quad$ $b)\ -\dfrac{x^2}{x+6}$

$c)\ -\dfrac{x}{x^2+6}$ $\quad$ $d)\ -\dfrac{x}{x+6}$

21 Find the expression that makes the following statement true for all values of x within the domain.

$$\frac{x-6}{x+3}\cdot\frac{?}{x-6}=x-2$$

a) $(x+3)^2$

b) $(x-2)(x-6)$

c) $(x+3)(x-2)$

d) $(x-6)(x+3)$

22 Write a rational expression that simplifies to

$$\frac{x-1}{x+4}$$

a) $\frac{x^2-1}{x^2+5x+4}$

b) $\frac{x^2-4x-5}{x^2+5x+4}$

c) $\frac{x^2-4x+5}{x^2+5x+4}$

d) $\frac{x^2+1}{x^2+5x+4}$

23 Write a rational expression that simplifies to

$$\frac{x+y}{x-y}\div\frac{4}{y-x}$$

a) $\frac{x+y}{4}$

b) $-\frac{x+y}{4}$

c) $\frac{4}{x+y}$

d) $-\frac{4}{x+y}$

24 **Identify the expression that does not belong with the other three.**

$a)\ \frac{1}{x-1}$ $b)\ \frac{x^2+3x+2}{x-5}$

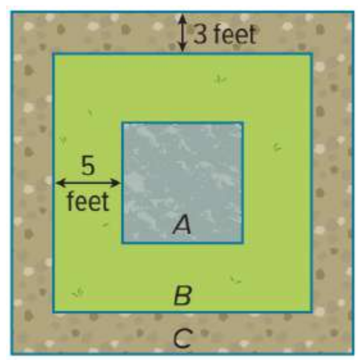
$c)\ \frac{x+1}{\sqrt{x+3}}$ $d)\ \frac{x^2+1}{3}$

25 Anita's yard is being professionally landscaped. The final design will consist of a circular fountain x feet in diameter in square A surrounded by a grassy area in square B and a gravel pathway in square C that borders the grassy area. The square areas will be centered on each other as shown in the diagram. Square A will have a side length of 2x feet

Anita would like the lengths of the sides to be proportional. For what values of x will the ratio of the lengths of a side of square C to a side of square B equal the ratio of the lengths of a side of square B to a side of square A? Explain your reasoning. What diameter could the fountain have?

$a)\ x=-12.5$ $b)\ x=12.5$

$c)\ x=5.25$ $d)$ *no solution*



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LESSON 7-2

Adding and Subtracting Rational Expressions

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Lesson (7-2)

Adding and Subtracting Rational Expressions

Outcomes

1. Simplify rational expressions by adding and subtracting.
2. Simplify complex fractions

Simplify each expression.

1. $\frac{7a}{4b} + \frac{4c^2}{10}$

2. $\frac{2x}{9y} - \frac{7y}{6z}$

3 $\frac{2x+1}{x^2+2x-15} - \frac{7}{5x-15}$.

4 $\frac{3x}{4x^2+4} - \frac{2x^2}{x^4-1}$.

5 $\frac{20}{x} + \frac{200x}{3x^2+20}$

6 $\frac{3t}{2-x}+\frac{5}{x-2}$

7 $\frac{n}{n-3}+\frac{2n+2}{n^2-2n-3}$

8 $\frac{\frac{x}{y}+1}{\frac{y}{x}-\frac{1}{y}}$

9 $\dfrac{\frac{x}{xy} - 2}{2xy - \frac{3}{y}}$.

10 $\dfrac{\frac{x}{y} + \frac{2}{x}}{\frac{x}{2} - \frac{y}{x}}$.

11 $\dfrac{1 + \frac{b^2}{a}}{\frac{a}{2} - \frac{a}{b}}$.

12 $$\frac{\frac{2}{x-3}+\frac{3x}{x^2-9}}{\frac{3}{x+3}-\frac{4x}{x^2-9}}$$

13 $$\frac{\frac{4}{x+5}+\frac{9}{x-6}}{\frac{5}{x-6}-\frac{8}{x+5}}$$

14 $\frac{12}{3y^2 - 10y - 8} - \frac{3}{y^2 - 6y + 8}$

15 **Simplify each expression** $\frac{3}{x} + \frac{5}{y}$

16 **Simplify each expression** $\frac{2c - 7}{3} + 4$

17 **Simplify each expression** $\frac{12}{5y^2} - \frac{2}{5yz}$

18 **Simplify each expression** $\frac{3}{w-3} - \frac{2}{w^2-9}$

19 **Simplify each expression** $\frac{k}{k-n} - \frac{k}{n-k}$

20 Simplify each expression

$$\frac{n}{n-3} + \frac{2n+2}{n^2-2n-3}$$

21 Simplify each expression

$$\frac{4z}{z-4} + \frac{z+4}{z+1}$$

22 Simplify each expression.

$$\frac{1}{12a} + 6 - \frac{3}{5a^2}$$

a) $\frac{5a^2+5a-36}{60a}$

b) $\frac{360a^2+5a-36}{60a^2}$

c) $\frac{360a^2+5a+36}{60a^2}$

d) $\frac{36a^2+5a-36}{60a^2}$

23 Simplify each expression.

$$\frac{5}{6x^2+46x-16}+\frac{5}{6x^2+57x+72}$$

a) $\frac{2x+41}{(3x-1)(x+8)(2x+3)}$

b) $\frac{41}{(3x-1)(x+8)(2x+3)}$

c) $\frac{42x+41}{6(3x-1)(x+8)(2x+3)}$

d) $\frac{42x+41}{(3x-1)(x+8)(2x+3)}$

24 Simplify each expression.

$$\frac{\frac{2}{a-1}+\frac{3}{a-4}}{\frac{6}{a^2-5a+4}}$$

a) $\frac{5a-36}{60}$

b) $\frac{5a-6}{a^2}$

c) $\frac{5a+36}{60a^2}$

d) $\frac{5a-11}{6}$

25 Find the slope of the line that passes through each p air of points.

$$A\left(\frac{2}{p},\frac{1}{2}\right) \text{ and } B\left(\frac{1}{3},\frac{3}{p}\right)$$

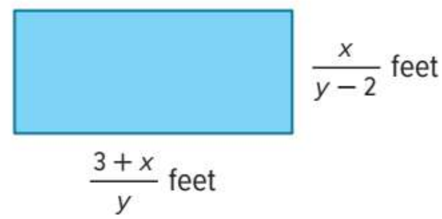
a) $-\frac{2}{3}$

b) $\frac{6}{5}$

c) $-\frac{3}{2}$

d) $\frac{11}{6}$

26 **Hachi needs to buy fencing for her rectangular garden. Write an expression, in simplest form, that represents the number of feet of fencing Hachi needs.**



a) $\dfrac{2xy + (6 + 2y)(y - 2)}{y(y - 2)}$

b) $\dfrac{2xy + (6 + 2x)(y - 2)}{y(y - 2)}$

c) $\dfrac{2xy + (6 + 2x)(y + 2)}{y(y - 2)}$

d) $\dfrac{2xy + (6 + 2x)(y - 2)}{(y - 2)}$

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LESSON 7-3

Graphing Reciprocal Functions

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Lesson (7-3)

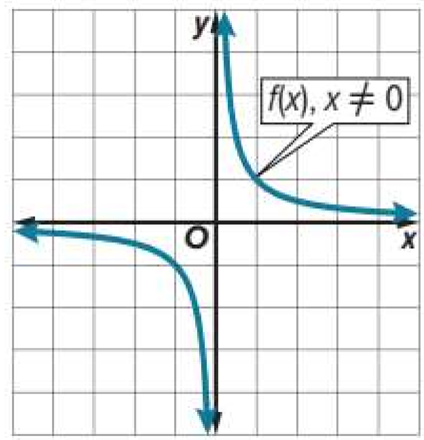
Graphing Reciprocal Functions

Outcomes

1. Graph reciprocal functions by making tables of values.
2. Graph and write reciprocal functions by using transformations

Key Concept • Reciprocal Functions

Parent function	$f(x) = \frac{1}{x}$
Type of graph	hyperbola
Domain and range	all nonzero real numbers
Asymptotes	$x = 0$ and $f(x) = 0$
Intercepts	none
Not defined	$x = 0$



1 Determine the excluded value of x for each function.

a) $g(x) = \frac{6}{x}$

b) $g(x) = \frac{2}{x-7}$

c) $g(x) = \frac{-5}{3x+4}$

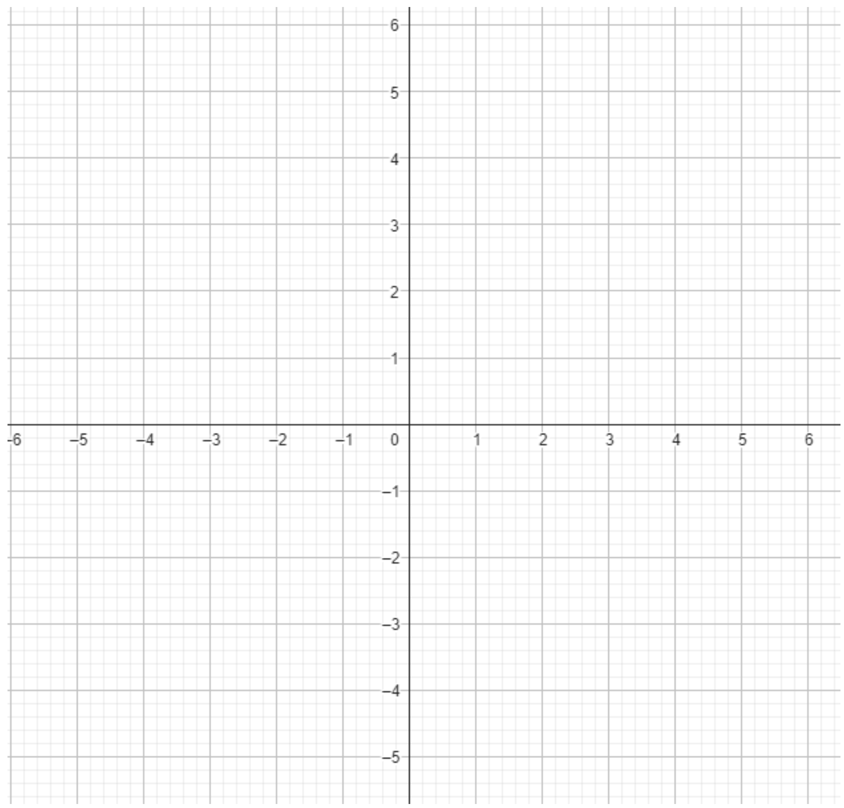
d) $g(x) = \frac{3}{x-11}$

e $g(x) = \frac{1}{2x+8}$

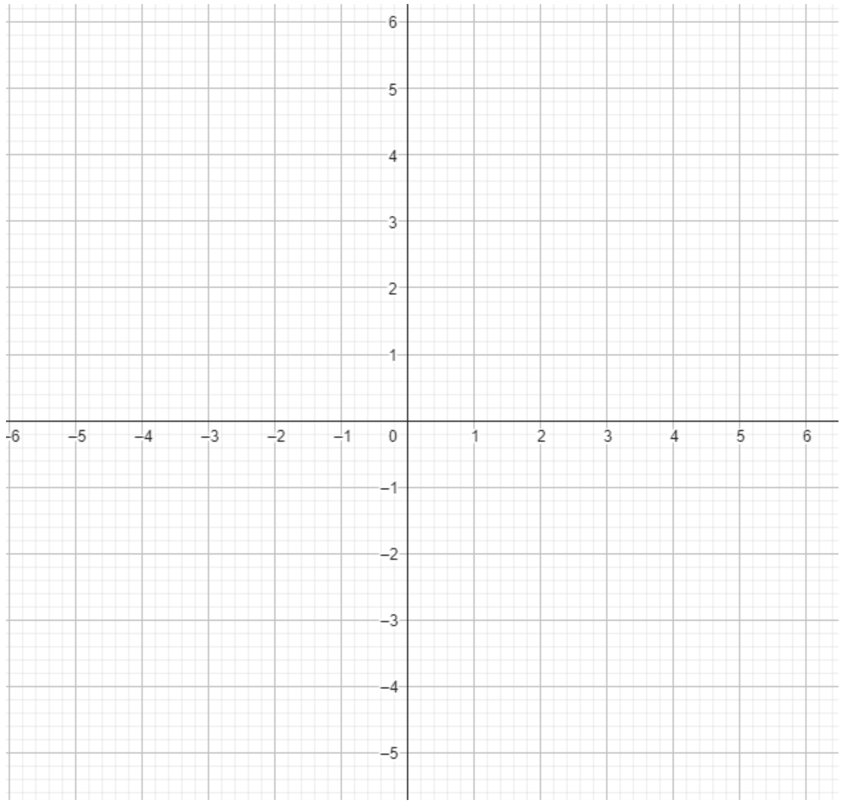
f $g(x) = \frac{5}{7x-9}$

Identify the asymptotes, domain, and range of each function. Then graph the function and identify its intercepts.

