ملخص دروس الوحدة الثانية functions polynominal and Polynominals منهج ريفيل

تم تحميل هذا الملف من موقع المناهج الإماراتية
موقع المناهج ⇦ المناهج الإماراتية ⇦ الصف الثاني عشر العام ⇦ رياضيات ⇦ الفصل الأول ⇦ ملفات متنوعة ⇦ الملف
تاريخ إضافة الملف على موقع المناهج: 17-10-2025 09:11:07
المزيد من مادة رياضيات: ملفات ا كتب للمعلم ا كتب للطالب ا اختبارات الكترونية ا اختبارات ا حلول ا عروض بوربوينت ا أوراق عمل منهج انجليزي ا ملخصات وتقارير ا مذكرات وبنوك ا الامتحان النهائي ا للمدرس
إعداد: عماد عودة

التواصل الاجتماعي بحسب الصف الثاني عشر العام					
صفحة المناهج الإماراتية على فيسبوك					
المواد على تلغرام	التربية الاسلامية	اللغة العربية	اللغة الانجليزية	الرياضيات	

المزيد من الملفات بحسب الصف الثاني عشر العام والمادة رياضيات في الفصل الأول	
1	أسئلة اختبار تجريبي الكتروني وكتابي
2	أوراق عمل مراجعة الدرس الثامن المتباينات التربيعية من الوحدة الأولى
3	أوراق عمل مراجعة الدرس السابع تحويلات الدوال التربيعية من الوحدة الأولى
4	أوراق عمل مراجعة الوحدة الخامسة functions Exponential الدوال الأسية منهج ريفيل
5	الخطة الفصلية للدروس المقررة في الفصل منهج ريفيل

الرياضيات

Mathematics

الصف الثاني عشر عام

12 General

Reveal

الفصل الأول T1

ملخص دروس الوحدة الثانية

Module 02

Polynomials and Polynomial Functions

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2025-2026

Reveal

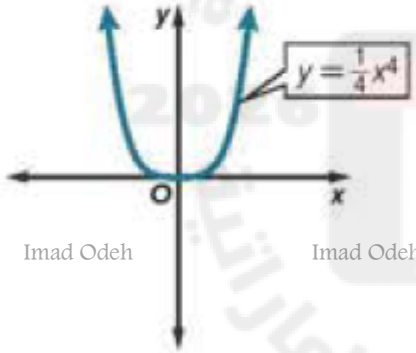
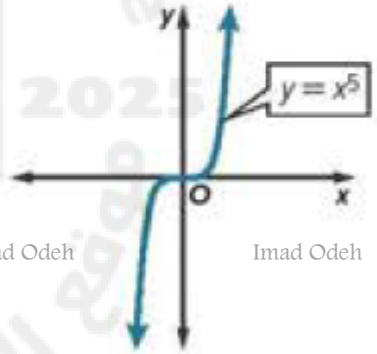
Lesson 2-1
Polynomial Functions

Learning Objectives

1. Graph and analysis power functions.
2. Graph and analyze polynomial functions

Power function is any function of the form $f(x) = ax^n$ where a and are nonzero real numbers. For a power function, a is the leading coefficient and n is the degree, which is the value of the exponent. A power function with positive integer n is called a monomial function.

Key Concept • End Behavior of a Monomial Function

Degree: even	Degree: odd
Leading Coefficient: positive	Leading Coefficient: positive
End Behavior:	End Behavior:
As $x \to -\infty$, $f(x) \to \infty$.	As $x \to -\infty$, $f(x) \to -\infty$.
As $x \to \infty$, $f(x) \to \infty$.	As $x \to \infty$, $f(x) \to \infty$.
	
Domain: all real numbers	Domain: all real numbers
Range: all real numbers ≥ 0	Range: all real numbers

Key Concept • Zeros of Even and Odd Degree Monomial Functions

Odd-degree functions will always have at least one real zero. Even-degree functions may have any number of real zeros or no real zeros at all.

Example 1 Describe the end behavior of using the leading coefficient and degree, and state the domain and range

$$f(x) = -2x^3$$

Leading coefficient

Degree

Domain

Range

End behavior

Ex1 Describe the end behavior of using the leading coefficient and degree, and state the domain and range

1) $f(x) = -10x^6$

Leading coefficient

Degree

Domain

Range

End behavior

2) $f(x) = 3x^4$

Leading coefficient

Degree

Domain

Range

End behavior

3) $f(x) = -2x^3$

Leading coefficient

Degree

Domain

Range

End behavior

4) $f(x) = -\frac{1}{2}x^5$

Leading coefficient

Degree

Domain

Range

End behavior

5) $f(x) = \frac{3}{4}x^6$

Leading coefficient

Degree

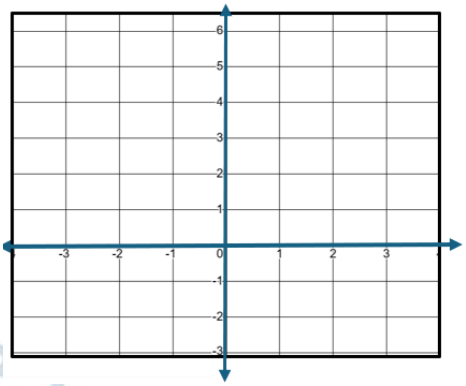
Domain

Range

End behavior

Example 2 PRESSURE For water to flow through a garden hose at a certain rate in gallons per minute (gpm), it needs to have a specific pressure in pounds per square inch (psi). Through testing and measurement, a company that produces garden hoses determines that the pressure P given the flow rate F is defined by $\boldsymbol{P(F)} = \frac{3}{2}\boldsymbol{F^2}$. Graph the function $\boldsymbol{P(F)}$, and state the domain and range.

F	$\frac{3}{2}F^2$	$P(F)$
-2		
-1		
0		
1		
2		

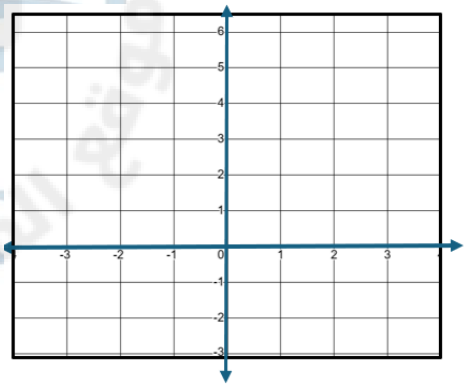


Ex2 The shape of a parabolic reflector inside a flashlight can be modeled by the function

$$\boldsymbol{f(x) = \frac{4}{3}x^2}$$

Graph the function $\boldsymbol{f(x)}$, and state the domain and range.

x	$f(x) = \frac{4}{3}x^2$	$f(x)$
-2		
-1		
0		
1		
2		

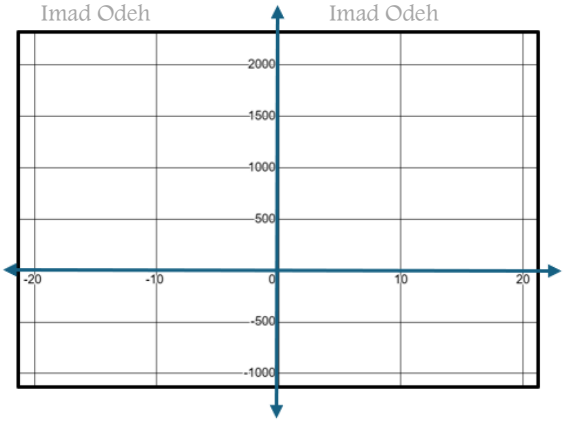


Ex3 MACHINE EFFICIENCY A company uses the function

$$f(x) = x^3 + 3x^2 - 218x - 40$$

To model the change in efficiency of a machine based on its position x. Graph the function and state the domain and range.

x	$f(x) = x^3 + 3x^2 - 218x - 40$	$f(x)$
-10		
-5		
5		
10		
15		
20		



Degrees and Leading Coefficients

Example3 State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1) $f(x) = 2x^4 - 3x^3 - 4x^2 - 5x + 6a$

2) $f(x) = 7x^3 - 2$

3) $f(x) = 4x^2 - 2xy + 8y^2$

4) $f(x) = x^5 + 12x^4 + 3x^3 + 8x^2 + 4$

Ex 4 State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

5) $f(n) = n + 8$

6) $f(x) = (2x - 1)(4x^2 + 3)$

7) $f(y) = 18 - 3y + 5y + 7y^6$

8) $f(x) = (5 - 2x)(4 + 3x)$

9) $f(x) = -5x^4 + 6x^2$

10) $h(x) = 9x^6 - 5x^7 + 3x^2$

11) $f(x) = -6x^6 - 4x^5 + 13x^{-2}$

12) $f(x) = 3x^7 - 4x^4 + \frac{3}{x}$

13) $f(x) = -5x^5 + 3x^3 - 8$

Ex 5 Select the degree and leading coefficient of

$$f(x) = 11x^3 + 5x^2 - 7x - \frac{6}{x}$$

a) degree: 3, leading coefficient: 11

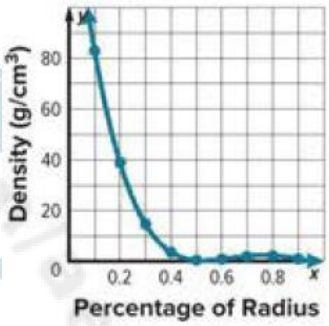
b) degree: 11, leading coefficient: 3

c) This is not a polynomial in one variable. There are two variables, x and y.

d) This is not a polynomial in one variable. The term $\frac{6}{x}$ has the variable with an exponent less than 0.

Ex 4 The density of the Sun, in grams per centimeter cubed, expressed as a percent of the distance from the core of the Sun to its surface can be modeled by the function $f(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155$ where x represents the percent as a decimal At the core x = 0, and at the surface $x = 1$. Graph the function.

x	f(x)
0.1	82.9619
0.2	38.7504
0.3	14.4539
0.4	3.4064
0.5	0.1875
0.7	1.7819
0.8	1.9824
0.9	0.7859



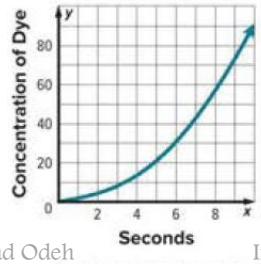
Ex 6 CARDIOLOGY To help predict heart attacks, doctors can inject a concentration of dye in a vein near the heart to measure the cardiac output in patients. In a normal heart, the change in the concentration of dye can be modeled by $f(x) = -0.006x^4 + 0.140x^3 - 0.053x^2 + 1.79x$, where x is the time in seconds.

Part A Find the concentration of dye after 5 seconds.

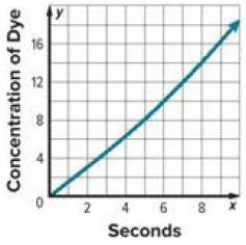
$f(5) =$

Part B Select the graph of the concentration of dye over 10 seconds.

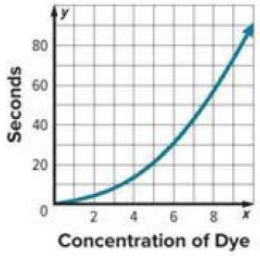
a)



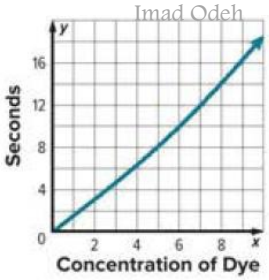
b)



c)



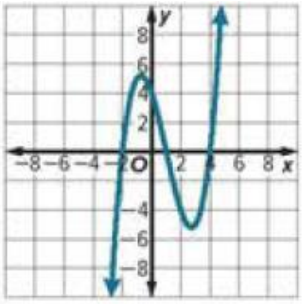
d)



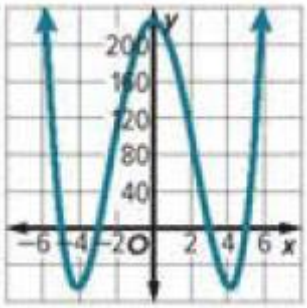
Zeros of a Polynomial Function

Use the graph to state the number of real zeros of the function.

Example

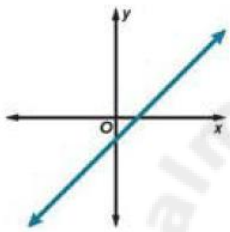


Ex 7

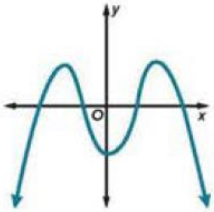


Ex8 Use the graph to state the number of real zeros of the function.

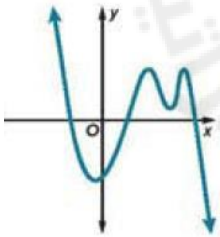
1)



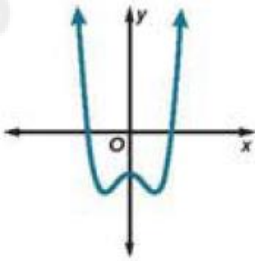
2)



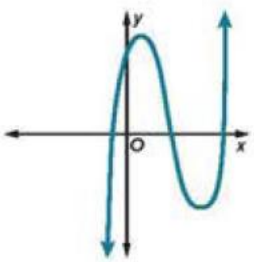
3)



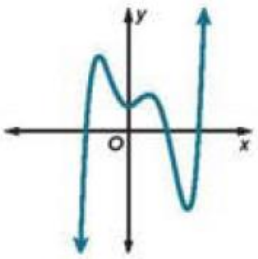
4)



5)

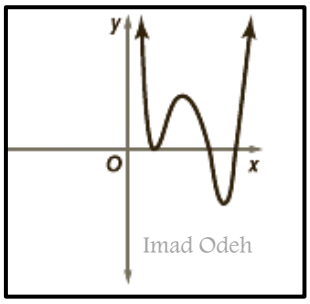


6)



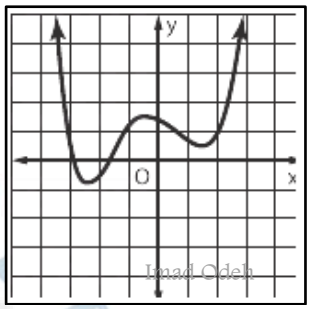
Ex 9	Use the graph to state the number of real zeros of the function.

a) 0
b) 3
c) 2
d) 5



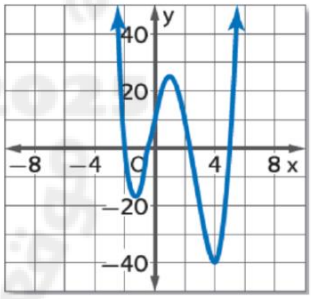
Ex 10	Use the graph to state the number of real zeros of the function.

a) 2
b) 3
c) 4
d) 5



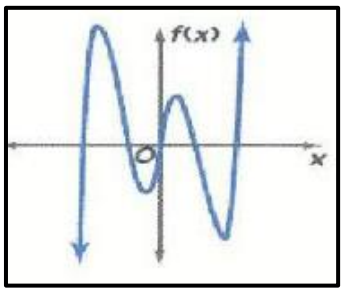
Ex 11	State the number of real zeros of the function with graph shown

a) 1
b) 4
c) 3
d) 2



Ex 12	State the number of real zeros of the function with graph shown

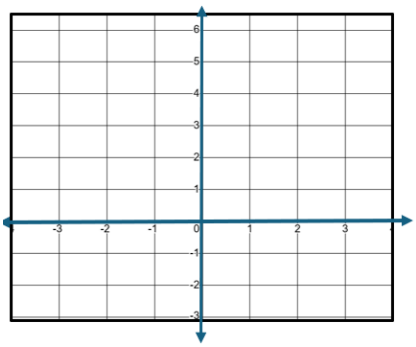
a) 5
b) 4
c) 3
d) 2

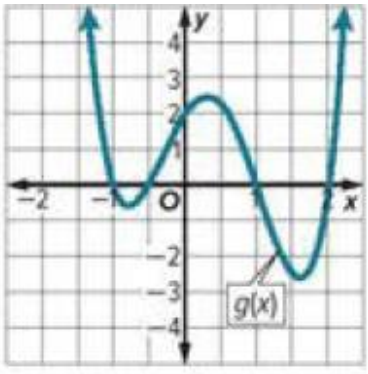


Compare Polynomial Functions

Example Examine $f(x) = x^3 + 2x^2 - 3x$ and $g(x)$ shown in the graph.