2 $f(x) = \frac{1}{x-1}$

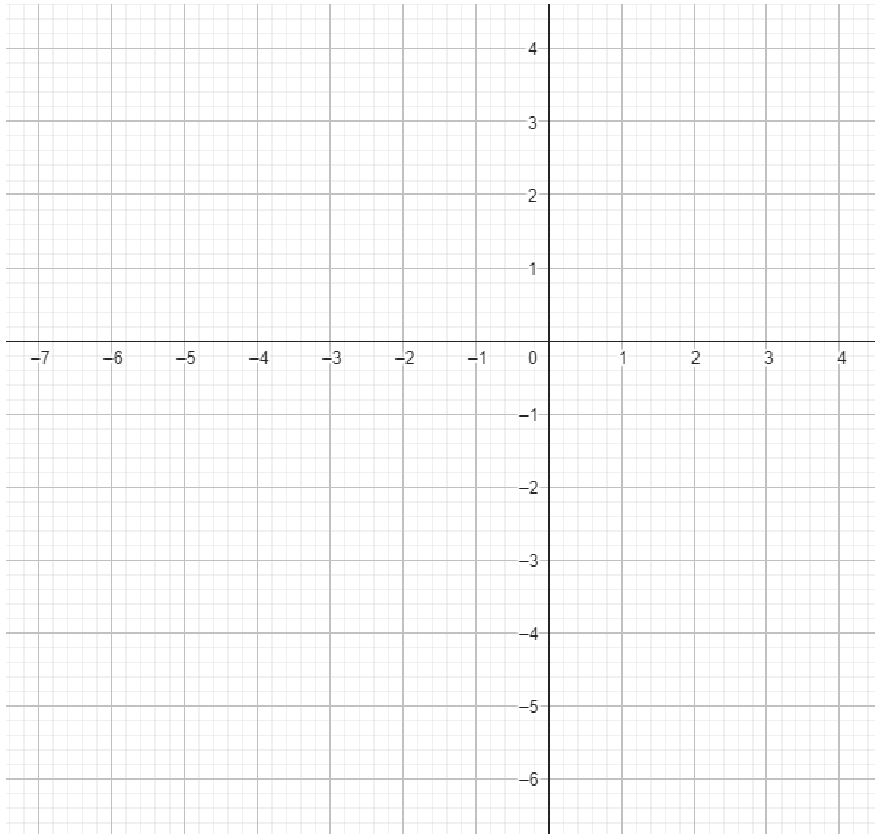


3 $g(x) = \frac{1}{2x - 5} + 2.$



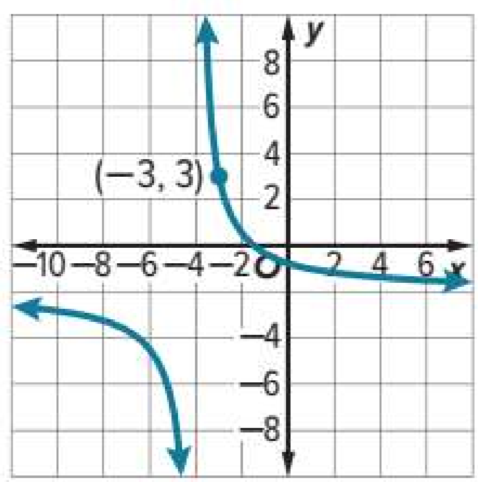
Key Concept • Transformations of Reciprocal Functions	
$g(x) = \frac{a}{x - h} + k$	
h – horizontal translation	If $h > 0$, the graph of $f(x)$ is translated h units right. If $h < 0$, the graph of $f(x)$ is translated $\lvert h \rvert$ units left. The *vertical asymptote* is at $x = h$.
k – vertical translation	If $k > 0$, the graph of $f(x)$ is translated k units up. If $k < 0$, the graph of $f(x)$ is translated $\lvert k \rvert$ units down. The *horizontal asymptote* is at $f(x) = k$.
a – orientation and shape	If $\lvert a \rvert > 1$, the graph is stretched vertically. If $0 < \lvert a \rvert < 1$, the graph is compressed vertically.

4 Graph $g(x) = \frac{-4}{x+1} - 2$. State the domain and range.



Identify the values of a, h, and k. Then write a function for the graph $g(x) = \frac{a}{x-h} + k$.

5



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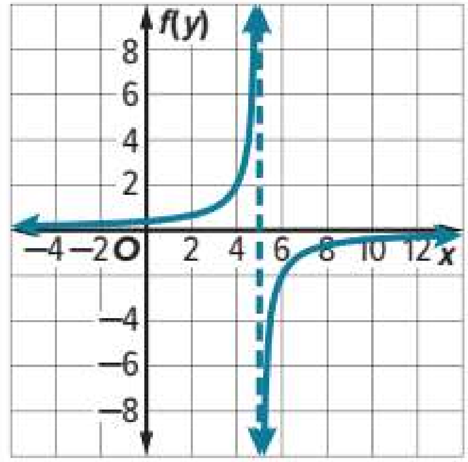
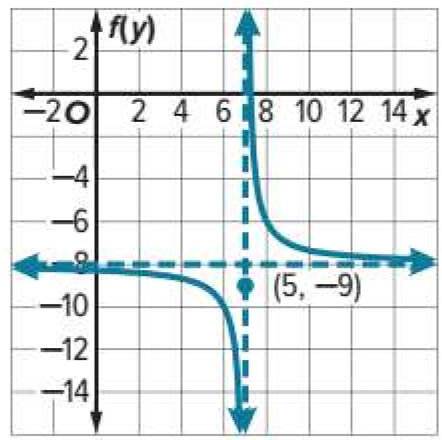
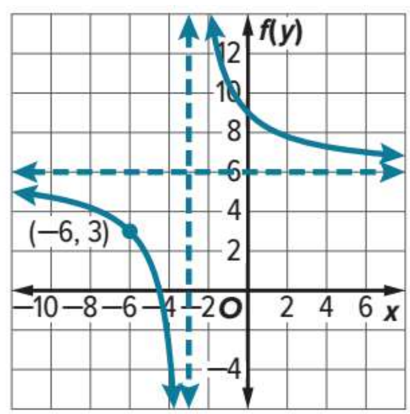

f(y)
8
6
4
2
−4 −2 O 2 4 6 8 10 12 x
−4
−6
−8

7


f(y)
2
−2 O 2 4 6 8 10 12 14 x
−4
−6
−10
−12
−14
(5, −9)

8


f(y)
12
10
8
6
4
2
(−6, 3)
−10 −8 −6 −4 −2 O 2 4 6 x
−4

9 **Determine the excluded value of *x* for each function.**

$g(x) = \frac{-2}{x+2}$

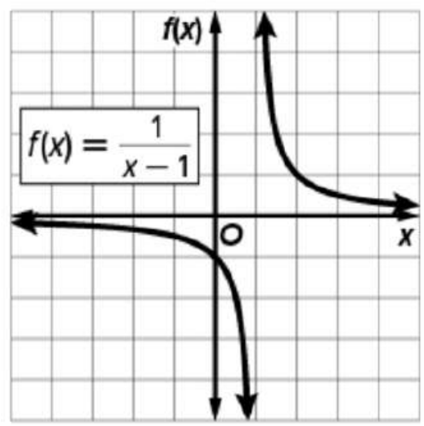
$f(x) = \frac{10}{x-3}$

$f(x) = \frac{5}{2x+3}$

$g(x) = \frac{5}{7x-9}$

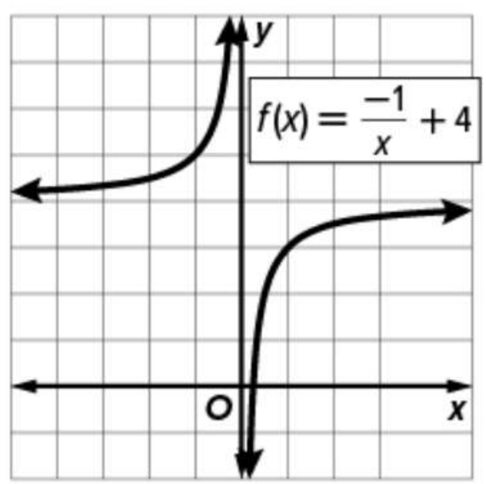
10 **Identify the asymptotes, domain, and range of each function.**

$f(x) = \frac{1}{x-1}$



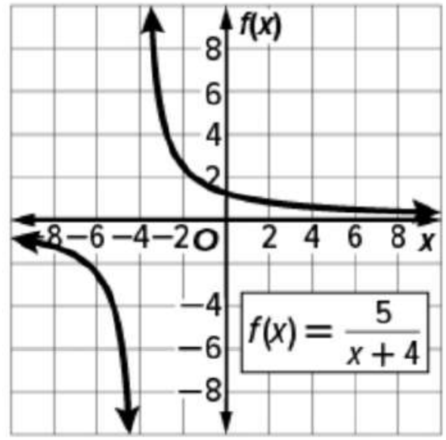
11 **Identify the asymptotes, domain, and range of each function.**

$f(x) = -\frac{1}{x} + 4$



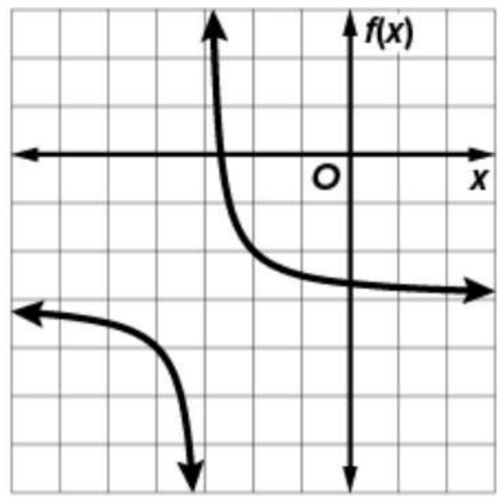
12 Identify the asymptotes, domain, and range of each function.

$f(x) = \frac{5}{x+4}$



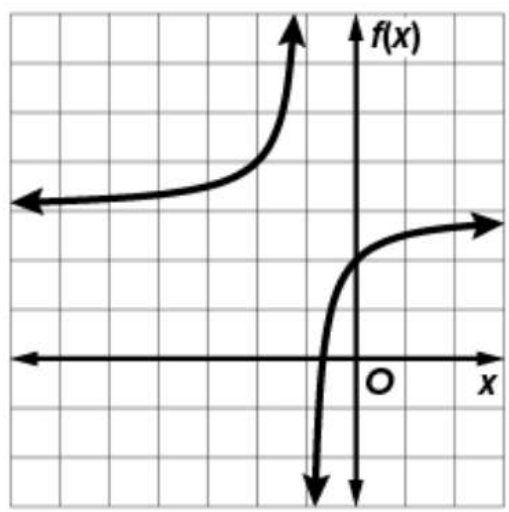
13 Graph each function. State the domain and range.

$f(x) = \frac{1}{x+3} - 3$



14 Graph each function. State the domain and range.

$f(x) = \frac{-1}{x+1} + 3$



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LESSON 7-4

Graphing Rational Functions

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Lesson (7-4)

Graphing Rational Functions

Outcomes

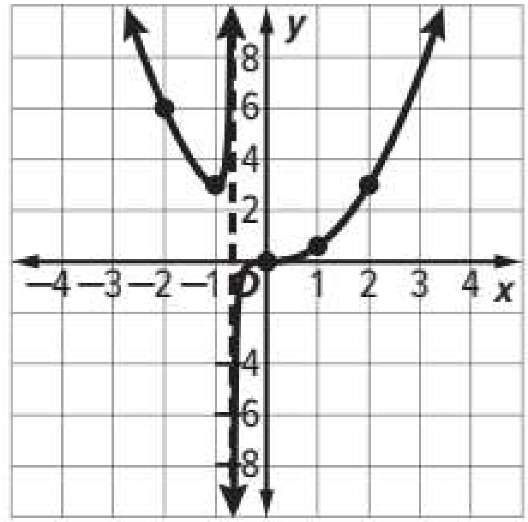
1. Graph and analyze rational functions with vertical and horizontal asymptotes.
2. Graph and analyze rational functions with oblique asymptotes.

Key Concept • Vertical and Horizontal Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

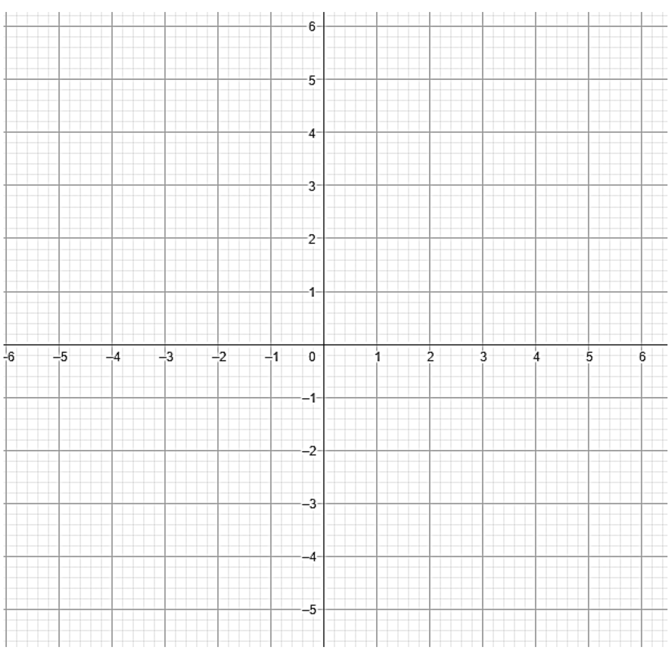
- $f(x)$ has a vertical asymptote whenever $b(x) = 0$.
- $f(x)$ has at most one horizontal asymptote.
  - If the degree of $a(x)$ is greater than the degree of $b(x)$, there is no horizontal asymptote.
  - If the degree of $a(x)$ is less than the degree of $b(x)$, the horizontal asymptote is the line $y = 0$.
  - If the degree of $a(x)$ equals the degree of $b(x)$, the horizontal asymptote is the line $y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}$.

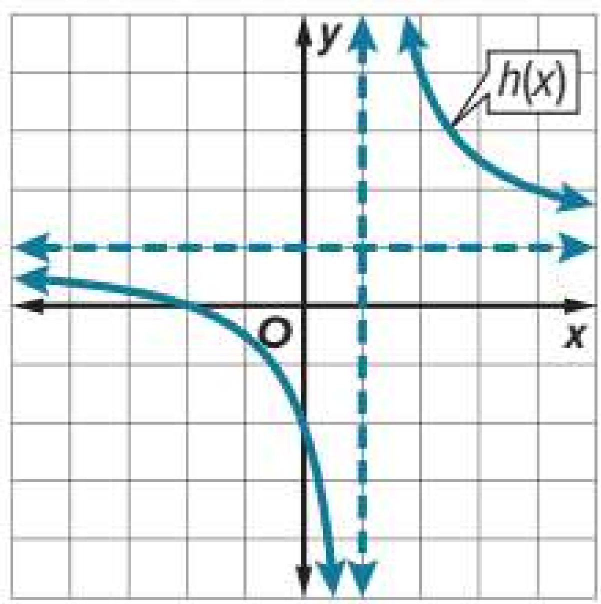
1 Graph $f(x) = \dfrac{x^3}{x + \frac{2}{3}}$.



2 Consider $g(x) = \dfrac{x - 2}{2x + 2}$ and rational function $h(x)$ shown in the graph.

Part A Graph $g(x)$.





Part B Which function has the greater *y*-intercept?

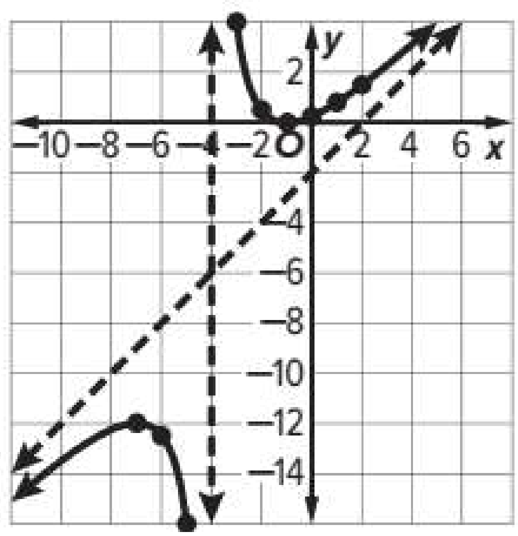
Part C Compare the asymptotes of $g(x)$ and $h(x)$.