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	

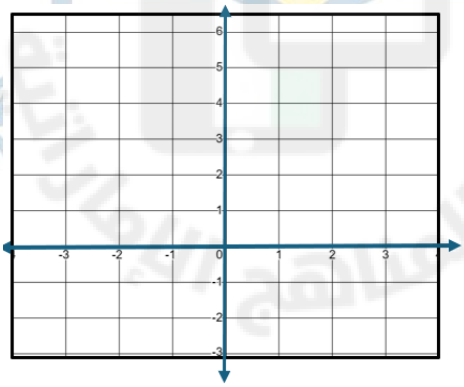


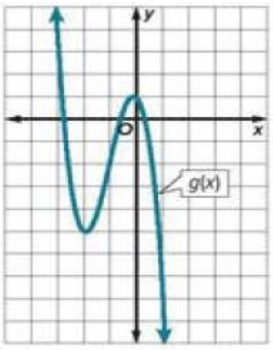


	$f(x)$	$g(x)$
Greater relative maximum		
Zeros		
x-intercepts		
y-intercepts		
End behavior		

Ex13 Examine $f(x) = x^3 - 2x^2 - 4x + 1$ and $g(x)$ shown in the graph.

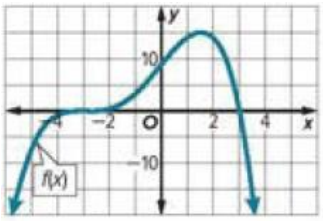
x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	





	$f(x)$	$g(x)$
Greater relative maximum		
Zeros		
x-intercepts		
y-intercepts		
End behavior		

Ex14 Examine the graph of $f(x)$ and $g(x)$shown in the table.

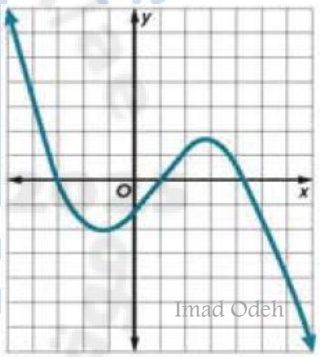


x	−5	−3	0	1.5	3
$g(x)$	7.5	0	−9	−15	0

a) Which function has the greater relative maximum?

b) Compare the zeros, x- and y-intercepts, and end behavior of $f(x)$ and $g(x)$.

Ex15 REASONING Describe the end behavior, and the possible degree and sign of the leading coefficient of the graph shown.



Ex 16 The weight of an ideal cut round diamond can be modeled by $f(d) = 0.0071d^3 - 0.090d^2 + 0.48d$, where d is the diameter of the diamond. Find the domain of the function in the context of the situation.

a) The domain is all real numbers.

b) The domain is $\{d|d \geq 0\}$.

c) The domain is $\{d|d \leq 0\}$.

d) The domain is $\{d|d \geq 0.48\}$.

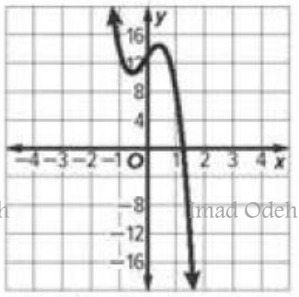
Ex 17 Use the function

$$f(x) = 13 - 2x^2 + 6x - 9x^3$$

to answer the following questions.

a) What is the degree?

b) What is the leading coefficient?



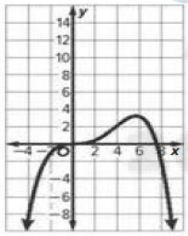
c) Describe the end behavior using the leading coefficient and degree.

Ex 18 The revenue of a certain business can be modeled using

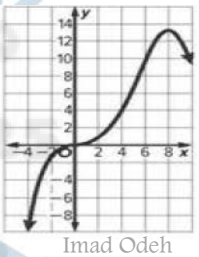
$f(x) = -0.01(x^4 - 11x^3 + 4x^2 - 5x + 7)$,

where x is the number of years since the business was started and f(x) is the revenue in hundred-thousands of dollars. Which graph represents the function?

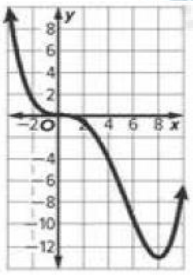
a)



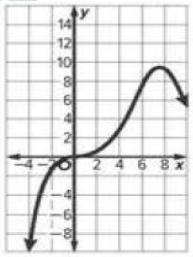
b)



c)



d)



Lesson 2-2
Analyzing Graphs of Polynomial Functions

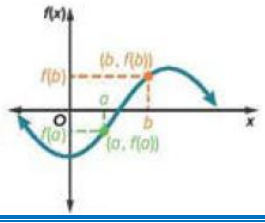
Today's Goals

- Approximate zeros by graphing polynomial functions.
- Find extrema of polynomial functions.

The Location Principle

Key Concept • Location Principle

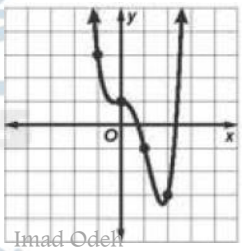
Suppose $y = f(x)$ represents a polynomial function, and a and b are two real numbers such that $f(a) < 0$ and $f(b) > 0$. Then the function has at least one real zero between **a** and **b**.



Locate Zeros of a Function

Example Determine the consecutive integer values of x between which each real zero of $f(x) = x^4 - 2x^3 - x^2 + 1$ is located. Then draw the graph.

x	-2	-1	0	1	2	3	4
$f(x)$							



Using the Location Principle, there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

Ex1 Determine the consecutive integer values of x between which each real zero of each function is located. Then draw the graph.

1) $f(x) = 2x^4 + x^3 - 2x^2 - 2$

x	-2	-1	0	1	2	3	4
$f(x)$							

Using the Location Principle, there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

2) $f(x) = x^2 + 3x - 1$

x	-2	-1	0	1	2	3	4
$f(x)$							

Using the Location Principle, there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

Ex1 Determine the consecutive integer values of x between which each real zero of each function is located. Then draw the graph.

3) $f(x) = 2x^4 + x^3 - 2x^2 - 2$

x	-2	-1	0	1	2	3	4
$f(x)$							

Imad Odeh Imad Odeh

Using the Location Principle, there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

4) $f(x) = x^2 + 3x - 1$

x	-2	-1	0	1	2	3	4
$f(x)$							

Imad Odeh Imad Odeh

Using the Location Principle, there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

5) $f(x) = -x^3 + 2x^2 - 4$

x	-2	-1	0	1	2	3	4
$f(x)$							

Imad Odeh Imad Odeh

Using the Location Principle, there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

6) $f(x) = x^3 + 4x^2 - 5x + 5$

x	-2	-1	0	1	2	3	4
$f(x)$							

Imad Odeh Imad Odeh

Using the Location Principle, there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

7) $f(x) = -x^4 - x^3 + 4$

x	-2	-1	0	1	2	3	4
$f(x)$							

Using the Location Principle, there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

Imad Odeh Imad Odeh

Ex2 Determine consecutive integer values if x between which real zero is located of

تمرين حدد القيم المتعاقبة للعدد الصحيح x التي يقع بينها صفر حقيقي ل

$$f(x) = x^3 - 2x^2 + 5$$

a) Between $x = -1, x = 0$ بين

b) Between $x = 1, x = 2$ بين

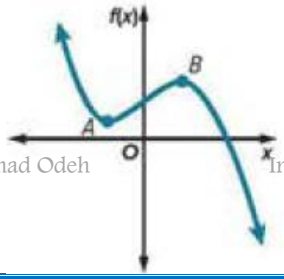
c) Between $x = -2, x = -1$ بين

Imad Odeh Imad Odeh Imad Odeh Imad Odeh

d) Between $x = -3, x = -2$ بين

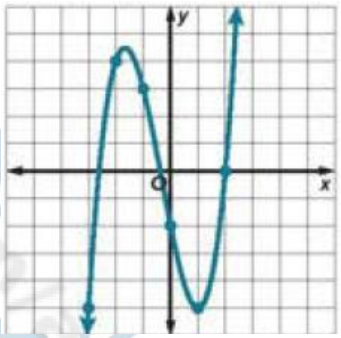
Extrema of Polynomials

Extrema occur at relative maxima or minima of the function.
Point **A** is a relative minimum, and point **B** is a relative maximum. Both points **A** and **B** are extrema.
The graph of a polynomial of degree n has at most $n-1$ extrema.



Example Use a table to graph $f(x) = x^3 + x^2 - 5x - 2$. Estimate the x-coordinates at which the relative maxima and relative minima occur.

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	

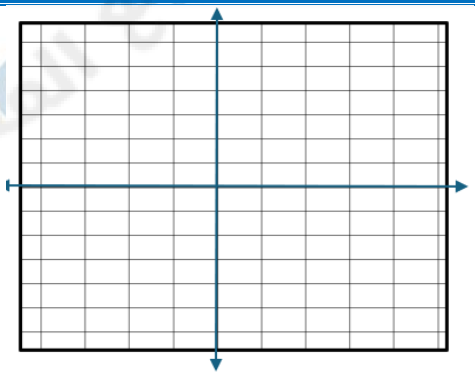


The value of $f(x)$ at $x =$ .....is greater than the surrounding points indicating a maximum near $x =$ .....

The value of $f(x)$ at $x =$..... is less than the surrounding points indicating a minimum near $x =$ .....

Ex3 Use a table to graph $f(x) = -x^4 - x^3 + x^5 + x - 3$. Estimate the x-coordinates at which the relative maxima and relative minima occur.

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



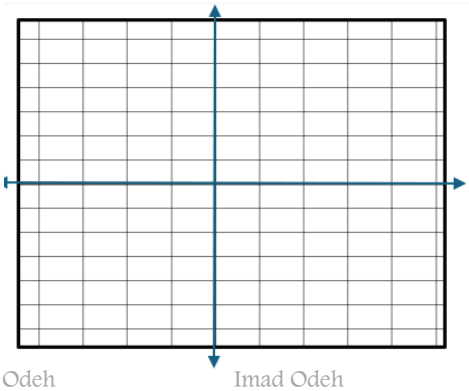
The relative maxima occur near $x =$..... and $x =$.....

The relative minimum occurs near $x =$..... and $x =$.....

Ex4 Determine the consecutive integer values of x between which each real zero of the function $f(x)$ is located by using a table. Then sketch the graph.

$f(x) = -2x^3 + 12x^2 - 8x$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



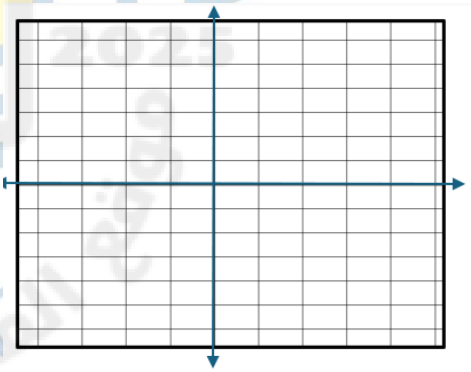
The relative maxima occur near x =..... and x =.....

The relative minimum occurs near x =..... and x =.....

Ex5 Determine the consecutive integer values of x between which each real zero of the function $f(x)$ is located by using a table. Then sketch the graph.

$f(x) = 2x^3 - 4x^2 - 3x + 4$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



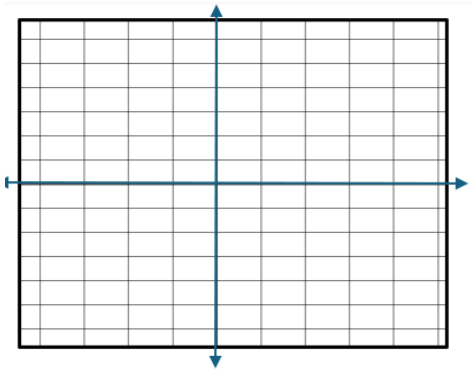
The relative maxima occur near x =..... and x =.....

The relative minimum occurs near x =..... and x =.....

Ex6 Determine the consecutive integer values of x between which each real zero of the function $f(x)$ is located by using a table. Then sketch the graph.

$f(x) = x^4 + 2x - 1$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



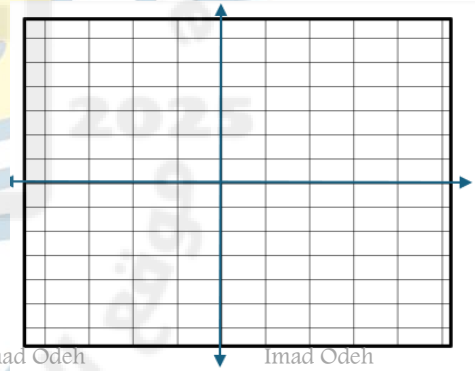
The relative maxima occur near x =..... and x =.....

The relative minimum occurs near x =..... and x =.....

Ex7 Determine the consecutive integer values of x between which each real zero of the function $f(x)$ is located by using a table. Then sketch the graph.

$f(x) = x^4 + 8x^2 - 12$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	

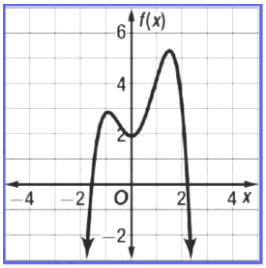


The relative maxima occur near x =..... and x =.....

The relative minimum occurs near x =..... and x =.....

Ex8 Estimate the x-coordinate at which a relative maximum occurs

a) -1.5
b) 2.2
c) 1.5
d) 0



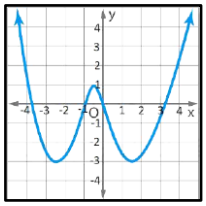
Ex9 Estimate the x-coordinate at which a relative maximum occurs

a) 1

b) 1.5

c) $-\frac{1}{2}$

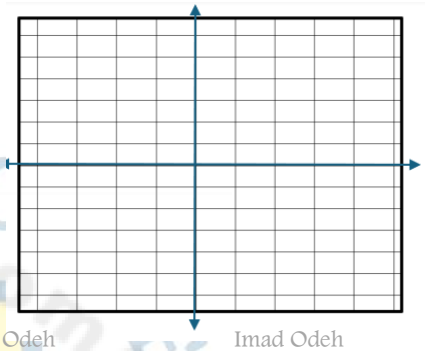
d) -2.5



Ex10 Graph each function by using a table of values. Then, estimate the x −coordinates at which each zero and relative extrema occur and state the domain and range.

$$f(x) = x^3 - 3x + 1$$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

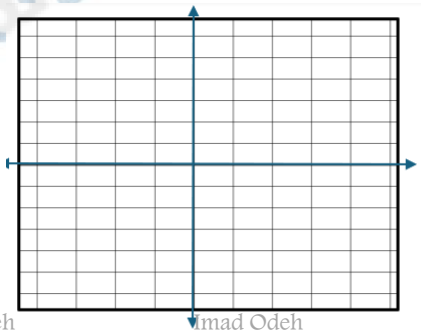
The relative maxima occur near
x =..... and x =.....
The relative minimum occurs near
x =..... and x =.....

Domain .....
Range .....

Exercise Graph each function by using a table of values. Then, estimate the x −coordinates at which each zero and relative extrema occur and state the domain and range.

$$f(x) = 2x^3 + 9x^2 + 12x + 2$$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

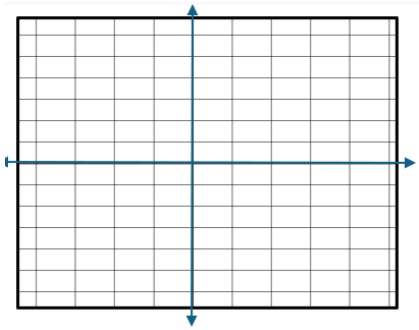
The relative maxima occur near
x =..... and x =.....
The relative minimum occurs near
x =..... and x =.....