Key Concept • Oblique Asymptotes

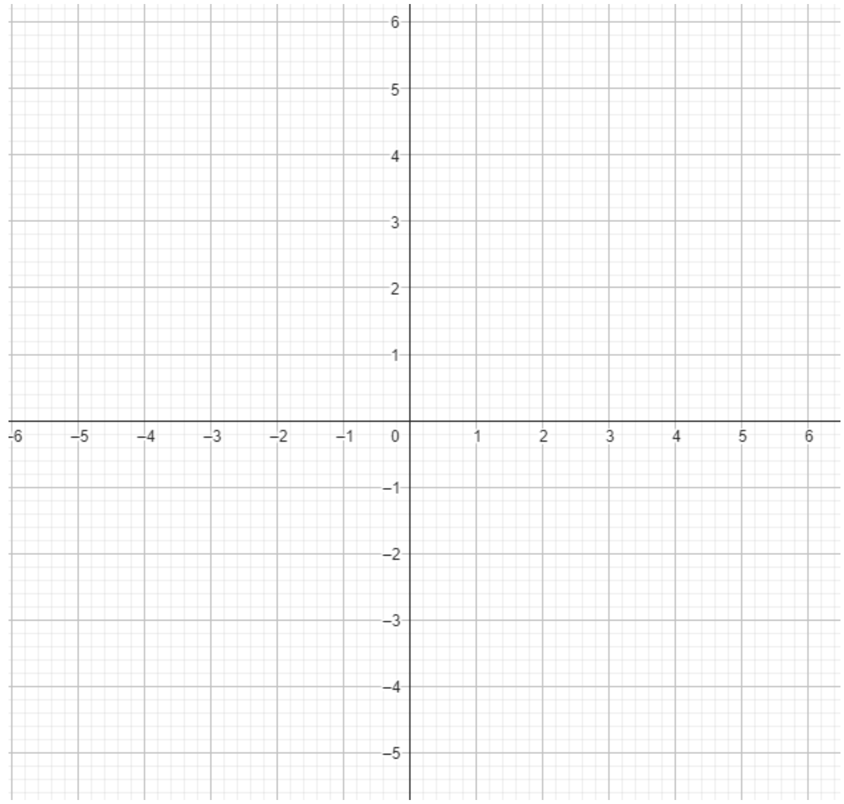
If $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1 and $b(x) \neq 0$, then $f(x)$ has an oblique asymptote if the degree of $a(x)$ minus the degree of $b(x)$ equals 1. The equation of the asymptote is $f(x) = \frac{a(x)}{b(x)}$ with no remainder.

Find the zeros and asymptotes of each function. Then graph each function.

3 $f(x) = \frac{x^2 + 2x + 1}{x + 4}$.

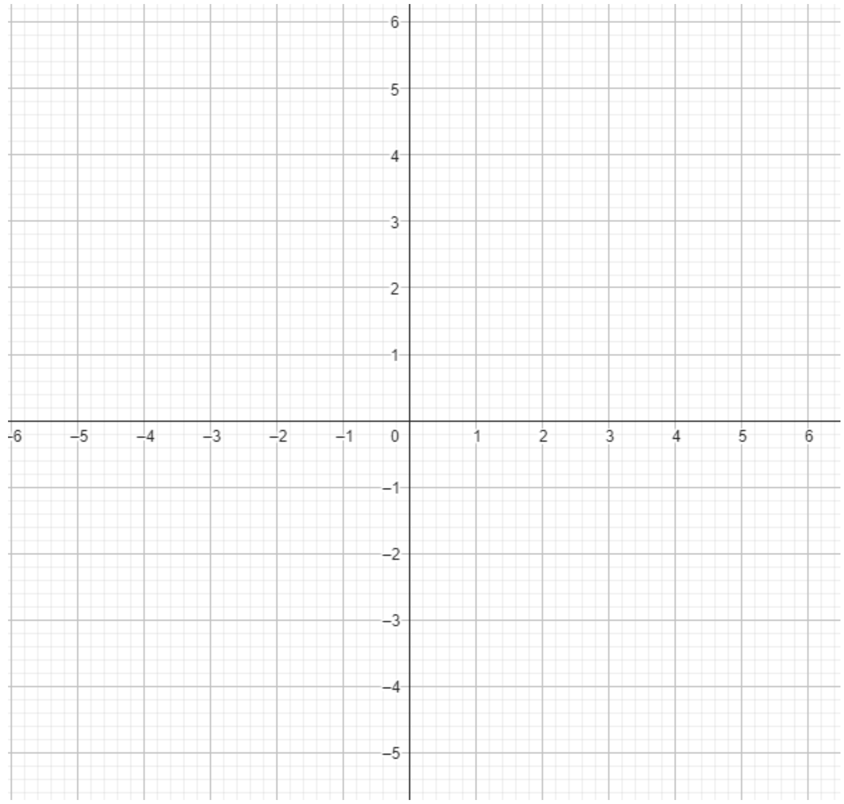


4 Graph $f(x) = \dfrac{x^2 - 4}{x + 2}$. Find the point discontinuity.



5 Graph each function. Find the point discontinuity.

$$f(x) = \frac{x^2 - 2x - 8}{x - 4}$$



6 Find the zeros and asymptotes of each function.

$$f(x) = \frac{(x-4)^2}{x+2}$$

a) $zero\ at\ x = 4\ \ vertical\ asymptote x = -2$

b) $zero\ at\ x = 4\ \ vertical\ asymptote x = 2$

c) $zero\ at\ x = -4\ \ vertical\ asymptote x = -2$

d) $zero\ at\ x = -4\ \ vertical\ asymptote x = -2$

7 **Find the asymptotes of each function.**

$$f(x) = \frac{(x+3)^2}{x-5}$$

a) *vertical asymptote* $x = -5$ *and oblique asymptote at* $y = x + 11$

b) *vertical asymptote* $x = 5$ *and oblique asymptote at* $y = x + 11$

c) *vertical asymptote* $x = 5$ *and oblique asymptote at* $y = x - 11$

d) *vertical asymptote* $x = 5$ *and oblique asymptote at* $y = 2x$

8 **Find the zeros and asymptotes of each function.**

$$f(x) = \frac{6x^2 + 4x + 2}{x+2}$$

a) *oblique at* $y = x - 8$, *vertical asymptote* $x = -2$

b) *oblique at* $y = 6x - 8$, *vertical asymptote* $x = -2$

c) *oblique at* $y = 6x + 8$ *vertical asymptote* $x = -2$

d) *oblique at* $y = 6x - 2$ *vertical asymptote* $x = -2$

9 **Find the asymptotes of each function.**

$$f(x) = \frac{2x^2 + 7x}{x-2}$$

a) *vertical asymptote* $x = -2$ *and oblique asymptote at* $y = 2x + 11$

b) *vertical asymptote* $x = 2$ *and oblique asymptote at* $y = x + 11$

c) *vertical asymptote* $x = 2$ *and oblique asymptote at* $y = x - 11$

d) *vertical asymptote* $x = 2$ *and oblique asymptote at* $y = 2x + 11$

10 **Find the zeros and asymptotes of each function.**

$$f(x) = \frac{3x^2 + 8}{2x - 1}$$

$a)$ $zero\ at\ x = -\frac{8}{3}$ $vertical\ asymptote x = \frac{1}{2}$

$b)$ $zero\ at\ x = -3$ $vertical\ asymptote x = -2$

$c)$ $no\ zeros\ and$ $vertical\ asymptote x = \frac{1}{2}$

$d)$ $zero\ at\ x = 3$ $vertical\ asymptote x = 2$

11 **Find the zeros and asymptotes of each function.**

$$f(x) = \frac{2x^2 + 5}{3x + 4}$$

$a)$ $zero\ at\ x = 8$ $vertical\ asymptote x = -3$

$b)$ $no\ zeros$ and $vertical\ asymptote x = -\frac{4}{3}$

$c)$ $zero\ at\ x = -2$ $vertical\ asymptote x = -1$

$d)$ $zero\ at\ x = 3$ $vertical\ asymptote x = 2$

THE FEATURED
PROGRAM EDUCATION

البرنامج المميز

12 GENERAL

MATH ENG

LESSON 9-1

Angles and Angle Measure

Mr. Ahmed Ata
The Featured Program

2025-2026

البرنامج المميز طريقك للتميز

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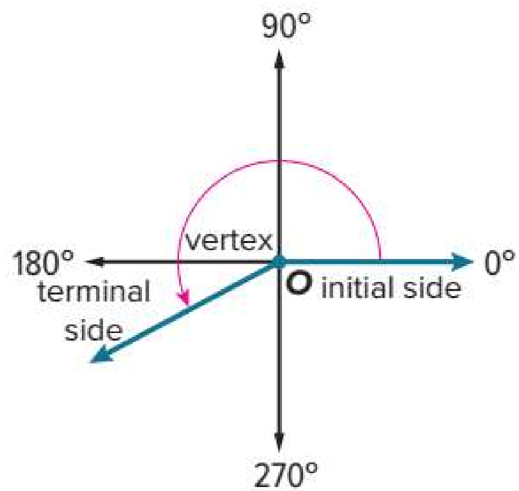
Lesson (9-1)

Angles and Angle Measure

Outcomes

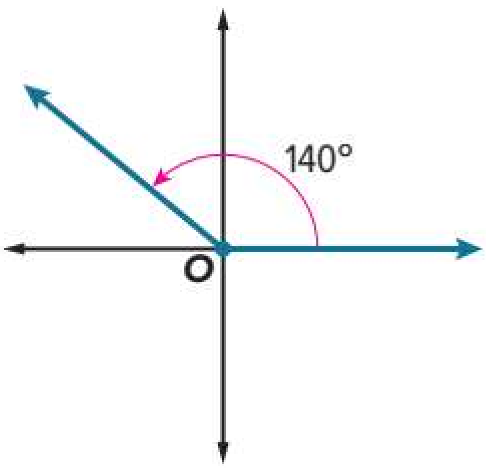
1. Draw angles in standard position and identify coterminal angles.
2. Convert between degree and radian measures and find arc lengths by using central angles.

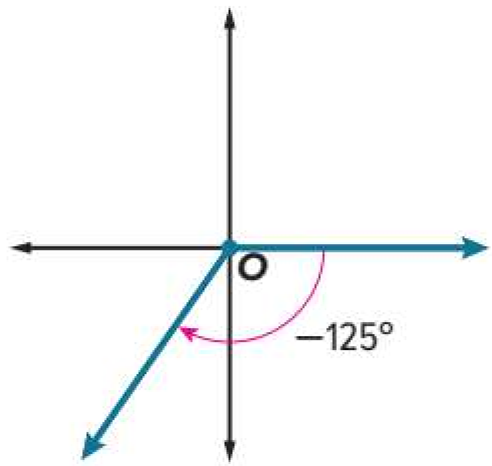
standard position if the vertex is at the origin and one ray is on the positive x-axis. The initial side of the angle is the ray that is fixed on the x-axis. The terminal side is the ray that rotates about the center.



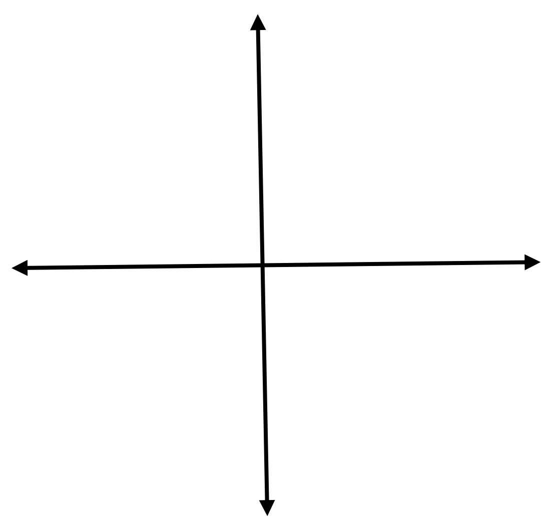
Key Concept • Angle Measures

Positive Angle Measures	Negative Angle Measures
If the measure of an angle is positive, the terminal side is rotated counterclockwise.	If the measure of an angle is negative, the terminal side is rotated clockwise.

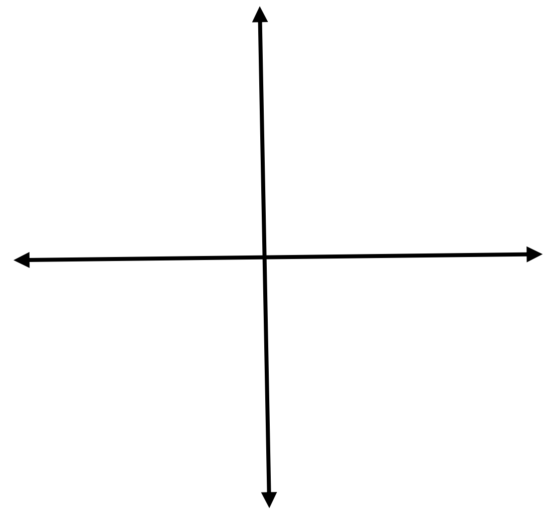




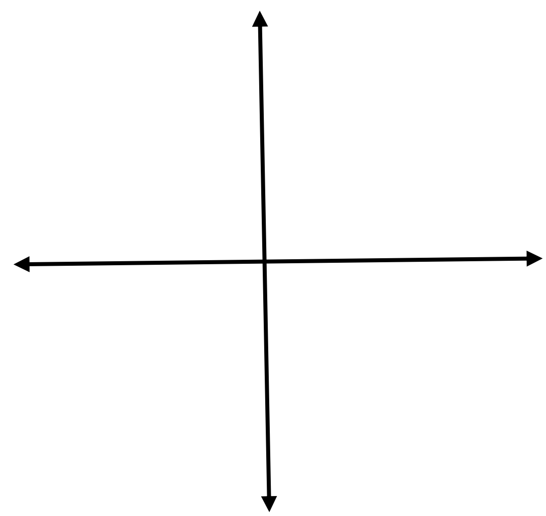
1 Draw an angle in standard position that measures 200°



2 **Draw an angle in standard position that measures 475°.**



3 **Draw an angle in standard position that measures -400°.**



4 **Find an angle with a positive measure and an angle with a negative measure that are coterminal with a 35° angle.**

Positive angle:

Negative angle:

5 Find an angle with a positive measure and an angle with a negative measure that are coterminal with a -50° angle.

Positive angle:

Negative angle:

6 Find an angle with a positive measure and an angle with a negative measure that are coterminal with a 230° angle.

Positive angle:

Negative angle:

7 Find an angle with a positive measure and an angle with a negative measure that are coterminal with a -90° angle.

Positive angle:

Negative angle:

8 Find an angle with a positive measure and an angle with a negative measure that are coterminal with a 420° angle.

Positive angle:

Negative angle:

9 Find an angle with a positive measure and an angle with a negative measure that are coterminal with a $\frac{2\pi}{5}$ angle.

Positive angle:

Negative angle:

10 Find an angle with a positive measure and an angle with a negative measure that are coterminal with a $-\frac{3\pi}{2}$ angle.