Domain .....
Range .....

Ex11 Graph each function by using a table of values. Then, estimate the x –coordinates at which each zero and relative extrema occur and state the domain and range.

$$f(x) = 2x^3 - 3x^2 + 2$$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

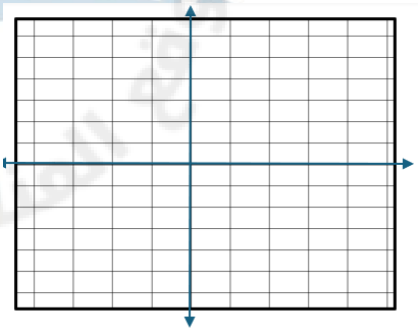
The relative maxima occur near
x =..... and x =.....
The relative minimum occurs near
x =..... and x =.....

Domain .....
Range .....

Ex 12 Graph each function by using a table of values. Then, estimate the x –coordinates at which each zero and relative extrema occur and state the domain and range.

$$f(x) = x^4 - 2x^2 - 2$$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



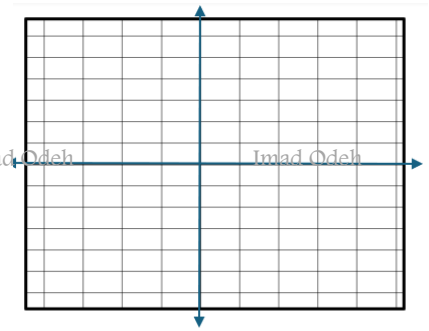
there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

The relative maxima occur near
x =..... and x =.....
The relative minimum occurs near
x =..... and x =.....

Domain .....
Range .....

Ex13 Determine the key features for

$$y = \begin{cases} x^2 & if\ x \le -4 \\ 5 & if\ -4 < x \le 0 \\ x^3 & if\ x > 0 \end{cases}$$



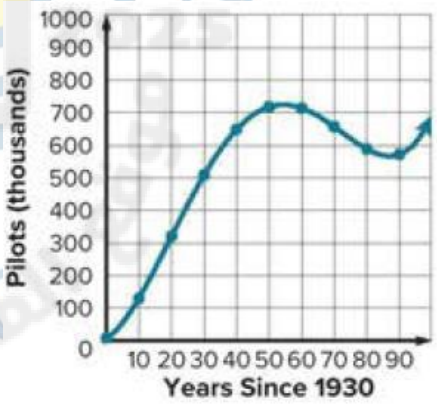
there are zeros
Between x = ... and x = ...
and
between x = ... and x = ...

The relative maxima occur near
x =..... and x =.....
The relative minimum occurs near
x =..... and x =.....

Domain .....
Range .....

Example PILOTS The total number of certified pilots in the United States is approximated by $f(x) = 0.0000903x^4 - 0.0166x^3 + 0.762x^2 + 6.317x + 7.708$, where x is the number of years after **1930** and $f(x)$ is the number of pilots in thousands. Graph the function and describe its key features over the relevant domain.

x	f(x)
0	7.708
10	131.381
20	320.496
30	507.961
40	648.356
50	717.933
60	714.616
70	658.001
80	589.356
90	571.621



The relevant domain is.....

The relevant range is.....

There is a relative minimum between ..... and .....

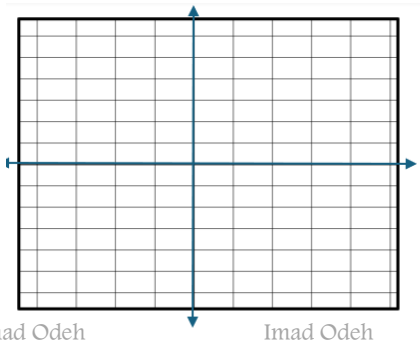
The y-intercept is.....

The graph of the function ..... have symmetry

It is ..... to assume that the trend will continue indefinitely

Ex 14 COINS The number of quarters produced by the United States Mint can be approximated by the function $f(x) = 16.4x^3 - 149.5x^2 - 148.9x + 3215.4$, where x is the number of years since **2005** and $f(x)$ is the total number of quarters produced in millions. Use the graph of the function to complete the table and describe its key features.

x, Years	f(x), Quarters (millions)
0	
2	
4	
6	
8	
10	



The relevant domain is

The relevant range is

There is a relative minimum between ..... and .....

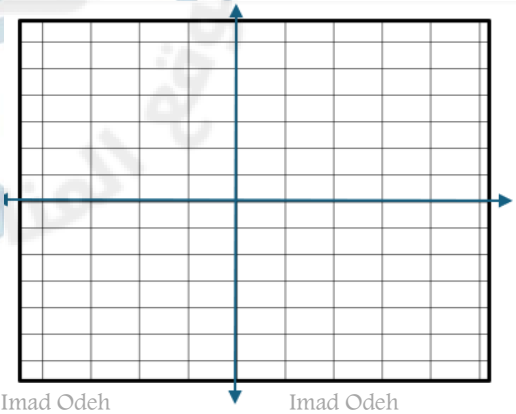
The y-intercept is

The graph of the function ..... have symmetry

It is ..... to assume that the trend will continue indefinitely

Ex15 **BUSINESS** A banker models the expected value v of a company in millions of dollars by using the formula $v = n^3 - 3n$, where n is the number of years in business. Graph the function and describe its key features over the relevant domain.

x	$f(x)$



The relevant domain is

The relevant range is

There is a relative minimum between ..... and .....

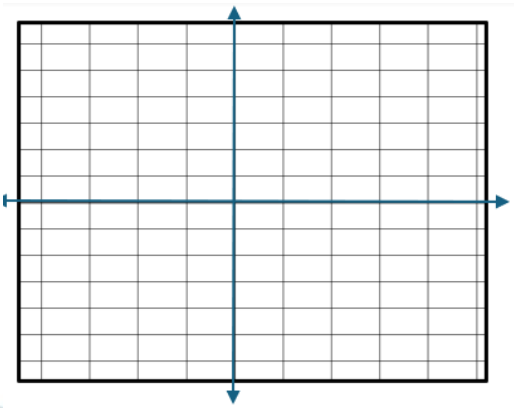
The y-intercept is

The graph of the function ..... have symmetry

It is ..... to assume that the trend will continue indefinitely

Ex 16 **BUSINESS** HEIGHT A plant's height is modeled by the function $f(x) = 1.5x^3 - 20x^2 + 85x - 84$ where x is the number of weeks since the seed was planted and f(x) is the height of the plant. Graph the function and describe its key features over its relevant domain.

x	$f(x)$



The relevant domain is.....

The relevant range is.....

There is a relative minimum between ..... and .....

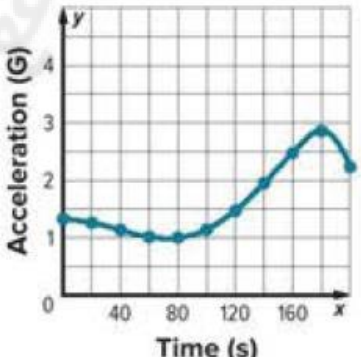
The y-intercept is.....

The graph of the function ..... have symmetry

It is ..... to assume that the trend will continue indefinitely

Example ROCKETS The Ares-V rocket was designed to carry as much as 75 tons of supplies and 4 astronauts to the Moon and possibly even to Mars. The table shows the expected g-force on the rocket over the course of its 200-second launch.

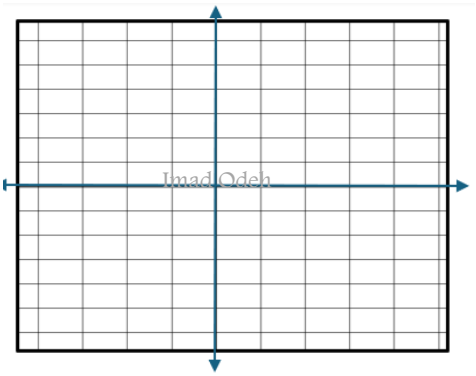
Time (s)	Acceleration (Gs)	Time (s)	Acceleration (Gs)
0	1.34	120	1.46
20	1.26	140	1.93
40	1.12	160	2.47
60	1.01	180	2.84
80	1	200	2.2
100	1.15		



Part A Find the average rate of change Estimate: Check algebraically:

Part B Interpret the results.

Ex17 **PRECISION** Sketch the graph of a third-degree polynomial function that has a **relative minimum** at $x = -3$, passes through the **origin**, and has **a relative maximum** at $x = 2$. Describe the end behavior of the graph. Based on the sketch, determine whether the leading coefficient is negative or positive.



Ex18 Describe the end behavior for

$g(x) = -2x^4 - 6x^3 + 11x - 18 \; as \; x \to \infty.$

Ex 19 Marshall claims that there is only one real zero in the function $f(x) = 4x^3 + 7x^2 - 5x + 3$. Based on the table provided, determine whether you agree with Marshall. Then name the interval(s) in which the zero(s) is/are located.

x	f(x)
−3	−27
−2	9
1	11
0	3
1	9
2	53

Ex 20 Helen started a business several years ago. The table shows her profits, in millions of dollars, for the first **7** years. Select the polynomial function of best fit that could be used to model Helen's profits

a) $f(x) = 0.001(-3.27x^2 + 53.51x + 1371)$

b) $f(x) = 0.0001(-6.944x^3 + 50.6x^2 + 250.4x + 13{,}957)$

c) $f(x) = 0.00001(9.47x^4 + 82.07x^3 - 312.5x^2 + 4203x + 138{,}500)$

d) $f(x) = 0.0001(-x^4 + 12x^3 - 77x^2 + 600x + 13{,}650)$

x	f(x)
1	1.425
2	1.46
3	1.5
4	1.53
5	1.56
6	1.58
7	1.58

Lesson 2-3
Operations with Polynomials

Today's Goals

- Add and subtract polynomials.
- Multiply polynomials.

Adding and Subtracting Polynomials

A polynomial is a **monomial** or the sum of two or more monomials.

A binomial is the sum of two monomials, and a trinomial is the sum of three monomials.

The degree of a polynomial is the greatest degree of any term in the polynomial.

Polynomials can be added or subtracted by performing the operations indicated and combining like terms. You can **subtract** a polynomial by adding its additive inverse.

The sum or difference of polynomials will have the same variables and exponents as the original polynomials, but possibly different coefficients.
Thus**, the sum or difference** of two polynomials is also a polynomial.

A set is **closed** if and only if an operation on any two elements of the set produces another element of the same set. Because adding or subtracting polynomials results in a polynomial, the set of polynomials is **closed under the operations of addition and subtraction**.

Identify Polynomials

Example Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

1) $x^6 + \sqrt[3]{x} - 4$

2) $5ab^4 + 3a^2b^7 - 9$

3) $\frac{2}{3}x^{-5} - 6x^{-3} - x$

Ex 1 State the degree of each polynomial.

1) $x^7 + 6x^5 - \frac{1}{3}$

2) $3c^7d^2 + 5cd - 9$

3) P^{10}

4) 25

5) $2x^2 - 3x + 5$

6) $a^3 - 11$

7) $\frac{5np}{n^2} - \frac{2g}{h}$

8) $\sqrt{m-7}$

Ex 2 Polynomial with degree 3

a) $4 - 2x^2 - 3x$

b) $x^2 + x + 12^3$

c) $x^3 + x^4 - x^2$

d) $1 + x + x^3$

Ex 3 Polynomial with degree 3

a) $x^3 + x^2 - 2x^4$

b) $x^2 + x + 5^3$

c) $-2x^3 - 3x + 4$

d) $3x - 3$

Ex 4 Polynomial with degree 3

x	-24	-18	-12	-6	0	6	12	18	24
$f(x)$	-8	-1	3	-2	4	7	-1	-8	5

a) 2
b) 3
c) 4
d) 5

Example Find

$$(6x^3 + 7x^2 - 2x + 5) + (x^3 - 4x^2 - 82x + 1).$$

Ex 5 Find

1) $(2x^3 + 9x^2 + 6x - 3) + (4x^3 - 7x^2 + 5x).$

2) $(6a^2 + 5a + 10) - (4a^2 + 6a + 12)$

3) $(g + 5) + (2g + 7)$

4) $(x^2 - 3x - 3) + (2x^2 + 7x - 2)$

5) $(-2f^2 - 3f - 5) + (-2f^2 - 3f + 8)$

Example Find

$$(2x^5 + 11x^4 + 7x - 8) - (5x^4 + 9x^3 - 3x + 4).$$

Ex 6 Find

$$(8x^2 - 3x + 1) - (5x^3 + 2x^2 - 6x - 9)$$

Ex 7 Find

1) $(6a^2 + 5a + 10) - (4a^2 + 6a + 12)$

2) $(5d + 5) - (d + 1)$

3) $(2x - 3) - (5x - 6)$

4) $(x^2 + 2x - 5) - (3x^2 - 4x + 7)$

Ex 8 Use the polynomials

$$f(x) = -6x^3 + 2x^2 + 4 \text{ and } g(x) = x^4 - 6x^3 - 2x$$

to evaluate and simplify the given expression. Determine the degree of the resulting polynomial. Show your work.

1) $f(x) + g(x)$

2) $g(x) - f(x)$

Ex 9 Find

$$(-4x^2 + 2x + 3) - 3(2x^2 - 5x + 1)$$

a) $2x^2$

b) $-10x^2 + 17x$

c) $-10x^2$

d) $2x^2 + 17x$

Ex 10 Find

$$4(4x - 9y) + 8(3x + 2y)$$

a) $16x - 20y$

b) $40x - 20y$

c) $20x - 40y$

d) $24x + 16y$

Ex 11 Find

$$(-2)(2x + y) - 3(x - 2y)$$

a) $7x - 4y$

b) $-7x - 4y$

c) $4y - 7x$

d) $4y + 7x$

Ex 12 Find

$$-2(-5x + 6y) - 9(-2x + 4y)$$

a) $28x - 48y$

b) $-18x + 48y$

c) $28x + 48y$

d) $-8x + 24y$

Ex 13 Find

$$3(2y + 5x) - 7(-2 + y)$$

a) $y + 15x + 14$

b) $y + 15x - 14$

c) $-y + 15x + 14$

d) $-y + 15x - 14$

Ex 14 Find

$$3(4x-2y)-2(3x+y)$$

a) $18x-18y$

b) $6x+4y$

c) $8x-4y$

d) $6x-8y$

Ex 15 Find

$$4(4x-9y)+8(3x+2y)$$

a) $16x-20y$

b) $40x-20y$

c) $20x-40y$

d) $24x+16y$

Ex 16 Find

$$(-4x^2+2x+3)-3(2x^2-5x+1)$$

a) $2x^2$

b) $-10x^2+17x$

c) $-10x^2$

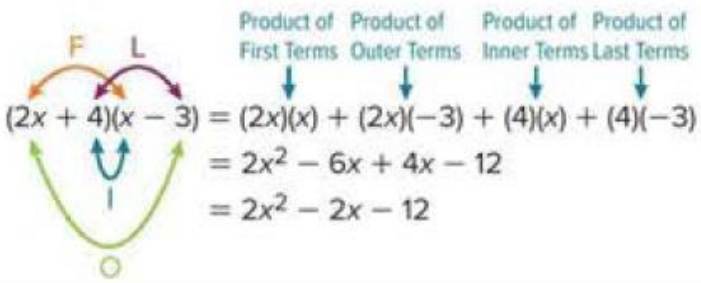
d) $2x^2+17x$

Multiplying Polynomials

Key Concept • FOIL Method

Words: Find the sum of the products of ***F*** the *First* terms, ***O*** the *Outer* terms, ***I*** the *Inner* terms, and ***L*** the *Last* terms.