Positive angle:

Negative angle:

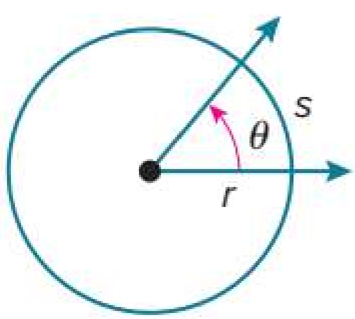
Key Concept • Convert Between Degrees and Radians

Degrees to Radians	Radians to Degrees
To convert a degree measure to radians, multiply the number of degrees by $\frac{\pi \text{ radians}}{180°}$.	To convert a radian measure to degrees, multiply the number of radians by $\frac{180°}{\pi \text{ radians}}$.

Key Concept • Arc Length

Words: For a circle with radius r and central angle θ (in radians), the arc length s equals the product of r and θ.

Symbols: $s = r\theta$



11 Rewrite each degree measure in radians.

140°   -260°   -75°

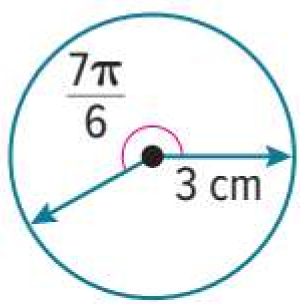
380°   130°   720°

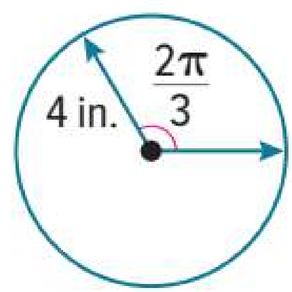
12 **Rewrite each radian measure in degrees.**

$-\frac{3\pi}{5}$ $\frac{7\pi}{6}$ $\frac{\pi}{3}$

$\frac{5\pi}{6}$ $\frac{2\pi}{3}$ $\frac{5\pi}{4}$

13 **Find the length of each arc. Round to the nearest tenth.**





14 **A traffic circle, or roundabout, is a circular roadway at the intersection of two or more streets that allows cars to travel through more continuously than a traffic light or stop sign. The diameter of a traffic circle is 160 feet. How far does a car travel in the roundabout if it goes three-fourths of the way around?**

15 If a surveyor's wheel with a radius of 15 inches completes $\frac{13}{20}$ of a rotation, what is the total distance traveled in feet? Round to the nearest hundredth if necessary.

16 A traffic roundabout has a diameter of 200 meters. How far does an automobile travel in the roundabout if it goes one-fourth of the way around?

17 A truck with 16-inch radius wheels is driven at 77 feet per second (52.5 miles per hour). Find the measure of the angle through which a point on the outside of the wheel travels each second. Round to the nearest degree and nearest radian.

18 Earth makes one full rotation on its axis every 24 hours. How long does it take Earth to rotate through 150°? Neptune makes one full rotation on its axis every 16 hours. How long does it take Neptune to rotate through 150°?

19 If a surveyor's wheel with a diameter of 19 inches completes $\frac{5}{6}$ of a rotation, what is the total distance traveled in inches? Round to the nearest hundredth if necessary

20 Find both the degree and radian measures of the angle through which the hour hand on a clock rotates from 5 a.m. to 10 p.m.

$a)\ -510°, -\frac{17\pi}{6}$

$b)\ 510°, \frac{17\pi}{6}$

$c)\ -360°, -2\pi$

$d)\ -150°, -\frac{5\pi}{6}$

21 A truck with 16-inch radius wheels is driven at 77 feet per second (52.5 miles per hour). Find the measure of the angle through which a point on the outside of the wheel travels each second. Round to the nearest degree.

$a)$ $3009°$

$b)$ $3039°$

$c)$ $3319°$

$d)$ $3309°$

22 Earth makes one full rotation on its axis every 24 hours. How long does it take Earth to rotate through 150°? Neptune makes one full rotation on its axis every 16 hours. How long does it take Neptune to rotate through 150°?

$a)$ Earth $= 10$ *hours*, Neptune $= 8$ *hours*

$b)$ Earth $= 10$ *hours*, Neptune $= 6.67$ *hours*

$c)$ Earth $= 11$ *hours*, Neptune $= 6.67$ *hours*

$d)$ Earth $= 19$ *hours*, Neptune $= 7$ *hours*

23 If a surveyor's wheel with a diameter of 19 inches completes $\frac{5}{6}$ of a rotation, what is the total distance traveled in inches? Round to the nearest hundredth if necessary.

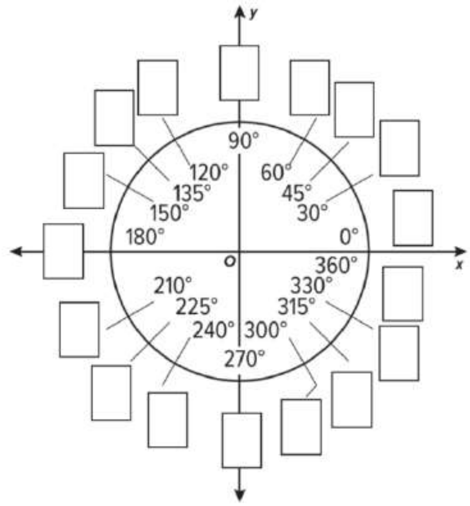
$a)$ 39.74

$b)$ 49.44

$c)$ 49.14

$d)$ 49.74

23 It is convenient to know the measures of some specific angles in both degree and radian measures
Copy and complete the figure at the right by writing the radian measure for each angle.



24 Through what angle, radians, does the hour hand on a clock rotate between 4 p.m. and 7 p.m.? Assuming the length of the hour hand is 6 inches, find the arc length of the circle made by the hour hand during that time.

a) $\theta = -\frac{\pi}{6}$ and $S = 3\pi$

b) $\theta = -\frac{\pi}{3}$ and $S = 3\pi$

c) $\theta = -\frac{\pi}{2}$ and $S = 3\pi$

d) $\theta = -\frac{\pi}{2}$ and $S = 2\pi$

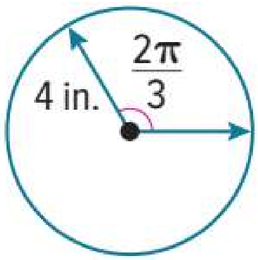
25 Rewrite each degree measure in radians and each radian measure in degrees.

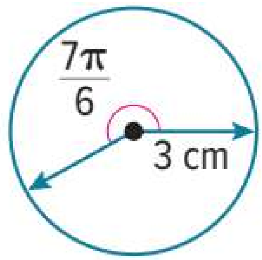
$18°$ $6°$

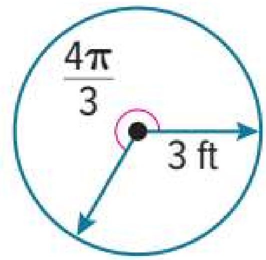
$-820°$ 4π

$-\frac{9\pi}{2}$ $-\frac{7\pi}{12}$

26 **Find the length of each arc. Round to the nearest tenth.**







THE FEATURED
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البرنامج المميز

12 GENERAL

MATH ENG

LESSON 9-2

Trigonometric Functions of General Angles

Mr. Ahmed Ata
The Featured Program

2025-2026

البرنامج المميز طريقك للتميز

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Lesson (9-2)

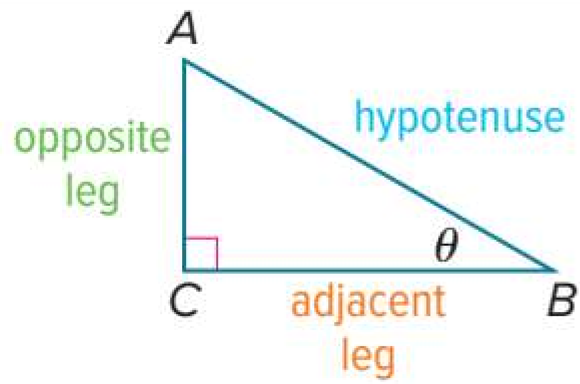
Trigonometric Functions of General Angles

Outcomes

1. **Find values of trigonometric functions for acute angles.**
2. **Find values of trigonometric functions of general angles.**
3. **Find values of trigonometric functions by using reference angles.**

Key Concepts • Trigonometric Functions in Right Triangle

If θ is the measure of an acute angle of a right triangle, then the trigonometric functions involving the opposite side *opp*, the adjacent side *adj*, and the hypotenuse *hyp* are defined as follows.



sine: $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

cosine: $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

tangent: $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

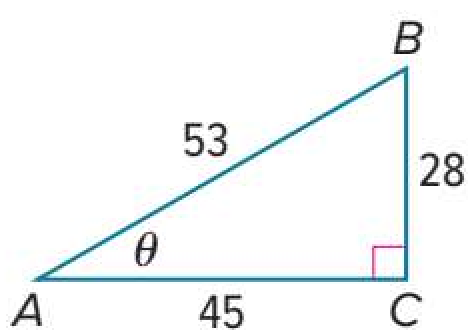
cosecant: $\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

secant: $\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

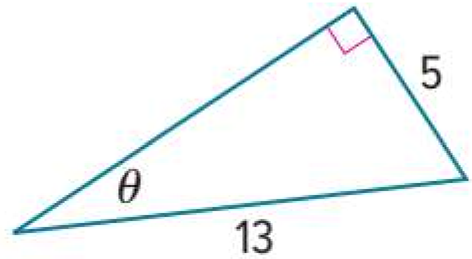
cotangent: $\cot\theta = \dfrac{\text{adj}}{\text{opp}}$

1 **Find the exact values of the six trigonometric functions for angle θ.**

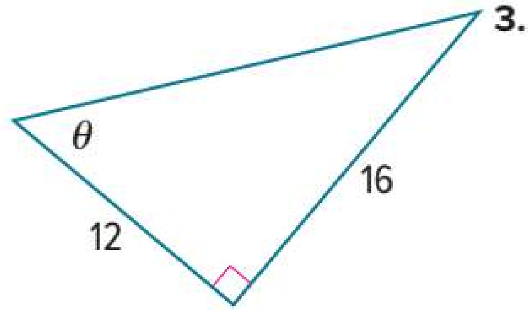
a



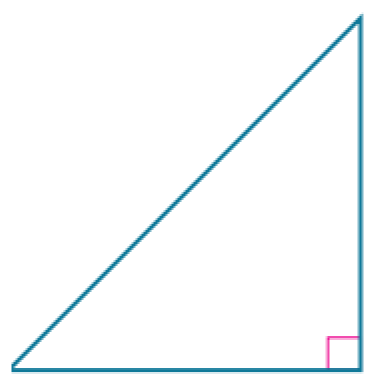
b



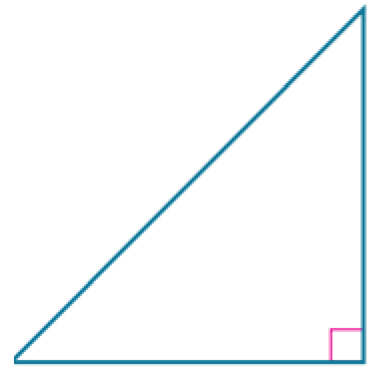
c



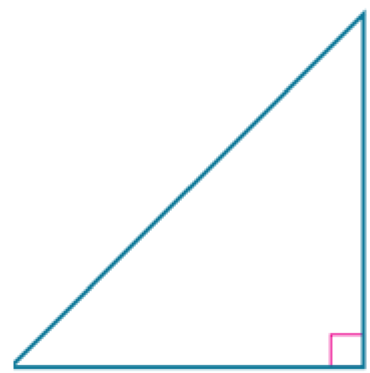
2 If $\cos A = \frac{9}{13}$, find the exact values of the five remaining trigonometric functions for A.



3 If $\sec B = \frac{11}{3}$, find the exact values of the five remaining trigonometric functions for B.

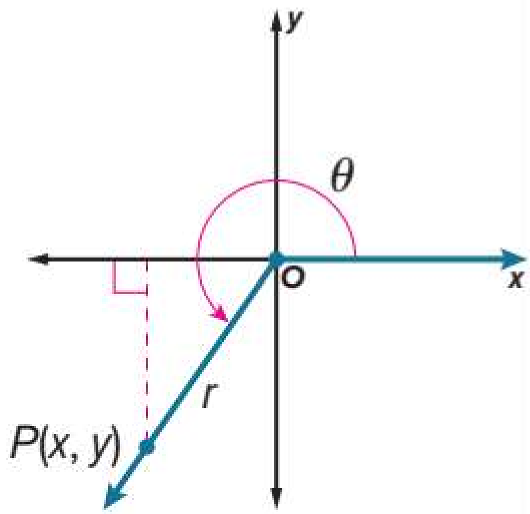


4 **In a right triangle, $\angle A$ and $\angle B$ are acute. Find the values of the five remaining trigonometric functions.** $\tan B = 3$



Key Concept • Trigonometric Functions of General Angles

Let θ be an angle in standard position, and let $P(x, y)$ be a point on its terminal side. By the Pythagorean Theorem, $r = \sqrt{x^2 + y^2}$, where r is the distance from the origin to point P along the terminal side. Using the coordinates of point P and r, the six trigonometric functions of θ are defined below.



$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$

$\tan \theta = \frac{y}{x}, x \neq 0$ $\quad$ $\csc \theta = \frac{r}{y}, y \neq 0$

$\sec \theta = \frac{r}{x}, x \neq 0$ $\quad$ $\cot \theta = \frac{x}{y}, y \neq 0$

- if $\theta = 0°$ or $360°$, then $r = x$.
- if $\theta = 90°$, then $r = y$.
- if $\theta = 180°$, then $r = |x|$.
- if $\theta = 270°$, then $r = |y|$.

The terminal side of θ in standard position contains each point. Find the exact values of the six trigonometric functions of θ.

5 (5, 12)

6 $(8, -15)$

7 $(-4, 3)$

8 $(-9, -40)$

Learn Trigonometric Functions with Reference Angles

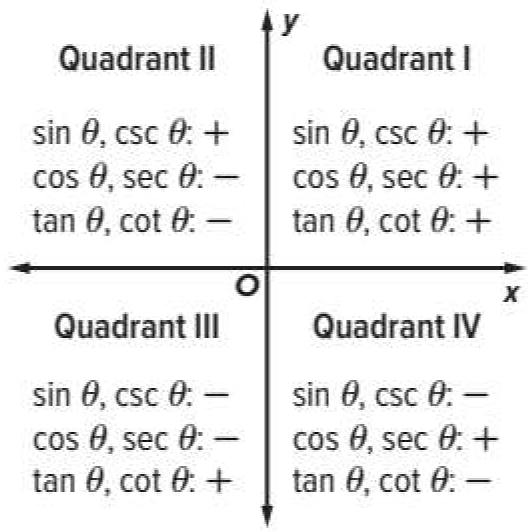
For a nonquadrantal angle θ in standard position, its **reference angle** is the acute angle θ' formed by the terminal side and the x-axis. The rules for finding the measures of reference angles vary depending on the quadrant in which the terminal side is located.

If the terminal side is in:

- Quadrant I, then $\theta' = \theta$.
- Quadrant II, then $\theta' = 180° - \theta$ or $\theta' = \pi - \theta$.
- Quadrant III, then $\theta' = \theta - 180°$ or $\theta' = \theta - \pi$.
- Quadrant IV, then $\theta' = 360° - \theta$ or $\theta' = 2\pi - \theta$.

Key Concept • Evaluating Trigonometric Functions

Step 1 Find the measure of the reference angle θ'.
Step 2 Evaluate the trigonometric function for θ'.
Step 3 Determine the sign of the trigonometric function values. Use the quadrant in which the terminal side of θ lies.



Trigonometric Values for Special Angles		
$\sin 30° = \frac{1}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$	$\sin 60° = \frac{\sqrt{3}}{2}$
$\cos 30° = \frac{\sqrt{3}}{2}$	$\cos 45° = \frac{\sqrt{2}}{2}$	$\cos 60° = \frac{1}{2}$
$\tan 30° = \frac{\sqrt{3}}{3}$	$\tan 45° = 1$	$\tan 60° = \sqrt{3}$
$\csc 30° = 2$	$\csc 45° = \sqrt{2}$	$\csc 60° = \frac{2\sqrt{3}}{3}$
$\sec 30° = \frac{2\sqrt{3}}{3}$	$\sec 45° = \sqrt{2}$	$\sec 60° = 2$
$\cot 30° = \sqrt{3}$	$\cot 45° = 1$	$\cot 60° = \frac{\sqrt{3}}{3}$

9 Sketch each angle. Then find the measure of its reference angle.

$155°$

$230°$

$-\frac{8\pi}{3}$

$-\frac{\pi}{6}$

$\frac{5\pi}{3}$

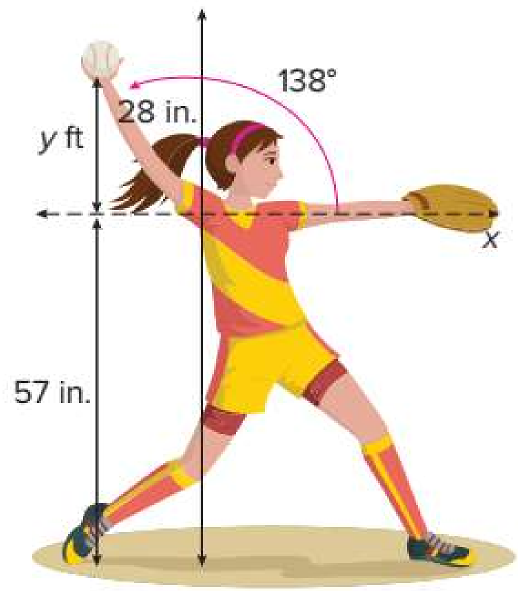
$200°$

10 **Find the exact value of each trigonometric function.**

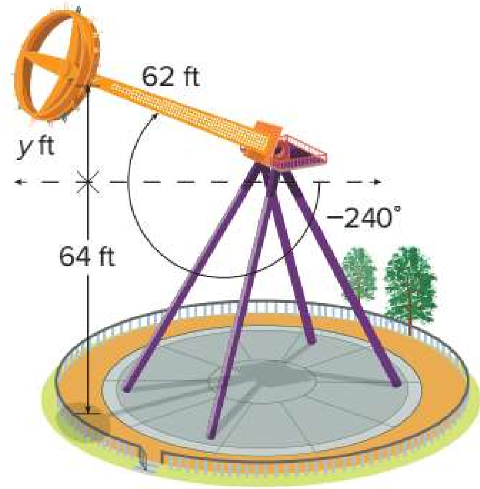
$\tan 330°$ $\cos\left(-\frac{11\pi}{4}\right)$ $\cot 30°$

$\csc \frac{\pi}{4}$ $\sin(-150°)$ $\tan\left(-\frac{\pi}{4}\right)$

11 **SOFTBALL** **During a windmill pitch in fastpitch softball, a pitcher's arm makes a complete clockwise rotation before releasing the ball. Suppose a pitcher has an arm length of 28 inches, and the axis from which the pitcher's arm swings is at her shoulder height of 57 inches. What is the height of the ball when the pitcher's arm is in the position shown?**

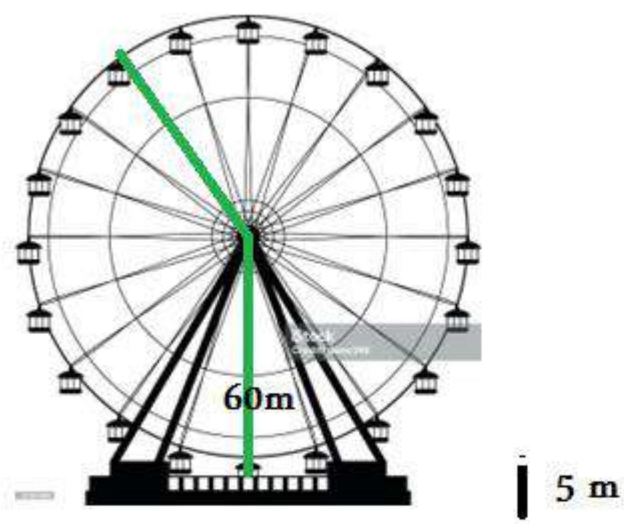


12 **AMUSEMENT RIDES** An amusement park thrill ride swings its riders back and forth on a pendulum that spins. Suppose the swing arm of the ride is 62 feet in length, and the axis from which the arm swings is about 64 feet above the ground. What is the height of the riders above the ground at the peak of the arc? Round to the nearest foot if necessary.



13 **FERRIS WHEELS Luis rides a Ferris wheel in Japan called the Sky Dream Fukuoka, which has a radius of about 60 m and is 5 m off the ground. After he enters the bottom car, the wheel rotates 210.5° counterclockwise before stopping. How high above the ground is Luis when the car has stopped?**

(116.7 m)



14 The terminal side of θ in standard position contains point (5,12). Find $sin\ \theta$ and $cos\ \theta$

$a)\ sin\ \theta = \frac{12}{13}, cos\ \theta = \frac{13}{12}$

$b)\ sin\ \theta = \frac{12}{13}, cos\ \theta = \frac{5}{13}$

$c)\ sin\ \theta = \frac{5}{12}, cos\ \theta = \frac{5}{13}$

$d)\ sin\ \theta = \frac{12}{5}, cos\ \theta = \frac{12}{13}$

15 The terminal side of θ in standard position contains point (8,-15). Find $tan\ \theta$ and $cos\ \theta$

$a)\ tan\ \theta = -\frac{8}{15}, cos\ \theta = \frac{8}{17}$

$b)\ tan\ \theta = \frac{15}{8}, cos\ \theta = -\frac{8}{17}$

$c)\ tan\ \theta = -\frac{15}{8}, cos\ \theta = \frac{8}{17}$

$d)\ tan\ \theta = -\frac{15}{17}, cos\ \theta = \frac{8}{15}$

16 The terminal side of θ in standard position contains point (-4,3). Find $cos\ \theta$ and $sec\ \theta$

$a)\ cos\ \theta = \frac{3}{5}, sec\ \theta = \frac{5}{3}$

$b)\ cos\ \theta = -\frac{4}{5}, sec\ \theta = -\frac{5}{4}$

$c)\ cos\ \theta = -\frac{5}{4}, sec\ \theta = -\frac{4}{5}$

$d)\ cos\ \theta = -\frac{3}{4}, sec\ \theta = -\frac{4}{3}$

17 The terminal side of θ in standard position contains point (-9,-40). Find $\sin\theta$ and $\tan\theta$

a) $\sin\theta = \frac{40}{41}, \tan\theta = \frac{40}{9}$

b) $\sin\theta = -\frac{40}{41}, \tan\theta = \frac{9}{40}$

c) $\sin\theta = -\frac{41}{40}, \tan\theta = \frac{40}{9}$

d) $\sin\theta = -\frac{40}{41}, \tan\theta = \frac{40}{9}$

18 Find the measure of each reference angle. $\frac{31\pi}{36}$

a) $\theta' = \frac{\pi}{3}$

b) $\theta' = \frac{5\pi}{36}$

c) $\theta' = \frac{3\pi}{8}$

d) $\theta' = \frac{\pi}{6}$

19 Find the measure of each reference angle.

$$\frac{4\pi}{3}$$

a) $\theta' = \frac{\pi}{3}$

b) $\theta' = \frac{5\pi}{36}$

c) $\theta' = \frac{3\pi}{8}$

d) $\theta' = \frac{\pi}{6}$

20 Find the measure of each reference angle.

$$200°$$

a) $\theta' = 60°$

b) $\theta' = 45°$

c) $\theta' = 30°$

d) $\theta' = 20°$

21 Find the measure of each reference angle.

$$-210°$$

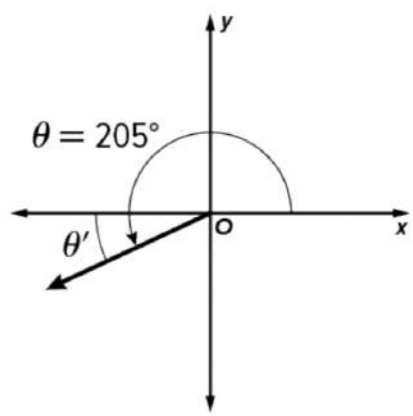
a) $\theta' = 60°$ b) $\theta' = 45°$

c) $\theta' = 30°$ d) $\theta' = 20°$

22 Find the measure of each reference angle.

a) $\theta' = 35°$ b) $\theta' = 25°$

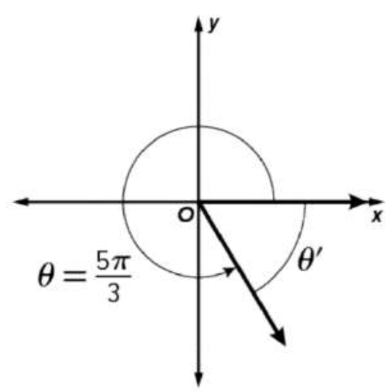
c) $\theta' = 15°$ d) $\theta' = 20°$



23 Find the measure of each reference angle.

a) $\theta' = \frac{\pi}{3}$ b) $\theta' = \frac{5\pi}{36}$

c) $\theta' = \frac{3\pi}{8}$ d) $\theta' = \frac{\pi}{6}$



THE FEATURED PROGRAM EDUCATION

البرنامج المميز

MATH ENG

12 GENERAL

LESSON 9-3

Circular and Periodic Functions

Mr. Ahmed Ata
The Featured Program

2025-2026

البرنامج المميز طريقك للتميز

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Lesson (9-3)

Circular and Periodic Functions

Outcomes

1. Find values of trigonometric functions given a point on a unit circle or the measure of a special angle.
2. Find values of trigonometric functions that model periodic events.

Key Concept • Functions on a Unit Circle

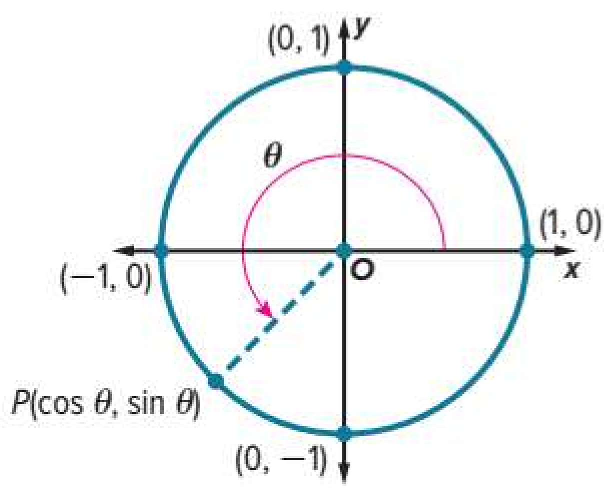
Words: If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos\theta = x$ and $\sin\theta = y$.

Symbols: $P(x, y) = P(\cos\theta, \sin\theta)$

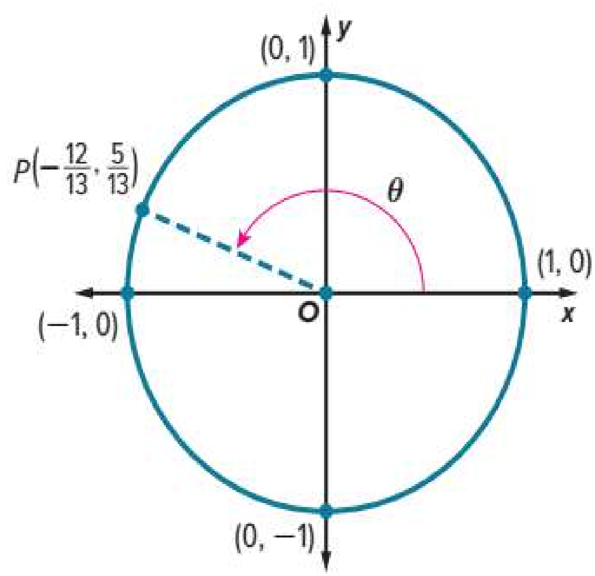
Example:

If $\theta = \frac{5\pi}{4}$,

$P(x, y) = P\left(\cos\frac{5\pi}{4}, \sin\frac{5\pi}{4}\right)$.



1 The terminal side of θ in standard position intersects the unit circle at $P\left(-\frac{12}{13}, \frac{5}{13}\right)$. Find $\cos\theta$ and $\sin\theta$.



2 The terminal side of θ in standard position intersects the unit circle at $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$. Find $\cos\theta$ and $\sin\theta$. Write the solutions as decimals.

$\cos\theta =$ $\sin\theta =$

3 **The terminal side of angle θ in standard position intersects the unit circle at each point P. Find cos θ and sin θ.**

$P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $P(0, -1)$ $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

$\cos\theta =$ $\cos\theta =$ $\cos\theta =$

$\sin\theta =$ $\sin\theta =$ $\sin\theta =$

4 **Find the exact values of the six trigonometric functions for an angle that measures $\frac{5\pi}{4}$ radians.**

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$ $\cot\theta =$

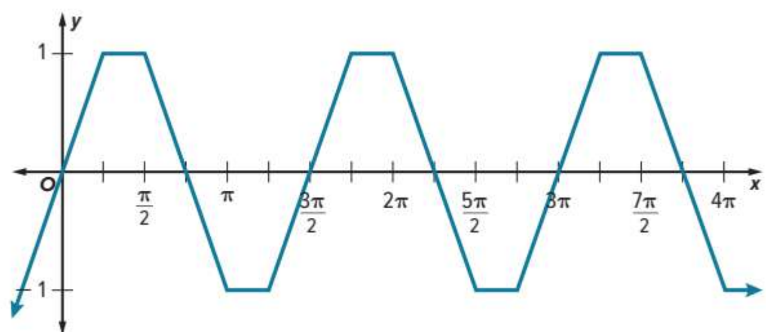
5 Find the exact values of the six trigonometric functions for an angle that measures $\frac{4\pi}{3}$ radians.