Symbols:



Example Find

$$2x(4x^3 + 5x^2 - x - 7).$$

Ex 17 Find

1) $3p(np - z)$

2) $4x(2x^2 + y)$

3) $x^2(2x + 9)$

Example Find

$$(3a + 5)(a - 7)(4a + 1).$$

Ex 18 Find

4) $(-2r - 3)(5r - 1)(r + 4).$

5) $-5(2c - d)^2$

6) $(x - y)(x^2 + 2xy + y)^2$

7) $(a + b)(a^3 - 3ab - b)^2$

8) $(x - y)(x + y)(2x + y)$

9) $(a + b)(2a + 3b)(2x - y)$

10) $(r - 2t)(r + 2t)$

11) $(3y + 4)(2y - 3)$

12) $(x^3 - 3x^2 + 1)(2x^2 - x + 2)$

13) $(4x^5 + x^3 - 7x^2 + 2)(3x - 1)$

Ex 19 Use the polynomials

$$f(x) = 3x^2 - 1, g(x) = x + 2, and\ h(x) = -x^2 - x$$

to evaluate and simplify the given expression. Determine the degree of the resulting polynomial. Show your work.

1) $f(x)g(x)$

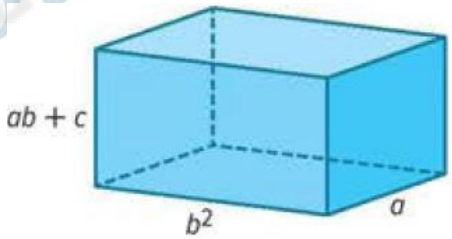
2) $h(x)f(x)$

3) $[f(x)]^2$

4)

Ex 20 CONSTRUCTION A rectangular deck is built around a square pool. The pool has side length s. The length of the deck is 5 units longer than twice the side length of the pool. The width of the deck is 3 units longer than the side length of the pool. What is the area of the deck in terms of s?

Ex 21 VOLUME The volume of a rectangular prism is given by the product of its length, width, and height. A rectangular prism has a length of b^2 units, a width of a units, and a height of $ab + c$ units. What is the volume of the rectangular prism? Express your answer in simplified form.



Ex 22 SAIL BOATS Tamara is making a sail for her sailboat.

a. Refer to the diagram to find the area of the sail.

b. If Tamara wants fabric on each side of her sail, write a polynomial to represent the total amount of fabric she will need to make the sail.

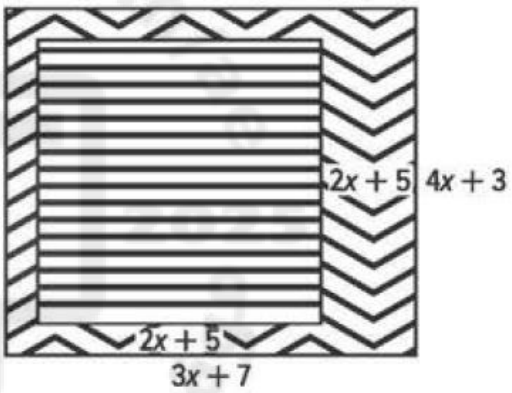
Ex 23 Enrique is designing a flag for a new school club. A smaller striped square is placed as part of the design and the rest of the flag will have chevrons
Which expression can be used to represent the area of the flag that is not striped?

a) $16x^2 + 57x + 46$

b) $8x^2 + 35x + 16$

c) $8x^2 + 17x + 4$

d) $8x^2 + 17x - 4$



Lesson 2-4

Dividing Polynomials

Today's Goals

- Divide polynomials by using long division.
- Divide polynomials by using synthetic division.

Dividing Polynomials by Using Long Division

Example Find $(24a^4b^3 + 18a^2b^2 - 30ab)(6ab^3)^{-1}$.

Ex 1 Find $(9x^9y^5 + 21x^4y^4 - 12x^3y^2) \div (3x^2y^2)$.

Ex 2 Simplify each expression.

1) $\dfrac{15y^3 + 6y^2 + 3y}{3y}$.

2) $(4f^5 - 6f^4 + 12f^3 - 8f^2)(4f^2)^{-1}$.

3) $(6j^2k - 9jk^2) \div (3jk)$.

4) $(4a^2h^2 - 8ah^3 + 3a^4) \div (2a^2)$

Example Find $(x^2 - 5x - 36) \div (x + 4)$.

Ex 3 Find

$$\frac{x^2 + 6x - 112}{x - 8}$$

Example Find

$$\frac{3z^3 - 14z^2 - 7z + 3}{z - 5}$$

Ex 4 Find

$(-4x^3 + 5x^2 - 2x - 9)(x - 2)^{-1}$.

Ex 5 Simplify by using long division.

1) $(n^2 + 7n + 10) \div (n + 5)$.

2) $(d^2 + 4d + 3)(d + 1)^{-1}$.

3) $(4t^2 + 13t + 15) \div (t + 6)$.

4) $(6y^2 + y - 2)(2y - 1)^{-1}$

5) $(4g^2 - 9) \div (2g + 3)$

6) $(2x^2 - 5x - 4) \div (x - 3)$

Dividing Polynomials by Using Synthetic Division

Example Find

$(3x^3 - 2x^2 - 53x - 60) \div (x + 3)$.

Ex6 Find

$$\frac{4x^4 - 37x^2 + 4x + 9}{2x - 1}$$

Ex7 Find

$(4x^4 + 3x^3 - 12x^2 - x + 6)(4x + 3)^{-1}$

Ex 8 Simplify using synthetic division.

1) $(3v^2 - 7v - 10)(v - 4)^{-1}$

2) $(3t^2 + 4t^3 - 32t^2 - 5t - 20)(t + 4)^{-1}$

3) $\dfrac{(y^3 + 6)}{(y + 2)}$

4) $\dfrac{(2x^3 - x^2 - 18x + 32)}{(2x - 6)}$

5) $(4p^3 - p^2 + 2p) \div (3p - 1)$

6) $(3c^4 + 6c^3 - 2c + 4)(c + 2)^{-1}$

Ex 9 STRUCTURE Jada used long division to divide $x^4 + x^3 + x^2 + x + 1$ by $x + 2$. Her work is shown below with three terms missing. What are **A**, **B**, and **C**?

1)

$$
\begin{array}{r|l}
x+2 & x^3 - x^2 - 3x - 5 \\
\hline
 & x^4 + x^3 + x^2 + x + 1 \\
 & \underline{x^4 + 2x^3} \\
 & -x^3 + A \\
 & \underline{-x^3 - 2x^2} \\
 & 3x^2 + x \\
 & \underline{3x^2 + B} \\
 & -5x + 1 \\
 & \underline{-5x - 10} \\
 & C
\end{array}
$$

Ex 10 Find

$$(-x^{-2}y^5)(2x^4y^{-3})$$

a) $-2x^2y^2$

b) $-\frac{2y^2}{x^2}$

c) $-\frac{2x^2}{y^2}$

d) $-2x^{-8}y^{-15}$

Ex 11 Find

$$\frac{3x^3y^4z}{9xy^5z}$$

a) $\frac{3x^2z^2}{y}$

b) $3x^4y^6z^2$

c) $\frac{x^2}{3y}$

d) $\frac{x^4z^2}{3y}$

Ex 12 Find

$$(-2x^{-4}y^3)(-7x^6y^{-2})$$

a) $-\frac{14y}{x^2}$

b) $14x^2y$

c) $-\frac{2x^2}{7y}$

d) $14xy^2$

Ex 13 Simplify

$$(-x^{-2}y^5)(2x^4y^{-3})$$

a) $-2x^2y^2$

b) $-\frac{2y^2}{x^2}$

c) $-\frac{2x^2}{y^2}$

d) $-2x^{-8}y^{-15}$

Ex 14 **simplify**

$$(3x^0)(2x^4)$$

a) $2x^4$

b) $18x^4$

c) $12x^4$

d) $18x^6$

Ex 15 Find

$$\frac{a^4n^7}{an^{-1}}$$

a) $\frac{a^2}{n^3}$

b) a^2n^8

c) a^3n^6

d) a^3n^8

Ex 16 Determine the quotient.

$$(5x^4 + 12x^3 - 64x^2 - 95x + 132) \div (x - 3)$$

a) $3x^3 + 21x^2 + x - 31$

b) $3x - 3\,21x - 2\,x + 31$

c) $5x^3 + 27x^2 + 17x - 44$

d) $5x^3 - 27x^2 - 17x + 44$

Ex 17 Determine the quotient.