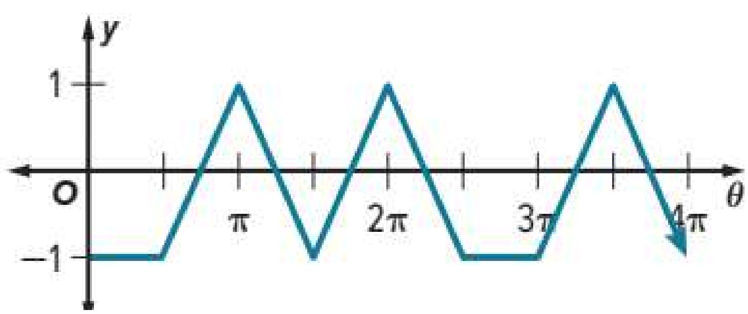
$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

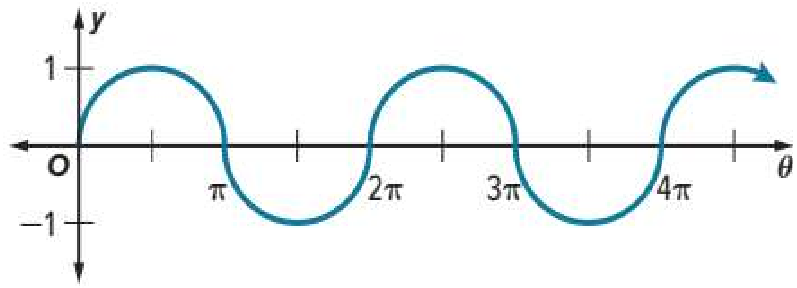
$\csc\theta =$ $\sec\theta =$ $\cot\theta =$

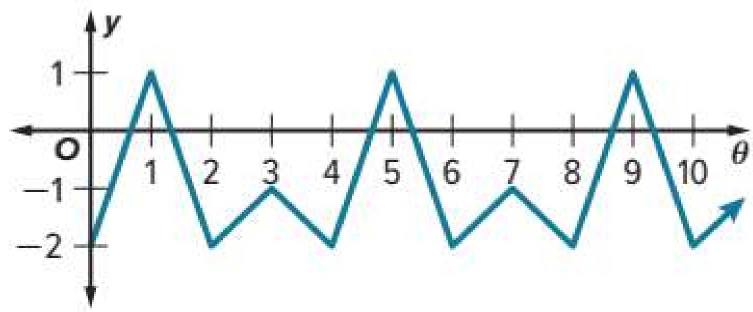
A periodic function has y-values that repeat at regular intervals. One complete pattern of a periodic function is called a cycle, and the horizontal length of one cycle is called the period.

6 **Determine the period of the function.**









7 **Find the exact value of each expression.**

$\sin \frac{8\pi}{3} =$

$\cos \frac{5\pi}{6} =$

$\sin \frac{21\pi}{4} =$

$\cos \frac{11\pi}{3} =$

$\cos 60° =$

$\sin 600° =$

8 Find the exact value of each expression.

$\cos 45° - \cos 30°$

$6(\sin 30°)(\sin 60°)$

$2 \sin \frac{4\pi}{3} - 3 \cos \frac{11\pi}{6}$

$\cos\left(-\frac{2\pi}{3}\right) + \frac{1}{3} \sin 3\pi$

$(\sin 45°)^2 + (\cos 45°)^2$

$\frac{(\cos 30°)(\cos 150°)}{\sin 315°}$

9 **The terminal side of angle θ in standard position intersects the unit circle at each point P. Find cos θ and sin θ.**

$P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$P\left(-\frac{4}{5}, -\frac{3}{5}\right)$

$P(0, -1)$

$P\left(\frac{1}{6}, -\frac{\sqrt{35}}{6}\right)$

10 Determine whether each statement is always true

a. If k is a real number, then there is a value of θ such that cos θ = k.

b. sin θ = sin (θ + 2π)

c. If θ = nπ, where n is a whole number, then cos θ = 1.

d. If θ is an angle in standard position in which the terminal side lies in Quadrant IV, then sin θ is positive

11 Point P lies on the unit circle and on the line y = x. If θ is an angle in standard position in which the terminal side contains P, what can you conclude about sin θ and cos θ?

$a)\ \sin\theta = 2\cos\theta$

$b)\ -\sin\theta = \cos\theta$

$c)\ \sin\theta = -2\cos\theta$

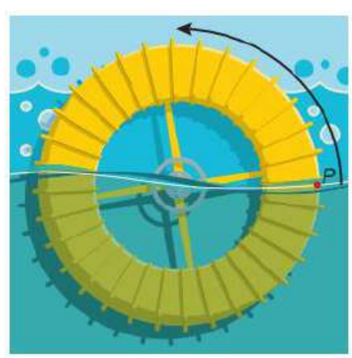
$d)\ \sin\theta = \cos\theta$

12 The wheel at a water park has a radius of 1 meter. As the water flows, the wheel turns counterclockwise, as shown. A point P on the edge of the wheel begins at the surface of the water. The function $f(x) = \sin x$ represents the height of P above or below the surface of the water as the wheel rotates through an angle of x radians.

How far does point P travel as the wheel rotates through an angle of $\frac{3\pi}{4}$

$a)\ \frac{5\pi}{4}$ *meters* $b)\ \frac{11\pi}{4}$ *meters*

$c)\ \frac{3\pi}{4}$ *meters* $d)\ \frac{\pi}{4}$ *meters*



13 A point on the edge of a car tire is marked with paint. As the car moves slowly, the marked point on the tire varies in distance from the surface of the road. The height in inches of the point is given by the function $h = -8\cos t + 8$, where t is the time in seconds.

a. What is the maximum and minimum height above ground that the point on the tire reaches?

$a)$ maximum 8 $inches$, minimum -8 $inches$

$b)$ maximum 8 $inches$, minimum 0 $inches$

$c)$ maximum 16 $inches$, minimum 0 $inches$

$d)$ maximum 16 $inches$, minimum -16 $inches$

THE FEATURED PROGRAM EDUCATION

البرنامج المميز

MATH ENG

12 GENERAL

LESSON 9-4

Graphing Sine and Cosine Functions

Mr. Ahmed Ata
The Featured Program

2025-2026

البرنامج المميز طريقك للتميز

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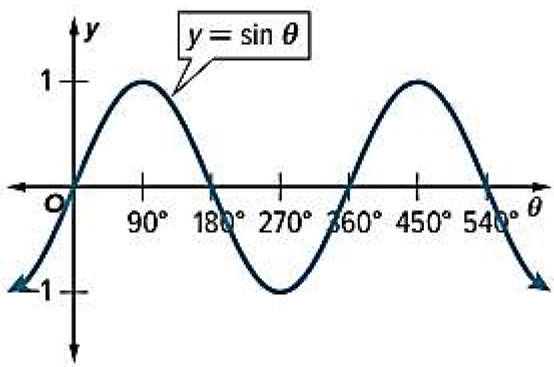
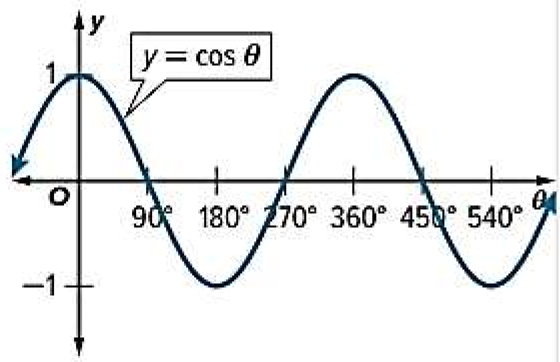
Lesson (9-4)

Graphing Sine and Cosine Functions

Outcomes

1. Graph and analyze sine and cosine functions.
2. Model periodic real-world situations with sine and cosine functions.

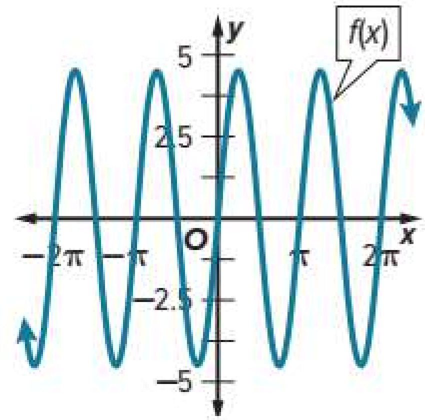
Key Concept • Sine and Cosine Functions

Parent	$y = \sin x$	$y = \cos x$
Graph		
Domain	all real numbers	all real numbers
Range	$\{y \mid -1 \le y \le 1\}$	$\{y \mid -1 \le y \le 1\}$
Amplitude	1	1
Midline	$y = 0$	$y = 0$
Period	360°	360°
Oscillation	between −1 and 1	between −1 and 1

You can also use x-intercepts to help graph the functions. The x-intercepts for one cycle of the sine and cosine functions are:

$y = a \sin bx$	$y = a \cos bx$
$(0, 0), \left(\frac{1}{2} \cdot \frac{360°}{b}, 0\right), \left(\frac{360°}{b}, 0\right)$	$\left(\frac{1}{4} \cdot \frac{360°}{b}, 0\right), \left(\frac{3}{4} \cdot \frac{360°}{b}, 0\right)$

1 **Identify the amplitude, midline, and period of $f(x)$.**

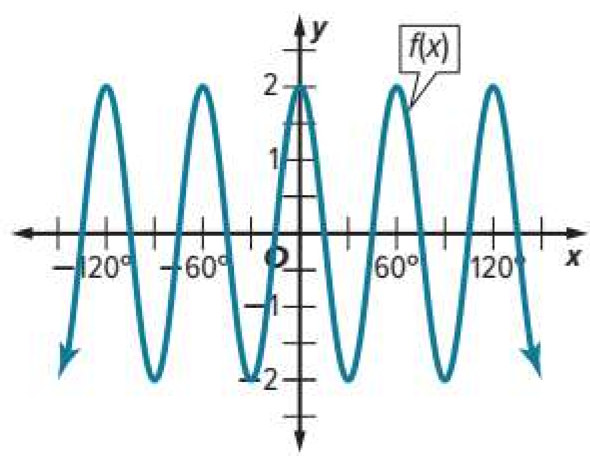


The amplitude is

The midline is at y =

The period is

2 **The amplitude is**



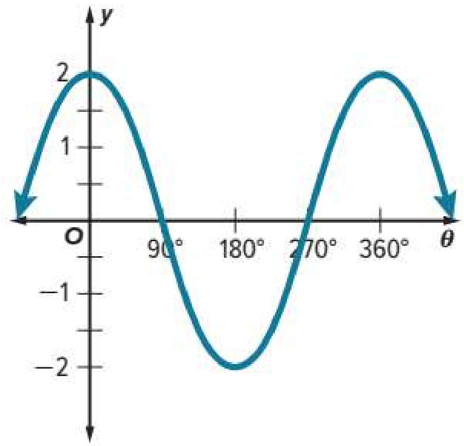
The midline is at y =

The period is

3 **The amplitude is**

The midline is at *y* =

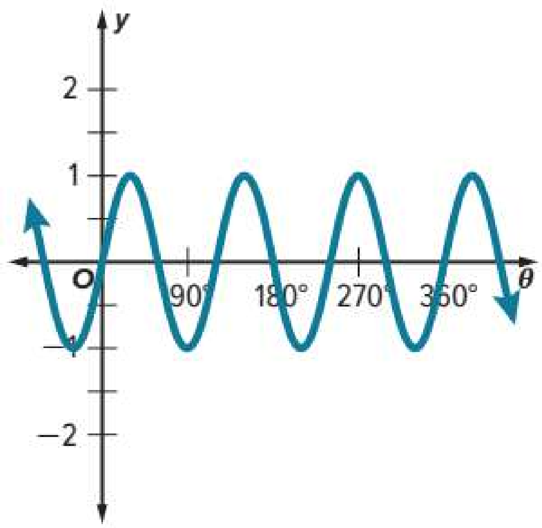
The period is



4 **The amplitude is**

The midline is at *y* =

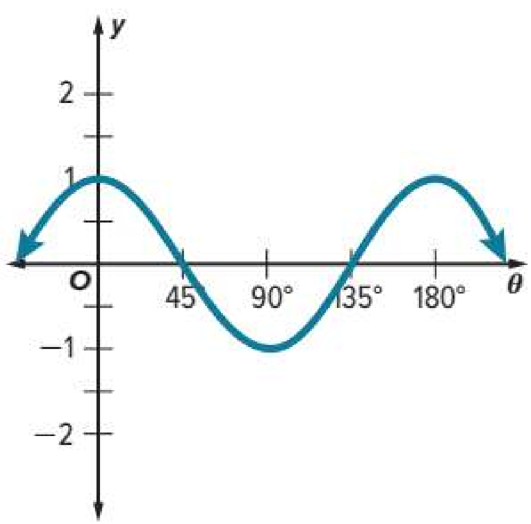
The period is



5 **The amplitude is**

The midline is at *y* =

The period is



6 Identify the amplitude and period of

$f(x) = 3\cos 5x$

The amplitude is

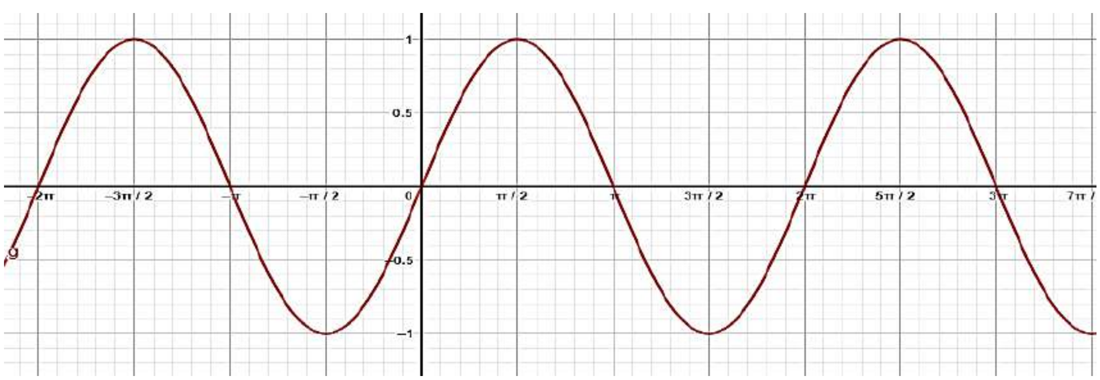
The period is

$f(x) = 7\sin 8x$

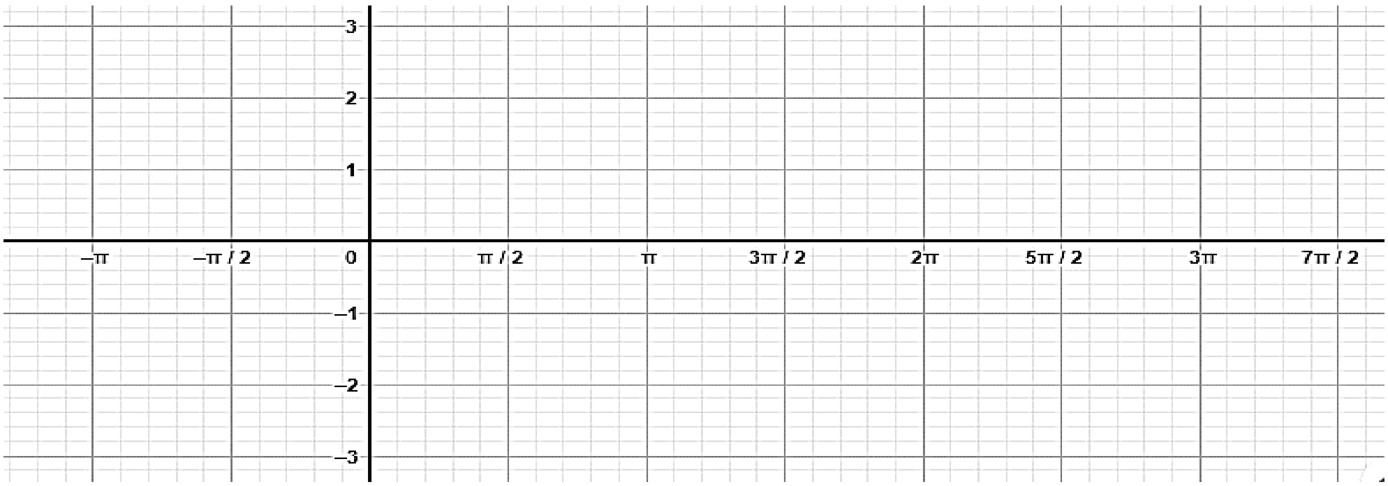
The amplitude is

The period is

7 Graph $y = 0.5\sin 2x$

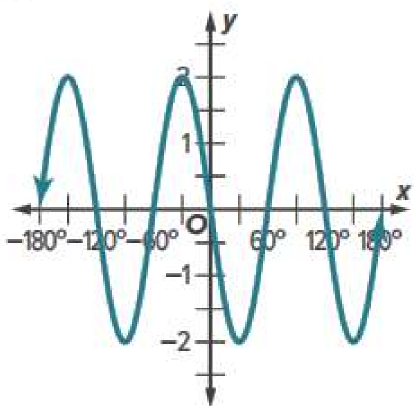


8 graph of $y = -2 \sin x$

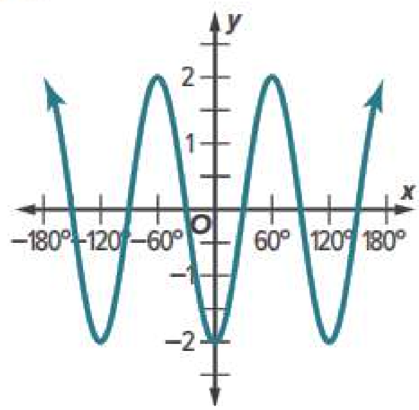


9 Select the graph of $y = -2 \sin 3x$

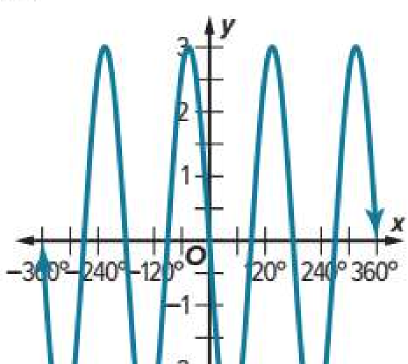
A.



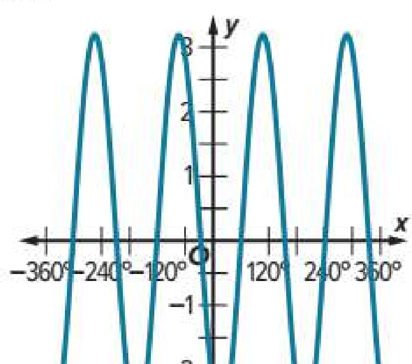
B.



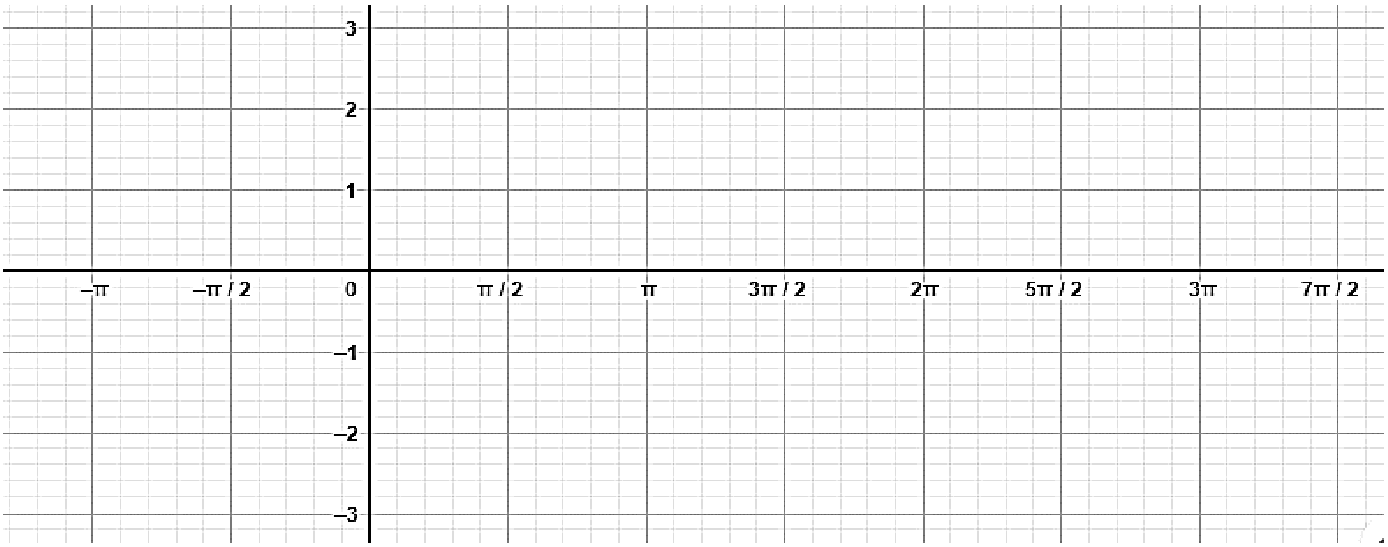
C.



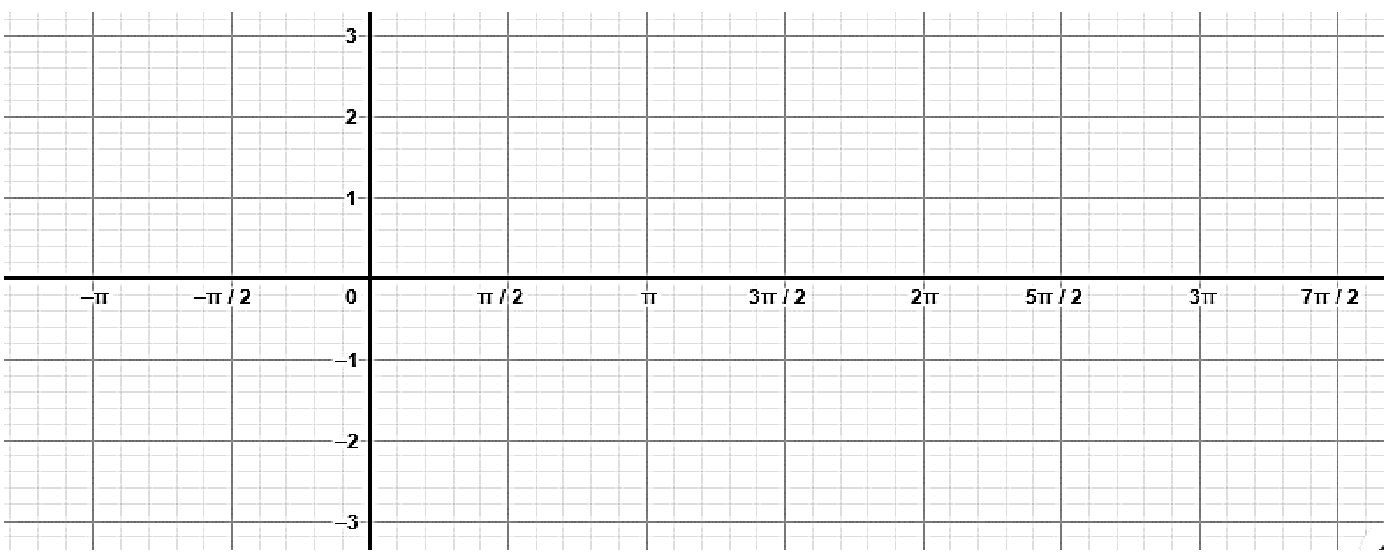
D.



10 Graph $y = 2\cos x$



11 Graph $y = -3\cos 2x$



12 An object on a spring oscillates according to the function $y = 40\cos \pi t$, where y is the distance in centimeters above its equilibrium position at time t in seconds.

Find the period and frequency

Identify the domain and range in the context of the situation.

13 The electric power delivered for household use in the United States is 110 volts. The function $y = 100\sqrt{2}\sin 120\pi t$ represents the effective current, where t is the time in seconds.

Find the period and frequency

14 The voltage supplied by an electrical outlet can be represented by a periodic function. The voltage oscillates between –120 and 120 volts, with a frequency of 50 cycles per second. Write a function for the voltage *v* as a function of time t

15 The voltage supplied by an electrical outlet can be represented by a periodic function. The voltage oscillates between –120 and 120 volts, with a frequency of 50 cycles per second. Write a function for the voltage *v* as a function of time *t*.

$a)$ $100\cos 120\pi t$

$b)$ $120\sin 100\pi t$

$c)$ $120\cos 50\pi t$

$d)$ $120\sin 50\pi t$

16 An object on a spring oscillates using the function $y = 25\cos\pi t$, where *y* is the distance in inches from its equilibrium position in *t* seconds.

a. Find and describe the period and frequency.
b. Identify the domain and range in the context of the situation.

17 Suppose a tire swing is rotated $\frac{\pi}{5}$ radians and released. The function $y = \frac{\pi}{5}cos2t$ represents the displacement of the swing at time t for a frequency of radians per second.

a. Find the period and frequency and describe them in the context of the situation.

b. Identify the domain and range in the context of the situation.

18 A Ferris wheel at a state fair has a diameter of 65 feet and makes 4 complete revolutions each minute. Santiago boards a car of the Ferris wheel at the car's lowest point, and he rides for 2 minutes. Write a trigonometric function that models his height above or below the axle of the Ferris wheel ϑ minutes after the ride starts.

$a)\ -65\sin 8\pi x$

$b)\ -65\cos 8\pi x$

$c)\ -32.5\sin 8\pi x$

$d)\ -32.5\cos 8\pi x$

THE FEATURED
PROGRAM EDUCATION

البرنامج المميز

12 GENERAL

MATH ENG

LESSON 9-5

Graphing Other Trigonometric Functions

Mr. Ahmed Ata
The Featured Program

2025-2026

البرنامج المميز طريقك للتميز

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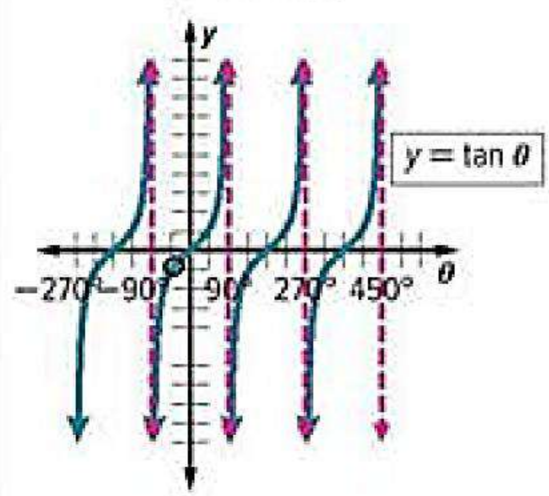
Lesson (9-5)