$$\frac{6x^3 - 71x^2 + 139x + 130}{3x + 2}$$

a) $2x^2 - 25x + 63 + \frac{8}{3}$

b) $2x - 25x + 63 + \frac{8}{3x+2}$

c) $2x - 25x + 63 + \frac{4}{3}$

d) $2x\,2 - 25x + 63 + \frac{4}{3x+2}$

Ex 18 Determine the quotient

$$(a^2 + 7a - 11)(3 - a)^{-1}$$

a) $a + 10 - \frac{19}{3-a}$

b) $-a - 10 + \frac{19}{3-a}$

c) $-a + 10$

d) $-a - 10 - \frac{19}{3-a}$

Ex 19 Determine the quotient

$$(r^2 + 5r + 7)(1 - r)^{-1}$$

a) $-r - 6 - \frac{13}{1-r}$

b) $r - 6 + \frac{13}{1-r}$

c) $r + 6$

d) $r + 6 - \frac{13}{1-r}$

Ex 20 Determine the quotient

$$(x^2 + 3x - 9)(4 - x)^{-1}$$

a) $-x - 7 + \frac{19}{4-x}$

b) $-x - 7$

c) $x + 7 - \frac{19}{4-x}$

d) $-x - 7 - \frac{19}{4-x}$

Ex 21 **Which expression below is equal to**

$$(30x^2 - 11x + 15)(5x - 6)^{-1}$$

a) $6x + 5 + \frac{45}{5x - 6}$

b) $6x - 5 + \frac{45}{5x + 6}$

c) $-6x - 5 + \frac{45}{5x-6}$

d) $6x + 5$

Ex 22 **Which expression below is equal to**

$$(b^3 + 2b^2 - b - 2) \div (b + 2)$$

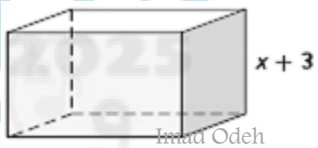
a) $b^2 + 1$

b) $b^2 - 1$

c) $b^2 + 2b + 1$

d) $b^2 + 3b + 2$

Ex 23 تمرين حجم المنشور المستطيل يساوي $6x^3 + 19x^2 + 2x - 3$ فأي تعبير كثيرة الحدود يمثل مساحة القاعدة؟



a) $6x^4 + 37x^3 + 59x^2 + 3x - 9$

b) $6x^2 + x + 1$

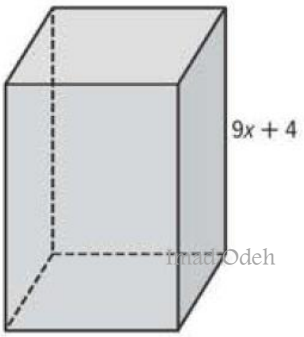
c) $6x^2 + x - 1$

d) $6x + 1$

Ex 24 The volume of the rectangular prism shown is

$$45x^3 + 83x^2 + x - 12.$$

What is the area of the base?



Lesson 2-5

Powers of Binomials

Today's Goal

● Expand powers of binomials by using Pascal's Triangle and the Binomial Theorem.

Powers of Binomials

Key Concept • Binomial Expansion

In the binomial expansion of $(\boldsymbol{a} + \boldsymbol{b})^{\boldsymbol{n}}$,
- there are $\boldsymbol{n} + \mathbf{1}$ terms.
- $\boldsymbol{n}$ is the exponent of $\boldsymbol{a}$ in the first term and $\boldsymbol{b}$ in the last term.
- in successive terms, the exponent of a decreases by **1**, and the exponent of b increases by 1.
- the sum of the exponents in each term is $\boldsymbol{n}$.
- the coefficients are symmetric.

Pascal's triangle is a triangle of numbers in which a row represents the coefficients of an expanded binomial $(\boldsymbol{a} + \boldsymbol{b})^{\boldsymbol{n}}$. Each row begins and ends with **1**. Each coefficient can be found by adding the two coefficients above it in the previous row. Instead of writing out the rows of **Pascal's triangle**, you can use the Binomial Theorem to expand a binomial. The Binomial Theorem uses combinations to calculate the coefficients of the binomial expansion.

			1				$(x+y)^0$
		1		1			$(x+y)^1$
	1		2		1		$(x+y)^2$
1		3		3		1	$(x+y)^3$
	4		6		4		$(x+y)^4$
1		5		10		5	$(x+y)^5$

Key Concept • Binomial Theorem

If n is a natural number, then

$$(a + b)^n = nC_0 a^n b^0 + nC_1 a^{n-1} b^1 + nC_2 a^{n-2} b^2 + \cdots + nC_n a^0 b^n$$

Example Use Pascal's triangle to expand.

$(x + y)^7$

Ex 1 Use Pascal's triangle to expand.

1) $(c+d)^4$

2) $(x-y)^3$

3) $(a+b)^4$

4) $(g-h)^4$

5) $(m+1)^4$

6) $(y-z)^6$

7) $(d+2)^8$

Example BASEBALL In **2016**, the Chicago Cubs won the World Series for the first time in **108** years. During the regular season, the Cubs played the Atlanta Braves **6** times, winning **3** games and losing **3** games. If the Cubs were as likely to win as to lose, find the probability of this outcome by expanding.

$(w + \ell)^6$

Ex 2 GAME SHOW A group of **8** contestants are selected from the audience of a television game show. If there are an equal number of men and women in the audience, find the probability of the contestants being **5** women and **3** men by expanding $(\boldsymbol{w} + \boldsymbol{m})^{\boldsymbol{8}}$. Round to the nearest percent if necessary.

Ex 3 BAND A school band went to **4** competitions during the year and received a superior rating **2** times. If the band is as likely to receive a superior rating as to not receive a superior rating, find the probability of this outcome by expanding$(\boldsymbol{s} + \boldsymbol{n})^{\boldsymbol{4}}$. Round to the nearest percent if necessary.

Ex 4 BASKETBALL Oliver shot **8** free throws at practice, making 6 free throws and missing **2** free throws. If Oliver is equally likely to make a free throw as he is to miss a free throw, find the probability of this outcome by expanding $(\boldsymbol{m} + \boldsymbol{n})^{\boldsymbol{8}}$. Round to the nearest percent if necessary.

Ex 1 Expand

$(2c - 6d)^4$

Ex 5 Expand

1) $(2c - 6d)^4$

2) $(3x + 4y)^5$

3) $(2c - 2d)^7$

4) $(8h - 3j)^4$

5) $(4a + 3b)^6$

Ex 6 Expand

1) $\left(x+\frac{1}{2}\right)^5$

2) $\left(x-\frac{1}{3}\right)^4$

3) $\left(2b+\frac{1}{4}\right)^5$

4) $\left(3c+\frac{1}{3}d\right)^3$

Ex 7 USE A MODEL Diego flips a fair coin **12** times. What is the probability that the coin lands on tails **3** times? **5** times? **9** times?

Ex 8 Use Pascal's Triangle to find the fourth term in the expansion of $(2x+7)^6$. Why is it the same as the fourth term in the expansion of $(7+2x)^6$?

Ex 9 Find the term in $(a + b)^{12}$ where the exponent of a is **5**.

Ex 10 Which of the following is the expansion of $(2h + f)^4$?

a) $2h^4 + 4h^3 f + 6h^2 f^2 + 4hf^3 + f^4$

b) $16h^4 + 32h^3 f + 24h^2 f^2 + 32hf^3 + 16f^4$

c) $16h^4 + 32h^3 f + 32h^2 f^2 + 8hf^3 + f^4$

d) $16h^4 + 32h^3 f + 24h^2 f^2 + 8hf^3 + f^4$

Ex 11 The first shelf on Hannah's bookshelf holds an equal number of fiction and nonfiction books. If Hannah selects **5** books randomly, what is the probability that **4** of the books will be fiction and 1 will be nonfiction? Round your answer to the nearest tenth of a percent.

a) 31.3%

b) 5.6%

c) 12.5%

d) 3.1%

Ex 12 Select all of the following that would be a coefficient of a term in the binomial expansion of $(x + y)^7$

a) 1

b) 3

c) 7

d) 14

e) 21

f) 28

g) 30

h) 35

اطيب التمنيات