Graphing Other Trigonometric Functions

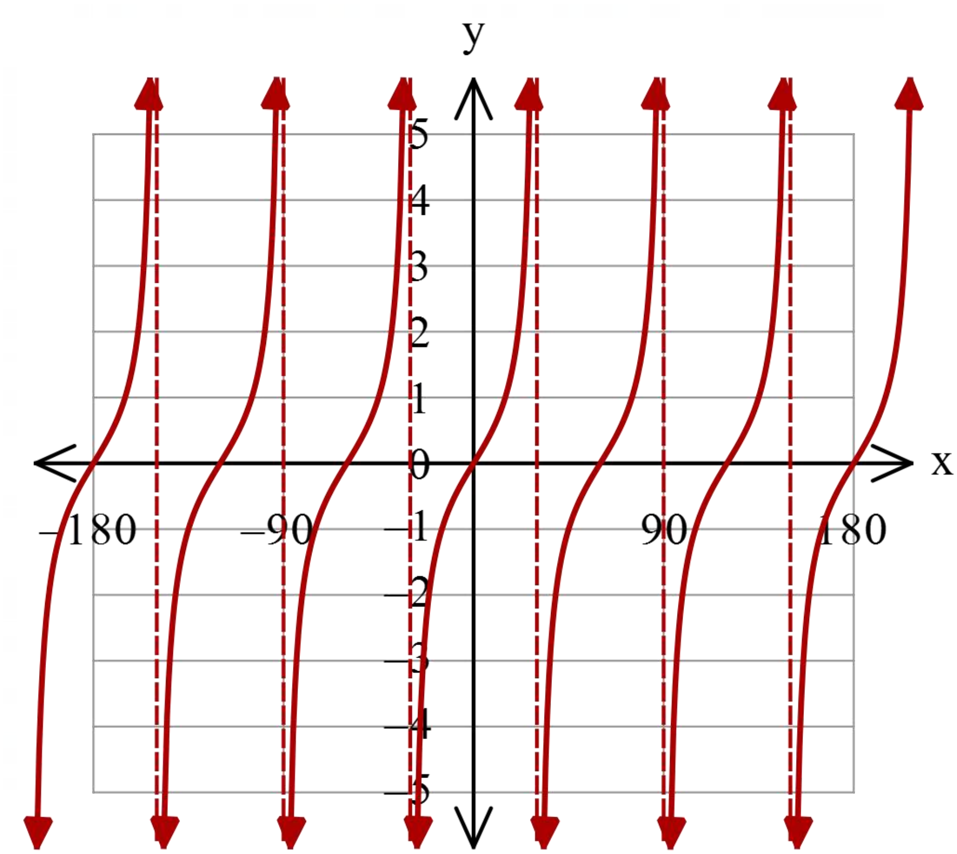
Outcomes

1. Graph and analyze tangent functions.
2. Graph and analyze reciprocal trigonometric functions.

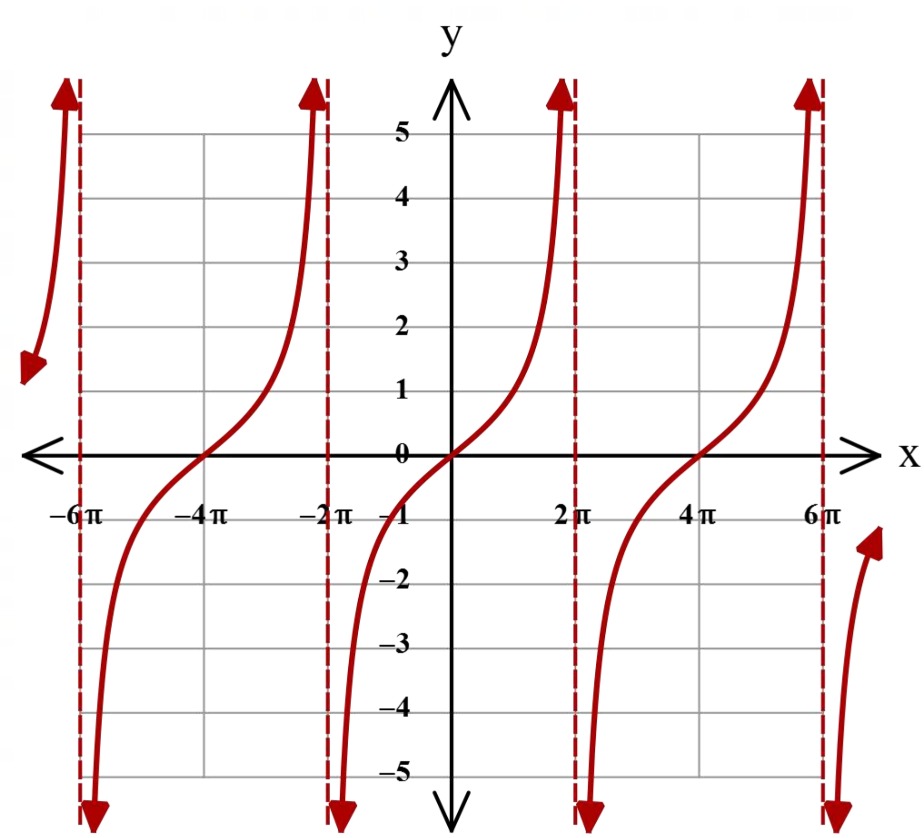
Key Concept • Graphs of Tangent Functions

		Graph
Parent Function	$y = \tan x$	
Domain	$\{x \mid x \neq (90 + 180n)°$, n is an integer$\}$	
Range	all real numbers	
Amplitude	undefined	
Period	180°	
Number of x-Intercepts in One Cycle	1	
Midline	$y = 0$	

1 **Find the period, asymptotes, x-intercepts, midline, and transformations of $y = tan\ 3x$ Then graph the function.**

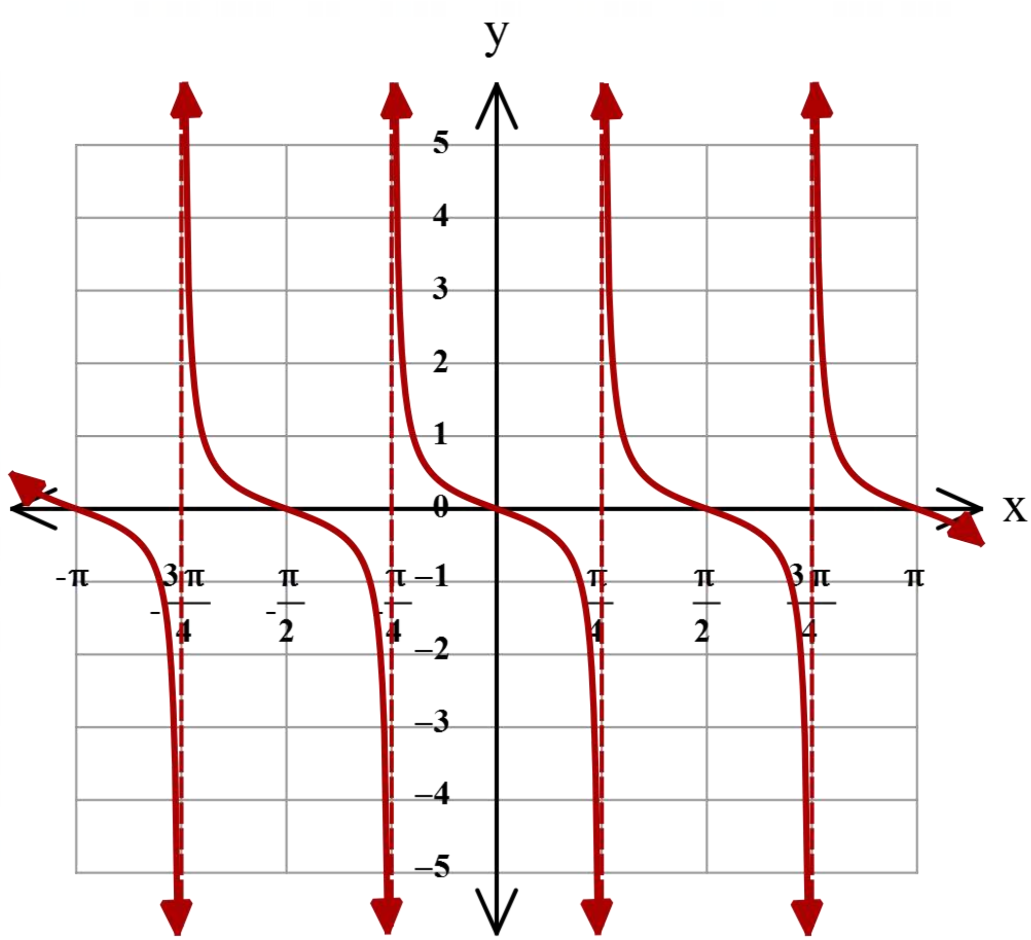


2 Find the period, asymptotes, x-intercepts, midline, and transformations of $y = tan\, 0.25x$ Then graph the function.

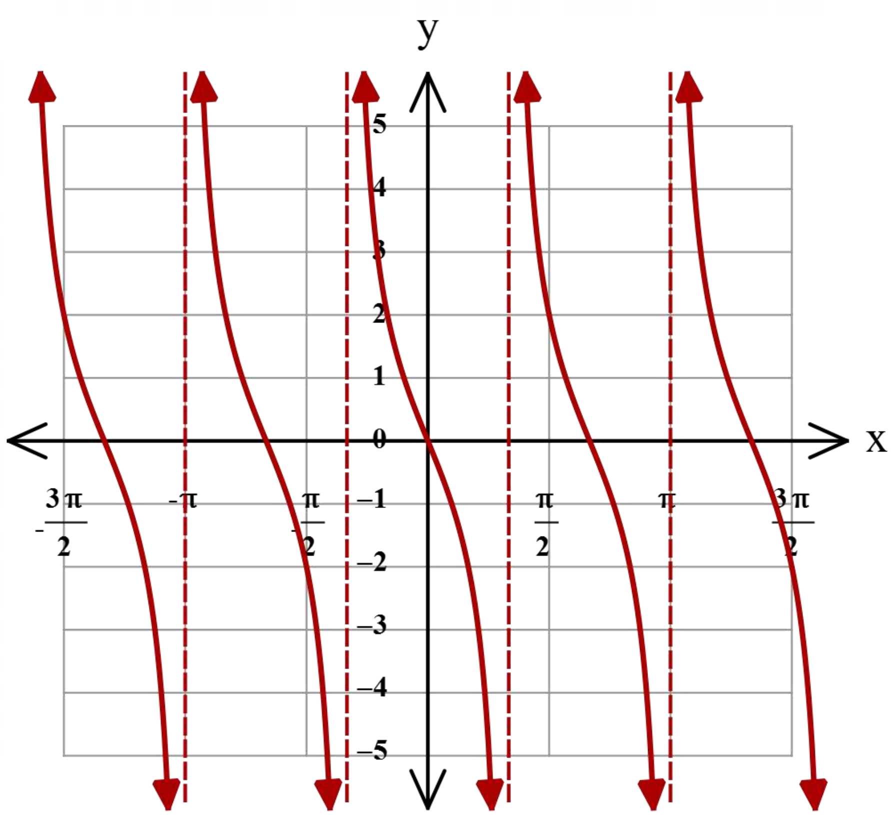


3 Find the period, asymptotes, x-intercepts, midline, and transformations of

$y = -\frac{1}{3}\ tan\ 2x$ Then graph the function.



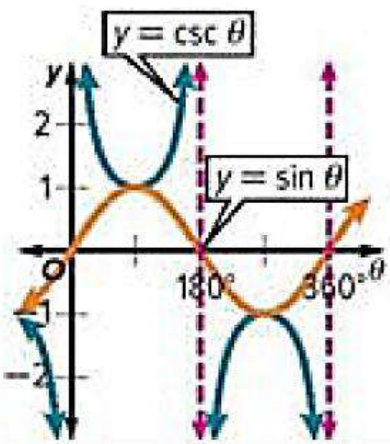
4 **Find the period, asymptotes, x-intercepts, midline, and transformations of** $y = -2\tan\frac{3}{2}x$ **Then graph the function.**

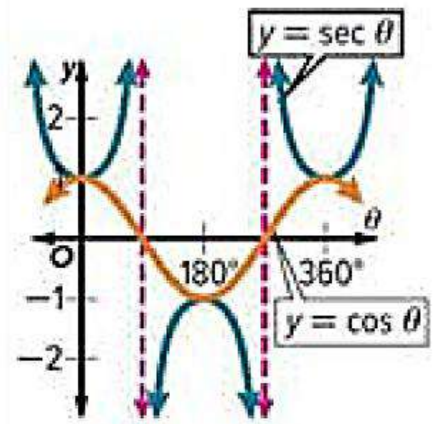


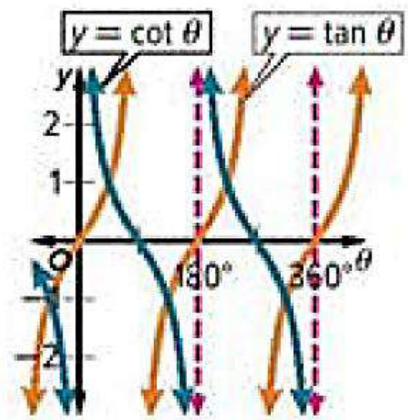
Key Concept • Graphs of Reciprocal Functions

Parent Function

$y = \csc x$	$y = \sec x$	$y = \cot x$







Domain		
$\{x \mid x \neq 180n°$, n is an integer$\}$	$\{x \mid x \neq (90 + 180n)°$, n is an integer$\}$	$\{x \mid x \neq 180n°$, n is an integer$\}$
Range		
$\{y \mid y \leq -1 \text{ or } y \geq 1\}$	$\{y \mid y \leq -1 \text{ or } y \geq 1\}$	all real numbers
Amplitude		
undefined	undefined	undefined
Period		
360°	360°	180°
Midline		
$y = 0$	$y = 0$	$y = 0$

1 **Find the period, asymptotes, relative extrema, and midline of**

$y = csc\, 0.5x$

6 $y = -4\, cot 2x$

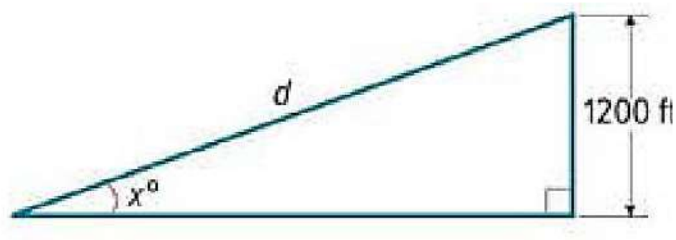
7 $y = csc\, \frac{3}{4}x$

8 $y = cot\frac{1}{2}x$

9 $y = 3secx$

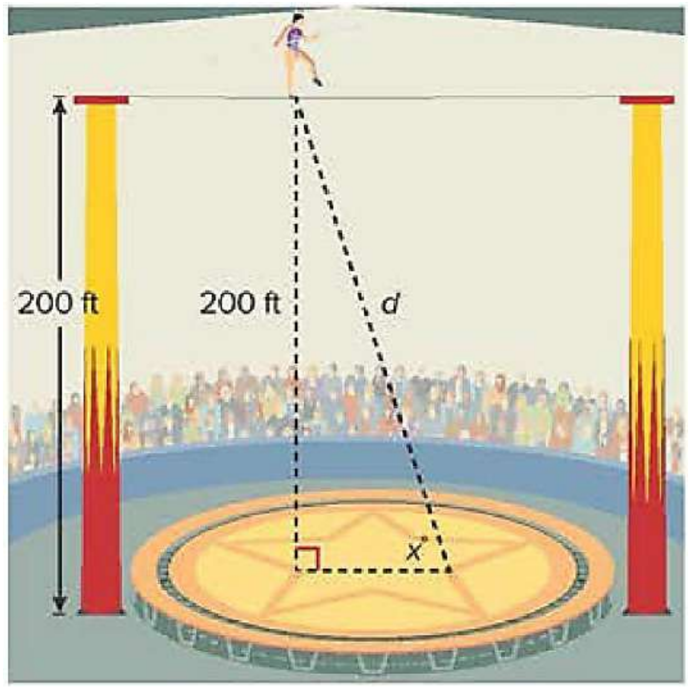
10 $y = sec\frac{1}{3}x$

11 **Suppose a banner towing plane is approaching a music festival at an elevation of 1200 feet above the crowd at the festival. The plane will eventually fly directly over the crowd. Let d be the distance in feet the banner is from the music festival and let x be the angle of elevation to the banner from the perspective of the crowd. Write a function that relates the distance as a function of an angle x. Then, graph the function and analyze its key features.**



12 Suppose an acrobat is walking from a high wire that is attached on its ends to two different towers at a height of 200 feet above the floor, as shown. The acrobat will eventually walk directly over the location of a camera in the floor. Let d be the distance in feet the acrobat is from the camera.

Write a function that relates the distance as a function of an angle x



13 **Ground-based air traffic controllers direct aircraft on the ground and in controlled airspace from airport control towers. Let y be the length of the shadow of a 300-foot control tower as the Sun moves across the sky at angle x.**

Write the function that relates the length of the shadow to the angle.